FREE CONVECTION OSCILLATORY FLOW BEHAVIOR PAST AN INFINITE VERTICAL POROUS PLATE

Introduction: Within the past few years, there has been remarkable interest in the flow of Newtonian fluid through porous walls. Familiar examples are boundary layer control, transpiration cooling and gaseous diffusion. In addition blowing is used to add reactants, prevent corrosion and reduce drag. Suction is applied to chemical processes to remove reactants. Much work has been done in order to understand the effects of fluid removal or injection through channel walls on the flow of Newtonian and non-Newtonian fluids. Important contributions in this field have been made by Berman [2], Brady [3], Combarnous [6], Cox [7], Skalak and Wang [10] and Wang [16]. The convective motion driven by buoyancy forces is well known natural phenomena and has attracted much attention Basak et al. [1], Perekattu and Balaji [8] and Prud’home and Jasmin [9].
The systematic studies of flow past a porous medium were enhanced by several authors. For many problems of practical interest, the flow may be unsteady. Soundalgekar and Bhatt [11] analysed oscillatory MHD channel flow and heat transfer under a transverse magnetic field. Umavathi and Palaniappan [12] studied oscillatory flow of unsteady convective fluid in an infinite vertical porous stratum. Chamkha [4, 5] dealt with unsteady MHD convective heat and mass transfers past a semi-infinite vertical permeable moving plate with heat absorption. Recently Umavathi et al. [13], Umavathi et al. [14] and Umavathi et al. [15] analysed oscillatory flow and heat transfer for viscous immiscible fluids in a horizontal channel.

In another new problem when subterranean water is forced by a pressure gradient through soil, each material element of the water traces out a devious path as it passes through the irregularly arranged interstices between the soil particles. Since a detailed knowledge of the shape of interstices occupied by the fluid is not available, it is customary to introduce dependent variables which are effectively averages over a large number of interstices. The equation governing the actual flow in and out the interstices are linear and flux of the fluid through a given piece of porous medium large enough to contain many interstices may be expected to be proportional to the pressure gradient applied across it and inversely proportional to $\mu$, just
as if the medium consisted of a number of tubes of small diameter in each of which the flow is of Poiseville type. If the porous medium has a structure which is statistically isotropic, so that pressure gradients applied in different directions produce the same flux, we may write

$$\nabla p^* = -\mu \frac{q^*}{K^*}$$

(2.1)

where $K^*$ is a constant called the permeability, which depends on the size and shape of the interstices. The above equation is known as Darcy law and has a long history of use in soil mechanics for a wide variety of porous media. Its justification rests partly on the above theoretical argument and partly on its agreement with measurements of the flow produced by an applied pressure gradient in homogeneous porous media like sand.

In the present chapter an unsteady two-dimensional free convective flow through a porous medium bounded by an infinite vertical plate is considered when the temperature of the plate is oscillatory with the time about a constant non-zero mean value. An analytical solution for the velocity and temperature fields is derived and the effect of permeability parameter $K$ and frequency parameter $\omega$ on velocity and temperature fields are discussed.
**Mathematical Analysis:**

Studies associated with flows through porous medium have been based on the Darcy’s empirical equation (2.1). We regard the porous medium as an assemblage of small identical particles of spherical shape fixed in the space. The equation (2.1) for incompressible fluid and unsteady flow takes the form

\[
\frac{\partial q^*}{\partial t^*} + (q^* \cdot \nabla) q^* = -\frac{1}{\rho} \nabla p^* - \frac{\mu}{\rho} q^* + \frac{\mu}{\rho} \nabla^2 q^* - g^*
\]

(2.2)

where t* is the time and g* is the acceleration of gravity.

The flow is bounded by an infinite vertical porous plate. The x* axis is taken along the plate in the direction of flow and the y* axis is normal to the plate. All the fluid properties are assumed to be constant except that the influence of the density variation with temperature is considered only in the body force term. Constant suction case through plate is considered. The temperature of the plate is oscillating with the time about a constant non-zero mean value where as the temperature away from the plate is constant.
Figure – 2.1
The continuity and momentum equations take the form

\[
\frac{\partial v^*}{\partial y^*} = 0 \quad (2.3)
\]

and

\[
\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = g^* \beta \left( T^* - T^*_{\infty} \right) + v^* \frac{\partial^2 u^*}{\partial y^*^2} - \frac{v^*}{K^*} u^* . \quad (2.4)
\]

Energy equation is

\[
\rho C_p \frac{d T^*}{d t^* d t^*} = \frac{dp^*}{d t^*} + \left\{ \frac{\partial}{\partial x^*} \left( k^* \frac{\partial T^*}{\partial x^*} \right) + \frac{\partial}{\partial y^*} \left( k^* \frac{\partial T^*}{\partial y^*} \right) \right\} + \frac{\partial}{\partial z^*} \left( k^* \frac{\partial T^*}{\partial z^*} \right) + \mu \phi \quad (2.5)
\]

where \(C_p\) represents the specific heat at constant pressure per unit mass.

