Chapter 2

SOME SIGNIFICANT PARAMETER AND BASIC EQUATIONS
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Every physical system is characterized by a large number of parameters. These parameters may depend on the geometrical shape of configurations and the forces acting on the system like magnetic field. They may also depend on the velocity field prevailing in the system or some other factors relevant to a particular problem. In a theoretical analysis of a physical problem it is, therefore, often necessary to make certain simplifications and idealization, due to geometrical considerations or mathematical simplifications or because of the physical applications of a specific problem in order to obtain the results with a reasonable amount of effort. Therefore, in order to explain the essential features of heat-transfer in magnetohydrodynamic flow under various conditions some important parameters and fundamentals are needed. Hence, only some of those are described here which are often directly or indirectly required in the subsequent chapters.

2.1 DEFINITIONS AND PARAMETERS:

(1) Reynolds Number $R_e$:

It is defined as $R_e = \frac{Ud}{\nu}$ where $U$ is the velocity, $d$ the characteristic length and $\nu$ the kinematic viscosity. It is measure of the ratio of inertial force to the viscous force. When the Reynolds Numbers of the system is small, the viscous force is predominant and the effect of the viscosity is important in the whole flow field. When the Reynolds Numbers of the system is large the inertial force is predominant and the effect of the viscosity is important only in the narrow region, known as boundary layer, adjacent to the solid boundary. If $R_e$ is very large, then the flow ceases to be laminar and becomes turbulent. The Reynolds Number at which the transition from laminar to turbulent occurs is known as critical Reynolds Number. For flow in a circular pipe $R_e$(critical) = 2300.
Significance of Reynolds Number (Re) :

Reynolds Number \( R_e \) is defined as ratio of inertial force to the viscous force. It is indicative of the relative importance of inertial and viscous effects in fluid motion. At low Reynolds Number, the viscous effects dominate and the fluid motion is laminar. At high Reynolds Number, the inertial effects lead to turbulent flow and the associated turbulence level dominates the momentum and energy flux.

Reynolds Number constitutes an important criterion of kinematic and dynamic similarity in forced convection heat transfer. Velocity within the given fields would be similar in magnitude, direction and turbulence pattern when their Reynolds Number is same.

(2) Magnetic field strength \( H \):

It is a force experienced by a unit north pole placed at the given point in the magnetic field assuming that the introduction of the pole does disturb the field.

(3) Magnetic permeability \( \mu_e \):

If a specimen is magnetized, the ratio of the flux density \( B \) in the material to the magnetized field \( H \) is called Magnetic permeability of the material. It is denoted by \( \mu_e \). Thus \( \mu_e = \frac{B}{H} \). The flux density \( B \) at a point in a medium is known as the magnetic induction at that point.

(4) Magnetic Reynold Number \( R_\sigma \):

This number shows how the magnetic field is influenced by the flow of the conducting fluid. It may be considered as the ratio of the flow velocity \( U \) to the characteristic velocity \( V_e \).
\[ R_\sigma = \frac{U}{V_e} \] where \( V_e = \frac{1}{\sigma \mu_e L} \)  

Magnetic Reynolds number is similar in form as the ordinary Reynolds number. We can write it as \( R_\sigma = \frac{UL}{\nu_\sigma} = \mu_e \sigma UL \) where \( \nu_\sigma = \frac{1}{\sigma \mu_e} \) is the magnetic viscosity. When \( R_\sigma \) is much larger than unity the magnetic field is greatly influenced by the motion of the fluid. On the other hand, if \( R_\sigma = 1 \), the magnetic field is not much influenced by the motion of the fluid, it is generally very small in magnetohydrodynamics. If the value of \( R_\sigma \) is small, the magnetic field \( H \) is practically independent of the motion.

(5) Magnetic Force Number (or Stommer \( \cdot \) s Number) \( S \):  

It is defined as \( S = \frac{H_0}{L} \sqrt{\frac{\mu_e}{\rho}} \) where \( H_0 \) = magnetic field, \( L \) = length, \( \rho \) = density, \( \mu_e \) = magnetic permeability. This gives the ratio of magnetic force to the inertia force. If this ratio is of order one then the magnetic force is important and the flow is to be considered as hydromagnetic flow. If this ratio is very much less than one, then the flow is to be considered as hydrodynamic flow.

