Chapter 7

Conclusion

Nonlinear phenomena play a central role in the evolution of all natural systems. Several physical systems, system of economics and social sciences also evolve in a nonlinear way. Hence studies related to the mechanism of evolution and the characterisation of the corresponding dynamical states are always relevant and rewarding. Recently the results and the techniques used in such studies are being applied to many engineering problems like computation, robotics, communication etc. as well as to bio engineering, macroeconomics and epidemiology.

Nonlinearity can enrich a system by providing multistability, chaotic states and several noise induced phenomena, that normally do not occur in linear systems. The chaotic state is well established for its strangeness and ‘SIC’ ness, which means the same system starting with slightly different ways. Hence techniques developed to drive two identical chaotic systems into a synchronous evolution is a very welcome aspect in the study and application of such systems.

In the work presented in this thesis, we have broadly reviewed the ba-
sic features of nonlinear systems and the possible ways or coupling schemes of producing synchronisation in them. We also describe the different types of synchronised states and the methods to clarify and characterise them. We take two dimensional discrete dynamical systems, viz GM map which is useful in producing fractal patterns and decision making algorithms and Predator-Prey map which is most often encountered in the population dynamics of evolutionary biology.

The dynamical states and bifurcation structure of GM map and its variant MGM map are analysed in detail. We find that the fractal GM patterns arise due to the occurrence of recurring periodic and self similar substructure in the bifurcation scenario, each cycle with its own window of intermittency, periodicity, quasiperiodicity and merging of bands leading to chaos. These windows are characterised using Lyapunov exponents and basin structure and the intermittency is established as Type I with an index 0.5. The MGM map, on the contrary follows a Hopf bifurcation sequence with limit cycles further undergoing period doubling bifurcation to chaos.

In addition to the usual schemes like linear feedback and linear difference coupling, we introduce a new scheme called additive parametric coupling which is especially suited for GM, where the coupling involves the parameter of the individual map. There three schemes are applied to GM and MGM and the important features of synchronisation in each case is briefed below.

(i) Linear feedback and mutual coupling is applied to two identical systems in the X variable alone. Synchronisation in periodic cycles with an average synchronisation time of 778 for the chosen parameters and a stabilisation time of 410 after synchronisation is disturbed by a random noise as perturbation.

The same scheme applied to MGM, results in synchronised fixed
points or limit cycles. The separate basins for both are isolated in the parameter plane and phase plane of the system. In the case of fixed point, stabilisation time is much less compared to the case of limit cycle.

(ii) Linear difference coupling is applied mutually to one of the variables. This leads to lag synchronisation in GM, which is restricted to a narrow region in the parameter plane. In the case of MGM, this gives rise to totally synchronised chaotic states but with larger synchronisation time and stabilisation time.

(iii) Additive parametric coupling is introduced for the GM type maps, which depends on the parameter of the individual map also and applied mutually to one of the variables. For GM, this scheme results in total synchronised four cycles and lag synchronisation is 10 cycles. In MGM, this leads to fully synchronised chaotic states. However in this case total synchronisation time $\tau_1$ and stabilisation time are less.

In all the above cases, the possible parameter values for achieving synchronisation are isolated in the $(\mu, \epsilon)$ plane and the initial values or basin of synchronisation is the $(X_1, X_2)$ plane are also displayed. It is to be noted that two GM maps always synchronise in periodic states while MGM maps can give rise to synchronised chaotic states. Thus control of chaos is achieved along with synchronisation for GM maps.

We use the GM maps as onsite dynamics and develop array systems, both vertical and horizontal and apply one way with a view to develop generalised synchronisation and complete synchronisation.

A vertical array working under drive-response mechanism, develops generalised synchronisation, the stability of the whole state is analysed using Maximum Conditional Lyapunov exponent. The dependence of response time for synchronisation to occur simultaneously in all units, on the
The coupling coefficient is studied for an array of 50 units driven by a similar unit. The coupling in this case is through the non-linear function which is a part of the map function of the GM system. For higher values of $\epsilon$ complete synchronisation is observed.

We introduce the concept of sequential synchronisation as a horizontal array of system driven in a direction as an open flow system. In an array of 51 units, synchronised chaotic states set in sequentially from one end to the other and the delay time required for the $n^{th}$ unit to synchronise after the $(n-1)^{th}$ has synchronised is found to saturate with the system size and the total response time of the whole array is analysed as a function of $\epsilon$.

An interesting observation in the horizontal array is a type of bunching effect that reflects in the total response time. Instead of fixing the same value of $\epsilon$ for all the units, its value is fixed the same for a bunch of units and increased step wise from bunch to bunch. Then the total response time is smaller compared to the previous case. Moreover there is a specific size for the bunch for which this time can be minimised. This makes sequential synchronisation flexible and controllable to suite specific applications.

Synchronisation of chaotic systems is a very active field of intense research especially because of its applications in control and secure communications. Many biological, social, economic and technological systems from complex networks of connected systems and hence synchronisation in spatially connected systems also is an interesting field of recent research. The onset of synchronisation in such systems has many aspects like scaling behaviour, cluster formation etc. In this work we analysed these aspects in unidirectionally coupled systems forming a horizontal array with generalised synchronisation. This can be extended to two dimensional arrays with vertical arrays at each point of a horizontal array. Moreover the associated critical properties and their scaling during synchronisation transition forms a rich field for future research.
We consider a typical case with Predator-Prey system as onsite dynamics forming a spatially coupled array. Similar studies with population dynamics will be relevant in models of epidemics spreading, immigration and self organisation of species etc.

Recently synchronisation in time delay systems leading to delay and anticipatory synchronisation has attracted much attention. Anticipatory synchronisation is claimed to be capable of predicting the behaviour of chaotic systems better than the usual prediction time given by the inverse of Lyapunov exponent. Such studies with unidirectionally coupled arrays of the type discussed in this thesis can lead to many interesting features associated with propagation of signals. Thus these studies on chaotic synchronisation opens up a variety of related studies both in characterisation of many coherence phenomena, in applications like faster propagation of information, synchronisation with controllable time delay and in critical properties near onset of synchronisation transition.