In general \(C_p\) depends on temperature. In case of incompressible fluid of constant thermal conductivity without viscous dissipation of energy, equation (2.5) for fully developed temperature flow takes the form

\[
\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^*^2} \quad (2.6)
\]

In equation (2.6) Darcy’s dissipation term is neglected because it is of the same order of magnitude with the viscous dissipation term.
For constant suction case, the integration of equation (2.3) gives

\[ v^* = -v_0 \]  

(2.7)

where \( v_0 \) is constant suction velocity and the negative sign indicates that it tends towards the plates.

The relevant boundary conditions are

at \( y^* = 0, \ u^* = 0; \ T^* = T^*_w + \varepsilon (T^*_w - T^*_e) e^{i\omega t} \) \]  

as \( y^* \to \infty, \ u^* \to 0; \ T^* \to T^*_e \) \]  

(2.8)

where \( \varepsilon \) (<< 1) is a small positive constant.

In view of equation (2.7), equations (2.4) and (2.6) reduce to the dimensionless form

\[ \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G, - \frac{1}{K} u \]  

(2.9)

\[ \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\rho} \frac{\partial^2 \theta}{\partial y^2} \]  

(2.10)

and boundary conditions (2.8) becomes

at \( y = 0, \ u = 0; \ \theta = 1 + \varepsilon e^{i\omega t} \) \]  

as \( y \to \infty, \ u \to 0; \ \theta \to 0 \) \]  

(2.11)
where the non-dimensional quantities without asterisk are defined as

\[ y = \frac{y^* v_0}{v}, \quad t = \frac{t^* v_0^2}{v^2}, \quad \omega = \frac{\omega^* v}{v_0^2}, \quad u = \frac{u^*}{v_0} \]

\[ \theta = \frac{T^* - T_{\infty}^*}{T_{\infty}^* - T_{\infty}^*}, \quad \rho = \frac{\mu C_p}{k}, \quad \text{(Prandtl number)} \]

\[ G_i = \frac{v g \beta (T_{\infty}^* - T_{\infty}^*)}{v_0^2}, \quad \text{(Grashoff number)} \]

\[ K = \frac{v_0^2}{v^2} K^*, \]

\[ \beta \] is the coefficient of volume expansion and for perfect gas \( \beta = \frac{1}{\theta}. \)

To solve the equations (2.9) and (2.10) subject to the boundary conditions (2.11), we look for the solution of the forms as

\[ \begin{align*}
  u &= u_o + e^{i\omega t} u_1 + \ldots \ldots \\
  \theta &= \theta_o + e^{i\omega t} \theta_1 + \ldots \ldots 
\end{align*} \]

Putting equation (2.12) into equations (2.9) and (2.10) and equating the steady and unsteady parts, we find

\[ u_o'' + u_o' - \frac{1}{K} u_o = -G_i \theta_o \]

(2.13)
\begin{equation}
\psi'' + u_1' - \left( i \omega + \frac{1}{K} \right) u_1 = -G_i \theta_i 
\end{equation}

(2.14)

\begin{equation}
\theta_0'' + P \theta_0' = 0 
\end{equation}

(2.15)

\begin{equation}
\theta_i'' + P \theta_i' - i \omega P \theta_i = 0.
\end{equation}

(2.16)

Boundary conditions for steady and unsteady parts of flow parameters become

\begin{align*}
\text{at } & y = 0; \quad u_0 = 0 = u_1; \quad \theta_0 = 1 = \theta_i \\
\text{as } & y \rightarrow \infty; \quad u_0 \rightarrow 0, \quad u_1 \rightarrow 0; \quad \theta_0 \rightarrow 0, \quad \theta_i \rightarrow 0
\end{align*}

(2.17)

Solving equations (2.13 – 2.16) under the boundary condition (2.17) and substituting the solutions into equation (2.12), we obtain equations (2.18) and (2.19) as

\begin{equation}
u = \frac{G_i}{P^2 - P - \frac{1}{K}} \left[ e^{-k_i y} - e^{-\epsilon y} \right]
\end{equation}

\begin{equation}
+ \epsilon e^{i \omega t} \left[ \frac{G_i}{C_i^2 - \epsilon_1 - \left( i \omega + \frac{1}{K} \right)} \left( e^{-k_i y} - e^{-\epsilon y} \right) \right].
\end{equation}

