Chapter 5

Group \(\{1, -1, i, -i\}\) cordial labeling of some special graphs

In this Chapter, we investigate the group \(\{1, -1, i, -i\}\) cordial labeling of some special graphs. We prove that Umbrella graph, Lotus inside a circle, Jewel graph, Jelly fish graph, Jahangir graph and Dumbbell graph are all group \(\{1, -1, i, -i\}\) cordial. We further characterize Butterfly graphs that are group \(\{1, -1, i, -i\}\) cordial.

5.1 Group \(\{1, -1, i, -i\}\) cordial labeling of Butterfly graph and Umbrella graph

Theorem 5.1.1. For a given positive integer \(m\), the Butterfly graph \(BF_{m,n}\) is group \(\{1, -1, i, -i\}\) cordial if and only if

\[
 n \leq \begin{cases} 
 2m + 3 & \text{if } m \text{ is even} \\
 2m - 1 & \text{if } m \text{ is odd}
\end{cases}
\]

Proof. Let \((u_1, u_2, \ldots, u_m)\) and \((v_1, v_2, \ldots, v_m)\) be the two Cycles of length \(m\) which are concatenated at vertices \(u_m\) and \(v_m\). Let \(w_1, w_2, \ldots, w_n\) be the end vertices of the \(n\) pendent edges attached at \(u_m\). Total number of vertices is \(2m + n - 1\) and total number of edges is \(2m + n\). For a given positive integer \(m\), assume that \(BF_{m,n}\) is group \(\{1, -1, i, -i\}\) cordial. Let \(f\) be a group \(\{1, -1, i, -i\}\) cordial labeling
of $BF_{m,n}$.

**Case 1.** $m$ is even.

We claim that $n \leq 2m + 3$. Suppose $n = 2m + 4$. We need to choose $m$ or $m + 1$ vertices to give label 1 so that $2m + 2$ edges get label 1. But we have only $m$ vertices say $u_1, u_3, ..., u_{m-1}, v_1, v_3, ..., v_{m-1}$ so that when they get label 1, we have $2m$ edges with label 1. There is no choice of one more vertex so that 2 more edges get label 1. A similar contradiction arises for every $n > 2m + 4$. Thus $n \leq 2m + 3$.

**Case 2.** $m$ is odd.

We claim that $n \leq 2m - 1$. Suppose $n = 2m$. We need to give label 1 to $m$ or $m - 1$ vertices so that $2m$ edges get label 1. But at most $m - 1$ vertices viz., $u_1, u_3, ..., u_{m-2}, v_1, v_3, ..., v_{m-2}$ can be given label 1 so that $2m - 2$ edges get label 1. There is no other choice of one vertex so that 2 more edges get label 1. A similar contradiction arises for every $n \geq 2m$. So $n \leq 2m - 1$.

Conversely, consider the following cases.

**Case 1.** $m$ is even.

Assume that $n \leq 2m + 3$. Consider the case when $n \leq 2m + 1$.

Now $\frac{2m+n-1}{4} \leq m$.

**Subcase(i).** $n - 1 \equiv 0 (mod 4)$

Let $f$ be a labeling defined as follows:
Choose $\frac{m}{2} + \frac{n-1}{4}$ vertices among $\{u_1, u_3, ..., u_{m-1}, v_1, v_3, ..., v_{m-1}\}$ and give them label 1. Label the remaining vertices arbitrarily so that $\frac{m}{2} + \frac{n-1}{4}$ vertices get label $-1$, $\frac{m}{2} + \frac{n-1}{4}$ vertices get label $i$ and $\frac{m}{2} + \frac{n-1}{4}$ vertices get label $-i$. 

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Subcase(ii). $n - 1 \equiv 1 (mod\ 4)$

Let $f$ be a labeling defined as follows:
Choose $\frac{m}{2} + \lceil \frac{n-1}{4} \rceil$ vertices among \{u_1, u_3, ..., u_{m-1}, v_1, v_3, ..., v_{m-1}\} and give them label 1. Also give label 1 to $w_1$. Label the remaining vertices arbitrarily so that $\frac{m}{2} + \lfloor \frac{n-1}{4} \rfloor$ vertices get label $-1$, $\frac{m}{2} + \lceil \frac{n-1}{4} \rceil$ vertices
vertices get label $i$ and $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices get label $-i$.

**Subcase(iii).** $n - 1 \equiv 2 \pmod{4}$

Let $f$ be a labeling defined as follows:
Choose $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices among $\{u_1, u_3, ..., u_{m-1}, v_1, v_3, ..., v_{m-1}\}$ and give them label 1. Label the remaining vertices arbitrarily so that $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices get label $-1$, $\frac{m}{2} + \left\lfloor \frac{n-1}{4} \right\rfloor$ vertices get label $i$ and $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices get label $-i$.