(6) Hartmann Number \( M \):

It is defined as the ratio of the magnetic force to the viscous force. It is denoted by \( M \) and is given by

\[ M = \mu_e H_0 L \sqrt{\frac{\sigma}{\mu}} \]  

or  

\[ M^2 = \mu_e^2 H_0^2 L^2 \frac{\sigma}{\mu} \]  

where \( H_0 \) = magnetic field \( \)  

\( L \) = length \n
\( \sigma \) = electrical conductivity \n
\( \mu \) = coefficient of viscosity \n
\( \mu_e \) = magnetic permeability
(7) Convection Heat Transfer Rates:

It is defined as \[ Q = hA \left( T_s - T_\infty \right) \]

where \( Q \) = Heat transfer from the surface to the surrounding fluid

\( A \) = Area of the surface

\( T_s \) = Temperature of the surface

\( T_\infty \) = Temperature of the surrounding fluid

The subscript \( \infty \) is used to identify that part of the fluid that is sufficiently away from the surface so as to be unaffected by the heat transfer.

\( h \) = Convective heat transfer coefficient.

(8) Momentum and Thermal diffusivity:

The molecular diffusivity of momentum and energy are defined as

momentum diffusivity \( \nu = \frac{\mu}{\rho} \)

and

thermal diffusivity \( \alpha = \frac{k}{\rho c_p} \)

where

\( \mu \) = coefficient of viscosity

\( \rho \) = density

\( k \) = thermal conductivity

\( c_p \) = specific heat at constant pressure.
(9) Prandtl Number \( P_r \):

It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of heat. It is denoted by \( P_r \) and is given by

\[
P_r = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}
\]

It is a measure of the relative importance of heat conduction and viscosity of fluid. The Prandtl Number is just a constant of the material and does not depend on the properties of the flow. It is a material property and it thus varies from fluid to fluid.

**Significance of Prandtl Number:**

Prandtl Number \( P_r \) is indicative of the relative ability of the fluid to diffuse momentum and internal energy by molecular mechanisms. From its mathematical formulation

\[
P_r = \frac{\mu c_p}{k} = \frac{\rho v c_p}{k} = \frac{\nu}{\left(\frac{k}{\rho c_p}\right)}
\]

Recalling that the parameter \( \frac{k}{\rho c_p} \) is thermal diffusivity \( \alpha \) of the fluid,

\[
P_r = \frac{\nu}{\alpha} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}
\]

Apparently \( P_r \) is the ratio of the kinematic viscosity to thermal diffusivity of the fluid. The kinematic viscosity indicates the momentum transport by molecular friction and thermal diffusivity represents the heat energy transport through conduction. Obviously \( P_r \) provides a measure of the relative effectiveness of momentum and energy transport by diffusion. For highly viscous oils, \( P_r \) is quiet
large (100 to 10,000) and that indicates rapid diffusion of momentum by viscous action compared to the diffusion of energy. Prandtl Number for gases is near unity and accordingly the momentum and energy transfer by diffusion are comparable. In contrast, the liquid metal have $P_r = 0.003$ to 0.01 and that indicates more rapid diffusion of energy compared to the momentum diffusion rate.

The Prandtl Number is connecting link between the velocity field and the temperature field, and its value strongly influences relative growth of velocity and thermal boundary layers. Mathematically, $\frac{\delta}{\delta_i} \approx (P_r)^n$ where $\delta$ and $\delta_i$ are the thickness of velocity and thermal boundary layers respectively, and $n$ is a positive exponent. For oils $\delta = \delta_i$. For gases $\delta_i = \delta$, and for liquid metals $\delta_i \approx \delta$.

(10) Heat Transfer Co-efficient $\alpha$ :

The heat transfer between solid bodies and the fluid is usually characterized by the heat transfer co-efficient $\alpha$, defined by

$$\alpha = \frac{q}{T_i - T_0}$$

where $q$ is the heat flux density through the surface and $T_i - T_0$ is the characteristic temperature difference between the solid body and the fluid.