(2.18)
where, 

\[ d_i = \frac{1 + \sqrt{1 + \frac{4}{K}}}{2} \]

\[ C_i = \frac{P + \sqrt{P^2 + 4i\omega P}}{2} \]

\[ L_i = \frac{1 + \sqrt{1 + 4 \left( \frac{i\omega + \frac{1}{K}}{1} \right)}}{2} \]

\[ \theta = e^{-py} + e^{i\omega t} - C_i y \] (2.19)

\[ u = u_o + \epsilon \left[ \left( M_i \cos \omega t - M_i \sin \omega t \right) + i \left( M_i \cos \omega t + M_i \sin \omega t \right) \right] \]

taking real part only

\[ u = u_o + \epsilon \sqrt{M_i^2 + M_i^2} \]

\[ \left\{ \frac{M_i}{\sqrt{M_i^2 + M_i^2}} \cos \omega t - \frac{M_i}{\sqrt{M_i^2 + M_i^2}} \sin \omega t \right\} \]

\[ u = u_o + \epsilon |B| \cos \alpha + (\omega t + \beta) \] (2.20)

where, \[ \beta = \tan^{-1} \frac{M_i}{M_i} \]
Skin friction at the plate is

\[ \tau_w = \left. \frac{du}{dy} \right|_{y=0} = \frac{G_i}{P^2 - P - \frac{1}{K}} \left( P - \alpha_i \right) \]

\[ + \epsilon e^{i\omega t} \left[ \frac{G_i}{C_i^2 - C_i^2 - \left( i\omega + \frac{1}{K} \right)} \left( C_i - L_i \right) \right] \]

\[ = A_o + \epsilon \left( \cos \omega t + i \sin \omega t \right) \left( B_r + B_i \right) \]

or \[ \tau_w = A_o + \epsilon |B| \cos (\omega t + \beta) \] (2.22)

where, \[ |B| = \sqrt{B_r^2 + B_i^2} \quad \text{and} \quad \beta = \tan^{-1} \left( \frac{B_i}{B_r} \right) \]

|B| is called the amplitude and \( \beta \) is called the phase lag of the fluctuating part of the skin friction.

\[ B_r = \frac{\left( P - 1 - \frac{1}{K} \right) \left( P^2 - \omega^2 - P - \frac{1}{K} \right)}{\left( P^2 - \omega^2 - P - \frac{1}{K} \right)^2 - 4\omega^2 \left( P - 1 \right)^2} \]

\[ B_i = \frac{\left( P - 1 - \frac{1}{K} \right) \left( 2\omega P - 2 \omega \right)}{\left( P^2 - \omega^2 - P - \frac{1}{K} \right)^2 - 4\omega^2 \left( P - 1 \right)^2} \]
Expression for transient velocity for $\omega t = \frac{\pi}{2}$ is given by

$$u = u_0 - \varepsilon M_i$$  \hspace{1cm} (2.23)

where, $M_i = \text{Imaginary part of } \left\{ \frac{G_i}{C_i^2 - C_i - i \omega - \frac{1}{K}} \left( e^{iC_i y} - e^{-C_i y} \right) \right\}$.

The numerical values of the transient velocity $u$ obtained from equation (2.23) are entered in the table for some values of the permeability parameter $K$ and the dimensionless frequency $\omega$ when $P = 0.71$, $\varepsilon = 0.2$ and $G_i = 4$ (which corresponds to the cooling of the plate by free convection currents).

Expression for transient temperature $\theta$ for $\omega t = \pi/2$ is obtained from equation (2.19) in the form

$$\theta = \theta_o - \varepsilon \theta_i$$  \hspace{1cm} (2.24)

where, $\theta_o = e^{-P y}$; \hspace{0.5cm} $\theta_i = -e^{-P y} \sin \omega y$.

Expression (2.24) shows that temperature field does not depend on permeability parameter $K$. Its variation in the medium depend on Prandtl number $P$ and frequency parameter $\omega$.  

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Heat transfer rate at the wall $\theta_w$ is given by

$$\theta_w = -\frac{d\theta}{dy} \bigg|_{y=0} = [-P + \epsilon e^{i\omega t} (-C_i)]$$

or

$$\theta_w = [P + \epsilon |D| \cos(\omega t + \delta)]$$  \hspace{1cm} (2.25)