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<thead>
<tr>
<th>Nature of $n$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
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<tr>
<td>$n - 1 \equiv 0 \pmod{4}$</td>
<td>$\frac{2m+n+1}{2}$</td>
<td>$\frac{2m+n-1}{2}$</td>
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<tr>
<td>$n - 1 \equiv 1 \pmod{4}$</td>
<td>$\frac{2m+n}{2}$</td>
<td>$\frac{2m+n}{2}$</td>
</tr>
<tr>
<td>$n - 1 \equiv 2 \pmod{4}$</td>
<td>$\frac{2m+n-1}{2}$</td>
<td>$\frac{2m+n+1}{2}$</td>
</tr>
<tr>
<td>$n - 1 \equiv 3 \pmod{4}$</td>
<td>$\frac{2m+n}{2}$</td>
<td>$\frac{2m+n}{2}$</td>
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<tr>
<td>$n = 2m + 2$</td>
<td>$2m + 1$</td>
<td>$2m + 1$</td>
</tr>
<tr>
<td>$n = 2m + 3$</td>
<td>$2m + 2$</td>
<td>$2m + 1$</td>
</tr>
</tbody>
</table>

Table 5.3

**Subcase(iv).** $n - 1 \equiv 3 \pmod{4}$

Let $f$ be a labeling defined as follows:
Choose $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices among $\{u_1, u_3, ..., u_{m-1}, v_1, v_3, ..., v_{m-1}\}$ and give them label 1. Label the remaining vertices arbitrarily so that $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices get label $-1$, $\frac{m}{2} + \left\lfloor \frac{n-1}{4} \right\rfloor$ vertices get label $i$ and $\frac{m}{2} + \left\lceil \frac{n-1}{4} \right\rceil$ vertices get label $-i$.

For $n = 2m + 2$ and $n = 2m + 3$, label $u_1, u_3, ..., u_{m-1}, v_1, v_3, ..., v_{m-1}$ and also $w_1$ by 1. The remaining vertices are labelled as in Table 5.1. That $f$ is a group $\{1, -1, i, -i\}$ cordial labeling is evident from Tables...
5.1, 5.2 and 5.3.

Case 2. $m$ is odd.

Assume that $n \leq 2m - 1$. Consider the case when $n \leq 2m - 2$.

Subcase(i). $n \equiv 0 (\text{mod} \ 4)$

Let $f$ be a labeling defined as follows:
Choose $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices among $\{u_1, u_3, ..., u_{m-2}, v_1, v_3, ..., v_{m-2}\}$ and give them label 1. Also give label 1 to $w_1$. Label the remaining vertices arbitrarily so that $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices get label $-1$, $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices get label $i$ and $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices get label $-i$.

Subcase(ii). $n \equiv 1 (\text{mod} \ 4)$

Let $f$ be a labeling defined as follows:
Choose $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices among $\{u_1, u_3, ..., u_{m-2}, v_1, v_3, ..., v_{m-2}\}$ and give them label 1. Label the remaining vertices arbitrarily so that $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $-1$, $\frac{m-1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $i$ and $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $-i$.

Subcase(iii). $n \equiv 2 (\text{mod} \ 4)$

Let $f$ be a labeling defined as follows:
Choose $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices among $\{u_1, u_3, ..., u_{m-2}, v_1, v_3, ..., v_{m-2}\}$ and give them label 1. Label the remaining vertices arbitrarily so that $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $-1$, $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $i$ and $\frac{m-1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $-i$.

Subcase(iv). $n \equiv 3 (\text{mod} \ 4)$

Let $f$ be a labeling defined as follows:
Choose $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices among $\{u_1, u_3, \ldots, u_{m-2}, v_1, v_3, \ldots, v_{m-2}\}$ and give them label 1. Label the remaining vertices arbitrarily so that $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $-1$, $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $i$ and $\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$ vertices get label $-i$.

Suppose $n = 2m - 1$. Choose $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices among $\{u_1, u_3, \ldots, u_{m-2}, v_1, v_3, \ldots, v_{m-2}\}$ and give them label 1. Also give $w_1$, label 1. Label the remaining vertices arbitrarily so that $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices get label $-1$, $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices get label $i$ and $\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$ vertices get label $-i$. Now $m + \left(\frac{n-1}{2}\right)$ edges get label 1 and $m + \left(\frac{n+1}{2}\right)$ edges get label 0. That f is a group $\{1, -1, i, -i\}$ cordial labeling is evident from above argument and also from Tables 5.4 and 5.5.

\[\square\]

<table>
<thead>
<tr>
<th>Nature of n</th>
<th>$v_f(1)$</th>
<th>$v_f(-1)$</th>
<th>$v_f(i)$</th>
<th>$v_f(-i)$</th>
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<tbody>
<tr>
<td>$n \equiv 0\pmod{4}$</td>
<td>$\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$</td>
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<td>$\frac{m+1}{2} + \left\lfloor \frac{n-4}{4} \right\rfloor$</td>
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<tr>
<td>$n \equiv 1\pmod{4}$</td>
<td>$\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$</td>
<td>$\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$</td>
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<tr>
<td>$n \equiv 2\pmod{4}$</td>
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<td>$\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$</td>
</tr>
<tr>
<td>$n \equiv 3\pmod{4}$</td>
<td>$\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$</td>
<td>$\frac{m+1}{2} + \left\lfloor \frac{n}{4} \right\rfloor$</td>
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Table 5.4

<table>
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<tr>
<th>Nature of n</th>
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<tr>
<td>$n \equiv 0\pmod{4}$</td>
<td>$m + \frac{n}{2}$</td>
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<td>$n \equiv 1\pmod{4}$</td>
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<td>$n \equiv 2\pmod{4}$</td>
<td>$m + \frac{n}{2}$</td>
<td>$m + \frac{n}{2}$</td>
</tr>
<tr>
<td>$n \equiv 3\pmod{4}$</td>
<td>$m + \frac{n+1}{2}$</td>
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</table>

Table 5.5
Corollary 5.1.2. The Butterfly graph $BF_{m,n}$ is group $\{1, -1, i, -i\}$ cordial for every $m$ if and only if $n \leq 5$.

