Chapter-7

SMALL PERTURBATIONS AND DYNAMIC INSTABILITY IN HIGH VOLTAGE SYSTEMS

7.0 Symbols used

R_a, X_a	Armature resistance, leakage reactance
X_d, X_{ds}	D-axis unsaturated/ saturated synchronous reactance
X_q, X_{qs}	Q-axis unsaturated/ saturated synchronous reactance
$X_d^{'}, X_{ds}^{'}$	D-axis unsaturated/ saturated transient reactance
$X_d^{''}, X_q^{''}$	D-axis/Q-axis sub-transient reactance
R_e, X_e, Z_e	External resistance, leakage reactance, impedance to infinite bus
$T_d^{'}, T_{do}^{'}$	D-axis S.C./ O.C. transient time constant.
$T_d^{''}, T_{do}^{''}$	D-axis S.C./ O.C. subtransient time constant
$T_q^{''}, T_{qo}^{''}$	Q-axis S.C./ O.C. subtransient time constant
I_d, I_q	D-axis & Q-axis components of armature current
V_d, V_q	D-axis & Q-axis components of armature voltage
E, E_{FD}	Generator Induced e.m.f, exciter output
V_t, V_b	Voltage at generator terminal, infinite bus
$K_1 - K_6$	Constants of D'Mello& Concordia
D, H	Damping/ Inertia constant
K_A, T_A, K_F, T_F	Constants related to exciter model IEEE, type 1s
$E^{'}, E_q^{'}$	Voltage behind transient reactance/ its q-axis component
ω, δ	Angular speed of the generator (rad/s), Power angle (rad)

All quantities are in per unit and all time-constants in sec. unless otherwise mentioned. The symbols are commonly used.

7.1 Introduction

Stability in power system means the ability of synchronous machines connected to the grid system to retain synchronism between each other. It may be pertaining to either steady state or transient conditions and may be either a power angle stability problem or a voltage stability problem. The power angle instability arises out of mismatch in active power. The voltage stability problem arises out of mismatch in reactive power [30,66,92].

Steady state stability can again be divided into static stability and dynamic stability [32]. While changes are very slow and in absence of control actions, the stability is called static. The dynamic stability problem is, in essence, the performance of a turbine-generator connected to grid system under small impacts, in presence of fast acting control systems viz. the AVR-excitation control. The small impact may be a small change in load, or in voltage or in network parameters [104].Auto-reclosing CBs also affect the stability [35]. The assessment is made by mathematical programming techniques [122,123].

In the following sections, the tools and techniques of dynamic stability assessment for a group of turbo generators operating in parallel in a high voltage grid system while a small perturbation is injected externally. The group of machines has been equivalenced to a single machine as they behave as a coherent group under system perturbation. The infinite bus idealisation has been made as the capacity of the plant is small compared to that of the regional/national power grid. The classical model based on Park-Gorev transformation has been used to represent the machines [111,112,113]. The current state space model has been developed from it. The dynamic stability has been assessed by examining the real parts of eigenvalues of the state matrix [32,36]. The formulation can be extended to multimachine system [37]. Reduced order model has also been developed for the same machine including exciter parameters and the stability has been examined by Routh's array on the characteristic equation [32,100].

7.2 Modelling approach

The first task is to model the machines and the system to which they are connected. Multivariable systems are now modelled in terms of state variables. Either current states or flux-linkage states are chosen by the modeler [100]. The general practice is to use the classical model of Park, Concordia and Adkins for the synchronous machines [112]. While subjected to small impact the state variables of the system change slightly from the initial states and assume new states. The initial states are called the quiescent states. They can be found out from the known and established performance equations of synchronous machine [33].

To examine the behaviour of the machine under small impact, the non-linear state space equations are linearised about the quiescent point. The stability is predicted from the eigenvalues of the linearised state matrix. If the eigenvalues are negative real or complex with negative real parts, then the system is stable, otherwise the system is unstable [32, 34,137].

There are two types of non-linearities in the state matrix of a machine viz. trigonometric and product nonlinearities. They can be eliminated from the system equations by trigonometric and algebraic manipulations [32]

The generator-transformer may be treated as one machine on infinite bus provided the capacity of the machine is small compared to that of the grid. If there be a local load e.g. the power house auxiliaries, then a Thevenin's equivalent should be used [134]. A multimachine problem can be reduced to a problem of one machine on infinite bus, by load flow analysis and network reduction. This approach hampers accuracy of the solution. However, it is frequently used in power system analysis for its simplicity and ease of programming[132].

7.2.1 The classical model of the synchronous machine

In a rotating machine, whether synchronous or induction, there are time-varying inductances due to relative motion of the stator and the rotor body. The existence of time-varying elements in the matrix makes the analysis of such machinesvary complicated. A great simplification was made by R.H. Park by using a transformation, known as Park's transformation. It transforms the 3-phase machine into an equivalent 2-phase machine ($\alpha-\beta$ transformation) and then to a machine with pseudo-stationary coils along the $d-q$ axes.

The transformed machine does not have any time-varying element. It consists of only mutually coupled coils along the fixed ($d-q$) axes. The pseudo-stationary coils are attributed with the property of generating rotational e.m.f.s by the speed action of the rotor. After computation using Park's transformation, the actual quantities can be found out by the inverse transformation [112,113].

