Preface

In this thesis entitled “Bayesian Inferences on Reliability for Life Time Distributions under Stress-Strength Setup”, a new dimension of stress-strength model known as augmented strength reliability problem is considered. In addition to augmentation problems, the multicomponent system's strength reliability problems were also attempted in the second phase of the thesis. The proposed research problems are carried out by me as research scholar in the Department of Statistics, Ramanujan School of Mathematical Sciences, Pondicherry University, Puducherry, India, under supervision of Dr. Navin Chandra, since August 2013.

This work mainly deals with gamma and Inverse Gaussian distributions under stress-strength set-up. The thesis consists of seven chapters, divided into two parts: the first part includes four chapters (Chapter 2 to 5) on estimation problem of augmented strength reliability under proposed ASP for above said life distributions by assuming identical and non-identical strengths. Whereas, the second part of the thesis consists two chapters (Chapter 6 and 7), which deal the problems of multicomponent system strength reliability for gamma distribution for each of identical and non-identical strengths. The ML and Bayes estimation of strength reliabilities in all the chapters are derived. In Bayesian context, two different types of informative viz. gamma and inverted gamma priors as well as two types of non-informative viz. uniform and Jeffreys priors are considered and the Bayes estimators are calculated under each of the squared error loss function (SELF) as well as LINEX loss function (LLF). A numerical comparison between ML and Bayes estimation has been done on the basis of their mean square errors (MSE) and absolute biases through simulated samples via MCMC algorithm.

In Chapter 2, the generalized augmented gamma strength reliability is developed and its Bayes and ML estimation are also carried out by assuming that the augmented strengths (X) of the components are independently and identically distributed (i.i.d.) as gamma(α, λ) distribution and the common stress (Y) imposed on it follows gamma(α, λ) model. In Bayesian context informative (gamma and inverted gamma) as well as non-informative (uniform and Jeffreys) types of prior are considered. The ML estimator as well as Bayes estimators under SELF and LLF for all the priors of generalized augmented strength reliability under ASP are obtained and compared through their mean square errors (MSEs) and absolute biases.

In Chapter 3, we consider a generalized augmented gamma strength reliability under the ASP by assuming that the strength (X) of the equipment and the common imposed stress (Y) are independently but not identically distributed (n.i.i.d.) as two parameter
gamma distribution with parameters \((\alpha_1, \lambda_1)\) and \((\alpha_2, \lambda_2)\) respectively. The ML and Bayes estimators of augmented strength reliability models are derived. In Bayesian paradigm gamma and inverted gamma priors under informative types of prior and uniform and Jeffreys priors under non-informative type of priors are considered. The Bayes estimators are calculated under SELF and LLF for each of the priors and compared with ML estimators on the basis of their MSEs and absolute biases.

In Chapter 4, a generalized augmented strength reliability under ASP is derived by assuming that the component strength \((X)\) and the imposed stress \((Y)\) are independently and identically distributed (i.i.d.) as two-parameter Inverse Gaussian distribution (IGD) with parameters \(\mu\) and \(\lambda\). The ML estimator as well as Bayes estimators for gamma, inverted gamma, uniform and Jeffreys priors under each of the loss functions viz. SELF and LLF are computed and compared through their MSEs and absolute biases.

In Chapter 5, the generalized form of augmented strength reliability under the proposed ASP is developed by considering the component strength \((X)\) and the common imposed stress \((Y)\) are independently but not identically distributed (i.n.i.d) as two-parameter Inverse Gaussian distributions (IGD) with parameters \((\mu_1, \lambda_1)\) and \((\mu_2, \lambda_2)\) respectively. The Bayes and ML estimators of augmented strength reliability are derived. In Bayesian paradigm, informative (gamma and inverted gamma) priors as well as non-informative (uniform and Jeffreys) priors are considered and the Bayes estimators are calculated under SELF and LLF. A numerical comparison between Bayes and ML estimators are carried out on the basis of their MSEs and absolute biases.

In Chapter 6, a comparison of ML and Bayes estimation of multicomponent system reliability assuming that the component strengths \((X_1, X_2,...,X_k)\) are independently and identically distributed (i.i.d.) as gamma distribution with parameters \((\alpha_1, \lambda_1)\) and each component of the system is subjected to face a common stress of a random magnitude \(Y\) which is independently follows as gamma distribution with parameter \((\alpha_2, \lambda_2)\). The Bayes estimators for gamma, inverted gamma, uniform and Jeffreys priors under each of the considered loss functions (SELF and LLF) are separately computed and compared with ML estimator through their MSEs and absolute biases.

In Chapter 7, multicomponent stress-strength reliability is derived by assuming that out of \(k\)-components, \(k_1\) components are of one category and their strengths are assumed to have a category of common distribution function \(F_1\) as \(\text{gamma}(\alpha_1, \lambda_1)\). The remaining \(k_2 = k - k_1\) components are of another category and their common strength
distribution is denoted by $F_2$ as $\text{gamma}(\alpha_2, \lambda)$. All the components are exposed to face a common stress $Y$ having the distribution $G$ as $\text{gamma}(\tau, \nu)$, and the system operates successfully if at least $s$ of the $k$ components withstand the stress. The ML and Bayes estimators of multicomponent stress-strength reliability are also derived. In Bayesian context gamma and inverted gamma priors under informative types of prior and uniform and Jeffreys priors under non-informative type of priors are considered. The Bayes estimators are calculated under SELF and LLF for each of the priors and compared with ML estimators on the basis of their MSEs and absolute biases.

In order to compare the proposed estimators, a simulation analysis through Markov-Chain-Monte-Carlo (MCMC) techniques (viz. Metropolis-Hasting Algorithm, Gibbs Sampling, Importance sampling etc.) are employed to solve the intractable integrals obtained in Bayes estimators for all the priors under both the loss functions. A comparative study between ML and Bayes estimators of strength reliabilities for single component as well as multicomponent system are carried out in all the chapters on the basis of their average estimates, mean square errors and absolute biases.

A part of the work contained here has been published and some are communicated for possible publication in the reputed national and international journals. The lists are given in the next page.