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HEAT TRANSFER IN THREE DIMENSIONAL HYDROMAGNETIC FLOW ALONG A POROUS INFINITE PLATE IN THE PRESENCE OF VISCOUS DISSIPATIVE HEAT

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Abstract: Gebhart has observed that in natural convection flow fields of extreme size or extremely low temperatures or in high gravity field, the heat due to viscous dissipation plays a dominant role. Hence taking the dissipative loss into account, the heat transfer in the hydromagnetic boundary layer flow of viscous incompressible and finitely conducting fluid along an infinite porous plate with slightly transverse sinusoidal suction has been analyzed. Expressions for velocity profile and temperature distribution are obtained. The rate of heat transfer coefficient on the surface of the plate is obtained and numerically computed to get physical insight of the problem.

AMS Subject Classification: 26A33
Key Words: heat transfer, dissipative heat, sinusoidal suction, Nusselt number

1. Introduction

Study of heat transfer in porous medium has paramount importance because of its potential applications in soil physics, geohydrology, filtration of solids from liquicks, chemical engineering and biological systems. The heat transfer in saturated porous medium in presence of transverse magnetic field was studied by Rao et al [18] and brought out the effects of porosity parameter on temperature.

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and Nusselt number. They have shown that the effects of increasing porous parameter is to increase the temperature and Nusselt number. But the effect of magnetic field on temperature is not discussed. Rao et al [14] made a similar analysis in a rotating straight pipe and derived the effects of porous parameter on the Nusselt number. It is shown that Nusselt number increases with increase in porosity. Gulab et al [8] investigated the unsteady flow in MHD porous media and concluded that the increase in porosity accelerates the flow. According to them unsteady motion converts into steady flow after considerable lapse of time and the flow becomes more stable near the wall. Gupta et al [9] studied three dimensional flow past a porous plate and established the effects of Hartmann number and suction parameter on velocity and skin friction.

Sikiadis [17] has shown that the flow past a continuously moving plate is different from the flow past a stationary plate studied by Blasius [2]. Goldstein et al [7] studied the heat transfer aspect of continuously moving plate without considering viscous dissipative heat. But when the velocity of the plate is rather higher in an incompressible fluid or when the Prandtl number of an incompressible fluid is high, the viscous dissipative heat does play an important role in heat transfer problem.


Chaudhari et al [3] investigated combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium.

Muthucumaraswamy et al [12] studied effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature.

The present paper deals with heat transfer aspect in three dimensional hydromagnetic flow along a porous infinite plate in the presence of viscous dissipative heat. Expression for velocity profile and temperature distribution are obtained. With the help of these the Nusselt number is obtained and numerically computed for various values of suction parameter S, Hartmann number M and Reynolds number Re.

Results of numerical calculations are analysed to observe the influence of these parameters on the flow.

2. Mathematical Model Equations

We consider the steady free convection laminar flow of an electrically conducting viscous, incompressible fluid past an infinite porous flat plate. The plate is subjected to a slightly sinusoidal transverse suction velocity distribution. The suction velocity profile leads to a cross-flow and consequently to a three dimensional flow over the surface. Let L be the wavelength of the weak super-imposed suction velocity distribution. The plate is taken horizontally as x-z plane. The x-axis is assumed to be along the horizontal porous infinite plate in the horizontal direction and z axis is taken perpendicular to it. The origin of coordinate system is taken to be at a point on the plate. Since the length of the plate is very large, all the physical variables are independent of x. The uniform magnetic field is applied perpendicularly to the plane x-z of the plate. The magnetic Reynolds number is taken very small. So the induced magnetic field is neglected. Hall effects, electrical and polarizations effect, Joule heating in the energy equation are also neglected. With these assumptions, the governing equations for the problem under consideration after boundary layer
simplifications in present notations in dimensionless form are:

\[
\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{1}
\]

\[
\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{z}} = \frac{1}{\text{Re}} \left[ \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{u}}{\text{Re}}, \tag{2}
\]

\[
\frac{\partial \bar{w}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{\text{Re}} \left[ \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right], \tag{3}
\]

\[
\frac{\partial \bar{w}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{\text{Re}} \left( \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) - \frac{M^2 \bar{w}}{\text{Re}}, \tag{4}
\]

\[
\frac{\partial \bar{T}}{\partial \bar{y}} + \frac{\partial \bar{T}}{\partial \bar{z}} = \frac{1}{Pr\text{Re}} \left( \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right) + \frac{E_{\text{CO}}}{\text{Re}}. \tag{5}
\]

Here

\[
\phi = 2 \left[ \left( \frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 + \left( \frac{\partial \bar{w}}{\partial \bar{z}} \right)^2 \right] + \left[ \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \left( \frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{z}} \right)^2 + \left( \frac{\partial \bar{w}}{\partial \bar{z}} \right)^2 \right]. \tag{6}
\]

The boundary conditions are:

\[
\bar{y} = 0 : \bar{u} = 0, \quad \bar{v} = -S(1 + \cos \pi \bar{z}), \quad \bar{w} = 0, \quad \bar{T} = 1,
\]

\[
\bar{y} \to \infty : \bar{u} = 1, \quad \bar{v} = -S, \quad \bar{w} = 0, \quad \bar{p} = \bar{p}_\infty, \quad \bar{T} = 0, \tag{7}
\]

where

\[
\bar{y} = \frac{y}{L}, \quad \bar{z} = \frac{z}{L}, \quad \bar{u} = \frac{U}{U}; \quad \bar{v} = \frac{v}{U}; \quad \bar{w} = \frac{w}{U}; \quad \bar{p} = \frac{p}{\rho U^2}; \quad \bar{T} = \frac{T - T_\infty}{T_\infty - T_\infty} \tag{8}
\]

\(\varepsilon (\ll 1)\) = Amplitude of the suction velocity distribution; \(\bar{u}, \bar{v}, \bar{w}\) are the dimensionless velocity components in \(x, y, z\) directions respectively, \(\bar{T}\) the non-dimensional temperature, \(\rho\) the constant density, \(\nu \left(= \frac{D}{L}\right)\) Kinematic viscosity, \(\mu\) the coefficient of viscosity, \(Re \left(= \frac{UL}{\nu}\right)\) the Reynolds number, \(Pr \left(= \frac{\nu}{\lambda}\right)\) the Prandtl number, \(c_p\) specific heat at constant pressure, assumed constant,

\[
\frac{E_{\text{co}}}{\frac{c_p\lambda}{\varepsilon(\nu^2 - T_\infty)}} \text{the Eckert number}, \quad M = -B_0L \left(\frac{\mu}{\nu}\right)^2 \text{the Hartmann number}, \quad \alpha \text{ the electrical conductivity}, \quad \bar{v} > 0 \text{the basic steady suction velocity}, \quad s \left(= \frac{L}{D}\right) \text{suction parameter}, \quad U \text{the free streamline velocity}, \quad L \text{characteristic length (wave length)}.
3. Method of Solution

For the solution of equations (1) to (5) we assume the following expressions for \( \bar{u}, \bar{v}, \bar{w}, \bar{p} \) and \( T \) near the plate:

\[
\begin{align*}
\bar{u}(\bar{y}, \bar{z}) &= \bar{u}_0(\bar{y}) + \varepsilon \bar{u}_1(\bar{y}, \bar{z}) + \varepsilon^2 \bar{u}_2(\bar{y}, \bar{z}) + \cdots, \\
\bar{v}(\bar{y}, \bar{z}) &= \bar{v}_0(\bar{y}) + \varepsilon \bar{v}_1(\bar{y}, \bar{z}) + \varepsilon^2 \bar{v}_2(\bar{y}, \bar{z}) + \cdots, \\
\bar{w}(\bar{y}, \bar{z}) &= \bar{w}_0(\bar{y}) + \varepsilon \bar{w}_1(\bar{y}, \bar{z}) + \varepsilon^2 \bar{w}_2(\bar{y}, \bar{z}) + \cdots, \\
\bar{p}(\bar{y}, \bar{z}) &= \bar{p}_0(\bar{y}) + \varepsilon \bar{p}_1(\bar{y}, \bar{z}) + \varepsilon^2 \bar{p}_2(\bar{y}, \bar{z}) + \cdots, \\
\bar{T}(\bar{y}, \bar{z}) &= \bar{T}_0(\bar{y}) + \varepsilon \bar{T}_1(\bar{y}, \bar{z}) + \varepsilon^2 \bar{T}_2(\bar{y}, \bar{z}) + \cdots.
\end{align*}
\]