The Park's transformation is based on an equivalent 2-pole machine with rotating armature. The transformation is given below:

$$\begin{bmatrix} V_d \\ V_q \\ V_o \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix} . \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad 7.1$$

Where, V_a, V_b, V_c are the phase voltages and V_d, V_q, V_o are the transformed voltages. The transformation matrix is same for the current variables. The inverse transformation is obtained from

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & \cos\theta & \sin\theta \\ 1/\sqrt{2} & \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) \\ 1/\sqrt{2} & \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix} . \begin{bmatrix} V_d \\ V_q \\ V_o \end{bmatrix} \qquad 7.2$$

In terms of the transformed variables, the d-q axes equations are given below:

$$v_d = r_a i_d + p\psi_d + \omega\psi_q \ ; \ v_q = r_a i_q + p\psi_q - \omega\psi_d \qquad 7.3$$

Where, $\psi_d = L_d i_d + L_{md}(i_f + i_{kd}) \ ; \ \psi_q = L_q i_q + L_{mq} i_{kq}$ 7.4

The effect of damper winding is accounted for by two fixed coils- *kd* along d-axis and *kq* along q-axis. The equations for the fixed field and damper windings are as given below:

$$v_f = (r_f + pL_f)i_f \ ; \ v_{kd} = (r_{kd} + pL_{kd})i_{kd} \ ; \ v_{kq} = (r_{kq} + pL_{kq})i_{kq} \qquad 7.5$$

Expanding equations 7.3 & 7.4, the following matrix equation for the machine is obtained:

$$\begin{bmatrix} v_d \\ v_f \\ v_{kd} \\ v_q \\ v_{kq} \end{bmatrix} = \begin{bmatrix} r_a & 0 & 0 & \omega L_q & \omega L_{mq} \\ 0 & r_f & 0 & 0 & 0 \\ 0 & 0 & r_{kd} & 0 & 0 \\ -\omega L_d & -\omega L_{md} & -\omega L_{md} & r_a & 0 \\ 0 & 0 & 0 & 0 & r_{kq} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \\ i_q \\ i_{kq} \end{bmatrix} + \begin{bmatrix} L_d & 0 & 0 & 0 & 0 \\ 0 & L_{md} & 0 & 0 & 0 \\ 0 & 0 & L_{md} & 0 & 0 \\ 0 & 0 & 0 & L_q & 0 \\ 0 & 0 & 0 & 0 & L_{mq} \end{bmatrix} \begin{bmatrix} \dot{i}_d \\ \dot{i}_f \\ \dot{i}_{kd} \\ \dot{i}_q \\ \dot{i}_{kq} \end{bmatrix} \qquad 7.6$$

For convenience in computing, all the parameters are expressed in p.u. The normalized swing equation with torque, time and angular velocity all in p.u. is given as:

$$2H\omega \frac{d\omega}{dt} = \tau_j \frac{d\omega}{dt} = T_a \qquad 7.7$$

The electrical torque is given as:

$$T_e = i_q\psi_d - i_d\psi_q = \left[L_d i_q \quad L_{md} i_q \quad L_{md} i_q \quad -L_q i_d \quad -L_{mq} i_d \right] . \left[i_d \quad i_f \quad i_{kd} \quad i_q \quad i_{kq} \right]^T \qquad 7.8$$

The torque balance equation (taking no of phases m =3 into consideration:

$$T_a = T_m - T_e/3 - T_d = T_m - T_e/3 - \omega D \qquad 7.9$$

where D is the damping constant. The power angle is obtained from:

$$\theta = \omega_o t + \delta \; ; \therefore \; \omega = \dot{\theta} = \omega_o + \dot{\delta} = 1 + \dot{\delta} \;\; ; \; \therefore \dot{\delta} = \omega - 1 \qquad 7.10$$

Incorporating, we get the following matrix equation for the electromechanical transient:

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_f \\ \dot{i}_{kd} \\ \dot{i}_q \\ \dot{i}_{kq} \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{vmatrix} RNI_{11} & RNI_{12} & RNI_{13} & RNI_{14} & RNI_{15} & 0 & 0 \\ RNI_{21} & RNI_{22} & RNI_{23} & RNI_{24} & RNI_{25} & 0 & 0 \\ RNI_{31} & RNI_{32} & RNI_{33} & RNI_{34} & RNI_{35} & 0 & 0 \\ RNI_{41} & RNI_{42} & RNI_{43} & RNI_{44} & RNI_{45} & 0 & 0 \\ RNI_{51} & RNI_{52} & RNI_{53} & RNI_{54} & RNI_{55} & 0 & 0 \\ \frac{-L_d i_q}{3\tau_j} & \frac{-L_{md} i_q}{3\tau_j} & \frac{-L_{md} i_q}{3\tau_j} & \frac{L_q i_d}{3\tau_j} & \frac{L_{mq} i_d}{3\tau_j} & \frac{-D}{\tau_j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \\ i_q \\ i_{kq} \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} LI_{11} v_d + LI_{12} v_f \\ LI_{21} v_d + LI_{22} v_f \\ LI_{31} v_d + LI_{32} v_f \\ LI_{44} v_q \\ LI_{54} v_q \\ \frac{T_m}{\tau_j} \\ -1 \end{bmatrix} \qquad 7.11$$

where RNI_{11} etc. are the elements of $\left[\mathbf{R} + \boldsymbol{\omega}\mathbf{N}\right]^{-1}$ and LI_{11} etc. are the elements of $\left[L\right]^{-1}$. Considering that the machine is connected to infinite bus through a series impedance ($R_e + j\omega L_e$), the $\left[\mathbf{R} + \boldsymbol{\omega}\mathbf{N}\right] \& \left[\mathbf{L}\right]$ matrices are modified to include the effect of series impedance, as shown below:

$$\hat{R} = r_a + R_e \; ; \; \hat{L}_d = L_d + L_e \; ; \; \hat{L}_q = L_q + L_e \qquad 7.11a$$

For convenience, we use the same symbol for the matrices $\left[\mathbf{R} + \boldsymbol{\omega}\mathbf{N}\right] \& \left[\mathbf{L}\right]$after including the series impedance. The infinite bus idealization yields the following matrix formwhere $K = \sqrt{3} V_{\inf}$