Example 5.1.3. A group $\{1, -1, i, -i\}$ cordial labeling of $BF_{5,7}$ is given in Fig 5.1.

Theorem 5.1.4. The Umbrella graph $U_{n,n}$ is group $\{1, -1, i, -i\}$ cordial for all $n$.

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path $P_n$ in the Fan $F_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the path $P_n$ where $v_1$ is identified with the vertex of $K_1$ in Fan $F_n$. Number of vertices in $U_{n,n}$ is $2n$ and number of edges is $3n - 2$.

Case 1. $n$ is even.

Let $n = 2k$, $k \geq 1$, $k \in \mathbb{Z}$.

Define a labeling $f$ as follows:

$f(u_2) = f(u_4) = \ldots = f(u_{2k}) = 1$;
$f(u_1) = f(u_3) = \ldots = f(u_{2k-1}) = -1$;
$f(v_i) = i$ for $1 \leq i \leq k$;
$f(v_i) = -i$ for $k + 1 \leq i \leq 2k$.

Number of edges with label $1 = 3(k - 1) + 2 = 3k - 1$. 

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Case 2. $n$ is odd.

Let $n = 2k + 1$, $k \geq 0, k \in \mathbb{Z}$. If $k = 0$, $U_{n,n}$ is $U_{1,1}$ which is trivially group $\{1,-1,i,-i\}$ cordial. Suppose $k \geq 1$.

Define a labeling $f$ as follows:
\[ f(u_2) = f(u_4) = \ldots = f(u_{2k}) = f(u_{2k+1}) = 1; \]
\[ f(u_1) = f(u_3) = \ldots = f(u_{2k-1}) = -1; \]
For $1 \leq i \leq k + 1$, $f(v_i) = i$;
For $k + 2 \leq i \leq 2k + 1$, $f(v_i) = -i$.

Number of edges with label 1 = $3k + 1$. Table 5.6 shows that $f$ is a group $\{1,-1,i,-i\}$ cordial labeling.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$v_f(1)$</th>
<th>$v_f(-1)$</th>
<th>$v_f(i)$</th>
<th>$v_f(-i)$</th>
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<td>$2k, k \geq 1, k \in \mathbb{Z}$</td>
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<tr>
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<td>$3k$</td>
<td>$3k + 1$</td>
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Table 5.6

Example 5.1.5. A group $\{1,-1,i,-i\}$ cordial labeling of $U_{5,5}$ is given in Fig. 5.2.
5.2 Group \( \{1, -1, i, -i\} \) cordial labeling of Jewel graph, Jelly fish graph and Jahangir graph

**Theorem 5.2.1.** The Jewel graph \( J_n \) is group \( \{1, -1, i, -i\} \) cordial if and only if \( n \leq 7 \).

**Proof.** Let \( V(J_n) = \{u, x, y, u_i(1 \leq i \leq n)\} \) and \( E(J_n) = \{ux, vx, uy, vy, xy, uu_i, vu_i(1 \leq i \leq n)\} \). \( J_n \) has \( n + 4 \) vertices and \( 2n + 5 \) edges. Suppose that \( J_n \) is group \( \{1, -1, i, -i\} \) cordial. Let \( f \) be a group \( \{1, -1, i, -i\} \) cordial labeling of \( J_n \).

**Case 1.** \( n \equiv 0 \pmod{4} \)

Let \( n = 4k \), \( k \geq 1, k \in \mathbb{Z} \). Each vertex label should appear \( k + 1 \) times. One edge label should appear \( 4k + 3 \) times and another should appear \( 4k + 2 \) times.

**Subcase (i).** Label of \( u \) or \( v \) is 1.

Without loss of generality, let label of \( u \) be 1. This induces edge label 1 to \( 4k + 2 \) edges. Hence, in order that the labeling is group \( \{1, -1, i, -i\} \) cordial, we need to have \( k + 1 = 1 \) or \( k + 1 = 2 \). So \( k = 0 \) or 1. Thus \( n = 0 \) or \( n = 4 \).

**Subcase (ii).** Neither \( u \) nor \( v \) has label 1.

Now, the possibilities are \( 2k + 3 = 4k + 2 \), \( 2k + 3 = 4k + 3 \), \( 2k + 2 = 4k + 2 \) and \( 2k + 2 = 4k + 3 \). In all these cases, either \( k = 0 \) or \( k \notin \mathbb{Z} \). Thus \( n = 0 \) or \( n = 4 \).

**Case 2.** \( n \equiv 1 \pmod{4} \)

Let \( n = 4k + 1 \), \( k \geq 0, k \in \mathbb{Z} \). In a group \( \{1, -1, i, -i\} \) cordial labeling, three vertex labels should appear \( k + 1 \) times and one vertex label should appear \( k + 2 \) times. One edge label should appear \( 4k + 3 \)
Table 5.7

<table>
<thead>
<tr>
<th>n</th>
<th>u</th>
<th>x</th>
<th>y</th>
<th>v</th>
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<td>i</td>
<td>-i</td>
<td>-i</td>
<td>-i</td>
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</tbody>
</table>

times and another $4k + 4$ times.

**Subcase(i).** Label of $u$ or $v$ is 1.

Without loss of generality, let label of $u$ be 1. This induces edge label 1 to $4k + 3$ edges. So the possibilities are $k + 1 = 1, k + 1 = 2, k + 2 = 1, k + 2 = 2$. Thus $k = 0$ or $k = 1$. So $n = 1$ or $n = 5$.

**Subcase(ii).** Neither $u$ nor $v$ has label 1.

As in Case 1, the only possible value for $k$ is 0.

