Chapter 6

Array synchronisation in Predator-Prey systems

6.1 Introduction

Predator-Prey model forms a class of dynamical systems that model the population of a species of prey and a species of predator and is governed by the equations given in 1.24, section 1.4.2. A discrete version of this system is capable of exhibiting fixed point, quasi-periodicity, chaos and escape [30]. In this chapter we report the studies on a horizontal array of such systems with one way coupling. Just as in the case of GM maps, here also we observe sequential synchronisation. In addition we find the formation of local clusters before global synchronisation. The scaling behaviour of three different order parameters and the corresponding critical exponents are also studied.
6.2 Synchronisation in horizontal array

A horizontal array of $N$ identical systems with open ends is constructed with the local dynamical unit as a predator prey map. Each unit is driven by the previous one. Thus the $i^{th}$ unit in the horizontal array follows the dynamics

$$X^i(n + 1) = aX^i(n) - \epsilon_1 X^i(n)Y^i(n) - KX^{2i}(n) - \epsilon_P(aX^{i-1}(n) - KX^{2i-1}(n) - aX^i(n) + KX^{2i}(n))$$

$$Y^i(n + 1) = \epsilon_2X^i(n)Y^i(n) - bY^i(n). \tag{6.1}$$

The coupling is on the $X$ equation and $\epsilon_P$ is the coupling coefficient.

The control parameter $\epsilon_2$ is the same for all the units such that the units are chaotic individually. The various parameters are fixed as follows. $a = 1.5, b = 0.4, K = 0.6, \epsilon_1 = 0.4$ and $\epsilon_2 = 3.2$.

As $\epsilon_p$ is increased more and more units synchronise in a sequential way and for $\epsilon_p = 0.00193$ complete synchronisation is observed which is the critical value of the coupling coefficient ($\epsilon_c$) i.e., $\epsilon_p = \epsilon_c = 0.00193$.

Total response time which is the total time taken for the last unit to synchronise is found to decrease with the increase in the coupling coefficient $\epsilon_p$. Fig 6.1 shows the variation of total response time $\langle \tau_s \rangle$ with the coupling coefficient $\epsilon_p$. This is drawn for 3 arrays consisting of 2 maps, 5 maps and 50 maps.

The delay time $\tau_l$ which is the additional time for the $N^{th}$ unit to synchronise after its previous one has synchronised is found to vary as shown in Fig 6.2.
Figure 6.1: Variation of total response time $\langle \tau_s \rangle$ with the coupling coefficient $\epsilon_p$ for these arrays continuing of 2 maps, 5 maps and 50 maps.

Figure 6.2: Variation of delay time which is the additional time for the $N$th unit to synchronise after its previous one has synchronised against the site number.
6.3 Cluster synchronisation

In large coupled systems, with local coupling, it is found that synchronisation occurs in small clusters which further develop into global synchronisation of the whole structure as the coupling strength is increased. This phenomena of cluster synchronisation has been reported in lattices of chaotic spikey-bursting neurons with local interactions and diffusive coupling [132–134] in coupled map networks [135] and scale free networks [138]. Cluster formation in the context of coupled nonidentical circle maps have also been studied [139].

In the present context of arrays of Predator-Pray maps, similar cluster formation is observed for lower values of the coupling constant. Thus for \( \epsilon_p < \epsilon_c \), clustering is observed in which two or more units in the linear array stabilise to a synchronised state. For \( \epsilon_p = 0.0005 \) some of the units stabilise to the same state as shown in Fig 6.3. Fig 6.5 gives the variation of \( X^i \)

![Figure 6.3: Clustering observed in the linear array for \( \epsilon_p = 0.0005 \). Iteration number is plotted along X axis and site number along Y axis.](image)

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Figure 6.4: Clustering observed in the linear array, 6.3 is zoomed for 1800 iterations.

Figure 6.5: The variation of $X^i$ and site number $i$ for different coupling strengths $\epsilon_p = 0.0003, 0.0004, 0.0005$ and .0006.
and site number \( i \) for different coupling strengths \( \epsilon_p, 0.0003, 0.0004, 0.0005 \) and 0.0006. The graph is plotted after 90,000 iterations. It is found that as \( \epsilon_p \) is increased clustering of various units moves to the right. Here iteration number is taken along the \( X \) axis and site number is along the \( Y \) axis. The graph is drawn for 10,000 iterations Fig 6.4 shows the same graph which is zoomed for 1800 iterations.

### 6.4 Scaling behaviour

The synchronisation transition is studied as a non equilibrium phase transition and the critical properties can be analysed for spatially extended systems [123]. Such a transition for two replicas of Coupled Map Lattices with short range interaction [124–131] is found continuous and the associated order parameters have a powerlaw exponent [136, 137].

In analogy with phase transition, different order parameters can be defined as [123].

\[
\begin{align*}
\rho(N, t, \epsilon_c - \epsilon) &= \frac{1}{N} \sum_{i=1}^{N} |X_i - \langle X \rangle| \\
\sigma^2(N, t, \epsilon_c - \epsilon) &= \frac{1}{N} \sum_{i=1}^{N} |X_i - \langle X \rangle|^2
\end{align*}
\]

where \( \langle X \rangle = \frac{1}{N} |\sum X_i| \).

All the above quantities will be zero in the synchronous state and will have non zero value in the asynchronous state.
The quantities $\rho(N, t, \epsilon_c - \epsilon)$ and $\sigma^2(N, t, \epsilon_c - \epsilon)$ are the first and second moments of absolute difference of variable values at various sites from the synchronous state at given time $t$. The quantity $d(N, t, \epsilon_c - \epsilon)$ gives the local fluctuations in the variables.

Define exponents $\alpha_1$, $\alpha_2$ and $\alpha_3$ as the scaling factors through the equations:

\[
\begin{align*}
d &\sim t^{-\alpha_1} \\
\rho &\sim t^{-\alpha_2} \\
\sigma^2 &\sim t^{-\alpha_3}
\end{align*}
\]

These are computed numerically for the array of Predator-Prey systems by drawing graphs with log of the order parameter along $Y$ axis and log of iteration number along the $X$ axis. The calculation is done for 80000 iterations. Graph is plotted after every 5000 iterations. Fig 6.6 gives the $d$ vs $t$ graph from which $\alpha_1 = +1.06$. Fig 6.7 gives the $\rho$ vs $t$ graph from which $\alpha_2 = 0.45$. Fig 6.8, $\sigma^2$ vs $t$ is drawn and scaling factor $\alpha_3$ is obtained as $\alpha_3 = 0.5$.

![Figure 6.6: log $d$ is plotted against iteration number log $t$. Scaling factor $\alpha_1 = +1.06$.](image-url)
Figure 6.7: Order parameter $\log \rho$ is plotted against iteration number $\log t$. Scaling factor $\alpha_2 = 0.45$.

Figure 6.8: $\log \sigma^2$ is plotted against iteration number $\log t$. Scaling factor $\alpha_3 = 0.5$. 
6.5 Conclusion

The Predator-Prey system is taken as a discrete dynamical system and an array system is constructed using it as the local dynamics with one directional coupling. The onset of sequential synchronisation and associated quantities are studied. In this case, we also observe the formation of localised clusters, before the onset of global synchronisation. Three order parameters are defined and the corresponding critical exponents isolated.