Chapter - IV

STUDY OF BLOOD VISCOSITY VARIATION
THROUGH BINGHAM FLOW MODEL

Introduction:

Blood flow in capillaries of internal diameter equal to that red cell is quite different from that in the large vessels. Since the size of RBC is not negligible as compared to vessel diameter, it is necessary to consider the flow of blood in capillaries as two phase non homogeneous flow.

The blood consists of two layers one is plasma of Newtonian fluid and other is central core region of non-Newtonian fluid. The thickness of plasma layer is assumed to be constant and independent of time and location.

The apparent viscosity of blood depends on several factors such as plasma viscosity, hematocrit, size of the vessel, shear rate, rate of flow, rigidity and deformability etc. Also the apparent viscosity in capillaries is much lower than in large vessels e.g. Dintenfass [12]. Also it is well known that apparent viscosity of blood
decreases as the tube radius decreases (Fahraeus – Lindquist effect). Gupta and Seshadri [14], Lida and Murata [16], Charm et al. [5] have considered the two fluid models in which both layers (PPL and core hematocrit layers) are Newtonian fluids with different viscosities.

Many mathematical models have been proposed for the study of variation of blood viscosity for flows in narrow gaps. Gupta and Seshadri [14] studied the flow of red blood cell suspensions through narrow tubes. The event of plug flow was seen to increase with increase in hematocrit. Batra and Jena [2] studied the existence of cell free plasma layer near the wall and a core with varying red cell concentration have been observed by Mishra et al. [17] and Bugliarello and Sevilla [4]. Also several workers like Chaturani and Biswas [9], Chaturani [6] Bugliarello et al. [3], Das and Seshadri [11], Haynes [15], Ariman et al. [1] Chaturani [7]; Chaturani [8], Chaturani et al. [10], Yamada [19], Frigaard [13] and Ramkisson et al. [18] have worked in the same direction.

In the present chapter velocities in the core – regions, flow rate and effective viscosity have been obtained. Also, variations of flow rate and effective
viscosity with respect to PPL thickness have been discussed with the help of Tables and Graphs.

**Formulation and Solution:**

Consider a two layered Bingham flow model of blood in a rigid circular tube. \( R \) is the radius of the tube and \( \delta \) is the thickness of PPL.

![Diagram of Two Layered Bingham Flow Model](image)

The governing equation of motion are –

In the cell free peripheral region \( R - \delta \leq r \leq R \)

\[
\eta_f \left( \frac{d^2 u_{fp}}{dr^2} + \frac{1}{r} \frac{du_{fp}}{dr} \right) = \frac{dp}{dz} \tag{4.1}
\]

and in the core-region \( 0 \leq r \leq R - \delta \) for fluid plasma

\[
(1-\phi) \left[ \frac{dp}{dz} + \frac{1}{r} \frac{\partial}{\partial r} (r \tau) \right] + \phi F_d \left( u_{pc} - u_{fc} \right) = 0 \tag{4.2}
\]

For particular phase cells –
\[ \phi \left[ -\frac{dp}{dz} + \frac{1}{r} \frac{\partial}{\partial r} (r \tau) \right] + \phi F_d \left( u_{pc} - u_{fc} \right) + \mu_0 m_p \phi \frac{dH}{dz} = 0 \]

(4.3)

where \( \eta_f \) is fluid viscosity, \( \eta_s \) the suspension viscosity, \( \phi \) the volume occupied by the red cells per unit volume of the blood, \( u_{fp} \) and \( u_{fc} \) are the velocities of plasma in the peripheral layer and core regions respectively. \( u_{pc} \) the velocity of red cells in the core, \( P \) the pressure, \( \mu_0 \) the magnetic permeability of the blood and \( m_p \) the magnitude of magnetic intensity. Bingham fluid model is used for core fluid.