**Case 3.** $n \equiv 2 \pmod{4}$

Let $n = 4k + 2, k \geq 0, k \in \mathbb{Z}$. Two vertex labels should appear $k + 1$ times and two other vertex labels should appear $k + 2$ times. One edge label should appear $4k + 4$ times and another should appear $4k + 5$ times.

**Subcase(i).** Label of $u$ or $v$ is 1.

As in previous cases, $k = 0$ or $k = 1$. So $n = 2$ or $n = 6$.

**Subcase(ii).** Neither $u$ nor $v$ has label 1.

We have $k = 0$.
Case 4. \( n \equiv 3 \pmod{4} \)

Let \( n = 4k + 3 \), \( k \geq 0, k \in \mathbb{Z} \). Three vertex labels should appear \( k + 2 \) times and one vertex label should appear \( k + 1 \) times. One edge label should appear \( 4k + 5 \) times and another should appear \( 4k + 6 \) times.

**Subcase (i).** Label of \( u \) or \( v \) is 1.
As in previous cases, \( k = 0 \) or \( k = 1 \). So \( n = 3 \) or \( n = 7 \).

**Subcase (ii).** Neither \( u \) nor \( v \) has label 1.

We have \( k = 0 \). Thus \( 0 \leq n \leq 7 \). For \( 0 \leq n \leq 7 \), a group \( \{1, -1, i, -i\} \) cordial labeling is given in Table 5.7. \( \square \)

**Example 5.2.2.** A group \( \{1, -1, i, -i\} \) cordial labeling of \( J_4 \) is given in Fig. 5.3.

![Fig. 5.3](image)

**Theorem 5.2.3.** Jelly fish graphs \( J(m, n)(m \leq n) \) are group \( \{1, -1, i, -i\} \) cordial if and only if either \( m + n \leq 10 \) or \( 3m - 6 \leq n \leq 3m + 6 \).

**Proof.** Let the \( m \) pendent vertices adjacent to \( u \) be labeled as \( u_1, u_2, \ldots, u_m \) and the \( n \) pendent vertices adjacent to \( v \) be labeled as \( v_1, v_2, \ldots, v_n \).
Number of vertices in $J(m, n)$ is $m + n + 4$ and number of edges is $m + n + 5$. Let $f$ be a group $\{1, -1, i, -i\}$ cordial labeling of $J(m, n)$.

**Case 1.** $m + n \equiv 0 (\text{mod } 4)$.

Let $m + n = 4k, k \geq 1, k \in \mathbb{Z}$. Each vertex label should appear $k + 1$ times. One edge label should appear $2k + 2$ times and another should appear $2k + 3$ times.

**Subcase(i).** $f(u) \neq 1$ and $f(v) \neq 1$.

If $f(x) = 1$ and $f(y) \neq 1$, then every other vertex with label 1 will yield only one edge with label 1. So $k = 2k - 1$ or $k = 2k$ so that $k = 1$ or $k = 0$. If both $f(x) = 1$ and $f(y) = 1$, then $k - 1 = 2k - 3$ or $k - 1 = 2k - 2$ and so $k = 2$ or $k = 1$. If $k = 1, m + n = 4$ and if $k = 2, m + n = 8$. If $k = 1$, label $x$ and $v_1$ with 1 and remaining vertices arbitrarily so that 2 vertices get label $-1$, 2 vertices get label $i$ and 2 vertices get label $-i$. If $k = 2$, label $x, y$ and $v_1$ with 1 and remaining vertices arbitrarily so that each vertex label appears on 3 vertices.

**Subcase(ii).** $f(u) = 1$ and $f(v) = 1$.

This induces label 1 to $m + n + 4$ edges and so this case is impossible.

**Subcase(iii).** $f(u) = 1$ and $f(v) \neq 1$.

If both $f(x) \neq 1$ and $f(y) \neq 1$, then either $k = 2k - m$ or $k = 2k - m + 1$. So either $k = m$ or $k = m - 1$ and so $n = 3m$ or $n = 3m - 4$. In both the cases, label the vertices $v_1, v_2, \ldots, v_k$ with 1 and the remaining vertices arbitrarily so that each vertex label appears on exactly $k + 1$ vertices. Suppose either $f(x) = 1$ or $f(y) = 1$. Without loss of generality, let $f(x) = 1$. Then as above, $k = m + 1$ or $k = m$. In both the cases, label the vertices $v_1, v_2, \ldots, v_{k-1}$ with 1 and
the remaining vertices arbitrarily so that each vertex label appears on exactly \( k + 1 \) vertices. When \( k = m - 1, m \) or \( m + 1 \), we have \( n = 3m - 4, 3m, 3m + 4 \) accordingly.

**Subcase (iv).** \( f(u) \neq 1 \) and \( f(v) = 1 \).

As in Subcase (iii), by symmetry, we have \( k = n, n - 1 \) or \( n + 1 \). But, by assumption \( m \leq n \) and so in this case \( n \leq 2 \).

**Case 2.** \( m + n \equiv 1(\text{mod} \ 4) \).

Let \( m + n = 4k + 1, k \geq 0, k \in \mathbb{Z} \). Three vertex labels should appear \( k + 1 \) times and one vertex label should appear \( k + 2 \) times. Each edge label should appear \( 2k + 3 \) times.