Substitute (9) in equation (1) to (5) and compare the coefficients of identical power of \( \varepsilon \). The zeroth order equations give following ordinary differential equations:

\[
\begin{align*}
\frac{d\bar{v}_0}{d\bar{y}} &= 0, \\
\frac{d\bar{v}_0}{d\bar{y}} &= \frac{1}{Re} \frac{d^2\bar{w}_0}{d\bar{y}^2} - \frac{M^2 \bar{u}_0}{Re}, \\
\frac{d\bar{w}_0}{d\bar{y}} &= \frac{d\bar{p}_0}{d\bar{y}} + \frac{1}{Re} \frac{d^2\bar{v}_0}{d\bar{y}^2}, \\
\frac{d\bar{v}_0}{d\bar{y}} &= \frac{1}{Re} \frac{d^2\bar{w}_0}{d\bar{y}^2} - \frac{M^2 \bar{w}_0}{Re}, \\
\frac{d\bar{T}_0}{d\bar{y}} &= \frac{1}{Re P_r} \frac{d^2\bar{T}_0}{d\bar{y}^2} + \frac{E}{Re} \left[ \frac{d\bar{u}_0}{d\bar{y}} \right]^2
\end{align*}
\]

whose solutions give \( \bar{u}_0(\bar{y}) \) and \( \bar{T}_0(\bar{y}) \).

The first order equations are partial differential equations which describe three dimensional MHD flow. These equations are as follows:

\[
\begin{align*}
\frac{\partial \bar{u}_1}{\partial \bar{y}} + \frac{\partial \bar{w}_1}{\partial \bar{z}} &= 0, \\
\bar{v}_1 \frac{\partial \bar{u}_0}{\partial \bar{y}} + \bar{v}_0 \frac{\partial \bar{u}_1}{\partial \bar{y}} &= \frac{1}{Re} \left[ \frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}_1}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{u}_1}{Re}, \\
\bar{v}_1 \frac{\partial \bar{v}_0}{\partial \bar{y}} - S \frac{\partial \bar{v}_1}{\partial \bar{y}} &= \frac{1}{Re} \left[ \frac{\partial^2 \bar{v}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}_1}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{v}_1}{Re}, \\
\bar{v}_0 \frac{\partial \bar{u}_1}{\partial \bar{y}} &= \frac{\partial \bar{p}_1}{\partial \bar{y}} + \frac{1}{Re} \left[ \frac{\partial^2 \bar{v}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}_1}{\partial \bar{z}^2} \right], \\
-S \frac{\partial \bar{w}_1}{\partial \bar{y}} &= -\frac{\partial \bar{p}_1}{\partial \bar{z}} + \frac{1}{Re} \left[ \frac{\partial^2 \bar{v}_1}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}_1}{\partial \bar{z}^2} \right] - \frac{M^2 \bar{w}_1}{Re},
\end{align*}
\]
\[
\vec{u}_1 \frac{\partial T_0}{\partial \bar{y}} - S \frac{\partial T_1}{\partial \bar{y}} = \frac{1}{P_r Re} \left[ \frac{\partial^2 T_1}{\partial \bar{y}^2} + \frac{\partial^2 T_1}{\partial \bar{z}^2} \right] + \frac{2Ec}{Re} \frac{\partial \vec{u}_0}{\partial \bar{y}} \cdot \frac{\partial \vec{u}_1}{\partial \bar{y}}.
\tag{20}
\]

For the solution of above partial differential equations we assume:
\[
\vec{u}_1(\bar{y}, \bar{z}) = \bar{u}_{1,1}(\bar{y}) \cos \pi \bar{z}, \quad \bar{u}_1(\bar{y}, \bar{z}) = \bar{u}_{1,1}(\bar{y}) \cos \pi \bar{z},
\]
\[
\bar{w}_1(\bar{y}, \bar{z}) = -\frac{1}{\pi} \bar{v}_{1,1}(\bar{y}) \sin \pi \bar{z}, \quad \bar{P}_1(\bar{y}, \bar{z}) = \bar{P}_{1,1}(\bar{y}) \cos \pi \bar{z}, \quad \bar{T}_1(\bar{y}, \bar{z}) = \bar{T}_{1,1}(\bar{y}) \cos \pi \bar{z},
\tag{21}
\]

where the prime in \(v_{1,1}\) denotes the differentiation with respect to \(\bar{y}\).

4. Solution

Solving zeroth and first order equations, we get
\[
\bar{u}_0(\bar{y}) = 1 - e^{-A_1 \bar{y}},
\]
\[
\bar{T}_0(\bar{y}) = (1 + h_1)e^{-sP_r \bar{y}} - h_1 e^{-2A_1 \bar{y}},
\tag{22}
\tag{23}
\]
\[
\bar{T}_1(\bar{y}, \bar{z}) = \frac{sP_r Re \cos \pi \bar{z}}{A_3 - A_4} \left[ \frac{2A_1}{A_7} \left\{ A_3 h_1 + E_c A_{12} (A_1 + A_4) \right\} \left\{ e^{-A_6 \bar{y}} - e^{-(2A_1 + A_4) \bar{y}} \right\} 

- \frac{2A_1}{A_8} \left\{ A_4 h_1 + E_c A_3 (A_1 + A_3) \right\} \left\{ e^{-A_6 \bar{y}} - e^{-2(A_1 + A_3) \bar{y}} \right\} 

+ \frac{sP_r Re A_4 (1 + h_1)}{A_9} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + sP_r Re) \bar{y}} \right\} 

- \frac{sP_r Re A_3 (1 + h_1)}{A_{10}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + sP_r Re) \bar{y}} \right\} 

+ \frac{2A_1 A_6 E_c (A_{13} - A_{12})}{A_{11}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + A_3) \bar{y}} \right\} \right] + \frac{2A_1 A_6 E_c (A_{13} - A_{12})}{A_{11}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + A_3) \bar{y}} \right\} \right],
\tag{24}
\]

where
\[
h_1 = \frac{P_r E_c A_1}{2(2A_1 - sP_r Re)}.
\tag{25}
\]

From equation (9) taking \(\bar{T}(\bar{y}, \bar{z}) = \bar{T}_0(\bar{y}) + \bar{h}_1(\bar{y}, \bar{z})\) and substituting (23) and (24) in it, we get
\[
\bar{T}(\bar{y}, \bar{z}) = (1 + h_1)e^{-sP_r \bar{y}} - h_1 e^{-2A_1 \bar{y}}

+ \frac{e^{sP_r Re \cos \pi \bar{z}}}{A_3 - A_4} \left[ \frac{2A_1}{A_7} \left\{ A_3 h_1 + E_c A_{12} (A_1 + A_4) \right\} \left\{ e^{-A_6 \bar{y}} - e^{-(2A_1 + A_4) \bar{y}} \right\} 