(11) Nusselt Number $N_u$ :

Consider a fluid flowing over a body. If a surface temperature is $T_w$ and if the free stream temperature is $T_\infty$, the temperature of the fluid near the solid boundary will vary in some fashion as shown in the figure.
Temperature distribution in a flowing fluid near a solid boundary

\[ h(T_w - T_\infty) = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \]  \hspace{1cm} (1)

We can express the rate of heat transfer, \( Q \), in the following manner

\[ Q_y = -k_{y=0} A \left( \frac{\partial T}{\partial y} \right)_{y=0} \]  \hspace{1cm} (2)

where \( k \) = The thermal conductivity of the fluid, evaluated at \( y = 0 \), that is, the solid boundary fluid – interface, and

\[ \left( \frac{\partial T}{\partial y} \right)_{y=0} = \text{The value of the temperature gradient in the fluid at } y = 0. \text{ The coordinate } y \text{ is measured along the normal to the surface.} \]

If equations (1) and (2) are combined, we obtain

\[ hA(T_w - T_\infty) = -kA \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

or

\[ \frac{h}{k} = \frac{-1}{(T_w - T_\infty) \left( \frac{\partial T}{\partial y} \right)_{y=0}} \]
If a dimensionless distance $\eta$ is defined as $\eta = \left(\frac{y}{L_c}\right)$ where $L_c$ is a characteristic length, we obtain

$$h = -\frac{1}{k} \left(\frac{\partial T}{\partial \eta}\right)_{\eta=0}$$

or

$$N_u = \frac{hL}{k} = \frac{-1}{(T_w - T_\infty)} \left(\frac{\partial T}{\partial \eta}\right)_{\eta=0}$$

If a dimensionless temperature $\theta$, is defined as

$$\theta = \frac{T - T_w}{T_w - T_\infty},$$

Then

$$N_u = \frac{hL}{k} = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}$$

The quantity $N_u = \frac{hL}{k}$ is a dimensionless quantity called the Nusselt number.

**Significance of Nusselt number:**

Nusselt number $N_u$ establishes the relation between convective film coefficient $h$, thermal conductivity of the fluid $k$ and a significant length parameter $l$ of the physical system,

$$N_u = \left(\frac{hl}{k}\right)$$
An energy balance at the surface of a heated plate stipulates that energy transport by conduction must equal the convective heat transfer into the fluid flowing past the plate. Thus

\[
Q = -kA \left( \frac{\partial t}{\partial y} \right)_{y=0} = hA(t_s - t_x)
\]

\[
h = \frac{-k \left( \frac{\partial t}{\partial y} \right)_{y=0}}{t_s - t_x}
\]

\[
\frac{hl}{k} = \frac{-\left( \frac{\partial t}{\partial y} \right)_{y=0}}{(t_s - t_x)}
\]

Apparently the Nusselt number \( \frac{hl}{k} \) may be interpreted as the ratio of temperature gradient at the surface to an overall reference temperature gradient.

The Nusselt number is a convenient measure of the convective heat transfer coefficient. For a given value of the Nusselt number, the convective heat transfer coefficient is directly proportional to the thermal conductivity of the fluid and inversely proportional to the significant length parameter.

(12) Grashof Number \( G_r \):

It is defined as:

\[
G_r = \frac{(\text{Inertia force})(\text{bouyancy force})}{(\text{viscous force})^2}
\]

\[
= \frac{g \beta (\Delta T)_0 d^3}{\nu^2}
\]
where

\((\Delta T)_0 = T_w - T_\infty\) is temperature difference between the wall and the fluid at a large distance from the body.

\[d = \text{the representative length}\]

\[\beta = \text{the coefficient of volume expansion}\]

\[\nu = \text{the kinematic viscosity}\]

\[g = \text{the acceleration due to gravity}\]

The dimensionless quantity \(G_r\) characterizes the free convection (that is, flow in which the motion of the fluid is caused by the effect of gravity on heated fluids of variable density)