**Subcase (i).** \( f(u) \neq 1 \) and \( f(v) \neq 1 \).

If \( f(x) = 1 \) and \( f(y) \neq 1 \), then either \( k = 2k \) or \( k + 1 = 2k \) so that \( k = 0 \) or \( 1 \). If \( k = 0 \), take \( f(y) = -1, f(u) = f(v) = i \) and \( f(v_1) = -i \).

If both \( f(x) = 1 \) and \( f(y) = 1 \), then either \( k - 1 = 2k - 2 \) or \( k = 2k - 2 \) so that \( k = 1 \) or \( k = 2 \). If \( k = 1, f(x) = 1, f(v_1) = 1 \) and \( f(v_2) = 1 \). Label the remaining vertices arbitrarily so that each vertex label appears on 2 vertices. If \( k = 2 \), let \( f(x) = 1, f(y) = 1, f(v_1) = 1 \) and \( f(v_2) = 1 \). Label the remaining vertices arbitrarily so that each vertex label appears on 3 vertices. As \( k = 0, 1 \) or \( 2 \), we have \( m + n = 1, 5 \) or \( 9 \).

**Subcase (ii).** \( f(u) = 1 \) and \( f(v) = 1 \).

As in Subcase (ii) of Case (1), this is impossible.

**Subcase (iii).** \( f(u) = 1 \) and \( f(v) \neq 1 \).

If both \( f(x) \neq 1 \) and \( f(y) \neq 1 \), then either \( k = 2k - m + 1 \) or \( k + 1 = 2k - m + 1 \) so that \( k = m - 1 \) or \( k = m \). In the former case, label \( v_1, v_2, \ldots, v_k \) with 1 and the remaining vertices arbitrarily so
that \( k+1 \) vertices get label \(-1\), \( k+1 \) vertices get label \( i \) and \( k+2 \) vertices get label \(-i\). In the latter case, label \( v_1, v_2, \ldots, v_{k+1} \) with 1 and the remaining vertices arbitrarily so that each of the vertex labels \(-1, i\) and \(-i\) appear on \( k+1 \) vertices. Suppose \( f(x) = 1 \) and \( f(y) \neq 1 \). Then as above \( k = m \) or \( k = m + 1 \). If \( k = m + 1 \), label the vertices \( v_1, v_2, \ldots, v_k \) with 1 and the remaining vertices arbitrarily so that each of the vertex labels \(-1, i\) and \(-i\) appear on \( k+1 \) vertices. As \( k = m - 1, m, m+1 \), we have \( n = 3m - 3, 3m + 1 \) or \( 3m + 5 \).

**Subcase (iv).** \( f(u) \neq 1 \) and \( f(v) = 1 \).

As in Subcase (3), we get \( k = n - 1, n \) or \( n + 1 \). As \( m \leq n \), in these cases, \( n \leq 2 \).

**Case 3.** \( m + n \equiv 2 \pmod{4} \).

Let \( m + n = 4k + 2, k \geq 0, k \in \mathbb{Z} \). Two vertex labels should appear \( k+1 \) times and two other vertex labels should appear \( k+2 \) times. One edge label should appear \( 2k + 3 \) times and another should appear \( 2k + 4 \) times.

**Subcase (i).** \( f(u) \neq 1 \) and \( f(v) \neq 1 \).

If \( f(x) = 1 \) and \( f(y) \neq 1 \), then there are four possibilities; \( k = 2k, k = 2k + 1, k+1 = 2k \) and \( k + 1 = 2k + 1 \). Hence \( k = 0 \) or 1. If \( k = 0, f(x) = 1, f(y) = -1, f(u) = f(v) = i, f(u_1) = f(v_1) = -i \). If \( k = 1, f(x) = 1, f(v_1) = f(v_2) = 1 \). Label remaining vertices arbitrarily so that 3 vertices get label \(-1\), 2 vertices get label \( i \) and 2 vertices get label \(-i\). If both \( f(x) = 1 \) and \( f(y) = 1 \), then \( k = 0, 1 \) or 2. If \( k = 2, f(x) = f(y) = f(v_1) = f(v_2) = 1 \). Label the remaining vertices arbitrarily so that 4 vertices get label \(-1\), 3 vertices get label \( i \) and 3 vertices get label \(-i\). As \( k = 0, 1, 2, m+n = 2, 6 \) or 10.

**Subcase (ii).** \( f(u) = 1 \) and \( f(v) = 1 \).
As in previous cases, this is not possible.

**Subcase(iii).** \( f(u) = 1 \) and \( f(v) \neq 1 \).

If both \( f(x) \neq 1 \) and \( f(y) \neq 1 \) then \( k = m - 1, m - 2 \) or \( m \). If \( k = m - 1 \) or \( m - 2 \), label \( v_1,v_2,\ldots,v_k \) with 1. Label the remaining vertices arbitrarily so that \( k+1 \) vertices get label \(-1\), \( k+2 \) vertices get label \( i \) and \( k+2 \) vertices get label \(-i\). If \( k = m \), label \( v_1,v_2,\ldots,v_{k+1} \) with 1. Label the remaining vertices arbitrarily so that \( k+2 \) vertices get label \(-1\), \( k+1 \) vertices get label \( i \) and \( k+1 \) vertices get label \(-i\).

Suppose \( f(x) = 1 \) and \( f(y) \neq 1 \). As above, \( k = m - 1, m \) or \( m + 1 \). If \( k = m \), label \( v_1,v_2,\ldots,v_{k-1} \) with 1. Label the remaining vertices arbitrarily so that \( k+1 \) vertices get label \(-1\), \( k+2 \) vertices get label \( i \) and \( k+2 \) vertices get label \(-i\). As \( k = m - 1, m - 2, m, m + 1 \), we have \( n = 3m - 6, 3m - 2, 3m + 2 \) or \( 3m + 6 \).