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_f \\ \dot{i}_{kd} \\ \dot{i}_q \\ \dot{i}_{kq} \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{vmatrix} RNI_{11} & RNI_{12} & RNI_{13} & RNI_{14} & RNI_{15} & 0 & 0 \\ RNI_{21} & RNI_{22} & RNI_{23} & RNI_{24} & RNI_{25} & 0 & 0 \\ RNI_{31} & RNI_{32} & RNI_{33} & RNI_{34} & RNI_{35} & 0 & 0 \\ RNI_{41} & RNI_{42} & RNI_{43} & RNI_{44} & RNI_{45} & 0 & 0 \\ RNI_{51} & RNI_{52} & RNI_{53} & RNI_{54} & RNI_{55} & 0 & 0 \\ \frac{-L_d i_q}{3\tau_j} & \frac{-L_{md} i_q}{3\tau_j} & \frac{-L_{md} i_q}{3\tau_j} & \frac{L_q i_d}{3\tau_j} & \frac{L_{mq} i_d}{3\tau_j} & \frac{-D}{\tau_j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \\ i_q \\ i_{kq} \\ \omega \\ \delta \end{bmatrix} + \begin{vmatrix} LI_{11} & LI_{12} & RNI_{13} & RNI_{14} & RNI_{15} & 0 & 0 \\ LI_{21} & LI_{22} & LI_{23} & LI_{24} & LI_{25} & 0 & 0 \\ LI_{31} & LI_{32} & LI_{33} & LI_{34} & LI_{35} & 0 & 0 \\ LI_{41} & LI_{42} & LI_{43} & LI_{44} & LI_{45} & 0 & 0 \\ LI_{51} & LI_{52} & LI_{53} & LI_{54} & LI_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} -K \sin\gamma \\ -v_f \\ 0 \\ K \cos\gamma \\ 0 \\ \frac{T_m}{\tau_j} \\ -1 \end{bmatrix} \qquad 7.12$$

7.3 The initial conditions

In dynamic studies, the initial conditions of the system are required. A one machine connected to infinite bus through aseries impedance (generator-transformer and short transmission line) has been shown in fig. 7.1. For specified boundary conditions, the pertinent equations for estimation of steady state conditions and the corresponding phasor diagrams are given all standard text-books [31,132].

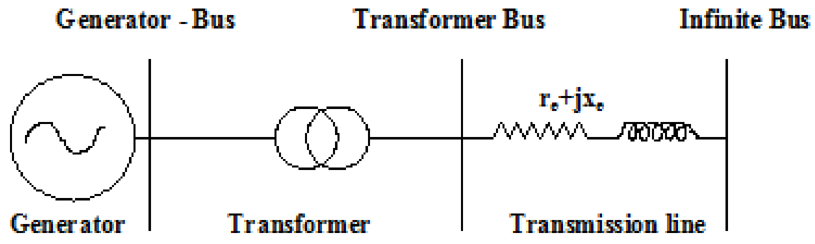

Fig. 7.1 generator-transformer connected to infinite bus through a short transmission line.

The effect of local load and transmission line capacitances can be included in the study by the application of Thevenin's theorem. For multimachine system, the bus voltages, active and the reactive power injected into the system by a generator or a group of generators are obtained from load flow study [104]. A group of generators operating in parallel is equivalenced to a single generator for convenience. The network is also reduced eliminating unimportant nodes to reduce the dimension of the problem. The initial states- currents, voltages and flux-linkages for individual machines are found out with reference to the slack bus. The initial conditions for a synchronous generator connected to an infinite bus through a series impedance are given as: $E_{qa}\angle\delta = V + I_a\angle-\phi(r_a + jX_q)$ 7.13

where, $E_{qa}\angle\delta$ is a fictitiousvoltage. From this expression, the value of the power angle is obtained. The axis quantities and the induced emf are given as:

$$V_d = V\sin\delta\ ;\ \ V_q = V\cos\delta \qquad 7.14$$

$$I_d = I\sin(\delta+\phi)\ ;\ \ I_q = I\cos(\delta+\phi) \qquad 7.15$$

$$E = V_q + I_q r_a + I_d X_d \qquad 7.16$$

Negative signs for the d-axis quantity and the suffix 'o' for initial conditions, have ben omitted. The flux-linkages are given as:

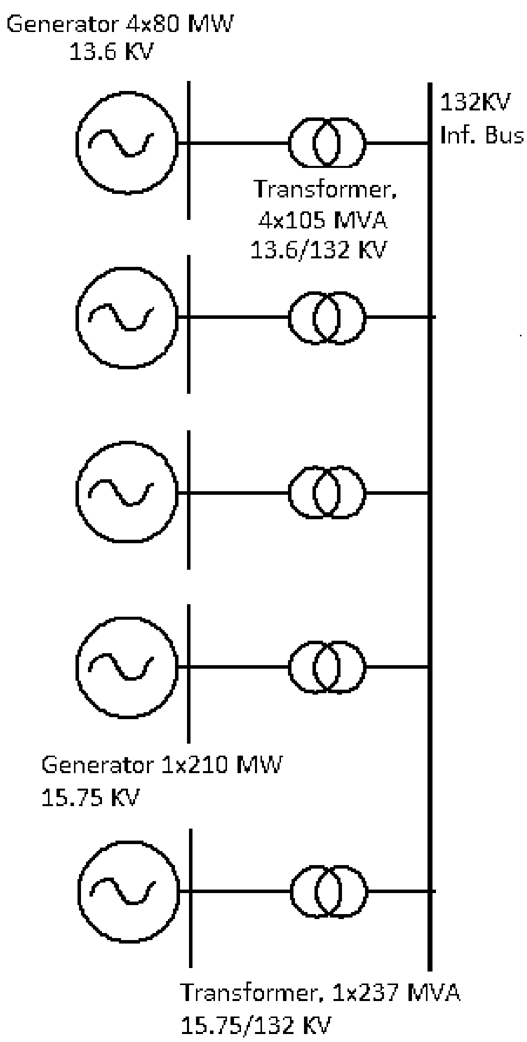

Fig. 7.2 Generator configuration in Bandel Thermal Power Station

$$\psi_d = L_d i_d + L_{md} i_f \; ; \psi_{md} = L_{md}(i_d + i_f) = \psi_{kd} ; \; \psi_f = L_f i_f + L_{md} i_d \; ; \psi_q = L_q i_q \; ; \psi_{mq} = L_{mq} i_q = \psi_{kq} \quad 7.17$$