**Significance of Grashof Number:**

Grashof Number \(G_r\) indicates the relative strength of the buoyant to viscous forces. From its mathematical formulation,

\[G_r = \frac{l^3 \rho^2 \beta g \Delta T}{\mu^2}\]

\[= \left(\rho l^3 \beta g \Delta T\right) \frac{\rho}{\mu^2}\]

\[= \left(\rho l^3 \beta g \Delta T\right) \times \frac{\rho \nu^2 l^2}{(\mu \nu l)^2}\]

\[= \text{buoyant force} \times \frac{\text{inertia force}}{(\text{viscous force})^2}\]
Obviously the Grashof number represents the ratio of the product of buoyant and inertia forces to the square of the viscous forces. Grashof number has a role in free convection similar to that played by the Reynolds number in forced convection. Free convection is usually suppressed at sufficiently small $G_r$, begins at some critical value of $G_r$ (depending upon the arrangement) and then becomes more and more effective as $G_r$ increases.

**Thermal conductivity $k$:**

The well-known Fourier’s heat conduction law states that the conductive heat flow per unit area (or heat flux) $q_n$ is proportional to the temperature decrease per unit distance in a direction normal to the area through which the heat is flowing. Thus, mathematically,

$$q_n \propto \frac{\partial T}{\partial n}$$

so that $q_n = -k \frac{\partial T}{\partial n}$, where $k$ is said to be thermal conductivity.

Thermal conductivity of a material is one of its transport properties. Others are the viscosity associated with the transport of momentum, and the diffusion coefficient associated with the transport of mass. Thermal conductivity provides an indication of the rate at which heat energy is transferred through a medium by the diffusion (conduction) process. For a prescribed temperature gradient and geometric parameters, the heat flow rate increases with increasing thermal conductivity. Thermal conductivity may be defined as the amount of heat conducted per unit time across unit area and through unit thickness, when a temperature difference of unit degree is maintained across the bounding surfaces.
(14) Electrical conductivity $\sigma$:

The electrical conductivity of a medium is the current density produced in it by a unit electric field when the medium is stationary. The electrical conductivity of various conducting fluids has a large range of values.

(15) Eckert Number $E_c$:

It is defined as

$$E_c = \frac{U_\infty^2}{C_p \left( T_w - T_\infty \right)}$$

where

$U_\infty$ = Potential flow velocity

$C_p$ = Specific heat at constant pressure

$T_w$ = Temperature at the wall

$T_\infty$ = Temperature of fluid far away from solid surface

It is a dimensionless quantity. In compressible fluids it determines the relative rise in temperature of the fluid due to adiabatic compression. It can also be retained in increase possible flow, if the frictional heat is to be considered.
2.2 EQUATIONS:

(I) THE GENERAL EQUATION OF HEAT TRANSFER:

We denote by \( \overline{q} \) the heat flux density due to thermal conduction. The flux \( \overline{q} \) is related to the variation of temperature through the fluid. This relation can be written down at once in cases where the temperature gradient in the fluid is not large. In phenomena of thermal conduction we are almost concerned with such cases. We can then expand \( \overline{q} \) as a series of powers of the temperature gradient, taking only the first terms of the expansion. The constant term is evidently zero, since \( \overline{q} \) must vanish when \( \text{grad} \ T \) does so. Thus, we have

\[
\overline{q} = -k \text{grad} \ T \quad (2.2.1)
\]

The constant \( k \) is called the thermal conductivity. It is always positive, as we see at once from the fact that the energy flux must be from points at a high temperature to those at a low temperature, that is, \( \overline{q} \) and \( \text{grad} \ T \) must be in opposite direction. The coefficient \( k \) is in general, a function of temperature and pressure. Thus the total energy flux in a fluid when there is viscosity and thermal conduction is

\[
\rho \overline{v} \left( \frac{1}{2} \rho \overline{v}^2 + w \right) - \overline{v} \overline{\sigma} - k \text{grad} \ T \quad (2.2.2)
\]