**Subcase(iv).** \( f(u) \neq 1 \) and \( f(v) = 1 \).

As in Subcase(iii), we get \( k = n - 2, n - 1, n \) or \( n + 1 \). As \( m \leq n \), we have \( n \leq 3 \).

**Case 4.** \( m + n \equiv 3 \) (mod 4)

Let \( m + n = 4k + 3, k \geq 0, k \in \mathbb{Z} \). Three vertex labels should appear \( k+2 \) times and one vertex label should appear \( k+1 \) vertices. Each edge label should appear \( 2k + 4 \) times.

**Subcase(i).** \( f(u) \neq 1 \) and \( f(v) \neq 1 \).

If \( f(x) = 1 \) and \( f(y) \neq 1 \), then either \( k + 1 = 2k + 1 \) or \( k = 2k + 1 \) so that \( k = 0 \) or \(-1\); If \( k = 0 \), \( f(v_1) = 1 \); Label the remaining vertices arbitrarily so that 2 vertices get label \(-1\), 2 vertices get label \( i \) and 1 vertex get label \(-i\). If both \( f(x) = 1 \) and \( f(y) = 1 \), then either \( k = 2k - 1 \) or \( k - 1 = 2k - 1 \) so that \( k = 1 \) or \( k = 0 \). If \( k = 1 \), \( f(v_1) = 1 \). Label the remaining vertices arbitrarily so that 3 vertices
get label $-1, 3$ vertices get label $i$ and $2$ vertices get label $-i$. As $k = 0, 1, m + n = 3$ or $m + n = 7$.

**Subcase (ii).** $f(u) = 1$ and $f(v) = 1$.

As in previous cases, this is impossible.

![Fig. 5.4](image)

**Subcase (iii).** $f(u) = 1$ and $f(v) \neq 1$.

If both $f(x) \neq 1$ and $f(y) \neq 1$, then $k = m - 1$ or $k = m - 2$. If $k = m - 1$, label $v_1, v_2, ..., v_{k+1}$ with $1$ and remaining vertices arbitrarily so that $k + 2$ vertices get label $-i, k + 2$ vertices get label $i$ and $k + 1$ vertices get label $-i$. If $k = m - 2$, label $v_1, v_2, ..., v_k$ with $1$ and remaining vertices arbitrarily so that $k + 2$ vertices get label $-1, k + 2$ vertices get label $i$ and $k + 2$ vertices get label $-i$. Suppose $f(x) = 1$ and $f(y) \neq 1$. Then $k = 2k - m$ or $k - 1 = 2k - m$ so that $k = m$ or $k = m - 1$. If $k = m$, label $v_1, v_2, ..., v_k$ with $1$. Label the remaining vertices arbitrarily so that $k + 2$ vertices get label $-1, k + 2$ vertices get label $i$ and $k + 1$ vertices get label $-i$. As $k = m - 2, m - 1$ or $m$, we have $n = 3m - 5, 3m - 1$ or $3m + 3$.

**Subcase (iv).** $f(u) \neq 1$ and $f(v) = 1$.

As in Subcase (iii), we get $k = n - 2, n - 1$ or $n$. As $m \leq n$, we have $n \leq 2$. □
Example 5.2.4. A group \( \{1, -1, i, -i\} \) cordial labeling of \( J(4, 5) \) is given in Fig. 5.4.

Theorem 5.2.5. The Jahangir graph \( J_{3,n}(n \geq 3) \) is group \( \{1, -1, i, -i\} \) cordial for all \( n \).

Proof. Let the vertices on the cycle be labeled as \( u_1, u_2, ..., u_{3n} \) and let the central vertex be labeled as \( w \). Assume that \( w \) is adjacent to \( u_i(i \equiv 1(mod 3)) \). Number of vertices = \( 3n + 1 \) and number of edges = \( 4n \). Group \( \{1, -1, i, -i\} \) cordial labelings for \( n = 3 \) and \( n = 4 \) are given in Table 5.8. Suppose \( n \geq 5 \). Let \( f : V(J_{3,n}) \to \{1, -1, i, -i\} \) be a function.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
<th>( u_5 )</th>
<th>( u_6 )</th>
<th>( u_7 )</th>
<th>( u_8 )</th>
<th>( u_9 )</th>
<th>( u_{10} )</th>
<th>( u_{11} )</th>
<th>( u_{12} )</th>
<th>( w )</th>
</tr>
</thead>
<tbody>
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<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>( i )</td>
<td>( i )</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
<td>(-i)</td>
</tr>
</tbody>
</table>

Table 5.8

Case 1. \( 3n + 1 \equiv 0(mod 4) \).

Let \( 3n + 1 = 4k(k \in \mathbb{Z}) \). Each vertex label should appear \( k \) times and each edge label should appear \( \frac{8k-2}{3} \) times in a group \( \{1, -1, i, -i\} \) cordial labeling. Note that \( k = 3r + 1(r \in \mathbb{Z}, r \geq 1) \). So the vertices on the Cycle are \( u_i(1 \leq i \leq 12r + 3) \) where \( u_i(i \equiv 1(mod 3), 1 \leq i \leq 12r + 3) \) are of degree 3 and others are of degree 2. Label the vertices \( u_i(1 \leq i \leq 6r - 2), i \equiv 1(mod 3) \) with 1. Also choose \( r + 1 \) vertices among \( u_i(6r \leq i \leq 12r + 3, i \not\equiv 1(mod 3)) \) and give them label 1. Label the remaining vertices arbitrarily so that \( k \) of them get label \(-1, k \) of them get label \( i \) and \( k \) of them get label \(-i\). Number of edges with label \( 1 = 3 \times 2r + (r + 1)2 = \frac{8k-2}{3} \).