The quantities: r_a, X_d, X_q includes the effect of external impedance to infinite bus. Using these equations, the initial conditions for the given operating conditions[33] are obtained.

7.4 Equivalencing technique and network reduction

While a group of generators swing similarly they form a coherent group. If there be **n**-number of generator-transformers in a coherent group, then their equivalenced parameters and time-constants are obtained by well-known methods.

Equivalencing may also be made by matching f-response curves.

If **n-**number of nodes is to be retained and**r**-number of nodes is to be eliminated from the equivalent load admittance matrix, network reduction techniques have to be used. Most often **n** is the number of equivalenced generators. The load is represented as constant impedance in this treatment.

Power station managers are provided with manufacturer’s data from which the equivalent circuit for the directand quadrature axes can be found out [139]. The parameters are generally given in p.u. and time-constants in sec.These parameters are with reference to the classical model of Park, Concordia and Adkins. The effect of saturation and distributed eddy currents in the solid iron rotor are not accounted for in the classical model. Such effects can also be included in the dynamic studies by extended mathematical models advanced by various authors [112,113].

7.4.1 Equivalencing technique applied to a power station having generators of different rating

In Kolaghat TPS, all the generators are of same rating and same make.So the equivalent parameters are same as the p.u. parameters of the individual machines. But in Bandel TPS there are 4 no. Westinghouse machines, originally of rating 89.25 MW and 1 no. 210 MW machine of BHEL, as shown in fig. 7.2. While under some disturbance they act as a coherent group, the equivalent parameters may be found outto a reasonable degree of approximation,using equations 7.18.

$$1/X_{de}=4/X_{de1}+1/X_{de2}\,;\;1/X'_{de}=4/X'_{de1}+1/X'_{de2}\;;\;1/X''_{de}=4/X''_{de1}+1/X''_{de2}$$
$$1/X_{qe}=4/X_{qe1}+1/X_{qe2}\,;\;1/X''_{qe}=4/X''_{qe1}+1/X''_{qe2}\,;\;H=4H_1+H_2 \qquad 7.18$$

where, X_{de} is the equivalent d-axis synchronous reactance, X'_{de} is the equivalent d-axis transient reactance, X''_{de} is the equivalent d-axis sub transient reactance, X_{qe} is the equivalent q-axis synchronous reactance, X''_{qe} is the equivalent q-axis sub transient reactance and H is the equivalent inertia constant. Before computing the equivalent parameters, the parameters are to be converted to a common MVA-base. The effect of resistance may be neglected except for calculating losses [139].

7.5 Linearisation- eigenvalue techniques for the prediction of dynamic stability

It has been already stated that for linearisation of non-linear state space equations of transient stability analysis, trigonometric and algebraic manipulations [32] are to be made. If X_0 be the initial states & X_Δ be the small departure, then the non-linear state space form

$$\dot{\mathbf{X}}=\mathbf{f(X,t)} \qquad 7.19$$

can be linearised as:

$$\dot{\mathbf{X}}_\Delta=\mathbf{A}(\mathbf{X}_o)\mathbf{X}_\Delta+\mathbf{B(X_o)U} \qquad 7.20$$

The elements of the $\mathbf{A}$-matrix depends on the initial states X_o which is assumed to be constant during the study. The dynamic properties of the system for the quiescent point depend on eigenvalues of the state matrix $\mathbf{A}$. The elements of the state matrix change for a change in quiescent conditions. Anderson and Fouad have derived the following matrix form by algebraic and trigonometric manipulations [69]:

$$\begin{bmatrix} v_d \\ -v_f \\ 0 \\ v_q \\ 0 \\ T_m \\ 0 \end{bmatrix} = -\begin{vmatrix} r_a & 0 & 0 & \omega_o L_q & \omega_o L_{mq} & \lambda_{qo} & 0 \\ 0 & r_f & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{kd} & 0 & 0 & 0 & 0 \\ -\omega_o L_d & -\omega_o L_{md} & -\omega_o L_{md} & r_a & 0 & -\lambda_{do} & 0 \\ 0 & 0 & 0 & 0 & r_{kq} & 0 & 0 \\ \frac{\lambda_{qo}-L_d i_{qo}}{3} & \frac{-L_{md} i_{qo}}{3} & \frac{-L_{md} i_{qo}}{3} & \frac{-\lambda_{do}+L_q i_d}{3\tau_j} & \frac{L_{mq} i_{do}}{3} & -D & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{vmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \\ i_q \\ i_{kq} \\ \omega \\ \delta \end{bmatrix} + \begin{vmatrix} L_d & L_{md} & L_{md} & 0 & 0 & 0 & 0 \\ L_{md} & L_f & L_{md} & 0 & 0 & 0 & 0 \\ L_{md} & L_{md} & L_{kd} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_q & L_{mq} & 0 & 0 \\ 0 & 0 & 0 & L_{mq} & L_{kq} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tau_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} \dot{i}_d \\ \dot{i}_f \\ \dot{i}_{kd} \\ \dot{i}_q \\ \dot{i}_{kq} \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} \qquad 7.21$$

i.e. in the matrix form: $\mathbf{M\dot{X}=-KX-V}$ Or, $\mathbf{\dot{X}=-M^{-1}K\,X-M^{-1}\,V}=\mathbf{AX+BU}$ 7.22

This is the current state space form. If the eigenvalues of the state matrix $\mathbf{A}$ are negative real quantities or they are complex with negative real components, then the system will remain dynamically stable. Otherwise the system will become unstable[147].