+ \frac{2A_1 A_6 E_c (A_{13} - A_{12})}{A_{11}} \left\{ e^{-A_6 \bar{y}} - e^{-(A_1 + A_3) \bar{y}} \right\} \right].
\]
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\[
- \frac{2A_1}{A_8} \left\{ A_9 h_1 + E_0 A_9 (A_1 + A_9) \right\} \left\{ e^{-A_8 y} - e^{-2A_1 + A_9 y} \right\} \\
+ \frac{SP_i R_e A_4 (1 + h_1)}{A_9} \left\{ e^{-A_8 y} - e^{-A_1 + SP_i R_e y} \right\} \\
- \frac{SP_i R_e A_3 (1 + h_1)}{A_{10}} \left\{ e^{-A_8 y} - e^{-A_1 + SP_i R_e y} \right\} \\
+ \frac{2A_1 A_2 E_0 (A_{13} - A_{12})}{A_{11}} \left\{ e^{-A_8 y} - e^{-A_1 + A_9 y} \right\}.
\]

(26)

This gives the temperature field. Knowing the temperature profile, the rate of heat transfer coefficient at the surface of the plate in terms of the Nusselt number \(Nu\) is obtained as:

\[
Nu = \frac{SP_i R_e}{1 + h_1 - \frac{2A_1 h_1}{SP_i R_e} + \varepsilon (1 - H) \cos \pi \frac{y}{L}}.
\]

(27)

where

\[
H = 1 - \frac{1}{A_3 - A_4} \left[ \frac{2A_1}{A_7} \left\{ A_9 h_1 + E_0 A_9 (A_1 + A_9) \right\} (A_6 - 2A_1 - A_4) \\
- \frac{2A_1}{A_5} \left\{ A_9 h_1 + E_0 A_9 (A_1 + A_9) \right\} (A_6 - 2A_5 - A_3) \\
+ \frac{SP_i R_e A_4}{A_9} (1 + h_1) (A_6 - SP_i R_e - A_3) \\
+ \frac{SP_i R_e A_3}{A_{10}} (A_6 - SP_i R_e - A_4) \\
+ \frac{2A_1 A_2 E_0 (A_{13} - A_{12})}{A_{11}} (A_6 - A_1 - A_3) \right].
\]

(28)

and

\[
A_1 = \frac{1}{2} \left[ \frac{SP_i R_e + (S^2 R_e^2 + 4M^2)^\frac{1}{2}}{y} \right], \\
A_2 = \frac{1}{2} \left[ \frac{SP_i R_e - (S^2 R_e^2 + 4M^2)^\frac{1}{2}}{y} \right], \\
A_3 = \frac{1}{2} \left[ A_1 + (A_1^2 + 4\pi^2)^\frac{1}{2} \right], \\
A_4 = \frac{1}{2} \left[ A_2 + (A_2^2 + 4\pi^2)^\frac{1}{2} \right], \\
A_5 = \frac{1}{2} \left[ \frac{SP_i R_e + \{S^2 R_e^2 + 4(\pi^2 + M^2)\}^\frac{1}{2}}{y} \right], \\
A_6 = \frac{1}{2} \left[ \frac{SP_i R_e + \{SP_i R_e^2 + 4\pi^2\}^\frac{1}{2}}{y} \right], \\
A_7 = (2A_1 + A_4)^2 - SP_i R_e (2A_1 + A_4) - \pi^2,
\]
Table 1: Values of $N_u$ for different values of $Pr, S, Re$ when $Ec = 0.01, \epsilon = 0$

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>$S$</th>
<th>$Re$</th>
<th>$N_u$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>0.9</td>
<td>0.1</td>
<td>0.950311999</td>
<td>0.06017177</td>
</tr>
<tr>
<td>0.71</td>
<td>0.5</td>
<td>1</td>
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</tr>
<tr>
<td>0.71</td>
<td>0.9</td>
<td>10</td>
<td>8.73483685</td>
<td>8.73483685</td>
</tr>
<tr>
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<td>0.00783792</td>
<td>0.00783792</td>
<td>0.00783792</td>
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<tr>
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<td>1</td>
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<td>0.963479</td>
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<td>10</td>
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<td>0.1</td>
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<td>1</td>
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<td>1.5</td>
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<td>10.549226</td>
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</table>

5. Results

For $\epsilon = 0$, equation (27) reduces to

$$N_u = \frac{SP, Re}{1 + h_1 - \frac{2A_1h_1}{SP, Re}},$$

and it is numerically computed for different values of magnetic parameter $M$, Reynolds number $Re$, suction parameter $S$, prandtl number $Pr$ and Eckert number $Ec$.

The calculated values are entered in Tables 1, 2 and 3.
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<table>
<thead>
<tr>
<th>Pr</th>
<th>S</th>
<th>Re</th>
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<th>$N_u$ 3</th>
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Table 2: Values of $N_u$ for different values of $Pr$, $S$, $Re$ when $Ec = 0.01$ and $\varepsilon = 0$

<table>
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<th>Re = 10</th>
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<th>Re = 50</th>
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</thead>
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<td>35.6</td>
</tr>
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<td>11.62818</td>
<td>35.1315968</td>
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<td>34.783851</td>
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<td>13.345631</td>
<td>32.5652322</td>
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</table>

Table 3: Variation of $N_u$ against $Ec$ when $S = 1$, $M = 10$, $Pr = 0.71$, $\varepsilon = 0$

6. Conclusions

A steady free convection laminar flow of an electrically conducting viscous incompressible fluid past an infinite porous plate in presence of a transverse magnetic field is analyzed numerically and the following conclusions are obtained:

1. The rate of heat transfer decreases with increase in the strength of magnetocity.
2. Nusselt number decreases with increase in Eckert number. That is rate of heat transfer decreases with increase in dissipative heat.
3. For fixed value of Hartmann number, Nusselt number increases with increase in Reynolds number.
4. For fixed value of Hartmann number, Nusselt number increases with increase in suction parameter.
5. For fixed value of Eckert number, Nusselt number increases with increase in Reynolds number.

6. For fixed value of Prandtl number, Reynolds number and suction parameter, Nusselt number decreases with increase in Hartmann number.

7. Dissipation heat plays a dominant role in the hydromagnetic flow.

8. Effects of dissipation heat as rate of heat transfer is quite important.

9. For correct prediction of temperature field the influence of dissipation heat should also be taken into account.

10. In absence of magnetic field (Hartmann number zero) and dissipative heat (Eckert number zero), the expression for suction parameter \( S = 1 \) reduces to

\[
H = \frac{1}{A_{14} - \pi} \left[ \left( A_{14} - \frac{\pi P_y}{A_{14}(1 + P_y)} \right) A_{15} \right.
\]

\[+ \left. \frac{\pi P_y^2 Re}{A_{14}(1 + P_y)} + \frac{\pi P_y}{1 + P_y} - \frac{A_{14} P_y Re}{\pi} - \pi \right], \quad (30)
\]

where

\[
A_{14} = \frac{1}{2} \left[ R_y + (R_y^2 + 4\pi^2)^{1/2} \right],
\]

\[
A_{15} = \frac{1}{2} \left[ P_y R_y + (P_y^2 R_y^2 + 4\pi^2)^{1/2} \right].
\]

In the notation of this paper the expression (30) is same as obtained by Geessen and Gross (1974). This confirms the correctness of the present method in which they did not consider magnetic field and dissipative heat.

Acknowledgements

I am very much thankful to Dr. N.P. Patil, Department of Mathematics, K.T.H.M. College, Gangapur Road, Nashik, 422002 (Maharashtra) for suggestions for improvements.

References


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An Investigation Into Effect of Electromagnetic Fields On Separation Tendency In Generalised Coutte Flow

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Abstract. One of the basic problem, the plane Coutte flow, has been a source of many research workers in dealing with the interplay of various fluid forces and their interaction with the electromagnetic forces. The combined effect of magnetic and electric fields on separation tendency in generalized Coutte flow has been studied. The velocity distribution has been numerically computed for various values of magnetic parameter, electric parameter and pressure gradient parameter. It is observed that for electric parameter k ≠ 0 (open circuited) the tendency of separation does not exist.