The boundary conditions are:

\[ u_{fp} = 0 \quad \text{at} \quad r = R \]
\[ u_{fc} \text{ is finite at} \quad r = 0 \]

(4.4)

\[ \begin{align*}
    u_{fp} &= u_{fc} \quad \text{at} \quad r = R - \delta \\
    \frac{\eta_s}{r} \frac{du_{fc}}{dr} &= \eta_f \frac{du_{fp}}{dr} \quad \text{at} \quad r = R - \delta \\
\end{align*} \]

(4.5)

\[ \tau = 0 \quad \text{at} \quad r = 0 \]

The Bingham constitutive equations may be given by

\[ \tau = \tau_0 + \eta \dot{\gamma} \]

(4.6)

Where \( \tau \) is the shear stress, \( \tau_0 \) the yield stress, \( \dot{\gamma} \) the strain rate and \( \eta \) is the coefficient of viscosity.

Equation (4.1) can be written as
\[ \eta_f \left[ \int \frac{\partial}{\partial r} \left( r \frac{du_{fp}}{dr} \right) \, dr \right] = \int \frac{dP}{dz} \, r \, dr \]

Integration above equation we get

\[ r \frac{du_{fp}}{dr} = \frac{1}{\eta_f} \left[ \frac{dP}{dz} \frac{r^2}{2} + C \right] \]  

(4.7)

Where \( C \) is constant of integration

or

\[ \frac{du_{fp}}{dr} = \frac{1}{\eta_f} \left[ \frac{dP}{dz} \frac{r}{2} + \frac{C}{r} \right] \]

On integration we get

\[ u_{fp} = \frac{1}{\eta_f} \left[ \frac{dP}{dz} \frac{r^2}{4} + C \log r \right] + C' \]

Using boundary condition at \( r = R, \ u_{fp} = 0 \)

\[ C' = -\left[ \frac{1}{\eta_f} \frac{dP}{dz} \frac{R^2}{4} + \frac{C}{\eta_f} \log R \right] \]

We get

\[ u_{fp} = \frac{1}{4\eta_f} \left[ \frac{dP}{dz} \left( r^2 - R^2 \right) \right] + \frac{C}{\eta_f} \left( \log r - \log R \right) \]

\[ u_{fp} = \frac{1}{4\eta_f} \left( r^2 - R^2 \right) + \frac{dP}{dz} + \frac{C}{\eta_f} \log \frac{r}{R} \]  

(4.8)

From equation (4.2) and (4.3), we get

\[ u_{pc} = u_{fc} + \left( 1 - \phi \right) \frac{\mu_0 \ m_p \ dH}{F_d} \]  

(4.9)

In view of equation (4.9), equation (4.3) becomes

\[ - \frac{dP}{dz} - \frac{1}{r} \frac{\partial}{\partial r} \left( r \ \tau \right) = - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \]  

(116)
or \[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \tau \right) = \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right)
\]

Integrating above equation we get
\[
r \ \tau = \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right) \frac{r^2}{2} + C_1
\]

Using boundary conditions \( \tau = 0 \) at \( r = 0 \) gives \( C_1 = 0 \)

Thus \[
\tau = \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right) \frac{r}{2}
\]

Now using Bingham constitutive equation in above equation we get
\[
\tau_0 + \eta_s \ \dot{\gamma} = \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right) \frac{r}{2}
\]

\[
\eta_s \ \frac{du_{fc}}{dr} = \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right) \frac{r}{2} - \tau_0
\]

or \[
\eta_s \ \frac{du_{fc}}{dr} = \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right) \frac{r}{2} - K_1
\]

\( ..(4.10) \)

where \( K_1 = \tau_0 \).

