Summary of the thesis

We begin with an introduction to frames, topological systems and topological groups. For more details one may refer to the books ‘Stone Spaces’ by P. T. Johnstone, ‘Topology via Logic’ by Steven Vickers and ‘Topological groups’ by L. Pontryagin.

Our main reference for information on topological systems and frames is [74], on topological groups [61] and [35] and on topological semi group is [19].

The thesis is divided into six chapters.

Basic definitions and a few examples in categories, frames and topological groups are given in Chapter 1. A few known results that are required in the study are also included.

In Chapter 2, we introduce the concept of a frame-group. We begin with the idea of a frame on a set with respect to a relation which, according to Steven Vickers, is a topological system. We then formulate and investigate the definition of a frame-group which is proved to be a reasonable generalization of the concept of a topological group. We also introduce frame-group homomorphisms. Further, we give examples of
frame-groups and frame-group morphisms and observe that the set of all frame-groups and frame-group morphisms is a category which we denote by $\text{Frm-Gp}$. We define multiplication of opens by points and product of two opens. We also prove a few results in frame-groups as analogues of results in topological groups.

Chapter 3 is devoted to construct more frame-groups from given frame-groups by defining sub groups and quotient groups for frame-groups. We follow the style of Pontryagin in developing sub groups and quotient groups. We verify that all these new constructions are frame-groups.

In Chapter 4, we introduce optimal frame-groups. We discuss properties like dense continuous homomorphisms, connectedness, compactness and regularity in optimal frame-groups. We also observe that optimal frame-groups are very close to topological groups in the sense that many questions about the frame-groups are answered by the natural topological groups associated with them. We prove a few results in optimal frame-groups which affirm the power of optimal frame-groups. Also we prove some results regarding connectedness, compactness and regularity in optimal frame-groups.
In Chapter 5, we construct a compactification of a Hausdorff frame-group analogous to the Bohr compactification of a Hausdorff topological semi group and we call it the Bohr-type compactification of the given frame-group. A proof for the existence and uniqueness of Bohr compactification for a Hausdorff semigroup is given in [19]. We observe that the same argument works for Hausdorff frame-groups also.

The goal of Chapter 6 is to define characters for frame-groups. Dual of a locally compact abelian group is studied by T. Husain in ‘Introduction to Topological groups’ [35]. We follow the style of T. Husain in developing characters for a frame-group. We give a group structure to the set of all characters. With a suitable frame on the character group we are able to prove that the dual of a frame group is also a frame-group.

The basic constructions and results of Chapter 2 and Chapter 3 and the constructions of Chapter 5 and Chapter 6 support our claim that the concept of frame-groups is a reasonable generalization of the concept of topological groups. The results of Chapter 4 show that optimal frame-groups are well behaved frame-groups and are close to topological groups.