Keywords: Open Circuit, back flow, Hartmann number.
PACS: 01.30.Cc.

1. INTRODUCTION

One of the basic problem, the plane Coutte flow, has been a source for investigation of many research workers in dealing with the interplay of various fluid forces and their interaction with the electromagnetic forces. In view of the applications of the Coutte flow results to modern technology, it has attracted the attention of several research workers. Lenth [1] investigated the behavior of an electrically conducting fluid in a magnetic field. The plane Coutte flow of a conducting gas in presence of transverse magnetic field has been studied by Leiden [2]. Morgan [3] analysed the Coutte flow of a compressible viscous heat conducting perfect gas. Blevins [4] studied magnetogas dynamics of hypersonic Coutte flow. Yen and Chang [5] discussed hydromagnetic Channel flow under time- dependent pressure. Agarwal [6] studied generalized incompressible Coutte flow in MHD. Rammurthy [7] investigated the generalized Coutte flow between two porous plates with suction at the stationary plate and injection at the other plate. Rath [8] discussed the same problem without neglecting the induced magnetic field. Sutan and Shermann [9] studied the joint influence of electric and magnetic fields on plane Coutte flow.


In the present analysis an attempt has been made to observe the joint effect of electric and magnetic fields on the tendency of separation (back flow) in the generalized Couette flow of viscous incompressible and electrically conducting fluid. The velocity distribution has been numerically computed for various values of magnetic parameter (M), electric parameter (K) and pressure gradient parameter (N). The numerical and graphical analysis led to several significant results.

2. Formulation Of The Problem And Governing Equations

We suppose that the steady laminar flow of viscous incompressible electrically conducting fluid is between two non-conducting parallel flat plates. The plates are infinite in extent in both x-direction and y-direction, the x-axis is chosen along lower stationary plate in the direction of flow and z-axis is chosen normal to the direction of flow.

We suppose that the steady laminar flow of viscous incompressible electrically conducting fluid is between two non-conducting parallel flat plates. The plates are infinite in extent in both x-direction and y-direction, the x-axis is chosen along lower stationary plate in the direction of flow and z-axis is chosen normal to the direction of flow.

Under these assumptions the governing equation of motion for the steady flow of a viscous incompressible and electrically conducting fluid is

$$\mu \frac{d^2 u}{dx^2} + \sigma (E_x - u B_y) B_y = \frac{\partial p}{\partial x}$$

(1)

where $\bar{E} = (0, E_y, 0)$ is the electric field, $\sigma$ the electric conductivity of the fluid.
$\vec{B} = (B_x, 0, B_0)$ the magnetic induction vector, $\vec{q} = (u, 0, 0)$ the velocity, $p$ the pressure, $\rho$ the density, $\mu$ is the coefficient of viscosity.

The boundary conditions are

$$z = d, \quad u = U$$
$$z = 0, \quad u = 0$$

(2)

where $d$ is the separation distance between the two parallel plates.

3. Solution For Velocity Distribution $\vec{u}$

In order to reduce equation (1) in dimensionless form, the following non-dimensional quantities are introduced.

$$\vec{u} = \frac{u}{U}$$
$$\vec{x} = \frac{x}{d}$$
$$\vec{z} = \frac{z}{d}$$

$$p = \frac{p}{\rho U^2}$$
\[ K = \frac{E_z}{UB_0} \]

\[ M^2 = \frac{\sigma B_0^2 d^4}{\mu} \]

\[ \dot{u} = \frac{u}{U} \]

\[ x = \frac{x}{d} \]

\[ z = \frac{z}{d} \]

\[ p = \frac{p}{\rho U^2} \]

\[ K = \frac{E_z}{UB_0} \]

\[ R_x = \frac{\rho Ud}{\mu} = \frac{Ud}{\nu} \]

\[ N = R_x p_x \]
\[
p = \frac{\partial p}{\partial x}
\]

These substitutions reduce equation (1) to
\[
\frac{d^2 \bar{u}}{d \bar{z}^2} - M^2 \bar{u} = N - M^2 K
\]
(3)

where \(M\) is the Hartmann number, \(K\) is the electric parameter and \(N\) is the pressure gradient parameter respectively.

The boundary conditions (2) become
\[
\bar{z} = 1 \quad ; \quad \bar{u} = 1
\]
\[
\bar{z} = 0 \quad ; \quad \bar{u} = 0
\]
(4)

Solving equation (3) we get
\[
\bar{u} = c_1 \cosh M \bar{z} + c_2 \sinh M \bar{z} - \frac{(N - M^2 K)}{M^2}
\]
(5)

where \(c_1\) and \(c_2\) are constants.

After further calculation the solution of equation (5) under the boundary conditions (4) give velocity distribution \(\bar{u}\) as
\[
\bar{u} = \frac{M^2 \sinh M \bar{z} + (N - M^2 K) \left[ \sinh M \bar{z} + \sinh M \left( 1 - \bar{z} \right) - \sinh M \right]}{M^2 \sinh M}
\]
(6)

The velocity profile \(\bar{u}\) has been numerically computed for various values of \(M, N\) and \(K\).
The results of calculations are entered in the following table.

**TABLE 1**

Values of $H$ for different Parameters

<table>
<thead>
<tr>
<th>K</th>
<th>M</th>
<th>N</th>
<th>Z→0.0</th>
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<th>0.3</th>
<th>0.4</th>
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</table>
4. Numerical Discussion And Conclusions

When the electric parameter $K=0$ (open circuited) the velocity profile $\bar{u}$ is positive over the entire range for all values of magnetic parameter $M$ and pressure gradient $N$, so the tendency of separation does not exist.

1. For a pressure decreasing ($N < 0$) in the direction of motion, $\bar{u}$ is positive over the whole width of the channel for all values of $M$ or $K$. Therefore the back flow does not occur.

2. For a pressure increasing ($N > 0$) in the direction of motion the back flow occurs near the stationary plate in case of $K=0$ (Short Circuited) even in the presence of transverse magnetic field. Also, tendency of separation increases with increase in $M$ in the case $N > 0$, $K = 0$.

3. The Separation tendency in the generalized Couette flow with increasing pressure gradient can be completely prevented by the joint application of magnetic field and electric fields.

Expression (6) for velocity profile can be utilized for obtaining shearing stress at the lower plate and velocity gradient at the upper plate.

ACKNOWLEDGMENTS

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REFERENCES


SOME PROPERTIES OF STRESS-ENERGY MOMENTUM TENSOR FOR MAGNETOFLUID

B.S. MUDAGI

(Received 21 March 2006)

Abstract. Past four decades Magnetohydrodynamics has been studied extensively. The stress-energy momentum tensor for Magnetofluid reveals very interesting properties of Magnetofluid. For thermodynamical perfect fluid with infinite electric conductivity and constant magnetic permeability, some results are obtained and applied to gravitational collapse. The ratio of energy density due to magnetic field and energy density due to matter gives the result that, magnetic field dominates if the ratio is very very great as compared to unity. In case of Zel’dovich’s fluid, $\rho = \rho_0 e^{\gamma}$ and during collapse the decrease in magnetic field is compensated by the increase in matter energy density. In pulsar, magnetic field should not be orthogonal to the rotational vector.

1. Introduction. The classical magnetohydrodynamics is the combination of fluid dynamics and electromagnetic theory. It has been applied with considerable success to the astrophysical systems like magnetic variable stars, sun spots and spiral arms (4) by ignoring gravitation as fields. However, the astrophysical systems possess intense gravitational field. For such field the methods of relativistic magnetohydrodynamics are indispensable.