Anderson and Fouad have also linearised the alternative state space formulation in flux-linkages and have established the following equation[9]:

$$\dot{\boldsymbol{\lambda}} = \mathbf{T}^{-1}\mathbf{C}\boldsymbol{\lambda} + \mathbf{T}^{-1}\mathbf{D} = \mathbf{AX} + \mathbf{BU}\boldsymbol{\lambda} \qquad 7.23$$

The expression for the elements of $\mathbf{T}$ and $\mathbf{C}$-matrices have been developed and given by Anderson and Fouad. Both the forms are used for dynamic performance studies. These matrices can be extended to include the effects of:

i. Excitation control ii. Turbine-governor control iii. Power system stabilizer

Detailed discussion of such methods has been made in the text books and papers[33,38]. Protective schemes are to be provided to counteract instability [40]. The effect of autoreclosing CBs are also to be accounted for [48].

The dynamic stability is assessed by finding eigenvalues of the state matrix $\mathbf{A}$ [32]. The characteristic equation is found out by Fadeev-Leverrier method and the eigenvalues by Lin-Bairstow method [36,123].

7.6 Simplified Linear Models- The $E_q^{'}$- Equation and DeMello's Constants:

While some investigators have tried to model a machine elaborately to get more accurate predictions, others have tried to reduce the model through idealising assumptions and approximations. Obviously, there is loss of accuracy in prediction with this approach, but the dimension of the model gets reduced which saves computer memory space and run time. For example, inter-area oscillations are not revealed by the simplified model [39,41].It is a useful technique particularly for large interconnected systems. Individual machines or group of machines remote from the point of perturbation does not appreciably affect the system dynamics. Hence it is expedient to use a reduced model for them.

With this idea in view, a reduced order model was developed by Concordia and De Mello, based on $(E^{'} - E_q^{'})$ concept. Their analysis gave rise to a set of equations in terms of constants K_1 to K_6. The expressions for the constants are given below:

$$K_1 = K_i V_b [E_{qao}\{R_e \sin(\delta_o - \alpha) + (X_d^{'} + X_e)\cos(\delta_o - \alpha)\}$$
$$+ I_{qo}(X_q - X_d^{'})\{(X_e + X_q)\sin(\delta_o - \alpha) - R_e \cos(\delta_o - \alpha)\}] \quad 7.24$$

where, $1/K_i = R_e^{\,2} + (X_q + X_e)(X_d^{'} + X_e)$ 7.25

$$K_2 = K_i[\{R_e . E_{qao} + I_{qo}\{R_e^2 + (X_e + X_q)^2\}] \quad 7.26$$

$$1/K_3 = 1 + K_i(X_d - X_d^{'})(X_q + X_e) \quad 7.27$$

$$K_4 = V_b K_i (X_d - X_d^{'})\{(X_q + X_e)\sin(\delta_o - \alpha) - R_e \cos(\delta_o - \alpha)\} \quad 7.28$$

$$K_5 = (K_i V_b / V_{to})[X_d^{'} V_{qo}\{R_e \cos(\delta_o - \alpha) - (X_q + X_e)\sin(\delta_o - \alpha)\}$$
$$-X_q V_{do}\{(X_d^{'} + X_e)\cos(\delta_o - \alpha) + R_e \sin(\delta_o - \alpha)\}] \quad 7.29$$

$$K_6 = (V_{qo} / V_{to})(1 - K_i X_d^{'}) - (V_{do} / V_{to}) K_i X_q R_e \quad 7.30$$

$$\tau_j = 2H \quad 7.31$$

Extensive application of this model has been made in various power system problems for its simplicity. A comprehensive report of the applications has been produced by P.M. Anderson et al and P. Kundur[32, 33]. Additional equations are used to include the effects of the solid state exciter.

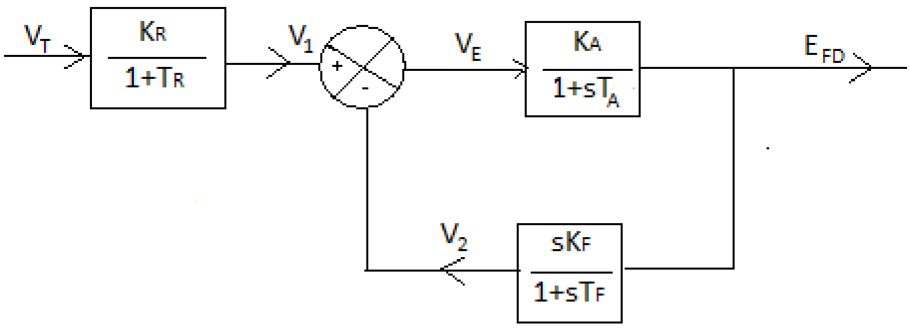

Fig. 7.3 Block diagram representation of exciter- model IEEE 1s.