Accordingly, the general law of conservation of energy is

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho \overline{v}^2 + \rho \epsilon \right) = -\text{div} \left[ \rho \overline{v} \left( \frac{1}{2} \overline{v}^2 + w \right) - \overline{v} \overline{\sigma} - k \text{grad} \ T \right] \quad (2.2.3)
\]

The equation could be taken to complete the system of fluid mechanical equations of a viscous fluid. It is convenient; however, to put in another form by transforming it with help of equation of motion. To do so, we calculate the time derivative of the energy in the unit volume of fluid, starting from the equation of motion. We have
\[
\frac{\partial}{\partial t} \left[ \frac{1}{2} \rho v^2 + \rho \varepsilon \right] = \frac{1}{2} v^2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial t} + \rho \varepsilon \frac{\partial}{\partial t} + \varepsilon \frac{\partial \rho}{\partial t} \quad (2.2.4)
\]

Substituting for \(\frac{\partial \rho}{\partial t} = -\text{div}(\rho \vec{v})\) from equation of continuity and for \(\frac{\partial v}{\partial t}\) from the Navier-Stokes equation, we have

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) = -\frac{1}{2} v^2 \text{div}(\rho \vec{v}) - \rho \vec{v} \cdot \text{grad} \frac{1}{2} v^2 - \vec{v} \cdot \text{grad} P + v_i \frac{\partial \rho'_{ik}}{\partial x_k} + \frac{\partial \varepsilon}{\partial t}
\]

\[-\varepsilon \text{div}(\rho \vec{v}) \quad (2.2.5)\]

Using now the thermodynamic relation

\[d \varepsilon = T ds - \rho dv \quad (2.2.6)\]

\[d \varepsilon = T ds + \frac{p}{\rho^2} d \rho \], we find

\[
\frac{\partial \varepsilon}{\partial t} = T \frac{\partial s}{\partial t} + \frac{p}{\rho^2} \frac{\partial \rho}{\partial t} = T \frac{\partial s}{\partial t} = \frac{p}{\rho^2} \text{div}(\rho \vec{v}) \quad (2.2.7)
\]

Substituting this and introducing the heat function

\[w = \varepsilon + \frac{p}{\rho} \quad (2.2.8)\]

We obtain

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \rho \varepsilon \right) = -\frac{1}{2} \left( \frac{1}{2} v^2 + w \right) \text{div}( \rho \vec{v}) - \rho \vec{v} \cdot \text{grad} \frac{1}{2} v^2 - \vec{v} \cdot \text{grad} p
\]

\[+ \rho \gamma \frac{\partial s}{\partial t} + v_i \frac{\partial \sigma'_{ik}}{\partial x_k} \quad (2.2.9)\]
Next, from thermodynamic relation

\[ dw = T \, ds + \frac{dp}{\rho}, \]  

(2.2.10)

We have

\[ \text{grad} \, p = \rho \, \text{grad} \, w - \rho T \, \text{grad} \, s \]  

(2.2.11)

The last term on the right of equation (2.2.9) can be written as

\[ v_i \frac{\partial \sigma'_{ik}}{\partial x_k} = \frac{\partial}{\partial x_k} (v_i \sigma'_{ik}) - \sigma'_{ik} \frac{\partial v_i}{\partial x_k} = \text{div} (v \cdot \sigma') - \sigma'_{ik} \frac{\partial v_i}{\partial x_k} \]  

(2.2.12)

Substituting these expressions and adding and subtracting \( \text{div} (k \, \text{grad} \, T) \), in the equation (2.29) we obtain

\[ \frac{\partial}{\partial t} \left[ \frac{1}{2} \rho v^2 + \rho \varepsilon \right] = -\text{div} \left[ \rho \bar{v} \left( \frac{1}{2} v^2 + w \right) - \bar{v} \cdot \sigma' - k \, \text{grad} \, T \right] + \rho T \left( \frac{\partial s}{\partial t} + \bar{v} \cdot \text{grad} \, s \right) - \sigma'_{ik} \frac{\partial v_i}{\partial x_k} - \text{div} (k \, \text{grad} \, T) \]  

(2.2.13)