The genesis of relativistic magnetohydrodynamics (RMHD) is in Minkowski’s electrodynamics of moving bodies. Lichnerowicz (1967) gave an elegant account of the basic equations of RMHD, with special reference to the existence and uniqueness of solutions.

Relativistic Magnetohydrodynamics is modern subject and its importance is being recognized by the astrophysicists and cosmologists and in the past four decades, it has been studied extensively. Broadly speaking the main object of these studies is to find out the behaviour of a magnetofluid in intergalactic space.

The discovery of pulsars in astrophysics due to Hewish (1969) is highly honoured by the award of Nobel physics prize in 1974. The discovery has added new dimensions to the study of magnetohydrodynamics since pulsar possess strong magnetic field and evolution of magnetic field in accreting neutron stars. Osiewath (1967) has shown that there exist two families of Lichnerowicz’s universes with geodesic flow and shear free fluid. Lichnerowicz’s formalism is applied by Yodzis (1971) in the study of galactic cosmology, gravitational collapse and pulsar theory. Bray (1972) and (1973) has found Godel type of universes by solving Lichnerowicz’s field equations. These field equations are developed by Date (1972) and (1973) to study the local behaviour of congruences in RMHD and has obtained a class of non-uniform cosmological models. While Shaha (1972) has found an exact solution which is a generalization of Einstein Universe. Besides these works based upon Lichnerowicz’s treatment mention should
be made of the significant contributions to RMHD by Coburn (1961), Taub (1970), Greenberg (1971), Verma (1997), S. Chatterjee and P.S. Joarder (1997). The effect of self induction on entropy as well as the basic cauchy problem in RMHD have been investigated by Coburn (1961). Taub (1970) has derived the equations of motion for a self gravitating, charged, electromagnetofuid with finite electrical conductivity. Greenberg (1971) derived the post Newtonian approximations to the equations of RMHD and obtained the conservation laws for mass, the linear momentum, the angular momentum and energy of the entire system.

In view of the frequent occurrence of magnetic fields in astronomical systems (for instance, stars, galaxies and pulsar) shown by Nobel Laureates Russel A Hulse, Joseph H. Taylor Jr. (1993) and Konar Sushan (1997), it is appropriate to consider the matter distribution as cosmic magnetofuids. The aim of this paper is to merge together the fields of hydrodynamics, electromagnetic theory and relativity by constructing a stress-energy momentum tensor for thermodynamical perfect fluid with infinite electrical conductivity and constant magnetic permeability Lichnerowics (1957). We call this type of fluid as a perfect magnetofuid. The assumption of infinite conductivity is convenient in analytical works and benefits the study of cosmic fluids which possess very high electrical conductivity. Nevertheless, the perfect magnetofuid is too ideal to describe the natural systems in which the matter consists of viscous compressible and thermally conducting fluids with strong magnetic fields.

It is, therefore desirable to study the properties of a viscous, compressible, charged fluid, with infinite electrical conductivity and constant magnetic permeability. We designate such fluid by magnetofuid.

In the course of the discussion, we make a number of remarks about the possible relevance of these results to various astrophysical problems. While our method is too general to permit very definite conclusions to be drawn from it, we do get good indications of the possible significance of magnetic effects in galactic cosmology and gravitational collapse, and of a gravitational effect in pulsar theory. The method should be useful for estimating the significance of other possible magnetohydrodynamics effects as well.

The aim of this paper is to study different fluids, its properties and applications of relativistic magnetohydrodynamics to astrophysics.

In Section I,

The stress energy tensors for the magnetofuid, for the perfect magnetofuid and for the Maugin's modified relativistic magnetofuid are discussed.

Section II is devoted for the properties of stress energy momentum tensor for (1) Perfect magnetofuid, (2) Magnetofuid, (3) Maugin's modified relativistic magnetofuid.

Application of Relativistic Magnetohydrodynamics to Gravitational Collapse is discussed in section III.

In section IV, magnetic field in astrophysical system is discussed.
Comparative study of different fluids and applications of RMHDs to Astrophysics.

(I) (A) A symmetric stress — energy tensor for electromagnetic field. The stress — energy tensor for electromagnetic field due to Minkowski has the form vide Moller.

\[
E_{a\beta} = \frac{1}{4} g_{a\beta} H^{\rho\sigma} G_{\rho\sigma} - H_{a\beta} G_{\rho}^{\rho}
\]

where \(H_{a\beta}\) is the skew symmetric electric field magnetic induction tensor and \(G_{\rho\sigma}\) is skew symmetric magnetic field electric induction tensor.

By taking

\[
e^a = H^{a\beta} u_{\beta}, \quad b^a = * H^{a\beta} u_{\beta}
\]
\[
d^a = G^{a\beta} u_{\beta}, \quad h^a = * G^{a\beta} u_{\beta}.
\]

The eqn. (1) becomes

\[
E_{a\beta} = (e_a d^x + h_a b^x)((1/2)\eta_{a\beta} - u_{\alpha} u_{\beta}) - (e_a d^x + h_a b^x) - (u_{\alpha} v_{\beta} + u_{\beta} w_{\alpha})
\]

where

\(u^a\) : the flow vector
\(e^a\) : electric field vector
\(d^a\) : electric induction vector
\(h^a\) : magnetic field vector
\(b^a\) : magnetic induction vector
\(v^a\) : \(\eta^{a\beta\sigma} u_{\beta} b_{\sigma}\)
  electromagnetics energy flux vector
\(w^a\) : \(\eta^{a\beta\sigma} u_{\beta} h_{\sigma}\)
  electromagnetics momentum vector

\(* H^{a\beta} = \frac{1}{2} \eta^{a\beta\sigma} H_{\rho\sigma}\)

\(* G^{a\beta} = \frac{1}{2} \eta^{a\beta\sigma} G_{\rho\sigma}\)

\(\eta^{a\beta\sigma}\) is the pseudo tensor with the properties

\(\eta^{a\beta\sigma} = \eta^{\sigma\beta\alpha} = (-g)^{-1/2} > 0\)

Consequently, we have

\(u_a v^a = e_a u^a = h_a v^a = 0\)
\(u_a w^a = e_a w^a = h_a w^a = 0\)
\(u_a e^a = 0\)

(3)
According to the theory of Maxwell, the linear dependence of inductions with fields are governed by the constitutive equations.

\[ d^\alpha = \lambda \varepsilon^\alpha, \quad h^\alpha = \mu \mu^\alpha \]  \hspace{0.5cm} (4)

where \( \lambda \) is the dielectric permittivity and \( \mu \) is the magnetic permeability. Usually these quantities are scalar functions of position or constant. Hence for an isotropic medium (2) and (4) imply

\[ E_{\alpha \beta} = (\lambda |\varepsilon|^2 + \mu |\mu|^2) (u_{\alpha} u_{\beta} - \frac{1}{2} g_{\alpha \beta}) - (\lambda e_{\alpha} e_{\beta} + \mu h_{\alpha} h_{\beta}) - (u_{\alpha} v_{\beta} + \lambda u_{\beta} v_{\alpha}) \]  \hspace{0.5cm} (5)

where \( u^\alpha u_{\alpha} = 1 \)

\[-\varepsilon_{\alpha} \varepsilon_{\alpha} = |\varepsilon|^2 \]

\[-\mu_{\alpha} \mu_{\alpha} = |\mu|^2 \]

The stress energy tensor (5) being asymmetric in nature, is not suitable for direct adoption in Einstein’s field equations.

For the case \( \lambda u = 1 \), eq. (5) becomes

\[ E_{\alpha \beta} = (\lambda |\varepsilon|^2 + \mu |\mu|^2) (u_{\alpha} u_{\beta} - \frac{1}{2} g_{\alpha \beta}) - (\lambda e_{\alpha} e_{\beta} + \mu h_{\alpha} h_{\beta}) - (u_{\alpha} v_{\beta} + h_{\beta} v_{\alpha}) \]  \hspace{0.5cm} (6)

**REMARK.** The form (6) of stress — energy tensor for the electromagnetic field is derived by Taub (1970) on the basis of variational principle.