Including the effect of a modern solid state exciter, the machine model in terms of these constants are given below: [type 1s model of IEEE has been used, ref. fig. 7.3]

$$\begin{bmatrix} \dot{E}_q' \\ \dot{\omega} \\ \dot{\delta} \\ \dot{V}_1 \\ \dot{E}_{FD} \end{bmatrix} = \begin{bmatrix} \frac{1}{K_3 T_{do}'} & 0 & \frac{-K_4}{T_{do}'} & 0 & \frac{1}{T_{do}'} \\ -K_2/\tau_j & -D/\tau_j & -K_1/\tau_j & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ & & & \frac{-(T_3 + K_f K_a)}{T_f T_a} & \frac{-K_f}{T_a T_f} \\ & & & \frac{-K_a}{T_a} & \frac{-1}{T_a} \end{bmatrix} . \begin{bmatrix} E_q \\ \omega \\ \delta \\ V_1 \\ E_{FD} \end{bmatrix} \qquad 7.32$$

A similar model has been developed for the type 1 of IEEE by Anderson and Fouad. The mathematical models based on constants K_1 to K_6 are sufficiently accurate. The pertinent equations are given below:

$$\dot{E}_q' = \frac{K_3 E_{FD\Delta}}{1 + K_3 T_{do\Delta}'} - \frac{K_3 K_4 \delta_\Delta}{1 + K_3 T_{do\Delta}'} \qquad 7.33$$

$$\dot{T}_{e\Delta} = K_1 \delta_\Delta + K_2 E_{q\Delta}' \qquad 7.34$$

$$\dot{V}_{t\Delta} = K_5 \delta_\Delta + K_6 E_{q\Delta}' \qquad 7.35$$

$\tau_j \, \dot{\omega}_\Delta = T_{m\Delta} - T_{e\Delta}$ - the linearised swing equation 7.36

These equations are used where great accuracy is not warranted.

7.7 Extension to multimachine systems

The approach of one machine on infinite bus holds good only for machines or group of machines the total generating capacity of which is insignificantly small with respect to the grid power. In such cases the changes in the operating conditions of the generator(s) does not appreciably affect the grid system. However, if the capacity of the generator(s) is comparable to the grid power then the approximation is no longer valid. Then all the synchronously coupled machines and the network must be modeled separately. To find out initial conditions, it is essential to make a load flow study beforehand. Then these are to be recast to a common reference frame. It is expedient to reduce the number of nodes and the number of machines for dynamic stability analysis to get a faster solution [37]. This is made by the techniques of network reduction and equivalencing. Interarea oscillations are revealed through this approach [39]. It has also been suggested by Anderson and Fouad to represent the machine(s) near to the location of perturbation in details and the remote machines approximately, thereby making a further saving of computer run-time [32]. The influence of large power grid on dynamic stability has been discussed in a paper by Z. Fang et al [45]. Implementation of detailed modeling has been emphasized by M. Watanabe in a paper [46].

7.8 Power system stabilizer for optimizing performance

Power system stabilizers are extensively used in modern power system as effective means of enhancing overall system stability [32,33,38]. It extends the system stability limit by modulating governor excitation so as to provide damping of the oscillations of synchronous machine rotors relative to one another. It produces a component of electrical torque on the rotor in phase with speed variation. The implementation details differ depending upon the input signal employed. However, for any input signal the transfer function of the PSS must compensate for the gain and phase characteristics of the excitation system, generator and the power system, which collectively determine the transfer function GEP(s) of the PSS. Two of the commonly used types of PSS are the delta-omega type and delta-T-Omega type. The structure in both the type is same, irrespective of the input signal. It comprises a pair of lead-lag network in cascade, which provides the desired phase lead and a gain to compensate for the phase lag caused by GEP(s).

The basic function of a power system stabilizer is to add damping to the generator rotor using auxiliary stabilising signals [42]. The stabilizer must produce a component of electrical torque in phase with the deviation in rotor speed. Since the purpose of a PSS is to introduce a damping torque component, a logical signal to use is the speed deviation [43].

7.9 Case studies on dynamic stability

Case studies have been made on dynamic stability of 210 MW turbo generator sets for different conditions of loading and different power factors. The first case-study has been made for the machine operating at continuous maximum rating (CMR)at rated lagging power factor. This is the characteristic of peak hours. The second and the third case-studies have been made for leading power factor conditions- one for full load condition and the other for half load. The third one is characteristic of lean hours in presence of highly capacitive long transmission lines. The effect of exciter has been neglected in these studies. Current state space model has been used. The stability has been predicted either by finding out eigenvalues of the state matrix or by applying Routh's criterion to the characteristic equation of the state matrix. Characteristic equation of the matrix has been found out by Fadeev-Leverrier method and the eigenvalues have been obtained by using Lin-Bairstow's method. Routh's criterion has been applied by using our own program. MATLAB-tools have been used for verification of the results obtained.

Another case study has been made on the same machine, based on reduced order model, in terms of the constants of Concordia and DeMello. In this case the exciter has been modeled according to IEEE 1s (solid state excitation). The system stability has been predicted from the characteristic equation of the state matrix using Routh's array [100].

Case-1: (for rated power at lagging p,f.)

Boundary conditions are given at generator node. The quantities are in p.u. on the base of machine rating. Time constants are in sec.

Machine parameters:

Armature Resistance, R_a =	0.001096
D-axis synchronous reactance, X_d =	2.225
Q-axis synchronous reactance, X_q =	2.11
Armature leakage reactance, X_a =	0.15
D-axis transient reactance, $X_d^{'}$ =	0.305
Inertia constant, H =	4.18 sec
Damping Constant, D =	1.0p.u.