Now using boundary condition
\[
\eta_s \ \frac{du_{fc}}{dr} = \eta_f \ \frac{du_{fc}}{dr}, \ at \ r = R - \delta
\]

We get from equations (4.7) and (4.10)
\[
\frac{dP}{dz} \ \frac{(R - \delta)^2}{2} + C
\]

\[
= \left( \frac{dP}{dz} - \mu_0 \ m_p \ \phi \ \frac{dH}{dz} \right) \frac{(R - \delta)^2}{2} - K_1 \ (R - \delta)
\]

(117)
Putting the value of $C$ in equation (4.8) we get

$$u_{fp} = \frac{1}{4\eta_f} \left( r^2 - R^2 \right) \frac{dP}{dz} - \frac{1}{\eta_f} \left[ \mu_0 m_p \phi \frac{dH}{dz} \frac{(R - \delta)^2}{2} + K_1 (R - \delta) \right] \log \frac{r}{R} \quad \text{(4.11)}$$

On integrating equation (4.10) and using boundary condition

$$u_{fc} = u_{fp} \text{ we get, at } r = R - \delta$$

$$= \frac{1}{4\eta_f} \left( (R - \delta)^2 - R^2 \right) \frac{dP}{dz} - \frac{1}{\eta_f} \left[ \mu_0 m_p \phi \frac{dH}{dz} \frac{(R - \delta)^2}{2} + K_1 (R - \delta) \right] \log \left( \frac{1 - \delta}{R} \right)$$

$$= \frac{1}{\eta_f} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{(R - \delta)^2}{4} - K_1 (R - \delta) \right] + C_2$$

Then

$$C_2 = \frac{1}{4 \eta_f} \left[ (R - \delta)^2 - R^2 \right] \frac{dP}{dz}$$

$$- \frac{1}{\eta_f} \left[ \mu_0 m_0 \phi \frac{dH}{dz} \frac{(R - \delta)^2}{2} + K_1 (R - \delta) \right] \log \left( \frac{1 - \delta}{R} \right)$$

$$- \frac{1}{\eta_s} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{(R - \delta)^2}{4} - K_1 (R - \delta) \right]$$

$$u_{fc} = \frac{1}{\eta_s} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{r^2}{4} - K_1 r \right] + C_2$$

$$u_{fc} = \frac{1}{\eta_s} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{r^2}{4} - K_1 r \right] + \frac{1}{4 \eta_f} \left[ (R - \delta)^2 - R^2 \right] \frac{dP}{dz}$$

(118)
\[- \frac{1}{\eta_f} \left[ \mu_0 m_p \phi \frac{dH}{dz} \frac{(R-\delta)^2}{2} + K_1(R-\delta) \right] \log \left( 1 - \frac{\delta}{R} \right) \]

\[- \frac{1}{\eta_s} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{(R-\delta)^2}{4} - K_1(r+R-\delta) \right] \]

\[u_{fc} = \frac{1}{\eta_s} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{r^2-(R-\delta)^2}{4} - K_1(r+R-\delta) \right] \]

\[- \frac{1}{4 \eta_f} \left[ (R-\delta)^2 - R^2 \right] \frac{dP}{dz} \]

\[- \frac{1}{\eta_f} \left[ \mu_0 m_p \phi \frac{dH}{dz} \frac{(R-\delta)^2}{2} + K_1(R-\delta) \right] \log \left( 1 - \frac{\delta}{R} \right) \]

\[(4.12)\]

Volume flow rate \(Q\) is given by

\[Q = 2\pi \phi \int_0^{R-\delta} r u_{pc} \, dr + 2\pi (1-\phi) \int_0^{R-\delta} r u_{fc} \, dr + 2\pi \int_{R-\delta}^R r u_{fp} \, dr \]

\[Q = 2\pi \phi \int_0^{R-\delta} r (u_{pc} - u_{fc}) \, dr + 2\pi \int_0^{R-\delta} r u_{fc} \, dr + 2\pi \int_{R-\delta}^R r u_{fp} \, dr \]

\[Q = Q_1 + Q_2 + Q_3 \]

\[(4.13)\]