**B**. A stress-energy tensor for the magnetofluid. With the assumption of infinite electrical conductivity, we get

\[ T_{\alpha \beta} = (\rho + p + \frac{1}{2} |\mu|^2) u_{\alpha} u_{\beta} - (p - \frac{1}{2} |\mu|^2) g_{\alpha \beta} + \sigma_{\alpha \beta} + 2 u_{\alpha} g_{\beta} - \mu h_{\alpha} h_{\beta} \]  \hspace{0.5cm} (7)

This is symmetric stress-energy tensor for the magnetofluid.

We infer from (7) that for the magnetofluid:

(i) The observable energy density is

\[ T_{\alpha \beta} u^\alpha u^\beta = \rho + \frac{1}{2} |\mu|^2 \]  \hspace{0.5cm} (8)

(ii) The space-like 3-dimensional stress is

\[ T_{\alpha \beta} p^\alpha p^\beta = - (p + \frac{1}{2} |\mu|^2) p_{\alpha \beta} + \sigma_{\alpha \beta} - \mu h_{\alpha} h_{\beta} \]  \hspace{0.5cm} (9)
(iii) The space—like 3 momentum density is

\[ T_{\alpha\beta}P^\alpha P^\beta = q_\rho \]  

(10)

(iv) The trace \( T \) is

\[ T_{\alpha\beta}g^{\alpha\beta} = T = \rho - 3p \]  

(11)

The well known energy condition [Hawking & Ellis]

\[ T_{\alpha\beta}u^\alpha u^\beta - \frac{1}{3}T \geq 0 \]  

(12)

to be satisfied by the stress — energy tensors.

For all known forms of matter leads, in the case of the magnetofluid to

\[ \rho + 3p + \mu |h|^2 \geq 0 \]  

(13)

which is always true. Consequently we say that the stress-energy tensor (7) for the magnetofluid is physically transparent.

Now let us consider an orthonormal frame \( \{ \phi_a \} \) such that \( \{ \phi_4 \} = \bar{\phi} \).

Hence in the rest frame of \( \{ \bar{\phi}_a \} \) we get \( u^a = (0, 0, 0, 1) \).

The components of \( T_{\alpha\beta} \) given by (7) in the rest frame are

\[ T_{44} = \rho + \frac{1}{2} \mu |h|^2 \]  

(14)

\[ T_{a4} = q_{a4} \]  

(15)

\[ T_{\alpha\beta} = -(\rho + p + \mu |h|^2) \delta_{\alpha\beta} + 2\mu u_a h_{\beta\alpha} - \mu h_{\alpha\beta} \]  

(16)

The expressions (14), (15), (16) give respectively the total energy density, poising vector and spatial stresses of the magnetofluid.

(C) A stress — energy tensor for the perfect magnetofluid. In the absence of viscosity and heat conduction \( [v = 0; \gamma_{00} = 0] \) equation (7) reduces to

\[ T_{\alpha\beta} = (\rho + p + \mu |h|^2) u_\alpha u_\beta - \left( p + \frac{1}{2} \mu |h|^2 \right) \delta_{\alpha\beta} - \mu h_{\alpha\beta} \]  

(17)

This characterizes the perfect magnetofluid and identical with the stress — energy tensor given by Lichnerowicz (1967). Also we conclude that for the perfect magnetofluid the flow vector \( u^\alpha \) is the time — like eigen vector and the magnetic field vector \( h^\alpha \) is the space — like
eigen vector with $|\hbar|^2 (p - \frac{1}{2} \mu |\hbar|^2)$ as it eigen value. In case of the perfect magnetofluid the expressions (14), (15) and (16) reduces to

$$T_{\alpha\alpha} = \rho + \frac{1}{2} \mu |\hbar|^2$$

$$T_{\alpha\beta} = 0$$

$$T_{\alpha\beta} = - \left( p + \frac{1}{2} \mu |\hbar|^2 \right) g_{\alpha\beta} - \mu h_{\alpha\beta}$$

(D) A stress-energy tensor for the Maugin’s modified relativistic magnetofluid.

A stress-energy tensor for the Maugin’s modified relativistic magnetofluid is

$$T_{\alpha\beta} = (\rho + p + \hbar^2) u_{\alpha} u_{\beta} - (p + \mu (1 - \mu/2) \hbar^2) g_{\alpha\beta} + \nu_{\alpha\beta} + 2 u_{\alpha} (\sigma_{\beta}) - \mu h_{\alpha\beta}$$

We infer from (21) that for the magnetofluid

(i) the observable energy density is

$$T_{\alpha\beta} u^\beta = \rho + \mu^2 / 2 \hbar^2$$

(ii) the eigen value corresponding to the time-like eigen vector $u_{\alpha}$ is

$$(\rho + \mu^2 / 2 \hbar^2)$$

(iii) the trace $T$ is

$$T_{\alpha\beta} u^\beta = T = \rho - 3p - 2\mu \hbar^2 (1 - \mu)$$

(iv) the space-like 3 dimensional stress is

$$T_{\alpha\beta} p^\alpha p^\beta = \left[ p + \mu \left( 1 - \frac{\mu}{2} \right) \hbar^2 \right] p_{\alpha\beta} + \nu_{\alpha\beta} - \mu h_{\alpha\beta}$$

(v) the space-like 3 momentum density is

$$T_{u_{\alpha}} p^\alpha u^\beta = \rho$$

(II) (A) Some properties of Date’s stress-energy momentum tensor for perfect magnetofluid. The stress-energy momentum tensor for perfect magnetofluid is

$$T_{\alpha\beta} = (\rho + p + \mu |\hbar|^2) u_{\alpha} u_{\beta} - \left( p + \frac{1}{2} \mu |\hbar|^2 \right) g_{\alpha\beta} - \mu h_{\alpha\beta}$$
With the help of above equation we obtained the following results

1) \( \sigma_{\alpha\beta} u^\alpha = (\rho + \frac{1}{2} \mu |\mathbf{h}|^2) u_\beta \)
   
   Thus corresponding to given eigen vector \( u_\beta \) we get \( \rho + \frac{1}{2} \mu |\mathbf{h}|^2 \) as the eigen value.

2) \( \sigma_{\alpha\beta} u^\alpha u^\beta = \rho + \frac{1}{2} \mu |\mathbf{h}|^2 \)
   
   This is called the observable energy density.

3) \( \sigma_{\alpha\beta} h^\alpha = - (\rho - \frac{1}{2} \mu |\mathbf{h}|^2) h_\beta \)
   
   Thus corresponding to given eigen vector \( h_\beta \) we get \( \rho - \frac{1}{2} \mu |\mathbf{h}|^2 \) as its eigen value.

4) \( \sigma_{\alpha\beta} \mathbf{h}^\alpha \mathbf{h}^\beta = (\rho - \frac{1}{2} \mu |\mathbf{h}|^2) |\mathbf{h}|^2 \)
   
   This gives total pressure of the magnetofluid.

5) The trace \( T \) is
   \[ T_{\alpha\beta} g^{\alpha\beta} = \rho - 3p \]

6) \( T_{\alpha\beta} u^\alpha h^\beta = 0 \)

7) \( T_{\alpha\beta} T^{\alpha\beta} = 5 (\rho + \frac{1}{2} \mu |\mathbf{h}|^2)^2 - 2 (\rho + \mu |\mathbf{h}|^2) (\rho + \frac{1}{2} \mu |\mathbf{h}|^2) + (\rho + \frac{1}{2} \mu |\mathbf{h}|^2)^2 + \mu \mathbf{h}^2 \).