Operating conditions (at continuous maximum rating):

The generator power, P =	0.85p.u.
The generator power factor =	0.85 lagging
The generator terminal voltage, V =	1.0p.u.
The infinite bus voltage, V_{inf} =	$0.9307\angle -7.003^o$
Power angle: δ =	40.307^o

D-Q axes components of the voltage and current:

V_d = -0.6469 ; V_q = 0.7626 ; $I_d = -0.9516$; $I_q = 0.3074$;

$E = E_{fd} = 2.8804$; $I_{fd} = 2.4067$

Flux-linkages:

D-axis armature linkage, ψ_d = 1.3218 ; Field flux linkage, ψ_f = 1.9700 ;

D-axis damper flux linkage, ψ_{kd} = 1.5724

Q-axis armature linkage, ψ_q = 1.1236 ; Q-axis damper flux linkage, ψ_{kq} = 1.0426

Infinite bus constant: K = 1.6121; Its axis components: $K_d = -1.1849$; K_q =1.0930

The elements of the state matrix are given in table 7.1

TABLE 7.1

(Elements of State-Matrix A(7,7)*1000)

-16.428	9.4412	6.1628	-878.42	246.28	-730.18	667.16
1.7833	-4.5154	2.6013	0	0	0	0
12.79	11.77	-25.49	0	0	0	0
1138.4	-166.43	-263.86	1.1717	-3.7078	762.04	823.34
0	0	0	59.08	-63.66	0	0
0.5988	-0.25	-0.3964	- 0.7486	0.4956	0	0
0	0	0	0	0	1000.0	0

Coefficients of the characteristic equation have been found out using Fadeev-Leverrier method:

$$s^7 + 1.1089s^6 + 1.0045s^5 + 7.399e-2s^4 + 2.325e-3s^3 + 6.1547e-5s^2$$
$$+8.463e-7s + 8.1277e-10 = 0$$

Stability analysis has been made in this case by applying Routh's criterion to the characteristic equation. The Routh's tabulation is given below:

TABLE 7.2

(Routh’s tabulation)

s^7	1	1.004536	0.00232515	0.0000008462933
s^6	0.1089272000000	0.07399189	0.00006154733	0.000000008128
s^5	0.3252573000000	0.001760118	0.0000008388317	
s^4	0.07340244	0.00006126641	0.0000000008128	
s^3	0.0014886370000	0.0000008352302		

s^2	0.0000200824800	0.0000000008128		
s^1	0.0000007749825			
s^0	0.0000000008128			

The system is stable as all elements of the 1st. column are positive.

Case-2: (Operation at full load, leading p.f.)

Operating conditions:

Rated power P =0.85 at 0.985 leading power factor.

Boundary conditions given at generator node

The generator terminal voltage, V = 1.05 $\angle 0^o$p.u.

The infinite bus voltage, V_b = 1.0711 $\angle 5.9638^o$

Power angle =66.23^o

The axis variables are given below:

V_d = -0.961 V_q = 0.4232 I_d = -0.6837 I_q = 0.456

Induced emf and field current: $E = E_{FD} = 1.9453$; I_{FD} = 1.6254

The flux-linkages and bus constants are given below:

D-axis/Q-axis armature linkage, ψ_d, ψ_q: 0.7344 /1.6667

Infinite bus constant, K = 1.8552

D-axis/Q-axis components of infinite bus const, K_d, K_q: -1.7664/ 0.5672

The elements of the state matrix are given for this operating condition in table 7.3

TABLE 7.3

Elements of the state-matrix A(7,7) x 1000

-19.652	1.0890	8.0550	-6407.9	-5584.7	-3754.4	3299.8
7.3760	-4.2560	14.072	2405.0	2096.1	1409.1	-1238.5
11.688	3.0150	-23.248	3811.2	3321.6	2233.0	-1962.6
5276.1	4631.4	4631.4	-15.393	20.523	2593.7	2934.7

-4847.7	-4255.2	-4255.3	14.143	-23.548	-2383.1	-2696.4
-2.6870	-48.428	-48.428	-81.727	-30.885	-119.62	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1000.0	0.0000

The characteristic equation is given below:

$$s^7 + 0.2057s^6 + 1.371s^5 + 0.1928s^4 + 0.3656s^3 + 1.598E-02s^2 + 1.776E-04s - 1.793E-07 = 0$$

As all the coefficients of the characteristic polynomial are not positive, the system is unstable. This is confirmed by finding out the real root of the polynomial by N-R method:

s = 9.302644E-04.

It is a pole in the right half of the s-plane showing instability of the system at the specified operating condition.

Case-3: (Operation at half load, leading p.f.)

Operating conditions:

Half rated power P = 0.425 at 0.985 leading power factor. Boundary conditions given at generator node

The generator terminal voltage: V_g = 1.05 $\angle 0^o$;

The infinite bus voltage: $V_{\inf}$ = 1.0591 $\angle$ 3.0116^o

Power angle: δ = 43.46^o

Axis components of voltage and current:

V_d = -0.7223 ; V_q = 0.7621 ; I_d = - 0.227 ; I_q = 0.3425

Induced emf and field current: $E = E_{FD}$ = 1.2678 ; I_{FD} = 1.0592

D-axis/Q-axis armature linkage ψ_d, ψ_q: 1.321/ 1.2519;

Infinite bus constant, K =1.8345;

Its D & Q-axes components- K_d, K_q : -1.3302/ 1.2632

The elements of the state matrix is given in table 7.4

TABLE 7.4

(Elements of State-Matrix A(7,7)*1000)

-19.652	1.089	8.055	-6407.9	-5584.7	-3801.8	3603.1
7.376	-4.256	14.072	2405.0	2096.1	1426.9	-1352.3
11.688	3.015	-23.248	3811.2	3321.6	2261.2	-2143.0
5276.1	4631.4	4631.4	-15.393	20.523	2831.4	2971.9
-4847.7	-4255.3	-4255.3	14.143	-23.548	-2601.5	-2730.5
-2.72	-49.040	-49.040	-85.750	-30.694	-119.62	0
0	0	0	0	0	1000.0	0