Where

\[Q_1 = 2\pi \phi \int_0^{R-\delta} r (u_{pc} - u_{fc}) \, dr , \]

\[Q_2 = 2\pi \int_0^{R-\delta} r u_{fc} \, dr , \]

\[Q_3 = 2\pi \int_{R-\delta}^R r u_{fp} \, dr , \]

\[Q_4 = 2\pi \phi \int_0^{R-\delta} r (u_{pc} - u_{fc}) \, dr \]

\[(119)\]
\[ Q_2 = 2\pi \int_0^{R-\delta} r u_{f,c} \, dr \]
\[ = \int_0^{R-\delta} r \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) \frac{r^2 - (R-\delta)^2}{4} - K_1 (r + R - \delta) \right] dr \]
\[ + \frac{2\pi}{4\eta_f} \int_0^{R-\delta} r \left[ (R-\delta)^2 - R^2 \right] \frac{dP}{dz} \, dr \]
\[ - \frac{2\pi}{\eta_f} \int_0^{R-\delta} r \left[ \mu_0 m_p \phi \frac{dH}{dz} \frac{(R-\delta)^2}{2} + K_1 (R - \delta) \right] \log \left( \frac{1-\delta}{R} \right) \, dr \]
or

\[ Q_2 = \frac{2\pi}{\eta_s} \left[ \left( \frac{dP}{dz} - \mu_0 m_p \phi \frac{dH}{dz} \right) - \frac{(R-\delta)^4}{16} - \frac{5}{3} K_1 (R-\delta)^3 \right] \]

\[ + \frac{2\pi}{4\eta_f} \left[ (R-\delta)^2 - R^2 \right] \frac{dP}{dz} \frac{(R-\delta)^2}{2} \]

\[ - \frac{2\pi}{\eta_f} \left[ \mu_0 m_p \phi \frac{dH}{dz} + K_1 (R-\delta) \right] \log \left( \frac{1-\delta}{R} \right) \frac{(R-\delta)^2}{2} \]

\[ Q_2 = \frac{\pi R^4}{8\eta_f} \left( -\frac{dP}{dz} \right) \left[ \frac{\eta_f}{\eta_s} (S_N - 1) \left( \frac{1-\delta}{R} \right)^4 - \frac{\eta_f}{\eta_s} \frac{80 K_1}{3 R} \left( \frac{1-\delta}{R} \right)^3 \right] - \frac{dP}{dz} \]

\[ + 2 \left( 1 - \frac{\delta}{R} \right)^2 \left( 1 - \left( \frac{1-\delta}{R} \right)^2 \right) + \frac{4 S_N \left( \frac{1-\delta}{R} \right)^4 \log \left( \frac{1-\delta}{R} \right)}{-\frac{dP}{dz}} \]

\[ + \frac{K_1}{R} \left( \frac{1-\delta}{R} \right)^3 \log \left( \frac{1-\delta}{R} \right) \frac{dP}{dz} \]

(4.15)

\[ Q_3 = 2\pi \int_{R-\delta}^{R} r u_{fp} \, dr = 2\pi \int_{R-\delta}^{R} r \left[ \frac{1}{4\eta_f} \left( r^2 - R^2 \right) \frac{dP}{dz} \right] \]

\[ - \frac{1}{\eta_f} \left\{ \mu_0 m_p \phi \frac{dH}{dz} \left( \frac{R-\delta)^2}{2} + K_1 (R-\delta) \right) \log \frac{r}{R} \right\} \, dr \]

(121)
or \[ Q_3 = \frac{\pi R^4 \left( \frac{-dP}{dz} \right)}{8 \eta_f} \left[ 1 - \left( 1 - \frac{\delta}{R} \right)^2 \right] - 2S_N \]

\[ \left\{ \left( 1 - \frac{\delta}{R} \right)^4 - 2 \left( 1 - \frac{\delta}{R} \right)^4 \log \left( 1 - \frac{\delta}{R} \right) - \left( 1 - \frac{\delta}{R} \right)^2 \right\} \]

\[ + \frac{4}{R} \left( 1 - \frac{\delta}{R} \right)^3 - \frac{8}{R} \left( 1 - \frac{\delta}{R} \right) \log \left( 1 - \frac{\delta}{R} \right) - \frac{4}{R} \left( 1 - \frac{\delta}{R} \right) \]

\[ \left( 4.16 \right) \]