(B) Some properties of stress-Energy momentum tensor for magnetofluid. The stress–energy momentum tensor for the magnetofluid is given by

\[ T_{\alpha\beta} = (\rho + \rho + \mu |\mathbf{h}|^2) u_\alpha u_\beta - \left( \rho + \frac{1}{2} \mu |\mathbf{h}|^2 \right) g_{\alpha\beta} + \sigma_{\alpha\beta} + 2 u_\alpha u_\beta - \mu h_\alpha h_\beta \]

For the above stress-energy momentum tensor, we obtained the following results,

1) \( \sigma_{\alpha\beta} u^\alpha = (\rho + \frac{1}{2} \mu |\mathbf{h}|^2) u_\beta + q_\beta \)

2) \( \sigma_{\alpha\beta} \mathbf{h}^\alpha = - (\rho + \frac{1}{2} \mu |\mathbf{h}|^2) h_\beta + \sigma_{\alpha\beta} \mathbf{h}^\alpha + (h^\alpha q_\alpha) u_\beta \)

3) \( \sigma_{\alpha\beta} \mathbf{q}^\alpha = - (\rho + \frac{1}{2} \mu |\mathbf{h}|^2) q_\beta + \sigma_{\alpha\beta} \mathbf{q}^\alpha - |\mathbf{q}|^2 u_\beta - \mu (h_\alpha q^\alpha) h_\beta \)

4) \( \sigma_{\alpha\beta} \mathbf{h}^\beta = q_\beta h_\beta \)

5) \( \sigma_{\alpha\beta} \mathbf{q}^\beta = - |\mathbf{q}|^2 \)

6) \( \sigma_{\alpha\beta} u^\alpha = \rho + \frac{1}{2} \mu |\mathbf{h}|^2 \)

7) \( \sigma_{\alpha\beta} \mathbf{h}^\alpha \mathbf{h}^\beta = |\mathbf{h}|^2 (\rho - \frac{1}{2} \mu |\mathbf{h}|^2) + \sigma_{\alpha\beta} \mathbf{h}^\alpha \mathbf{h}^\beta \)
(C) Some properties of stress-energy momentum tensor for the Maugin's modified relativistic magnetofluid. The stress-energy momentum tensor for the Maugin's modified relativistic magnetofluid is given by

\[ T_{\alpha \beta} = (\rho + p + h^2) u_{\alpha} u_{\beta} - \{ p + \mu \left( 1 - \frac{\mu}{2} \right) h^2 \} g_{\alpha \beta} + \nu_{\alpha \beta} + 2\mu_{\alpha} \epsilon_{\beta} - \mu h_{\alpha} h_{\beta}. \]

For the above stress-energy tensor, we obtained the following results.

1. \[ T_{\alpha \beta} u_{\alpha} = \left( p + \frac{\mu}{2} h^2 \right) u_{\beta} + q_{\beta} \]
2. \[ T_{\alpha \beta} u_{\beta} = \rho + \frac{\mu}{2} h^2 \]
3. \[ T_{\alpha \beta} h_{\alpha} = - \left( p - \frac{\mu}{2} h^2 \right) h_{\beta} + \sigma_{\alpha \beta} h^\alpha + h^\alpha q_{\alpha} \]
4. \[ T_{\alpha \beta} h_{\alpha} h_{\beta} = \left( p - \frac{\mu}{2} h^2 \right) |h|^2 - \frac{1}{2} w^2 h^2 - 2|\varepsilon| h^2 \]
5. \[ T_{\alpha \beta} p_{\alpha} = (-3p - 2\mu h^2 + 3/2 \mu^2 h^2) \]
6. \[ T_{\alpha \beta} q_{\alpha} = T = p - 3p - 2\mu h^2 (1 - \mu) \]
7. \[ T_{\alpha \beta} w_{\alpha} h_{\beta} = q_{\alpha} h_{\beta} \]
8. \[ T_{\alpha \beta} u_{\alpha} q_{\beta} = -|q|^2 \]
9. \[ T_{\alpha \beta} h_{\alpha} h_{\beta} = \left( p - \frac{\mu}{2} h^2 \right) |h|^2 + \nu_{\alpha \beta} h^\alpha h^\beta \]
10. \[ T_{\alpha \beta} p_{\alpha} u_{\beta} = q_{\beta} \]
11. \[ T_{\alpha \beta} p_{\alpha} q_{\beta} = - \{ p + \mu \left( 1 - \frac{\mu}{2} \right) h^2 \} P_{\alpha \beta} + \nu_{\alpha \beta} - \mu h_{\alpha} h_{\beta} \]
8) \( T_{\alpha\beta} h^{\alpha} q^\beta \) \( = - \left( p + \frac{1}{2} \mu |\vec{h}|^2 \right) q_{\alpha} q^\beta + \sigma_{\alpha\beta} h^{\alpha} q^\beta \)

9) \( T_{\alpha\beta} q^\alpha q^\beta \) \( = |\vec{q}|^2 \left( p + \frac{1}{2} \mu |\vec{h}|^2 \right) + \sigma_{\alpha\beta} q^\alpha q^\beta - \mu (h_{\alpha} q^\alpha)^2 \)

10) \( T_{\alpha\beta} \sigma^{\alpha\beta} \) \( = 2 \sigma \gamma^2 - \mu \sigma^{\alpha\beta} h_{\alpha} h_{\beta} \)

11) \( T_{\alpha\beta} q^\alpha \) \( = p - 3p \)

12) \( T_{\alpha\beta} q^\alpha \) \( = 5 \left( p + \frac{1}{2} \mu |\vec{h}|^2 \right) - 2 \left( p + \frac{1}{2} \mu |\vec{h}|^2 \right) \left( p + \frac{1}{2} \mu |\vec{h}|^2 \right) + \left( p + \frac{1}{2} \mu |\vec{h}|^2 \right)^2 - 2w^2 \alpha^2 - 2\mu \sigma_{\alpha\beta} h^{\alpha} h^{\beta} - 2q^2 \mu - \mu h_{\alpha} h_{\beta} \)

\( \langle C \rangle \) Some properties of stress-energy momentum tensor for the Maujin's modified relativistic magnetofluid. The stress-energy momentum tensor for the Maujin's modified relativistic magnetofluid is given by

\[ T_{\alpha\beta} = (p + \mu + h^2) u_{\alpha} u_{\beta} - \left( p + \mu \left( 1 - \frac{\mu}{2} \right) h^2 \right) \sigma_{\alpha\beta} + \nu_{\alpha\beta} + 2\mu (u_{\alpha} q_{\beta}) - \mu h_{\alpha} h_{\beta}. \]

For the above stress-energy tensor, we obtained the following results.

1) \( T_{\alpha\beta} u_{\alpha} = \left( p + \frac{w^2}{2} h^2 \right) u_{\beta} + q_{\beta} \)

2) \( T_{\alpha\beta} u_{\alpha} u_{\beta} = p + \frac{w^2}{2} h^2 \)

3) \( T_{\alpha\beta} h^{\alpha} = - \left( p + \frac{w^2}{2} h^2 \right) h_{\beta} + \nu_{\alpha\beta} h^{\alpha} + h^{\alpha} q_{\alpha} u_{\beta} \)

4) \( T_{\alpha\beta} h_{\alpha} ^{\gamma} = \left( p + \frac{w^2}{2} h^2 \right) \frac{|\vec{h}|^2}{2} - \frac{1}{2} w^2 h^2 \gamma - 2 \mu / 3 |\vec{h}|^2 \)

5) \( T_{\alpha\beta} p^{\alpha\beta} = (3p - 2u^2 h^2 + 3/2 \mu^2 h^2) \)

6) \( T_{\alpha\beta} q^{\alpha\beta} = T - 3p - 2p |\vec{h}|^2 (1 - \mu) \)

7) \( T_{\alpha\beta} u_{\alpha} h_{\beta} = q_{\alpha} h_{\beta} \)

8) \( T^{\alpha\beta} u_{\beta} q^{\alpha} = - |\vec{q}|^2 \)

9) \( T_{\alpha\beta} h_{\alpha} h_{\beta} = \left( p + \frac{w^2}{2} h^2 \right) |\vec{h}|^2 + \nu_{\alpha\beta} h^{\alpha} h^{\beta} \)

10) \( T_{\alpha\beta} q_{\alpha} u_{\beta} = q_{\beta} \)

11) \( T_{\alpha\beta} q_{\alpha} q_{\beta} = - \left( p + \mu \left( 1 - \frac{\mu}{2} \right) h^2 \right) P_{\alpha\beta} + \nu_{\alpha\beta} - \mu h_{\alpha} h_{\beta} \)

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(III) Application of relativistic magnetohydrodynamics to gravitational collapse.