Coefficients of characteristic polynomial (by Fadeev-Leverrier method) are given below:

$$s^7 + 0.2057 s^6 + 1.346 s^5 + 0.1903 s^4 + 0.3406 s^3 + 1.336E-02 s^2 + 1.279E-04 s + 7.743E-07 s = 0$$

The real roots by N-R method: s(1) = -2.949653e-02

No more real roots- 3 pairs of complex roots are being found out by Lin-Bairstow's method:

Complex pair(1): $-5.1354e-03 \pm j7.216e-03$

The frequency of oscillation in Hz. = 0.3608; the damping ratio= 0.6198

Complex pair(2): $-6.1265e-02 \pm j\ 0.5755$

The frequency of oscillation in Hz. = 28.78; the damping ratio= 5.196e-02

Complex pair(3): $-2.1709e-02 \pm j0.9992$

The frequency of oscillation in Hz.= 49.95 ; the damping ratio= 0.1466

This is a stable system as all the eigenvalues are either negative real or complex with negative real components.

Case-4: Dynamic stability analysis based on reduced order model

Now, the current state space model is being worked out in terms of Concordia and D'mello constants. Saturation and the effect of exciter have been included in the study.

Machine parameters:

Parameters are in p.u. including saturation. Time constants are in sec.

Armature Resistance, R_a = 0.001096

D-axis synchronous reactance, X_d = 1.7

Q-axis synchronous reactance, X_q = 1.64

Leakage reactance, X_a = 0.15

D-axis transient reactance, $X_d^{'}$ = 0.245

External impedance to infinite bus, Z_e = 0.02 + j. 0.4

Inertia constant in sec, H = 4.18

Damping constant in p.u. , D = 1.0

Exciter constants (IEEE model 1s):

$K_a = 300$ $T_a = 314$ $K_f = 1$ $T_f = 47.1$

Operating variables and initial conditions:

The generator power, P = 1.0 at 0.85 lagging power factor

$V_t = 1.0 \angle 0^o$; $V_b = 1.0106 \angle 3.1224^o$; $\delta = 45.51^o$; $V_d = -0.63062$; $V_q = 0.77609$

$I_d = -1.1116$; $I_q = 0.38527$; $E_{qa} = 2.5995$ $V_b \angle \alpha = 0.82838 \angle -27.9^o$

The Concordia &D'mello Constants for this condition of loading are given below:

$K_1 = 1.0751$ $K_2 = 1.2578$ $K_3 = 0.3072$ $K_4 = 1.7124$ $K_5 = -0.0477$ $K_6 = 0.4971$

The elements of the state matrix are given in table 7.5

Table 7.5

(State matrix for the reduced order model)

-0.46502	0	-0.24462	0	-0.14286
-0.00048	-0.00026	-0.00041	0	0
0	1	0	0	0
0	0	0	-0.04152	-0.00007
0	0	0	-0.00342	-0.00318

The corresponding B-matrix:

0	0.00038	0	0	0

The characteristic equation is given below:

$$s^5 + 0.51\ s^4 + 0.02146\ s^3 + 1.583E\text{-}04\ s^2 + 3.342\ s + 9.59E\text{-}09 = 0$$

Stability analysis has been made by using Routh's criterion on the characteristic equation. It is given in table 7.6

Table 7.6

(The Routh's tabulation)

s^5	1.0	0.02146044	0.000003341687
s^4	0.5099791	0.000158290	0.0000000095924
s^3	0.02115005	0.000003322877	
s^2	0.00007816764	0.0000000095924	
s^1	0.0000007274391		
s^0	0.0000000095924		

Changes of sign in the first column = 0 . The system is stable for the given initial conditions. The eigenvalues are being found out by MATLAB-tools:

s_1 = -0.4645 ; s_2 = -0.0415 ; s_3 = -0.0032 ; s_4, s_5 = -0.0004 $\pm$ j 0.0126

All the real eigenvalues are negative and the complex eigenvalues have negative real part. So the system is stable under given initial conditions.

7.10 Conclusion

Dynamic stability is a kind of steady state stability [32,132]. It applies to operating conditions of a machine or a group of machines, subjected to small perturbations. Both oscillatory and exponential modes are created under small impact. Their nature depends on the machine parameters and the initial conditions. If there be a growing oscillatory or exponential response, then the system is unstable. This phenomenon occurs if the eigenvalues of the state matrix are either positive real or they are imaginary with positive real parts i.e. situated in the right half of s-plane. The state matrix is found out either in terms of current states or flux-linkage states by applying linearizing techniques around the operating point. The model can be elaborated to include the effects of saturation, exciter and governor control etc. Sometimes reduced order models (first advanced by Concordia and de'Mello) are also used to reduce the dimension of the state matrix and run-time of the computer. The idealization of one machine on infinite bus is made if the rating of the machine is small compared to the total capacity of the grid. Otherwise multimachine dynamic stability analysis has to be made. If the addition of fast excitation controller gives rise to poor or negative damping, then a variable structure Power System Stabilizer (PSS) should be added.

Dynamic stability analysis has been made for 210 MW set using current state space model [32] for three different conditions of loading. While the machine delivers rated power at 0.85 lagging p.f., it is found to be stable. But when the power factor becomes slightly leading (0.985 in the case-study) the machine is found to be unstable. For the same power factor under light load condition (0.5 p.u.), the system is found to be stable. Each machine has been idealized as one machine on infinite bus. The effect of exciter has been neglected in these case-studies.

Another case study has been made on the same machine. This time the effect of saturation and the solid state exciter have been included. A reduced order model based on the constants K_1 to K_6 of Concordia and D'Mello has been made [32,33]. The exciter has been modelled according to IEEE model 1s. Dynamic stability study has been made for operation at continuous maximum rating (CMR), using Routh's array on the characteristic equation of the state matrix. It is found that the machine at this operating condition is dynamically stable.

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