From equations (4.13), (4.14), (4.15) and (4.16) we get

\[ Q = \frac{\pi R^4 \left( \frac{-dP}{dz} \right)}{8 \eta_f} \left[ \frac{8 \eta_f S_N (1 - \phi)}{F_d R^2} \left( 1 - \frac{\delta}{R} \right)^2 - \frac{\eta_f}{\eta_s} (S_N - 1) \left( 1 - \frac{\delta}{R} \right)^4 \right] \]

\[ - \frac{80 K_1}{3} \left( 1 - \frac{\delta}{R} \right)^3 \frac{\eta_f}{\eta_s} + 2 \left( 1 - \frac{\delta}{R} \right)^2 \left[ 1 - \left( 1 - \frac{\delta}{R} \right)^2 \right] \]

\[ + \frac{4 S_N \left( 1 - \frac{\delta}{R} \right)^4 \log \left( 1 - \frac{\delta}{R} \right)}{\frac{dP}{dz}} + \frac{K_1 \left( 1 - \frac{\delta}{R} \right)^3}{R} \frac{\log \left( 1 - \frac{\delta}{R} \right)}{\frac{dP}{dz}} \]

\[ + \left\{ 1 - \left( 1 - \frac{\delta}{R} \right)^2 \right\} - 2S_N \left\{ \left( 1 - \frac{\delta}{R} \right)^4 - 2 \left( 1 - \frac{\delta}{R} \right)^4 \log \left( 1 - \frac{\delta}{R} \right) - \left( 1 - \frac{\delta}{R} \right)^2 \right\} \]

\[ \left( 122 \right) \]
\[
\frac{4}{R} \left(1 - \frac{\delta}{R}\right)^3 - 8 \frac{\delta}{R} \log \left(1 - \frac{\delta}{R}\right) - \frac{4}{R} \left(1 - \frac{\delta}{R}\right)
\]

(4.17)

where \( S_N \) is a non-dimensional number and a ratio of magnetic force to pressure force. Effective viscosity can be derived by using the formula

\[
\eta = \frac{\pi R^4 \left( -\frac{dP}{dz} \right)}{8Q}
\]

For non-magnetic case \((m_p = 0)\) or when volume fraction \(\phi\) is zero the expression for total rate reduces to

\[
Q = \frac{\pi R^4}{8\eta_f} \left( -\frac{dP}{dz} \right) \left[ 1 - \left(1 - \frac{\delta}{R}\right)^4 \left(1 - \frac{\eta_f}{\eta_s}\right) \right]
\]

\[
+ \frac{K_1}{R} \left(1 - \frac{\delta}{R}\right)^3 \left[ 80 \frac{\eta_f}{3 \eta_s} - \log \left(1 - \frac{\delta}{R}\right) \right]
\]

\[
+ \frac{4}{R} \left(1 - \frac{\delta}{R}\right) \left\{ \left(1 - \frac{\delta}{R}\right)^2 - 2 \left(1 - \frac{\delta}{R}\right)^2 \log \left(1 - \frac{\delta}{R}\right) \right\} - 1
\]

(4.18)

When yield stress is zero i.e., \(K_1 = 0\),

(123)
Equation (4.17) reduces to

\[
Q = \frac{\pi R^4}{8 \eta_f} \left[ \frac{8 \eta_f S_N (1-\phi) \left( 1 - \frac{\delta}{R} \right)^2}{F_d R^2} - \frac{\eta_f}{\eta_s} (S-1) \left( 1 - \frac{\delta}{R} \right)^4 \right]
\]

\[
+ 2 \left( 1 - \frac{\delta}{R} \right)^2 \left\{ 1 - \left( 1 - \frac{\delta}{R} \right)^2 \right\} + \frac{4 S_N \left( 1 - \frac{\delta}{R} \right)^4}{dP \over dz} \log \left( 1 - \frac{\delta}{R} \right)
\]

\[
+ \left\{ 1 - \left( 1 - \frac{\delta}{R} \right)^2 \right\} + 2 S_N \left\{ \left( 1 - \frac{\delta}{R} \right)^2 - 2 \left( 1 - \frac{\delta}{R} \right)^2 \log \left( 1 - \frac{\delta}{R} \right) - \left( 1 - \frac{\delta}{R} \right)^2 \right\}
\]

\[
+ \frac{4}{R} \frac{\left( 1 - \frac{\delta}{R} \right)}{dP \over dz} \left\{ \left( 1 - \frac{\delta}{R} \right)^2 - 2 \left( 1 - \frac{\delta}{R} \right)^2 \log \left( 1 - \frac{\delta}{R} \right) - 1 \right\} \quad (4.19)
\]