In order to study gravitational collapse of magnetofluids we use a ratio defined as Yodzis (1971)

\[ R = \frac{\text{Energy density due to magnetic field}}{\text{Energy density due to matter}} \]

For the magnetofluid under investigation this ratio becomes

\[ R = \frac{\frac{1}{2} \mu |\vec{B}|^2}{\rho} \quad (1) \]

The relative importance of the magnetic field and the field created by matter can be determined on using the ratio \( R \).

If \( R \gg 1 \), magnetic field dominates.

From (1) we get,

\[ \frac{1}{R} D_\alpha R = \frac{2}{|\vec{B}|} D_\alpha (|\vec{B}|) + \frac{1}{\mu} D_\alpha \mu - \frac{1}{\rho} D_\alpha \rho \quad (2) \]

If the heat flux vector is zero

The following equation

\[ D_\alpha \left[ \rho + \frac{1}{2} \mu |\vec{B}|^2 \right] + 3 \left[ \rho + \mu |\vec{B}|^2 \right] \theta + \frac{3\rho}{c^2} \theta + \frac{4}{c^2} \theta + \frac{1}{c^2} = -\mu |\vec{B}|^2 \bar{\sigma}_{\alpha \beta} \bar{a}^\beta + \mu \vec{B}^2 \bar{a}^\alpha \bar{u}^\beta \]

becomes

\[ D_\alpha \rho = \frac{1}{2} |\vec{B}|^2 D_\alpha \mu + \frac{3}{2} \left[ \rho + \frac{1}{c^2} \left( \frac{1}{|\vec{B}|} D_\alpha (|\vec{B}|) + \frac{3}{c^2} \theta \right) + \frac{1}{2} \left( \rho + \frac{1}{c^2} \right) D_\alpha \mu \right] \]

\[ + \frac{3}{2} \left( \rho + \frac{1}{c^2} \right) \bar{a}^\alpha \bar{a}^\beta - \frac{2}{3} \theta \bar{a}^\beta \sigma_{\alpha \beta} \quad (3) \]

where

\[ \rho a^\beta = -\rho c^2 h a^\alpha - \frac{\rho}{c^2} a^\alpha \]

Eliminating \( |\vec{B}| \) from (2) and (3) we get

\[ \frac{1}{R} D_\alpha R = \left( \rho - \frac{3\rho}{c^2} \right) \frac{3\rho}{c^2} D_\alpha \rho - \frac{1}{\mu} \left\{ 1 + \frac{4R\rho}{3\left( \rho + \frac{1}{c^2} \right)} \right\} D_\alpha \mu \]

\[ - 2 \left( \delta^\alpha \bar{a}^\beta - \frac{2}{3} \theta a^\alpha \bar{a}^\beta \right) \sigma_{\alpha \beta} - \frac{4}{3} \theta^2 / c \left( \rho + \frac{1}{c^2} \right) \quad (5) \]
For the magnetofluid with constant magnetic permeability eq. (5) reduces to
\[ \frac{1}{R} D_\alpha R = \frac{\rho - \frac{3p}{c^2}}{3(\rho + p/c^2)} D_\alpha \rho - 2 \left\{ \alpha^\alpha a^\beta + \frac{2p a^\beta}{3(\rho c^2 + p)} \right\} \sigma_{\alpha\beta} \] (6)

For perfect magnetofluid eq. (6) further reduces to
\[ \frac{1}{R} D_\alpha R = \frac{\rho - \frac{3p}{c^2}}{3(\rho + p/c^2)} \frac{1}{\rho} D_\alpha \rho - 2 \left\{ \alpha^\alpha a^\beta + \frac{2p a^\beta}{3(\rho c^2 + p)} \right\} \sigma_{\alpha\beta} \]
(7)

Eq. (7) yields for shear free flow
\[ D_\alpha R = \frac{\rho - \frac{3p}{c^2}}{3(\rho + p/c^2)} ||\delta|| / \rho^2 \]
(8)

At an early stage of collapse of the magnetofluid
\[ \rho > \frac{3p}{c^2} \]

So that R.H.S. of eq. (8) is positive.

Thus R goes on increasing when the magnetofluid is collapsing as long as
\[ \rho > \frac{3p}{c^2} \]

Thus the magnetic field in the magnetofluid increases in collapsing magnetofluid.

In the limiting case, when magnetofluid becomes degenerate gas we have \( \rho = \frac{3p}{c^2} \) and R attains a stable value.

In case of Zeldovich’s fluid \( \rho = \frac{p}{c^2} \), and R goes on decreasing during collapse. Thus in Zeldovich’s fluid, during collapse the decrease in magnetic field is compensated by the increase in matter energy density.

(IV) Magnetic field in astrophysical system. Consider the Maxwell’s equation
\[ \eta^{\alpha\beta\gamma\delta} \left( \eta_{\gamma\delta} h_{\alpha\beta} + \eta_{\alpha\gamma} h_{\beta\delta} \right) = -J^\gamma \]
(1)

where \( J^\gamma = -\rho u^\gamma \).

The vorticity vector \( \omega^\gamma \) is defined as
\[ \omega^\gamma = \frac{1}{2} \eta^{\alpha\beta\gamma\delta} u_\beta u_\gamma \]

where \( \eta^{\alpha\beta\gamma\delta} \) is the Levi-Civita permutation tensor.

Contracting (1) with \( u_\alpha \) we get
\[ \eta^{\alpha\beta\gamma\delta} \left( \eta_{\gamma\delta} h_{\alpha\beta} + \eta_{\alpha\gamma} h_{\beta\delta} \right) = -\rho \omega^\gamma u_\alpha \]
Some properties of stress-energy momentum tensor for magnetofluid

\[ \text{i.e., } 2u^i \delta_{ij} + 0 = -\rho_d \]
\[ \text{i.e., } 2u^i \delta_{ij} = -\rho_d \]

(2)

It explains the orientation of fields in rotating stars. Thus the rotating stars which possess some electric charge of primordial origin Zel'dovich's fluid possess magnetic field orthogonal to the rotation vector if and only if the electric charge in the stars is zero. There are good indications that for the rotating disk, the magnetic field lines are parallel to the galactic plane Parker 1969 and hence the rotation vector \( \omega \) and the magnetic field vector \( \mathbf{B} \) are orthogonal to each other. Thus from (2) there may not be any electric charge in the rotating disk. Pulses discovered by Hewish and his group in 1967 are rotating neutron stars Thorne (1971). They were formed in supernova outbursts and may possess primordial charge. Therefore, according to eq. (2), we conclude that in pulses, the magnetic field should not be orthogonal to the rotation vector.

(V) Concluding remarks. The magnetic field in the magnetofluid increases in collapsing magnetofluid. In Zel'dovich's fluid, during collapse the decrease in magnetic field is compensated by the increase in matter energy density.

In pulsar, magnetic field should not be orthogonal to the rotation vector.

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