where,

$$|D| = \sqrt{P^2 + W^2}; \quad \text{amplitude of } \theta_w$$

$$\delta = \tan^{-1} \frac{W}{P}; \quad \text{phase lag of } \theta_w.$$ 

**Discussions:**

From Tables – 2.1 and 2.2 we observe that transient velocity and temperature fields for $\omega t = \pi/2$ decrease with frequency parameter $\omega$. Fluid velocity varies with permeability parameter $K$ where as $K$ has no effect on temperature variation in the medium. Increment in $K$ accelerates the fluid velocity in the medium both at lower and higher values of $\omega$. From Tables – 2.3 and 2.4 we observe that amplitudes of velocity and temperature fields increase with the increase in frequency $\omega$. The phase lags $\beta$ and $\delta$ have opposite trend of variation with respect to $\omega$. The fluctuating phase $\beta$ velocity decreases and $\delta$ that of temperature increases with frequency parameter. Variation of flow parameter for air ($P = 0.71$) and water ($P = 7.01$) are shown in the Tables.
Table – 2.1: Variation of Transient Velocity for \( \omega t = \pi / 2 \) with respect to Frequency and Permeability Parameters.

\[
\begin{align*}
\text{u} & = u_0 - \varepsilon \in M_i \\
\omega = 6 & \quad \omega = 10 \\
K = 0.5 & \quad K = 2.0 & K = 0.5 & \quad K = 2.0
\end{align*}
\]

<table>
<thead>
<tr>
<th>y</th>
<th>u = 0.000</th>
<th>u = 0.000</th>
<th>u = 0.000</th>
<th>u = 0.000</th>
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<tbody>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
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<td>0.742</td>
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<tr>
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<td>0.623</td>
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<td>0.75</td>
<td>0.685</td>
<td>1.319</td>
<td>0.672</td>
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</tr>
<tr>
<td>1.00</td>
<td>0.663</td>
<td>1.357</td>
<td>0.650</td>
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<tr>
<td>1.25</td>
<td>0.606</td>
<td>1.313</td>
<td>0.592</td>
<td>1.304</td>
</tr>
<tr>
<td>1.50</td>
<td>0.537</td>
<td>1.224</td>
<td>0.532</td>
<td>1.220</td>
</tr>
</tbody>
</table>
Table – 2.2: Variation of Amplitude and Phase Lag of the Skin Friction for Incompressible Fluid (Water at 20°C, \( P = 7.01 \)).

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( K = 0.5 )</th>
<th>( K = 2.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( B_r )</td>
<td>( B_i )</td>
</tr>
<tr>
<td>5</td>
<td>-0.0177</td>
<td>-0.0177</td>
</tr>
<tr>
<td>10</td>
<td>-0.0222</td>
<td>-0.0177</td>
</tr>
<tr>
<td>15</td>
<td>0.4054</td>
<td>0.3945</td>
</tr>
</tbody>
</table>
Table – 2.3 : Variation of Transient Temperature

\( \theta \) for \( \omega t = \pi/2 \).

<table>
<thead>
<tr>
<th>y</th>
<th>( \theta = \theta_0 - \varepsilon \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P = 0.71 ) (air), ( \varepsilon = 0.2 )</td>
</tr>
<tr>
<td></td>
<td>( \omega = 5 )</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0062</td>
</tr>
<tr>
<td>1.0</td>
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<tr>
<td>1.5</td>
<td>0.4022</td>
</tr>
<tr>
<td>2.0</td>
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</tr>
</tbody>
</table>
Table – 2.4 : Variation of Amplitude and Phase

Lag of the Heat Transfer Rate with respect to $P$ and $\omega$.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$P = 0.71$</th>
<th></th>
<th>$P = 7.01$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>D</td>
<td>$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>5</td>
<td>5.0501</td>
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<td>0.0124</td>
</tr>
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</tr>
<tr>
<td>15</td>
<td>15.0167</td>
<td>0.3864</td>
<td>16.5529</td>
<td>0.0374</td>
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</tbody>
</table>
Graph – 2.1 a: Variation of Transient Velocity for $\omega t = \pi/2$ with respect to Frequency and Permeability Parameters.
Graph – 2.1 b: Variation of Transient Velocity for $\omega t = \pi/2$ with respect to Frequency and Permeability Parameters.
Graph – 2.2 a: Variation of Amplitude and Phase Lag of the Skin Friction for Incompressible Fluid (Water at 20°C, P = 7.01).
Graph – 2.2 b: Variation of Amplitude and Phase Lag of the Skin Friction for Incompressible Fluid (Water at 20°C, P = 7.01).
Graph – 2.3 a : Variation of Transient Temperature $\theta$ for $\omega t = \pi/2$.
Graph – 2.3 b: Variation of Transient Temperature $\theta$ for $\omega t = \pi/2$. 

$P = 7.01$ (water)
Graph – 2.4 a: Variation of Amplitude and Phase Lag of the Heat Transfer Rate with respect to \( P \) and \( \omega \).
Graph – 2.4 b: Variation of Amplitude and Phase Lag of the Heat Transfer Rate with respect to P and ω.
References