Case 2. \( 3n + 1 \equiv 1(mod 4) \).

Let \( 3n + 1 = 4k + 1(k \in \mathbb{Z}) \). Three vertex labels should appear \( k \)
times and one vertex label should appear \(k + 1\) times. Each edge label should appear \(\frac{8k}{3}\) times in a group \(\{1, -1, i, -i\}\) cordial labeling. In this case \(k = 3r (r \in \mathbb{Z}, r \geq 2)\). Now the vertices on the Cycle are \(u_i (1 \leq i \leq 12r)\) where \(u_i (i \equiv 1 (mod 3))\) are of degree 3 and others are of degree 2. Label the vertices \(u_i (1 \leq i \leq 6r - 8)\) with 1. Also choose \(r + 3\) vertices \(u_i (6r - 6 \leq l \leq 12r, i \not\equiv 1 (mod 3))\) and give them label 1. Label the remaining vertices arbitrarily so that \(k\) of them get label \(-1\), \(k\) of them get label \(i\) and \(k\) of them get label \(-i\).

**Case 3.** \(3n + 1 \equiv 2 (mod 4)\)

Let \(3n + 1 = 4k + 2 (k \in \mathbb{Z})\). Two vertex labels should appear \(k\) times and 2 vertex labels should appear \(k + 1\) times. Each edge label should appear \(2n = \frac{8k + 2}{3}\) times. Now \(k = 3r + 2 (r \geq 1, r \in \mathbb{Z})\). The vertices on the Cycle are \(u_i (1 \leq i \leq 12r + 9)\). Label the vertices \(u_i (1 \leq i \leq 6r - 2, i \equiv 1 (mod 3))\) with 1. Also choose \(r + 3\) vertices among \(u_i (6r \leq i \leq 12r + 9, l \not\equiv 1 (mod 3))\) and give them label 1. Label the remaining vertices arbitrarily so that \(k + 1\) vertices get label \(-1\), \(k\) vertices get label \(i\) and \(k\) vertices get label \(-i\). Number of edges with label 1 = \(3 \times (2r - 2) + (r + 3)2 = 8r\).

**Case 4.** \(3n + 1 \equiv 3 (mod 4)\)

Let \(3n + 1 = 4k + 3 (k \in \mathbb{Z})\). Three vertex labels should appear \(k + 1\) times and 1 vertex label should appear \(k\) times. Each edge label should appear \(\frac{8k + 4}{3}\) times. Now \(k = 3r + 1 (r \geq 1, r \in \mathbb{Z})\). The vertices on the Cycle are \(u_i (1 \leq i \leq 12r + 6)\). Label the vertices \(u_i (1 \leq i \leq 6r - 2, l \equiv (mod 3))\) with 1. Also choose \(r + 2\) vertices among \(u_i (6r \leq i \leq 12r + 6, l \not\equiv i (mod 3))\) and give them label 1. Label the remaining vertices arbitrarily so that \(k + 1\) vertices get label \(-1\), \(k + 1\) vertices get label \(i\) and \(k\) vertices get label \(-i\). Number of edges with label 1 = \(2r \times 3 + 2(r + 2) = 8r + 4\). Table 5.9 shows that in all cases, the given labeling is group \(\{1, -1, i, -i\}\) cordial.

\[\square\]
\[ 3n + 1 \quad v_f(1) \quad v_f(-1) \quad v_f(i) \quad v_f(-i) \quad e_f(0) \quad e_f(1) \\
4k \quad k \quad k \quad k \quad \frac{8k-2}{3} \quad \frac{8k-2}{3} \\
4k + 1 \quad k + 1 \quad k \quad k \quad \frac{8k}{3} \quad \frac{8k}{3} \\
4k + 2 \quad k + 1 \quad k + 1 \quad k \quad \frac{8k+2}{3} \quad \frac{8k+2}{3} \\
4k + 3 \quad k + 1 \quad k + 1 \quad k \quad \frac{8k+4}{3} \quad \frac{8k+4}{3} \\

Table 5.9

Example 5.2.6. A group \{1, -1, i, -i\} cordial labeling of \(J_{3,5}\) is given in Fig. 5.5.

![Fig. 5.5](image)

5.3 Group \{1, -1, i, -i\} cordial labeling of Dumbbell graph and Lotus inside a circle

Theorem 5.3.1. The Dumbbell graph \(Db_n\) is group \{1, -1, i, -i\} cordial for all \(n\).

Proof. Number of vertices in \(Db_n\) is \(2n\) and number of edges is \(2n+1\).
Let $f : V(Db_n) \to \{1, -1, i, -i\}$ be a function.

**Case 1.** $n$ is even.

Let $n = 2k, k \geq 2, k \in \mathbb{Z}$.

In a group $\{1, -1, i, -i\}$ cordial labeling, each vertex label should appear $k$ times. One edge label should appear $2k$ times and another $2k + 1$ times. Define a labeling $f$ as follows:

Label $u_1, u_3, u_5, ..., u_{n-1}$ with 1. Label the remaining vertices arbitrarily so that $k$ of them get label $-1$, $k$ of them get label $i$ and $k$ of them get label $-i$. Number of edges with label 1 is $3 + 2(k-1) = 2k + 1$.