Effective viscosity

\[
\eta_e = \frac{\pi R^4 \left( - dP \over dz \right)}{8 Q} \quad (4.20)
\]

For non-magnetic case \( (m_p = 0) \) and zero volume fraction, the expression for total flow rate reduces to

\[
Q = \frac{\pi R^4 \left( - dP \over dz \right)}{8 \eta_f} \left[ 1 - \left( 1 - \frac{\delta}{R} \right)^4 \left( 1 - \frac{\eta_f}{\eta_s} \right) \right]
\]
\[ + \frac{4}{R} \left( \frac{1 - \delta}{R} \right) \frac{dP}{dz} \left\{ \left(1 - \frac{\delta}{R}\right)^2 - 2 \left(1 - \frac{\delta}{R}\right)^2 \log \left(1 - \frac{\delta}{R}\right) - 1 \right\} \] (4.21)

and the effective viscosity is given by

\[ \eta_e = \eta_f \left[ 1 - \left(1 - \frac{\delta}{R}\right)^4 \left(1 - \frac{\eta_f}{\eta_s}\right) \right] \]

\[ + \frac{4}{R} \left( \frac{1 - \delta}{R} \right) \frac{dP}{dz} \left\{ \left(1 - \frac{\delta}{R}\right)^2 - 2 \left(1 - \frac{\delta}{R}\right)^2 \log \left(1 - \frac{\delta}{R}\right) - 1 \right\} \] (4.22)

**Table – 4.1**

Variation of flow rate with PPL thickness when \( K_i \neq 0, S_N > 1, R=10 \)

\( \eta_f = 1.25, \eta_s = 2.5, \phi = 0.02, F = 0.1, S_N = 10 \) and \( -\frac{dP}{dz} = 65.31 \)

<table>
<thead>
<tr>
<th>( \delta / R )</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>121522</td>
</tr>
<tr>
<td>0.02</td>
<td>129893</td>
</tr>
</tbody>
</table>
Table – 4.2
Variation of effective viscosity with PPL thickness when $K_1 \neq 0$ and $S_N > 1$

<table>
<thead>
<tr>
<th>$\delta \setminus R$</th>
<th>$\eta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.2133</td>
</tr>
<tr>
<td>0.02</td>
<td>0.1973</td>
</tr>
<tr>
<td>0.03</td>
<td>0.1848</td>
</tr>
<tr>
<td>0.04</td>
<td>0.1702</td>
</tr>
<tr>
<td>0.05</td>
<td>0.1760</td>
</tr>
</tbody>
</table>

Table – 4.3
Variation of flow rate with PPL thickness when $K_1 = 0$ and $S_N > 1$

<table>
<thead>
<tr>
<th>$\delta \setminus R$</th>
<th>$Q$</th>
</tr>
</thead>
</table>

(126)
Table – 4.4
Variation of effective viscosity with PPL thickness when $K_1 = 0$ and $S_n > 1$

<table>
<thead>
<tr>
<th>$\delta$ \ $R$</th>
<th>$\eta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.2126</td>
</tr>
<tr>
<td>0.02</td>
<td>0.1967</td>
</tr>
<tr>
<td>0.03</td>
<td>0.1847</td>
</tr>
<tr>
<td>0.04</td>
<td>0.1691</td>
</tr>
</tbody>
</table>
**Table – 4.5**
Variation of flow rate with PPL thickness when $K_1 = 0$ and $S_N = 0.1$