Comparing this expression for the time derivative of the energy in unit volume with equation (2.2.3), we have

\[ \rho T \left( \frac{\partial s}{\partial t} + \bar{v} \cdot \text{grad} \, s \right) = \sigma'_{ik} \frac{\partial v_i}{\partial x_k} + \text{div} (k \, \text{grad} \, T) \]  

(2.2.14)

This equation is called the general equation of heat transfer.
(II) BASIC EQUATIONS IN MAGNETOHYDRODYNAMICS:

The Faraday’s Law says that if current $\vec{J}$ is passing through a conductor under a magnetic flux $\vec{B}$, then the conductor experiences a force perpendicular to both of them and is proportional to their magnitude. Thus we can write the force $\vec{F}$, acting on the conductor as

$$\vec{F} = \vec{J} \times \vec{B}$$

(2.2.15)

This force is sometimes called the Lorenz’s force and is body-force acting on the fluid.

Thus the momentum equation is

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla = -\nabla p + \mu \nabla^2 \vec{v} + \vec{J} \times \vec{B}$$

(2.2.16)

The electric dissipation which is the heat energy produced by the work done by the electric currents and is equal to $\vec{J} \cdot \vec{E}'$ where

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

(2.2.17)

is the effective electric field.

Thus the dissipation due to electric currents is $\sigma J^2$ where $J = |\vec{J}|$ and $\sigma$ is the electrical conductivity of the fluid. Thus the energy equation is

$$\rho C_v \frac{DT}{Dt} = k v^2 + \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]^2 \frac{1}{\sigma} J^2$$

(2.2.18)

The mass conservation equation remains the same.
In addition to these equations we have the Maxwell equations,

\[
\frac{\partial \vec{H}}{\partial t} = -\nabla \times \vec{E} \quad (2.2.19)
\]

\[
\nabla \times \vec{H} = \vec{J} \quad (2.2.20)
\]

\[
\nabla \cdot \vec{B} = 0 \quad (2.2.21)
\]

\[
\nabla \cdot \vec{D} = \rho_e \quad (2.2.22)
\]

Finally, we have generalized Ohm’s Law,

\[
\vec{J} = \sigma \left[ \vec{E} + \nabla \times \vec{B} \right] + \rho_e \vec{v} \quad (2.2.23)
\]
(III) FUNDAMENTAL EQUATIONS OF THE FLOW OF VISCOUS FLUID:

The fundamental equations of the flow of viscous incompressible fluids are

(a) Equation of state (one)

(b) Equation of continuity (one)

(c) Equations of motion (three)

(d) Equation of energy (one)

These equations are mathematical expressions of the basic physical concept.

For a viscous incompressible fluid with constant fluid properties in ordinary Cartesian co-ordinates under usual notations equations are

(i) Equation of continuity: \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \)  \hspace{1cm} (2.2.24)

(ii) Equation of motion:

x-component: \( \rho \frac{Du}{Dt} = \rho f_x - \frac{\partial p}{\partial x} + \mu \nabla^2 u \)  \hspace{1cm} (2.2.25)

y-component: \( \rho \frac{Dv}{Dt} = \rho f_y - \frac{\partial p}{\partial y} + \mu \nabla^2 v \)  \hspace{1cm} (2.2.26)

z-component: \( \rho \frac{Dw}{Dt} = \rho f_z - \frac{\partial p}{\partial z} + \mu \nabla^2 w \)  \hspace{1cm} (2.2.27)

(iii) Equation of energy:

\( \rho C_v \frac{DT}{Dt} = \frac{\partial Q}{\partial t} + k \nabla^2 T + \Phi \)  \hspace{1cm} (2.2.28)
where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}
\]  
(2.2.29)

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\]  
(2.2.30)

\[
\Phi = 2\mu \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right\}
\]

\[
+ \mu \left\{ \left( \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} + \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right)^2 \right\}
\]  
(2.2.31)

The components of heat-flux vectors are:

\[
q_x = -k \frac{\partial T}{\partial x}, \quad q_y = -k \frac{\partial T}{\partial y}, \quad q_z = -k \frac{\partial T}{\partial z}
\]  
(2.2.32)