**Case 2.** $n$ is odd.

Let $n = 2k + 1, k \geq 1, k \in \mathbb{Z}$.

In a group $\{1, -1, i, -i\}$ cordial labeling, two vertex labels should appear $k$ times and two vertex labels should appear $k + 1$ times. One edge label should appear $2k + 1$ times and another $2k + 2$ times. Define a labeling $f$ as follows:

Label $u_1, u_3, u_5, ..., u_{n-1}$ with 1. Label the remaining vertices arbitrarily so that $k$ of them get label $-1$, $k + 1$ of them get label $i$ and $k + 1$ of them get label $-i$. Number of edges with label 1 is $3 + 2(k-1) = 2k + 1$.

Table 5.10 shows that in all cases, the given labeling is group $\{1, -1, i, -i\}$ cordial.


<table>
<thead>
<tr>
<th>$n$</th>
<th>$f_1$</th>
<th>$f_{-1}$</th>
<th>$f_i$</th>
<th>$f_{-i}$</th>
<th>$f_0$</th>
<th>$f_1$</th>
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</thead>
<tbody>
<tr>
<td>$2k$</td>
<td>$k$</td>
<td>$k$</td>
<td>$k$</td>
<td>$k$</td>
<td>$2k$</td>
<td>$2k + 1$</td>
</tr>
<tr>
<td>$2k + 1$</td>
<td>$k$</td>
<td>$k$</td>
<td>$k + 1$</td>
<td>$k + 1$</td>
<td>$2k + 2$</td>
<td>$2k + 1$</td>
</tr>
</tbody>
</table>

Table 5.10

**Example 5.3.2.** A group $\{1, -1, i, -i\}$ cordial labeling of $Db_6$ is given in Fig. 5.6.
Theorem 5.3.3. The graph Lotus inside a circle $LC_n$ is group $\{1, -1, i, -i\}$ cordial for every $n$.

Proof. Let $u$ be the center of $K_{1,n}$ and $u_1, u_2, ..., u_n$ be the pendent vertices of $K_{1,n}$. Let $w_1, w_2, ..., w_n$ be the vertices of the cycle $C_n$ such that $u_i$ is adjacent with $w_i$ and $w_{(i+1)(mod \ n)}$. The graph $LC_n$ has $2n+1$ vertices and $4n$ edges.

Let $f : V(LC_n) \rightarrow \{1, -1, i, -i\}$ be a function.

Case 1. $n \equiv 0(mod \ 4)$

Let $n = 4k \ , k \geq 1, k \in \mathbb{Z}$. Label the vertices $w_1, w_3, ..., w_{n-1}$ with 1. This induces edge label 1 to $8k$ edges. Label the remaining vertices arbitrarily so that $2k$ vertices get label $-1$, $2k$ vertices get label $i$ and $2k+1$ vertices get label $-i$.

Case 2. $n \equiv 1(mod \ 4)$

Let $n = 4k + 1 \ , k \geq 1, k \in \mathbb{Z}$. Label the vertices $u_1, w_1, w_3, ..., w_{n-2}$ with 1. This induces edge label 1 to $4(\frac{n-1}{2}) + 2 = 8k + 2$ edges. Label the remaining vertices arbitrarily so that $2k+1$ vertices get label $-1$, $2k+1$ vertices get label $i$ and $2k + 1$ vertices get label $-i$.

Case 3. $n \equiv 2(mod \ 4)$

Let $n = 4k + 2 \ , k \geq 1, k \in \mathbb{Z}$. Label the vertices $w_1, w_3, ..., w_{n-1}$ with 1. This induces edge label 1 to $(2k+1)4 = 8k + 4$ edges. Label the remaining vertices arbitrarily so that $2k + 1$ vertices get label $-1$, $2k$ vertices get label $i$, $2k+1$ vertices get label $-i$, and $2k+1$ vertices get label $-i$.
2k + 1 vertices get label i and 2k + 2 vertices get label −i.

<table>
<thead>
<tr>
<th>n</th>
<th>v_f(1)</th>
<th>v_f(-1)</th>
<th>v_f(i)</th>
<th>v_f(-i)</th>
<th>e_f(0)</th>
<th>e_f(1)</th>
</tr>
</thead>
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<td>2k</td>
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<td>2k</td>
<td>2k + 1</td>
<td>8k</td>
<td>8k</td>
</tr>
<tr>
<td>4k + 1, k ≥ 1, k ∈ ℤ</td>
<td>2k + 1</td>
<td>2k + 1</td>
<td>2k + 1</td>
<td>2k</td>
<td>8k + 2</td>
<td>8k + 2</td>
</tr>
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<td>2k + 1</td>
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<td>2k + 2</td>
<td>8k + 4</td>
<td>8k + 4</td>
</tr>
<tr>
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<td>8k + 6</td>
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</tr>
</tbody>
</table>

Table 5.11

Case 4. n ≡ 3(mod 4)

Let n = 4k + 3, k ≥ 0, k ∈ ℤ. Label the vertices u_1, w_1, w_3, ..., w_{n-2} with 1. This induces edge label 1 to (2k+1)4+2 = 8k+6 edges. Label the remaining vertices arbitrarily so that 2k + 2 vertices get label −1, 2k + 2 vertices get label i and 2k + 1 vertices get label −i.

Table 5.11 shows that the above labelings are group \{1, −1, i, −i\} cordial.

Example 5.3.4. A group \{1, −1, i, −i\} cordial labeling of LC_b is given in Fig. 5.7.