<table>
<thead>
<tr>
<th>$\delta \setminus R$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1173</td>
</tr>
<tr>
<td>0.02</td>
<td>1222</td>
</tr>
<tr>
<td>0.03</td>
<td>1274</td>
</tr>
<tr>
<td>0.04</td>
<td>1317</td>
</tr>
<tr>
<td>0.05</td>
<td>1388</td>
</tr>
</tbody>
</table>

**Table – 4.6**
Variation of effective viscosity with PPL thickness when $K_1 = 0$ and $S_N < 1$

$\left(S_N = 0.1\right)$

<table>
<thead>
<tr>
<th>$\delta \setminus R$</th>
<th>$\eta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>2.185</td>
</tr>
<tr>
<td>0.02</td>
<td>2.097</td>
</tr>
<tr>
<td>0.03</td>
<td>2.012</td>
</tr>
<tr>
<td>0.04</td>
<td>1.947</td>
</tr>
<tr>
<td>0.05</td>
<td>1.888</td>
</tr>
</tbody>
</table>
**Table – 4.7**

Variation of flow rate with PPL thickness when $m_p = 0$ and $K_1 = 0$

<table>
<thead>
<tr>
<th>$\delta \backslash R$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>10643</td>
</tr>
<tr>
<td>0.02</td>
<td>11051</td>
</tr>
<tr>
<td>0.03</td>
<td>11431</td>
</tr>
<tr>
<td>0.04</td>
<td>11799</td>
</tr>
<tr>
<td>0.05</td>
<td>12157</td>
</tr>
</tbody>
</table>

**Table – 4.8**

Variation of effective viscosity with PPL thickness when $m_p = 0$ and $K_1 = 0$

<table>
<thead>
<tr>
<th>$\delta \backslash R$</th>
<th>$\eta_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>2.408</td>
</tr>
<tr>
<td>0.02</td>
<td>2.319</td>
</tr>
<tr>
<td>0.03</td>
<td>2.242</td>
</tr>
<tr>
<td>0.04</td>
<td>2.172</td>
</tr>
<tr>
<td>0.05</td>
<td>2.108</td>
</tr>
</tbody>
</table>
Graph 4.1: Variation of flow rate with PPL thickness when $K_1 \neq 0$, $S_N > 1$. 
Graph-4.2: Variation of effective viscosity with PPL thickness when $K_1 \neq 0$, $S_N > 1$. 
Graph 4.3: Variation of flow rate with PPL thickness when $K_1=0$, $S_N > 1$. 
Graph-4.4: Variation of effective viscosity with PPL thickness when $K_1=0$, $S_N > 1$. 

(133)
Graph 4.5: Variation of flow rate with PPL thickness when $K_1 = 0, S_N < 1$. 

(134)
Graph-4.6: Variation of effective viscosity with PPL thickness when $K_1=0$, $S_N < 1$. 
Graph 4.7: Variation of flow rate with PPL thickness
when $K_z = 0$, $M_P = 1$. 
Graph 4.8: Variation of effective viscosity with PPL thickness when $K_1 = 0$, $M_P = 0$. 
Result and Discussion:

1. It is shown from Table – 4.1, that flow rate rapidly increases with increasing PPL thickness at constant value of $S_N$ but it is maximum at $\delta/R=0.04$ and then decreases with increasing PPL thickness.

2. Table – 4.2 shows that the effective viscosity decreases with increasing PPL thickness but it is maximum at $\delta/R=0.04$ and then increases with increasing PPL thickness.

3. From Table – 4.3 the variations of flow rate and PPL thickness are same as in the Table 4.1.

4. From Table 4.4. the variations in effective viscosity and PPL thickness are shown similarly as Table – 4.2.

5. From Table – 4.5, the flow increases with increasing PPL thickness for a constant value of $S_N$

6. From Table – 4.6, the effective viscosity decreases with increasing PPL thickness for a constant value of $S_N$ less than 1.

7. From Table – 4.7, in case of non-magnetic field, flow rate varies with PPL thickness.

8. In case of non-magnetic field, the effective viscosity decreases with increasing PPL thickness as shown in Table – 4.8.
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