Chapter 6

Modeling

6.1 Introduction

In the changing technology environment and complex nature of the network traffic, effective management of traffic flow is necessary and essential for network provider and business environment. Internet service provider must have, on an average guess, of the future traffic so that network resources are managed effectively. It is an urgent need of the Internet service Provider to forecast the traffic flow as given by Frost in his study,(ref.[39]). Therefore it is essential to have input output relationship for the traffic which may be called as Traffic Model. A good traffic model will predict the traffic behavior to a high degree of certainty. There are two approaches of traffic modeling: (ref.[5]).

- Empirical models, obtained by the measurements and analysis of thousands or millions of connections over a day.

- Analytical models,

Further, the traffic modeling can be performed at different levels as given below:

- Packet traffic models.
6.2 Why Models Are Important?

- Useful while designing new systems for communication.
- Useful for performance evaluation of newly developed protocols.
- Useful for corrective measures in the present systems.
- Useful in simplification of complex problems and scientific understanding of performance and fundamental limit.
- For simulations of systems.
- Useful for comparison and calibration of algorithm.

Thus TRAFFIC MODELING IS THE KEY STEP for the development of communication networks.

6.3 Telephone Traffic Models

Telephone traffic models as given in [19] have been into existence since long back which does model the traditional telephone traffic efficiently. Following characteristics of telephone traffic are important when we attempt to model it:

- Arrival of calls.
- Duration of calls.
6.3.1 Model for Call Arrival Process

The studies like [40], [41], [42] have shown that call arrival at a switch is analogous to customer arrival process at a counter. It is shown in [43], [44] that the arrival of calls obeys the Poission Process. Thus it gives the inter-arrival time which is drawn from the exponential distribution as given by equation 6.1:

\[ P(X > x) = e^{-x/\lambda} \]  

(6.1)

Where, \( X \) is the inter-arrival time and \( \lambda \) is the arrival rate of the calls. In equation 6.1 we observe that the probability of next call arrival is longer than \( x \) which is given by \( e^{-x/\lambda} \). It is interesting to note that the time elapsed from the last arrival does not give any information about when the next call will arrive. Thus the Poission process exhibits memoryless property. A detailed description of Poisson distribution is given in appendix C.

6.3.2 Model for Call Duration/Holding Time

Traditionally it is shown that Call holding times in telephone networks are modeled as drawn from exponential distribution. Thus the probability that a call last longer than \( x \) decreases exponentially with \( x \). However the recent studies like [45] shows that the call duration times are heavy tailed. This indicates that many calls last for longer times. For detailed understanding of heavy tailed distribution we have included Figure 6.1 from [5].

6.4 Overview of Internet Traffic Modeling

We have seen previously that a good traffic model is required to simulate the network behavior and anticipate its performance. When it comes to network traffic, it can be considered as flow of discrete entities e.g. packets, cells etc.(ref.[39]). From mathematical point of view, packet flow can be considered as a point process as explained in [46]. This
point process can be described by the arrivals of packets at time instants \( T_1, T_2, \ldots, T_n \) with origin as reference point, by convention. Further the point processes have following variations:

- **Counting Process** \( \{N(t)\}_{t=0}^{\infty} \): It is a continuous time, non negative integer valued stochastic process. Where \( N(t) = \max\{n : T_n \leq t\} \) is the number of arrivals in the interval \( (0, t] \).

- **Inter-arrival Time Process** \( \{A_n\}_{n=0}^{\infty} \): It is a non negative random sequence where \( \{A_n\} = T_n - T_{n-1} \) is the length of the time interval between \( n \) and \( n - 1^{th} \) arrivals.

Inherently, computer network traffic is a discrete time traffic process. This means the random variable \( \{A_n\} \) can have only integer values and the random variables \( \{N(t)\} \) are allowed to increase at integer valued time instants \( T_n \).

Importantly, aggregate traffic may consists of more than one arrivals at a given time instant \( T_n \). Such traffic can be considered as a compound traffic. To characterise such a traffic one needs to define a non negative random sequence as follows:
\( \{B_n\}_{n=0}^{\infty} \), where \( B_n \) is the random number of entities in a batch.

Such compound processes are called as marked point processes [47], as depicted in Figure 6.2. Thus there may be more than one arrivals at an arrival instant \( T_n \). To characterise such a process we need to specify a non negative random sequence \( \{B_n\}_{n=0}^{\infty} \).

![Marked point process from [3]](image)

Other than the above stochastic process approach, *queuing models and models of networks of queues* have been successfully used to predict the behavior of packet switching networks which are explained in studies like [48],[49],[50],[51]. This theory is mainly employed to estimate transit delay, packet loss probability, line and buffer utilisations, network utilisations etc.

Also the Time series modeling approach has been discussed in chapter 7.

### 6.4.1 Burstiness In Network Traffic

Studies like [42],[2] have shown that the packet arrival process is highly variable further the researchers have described the computer traffic as *Bursty*. As described by Frost *et al* in [39] two major causes of burstiness are:

- Shapes of marginal distribution
6.4.2 Measures of Burstiness

The easiest measure of burstiness is the ratio of peak rate to mean rate. The drawback of this method is its dependence on the interval length utilised for rate measurement. A little improved measure of the burstiness is the coefficient of variation, which is defined as the ratio of standard deviation to mean as given by equation 6.2:

\[ c_A = \frac{\sigma[A_n]}{E[A_n]} \] (6.2)

Other few studies like [52] and [53], [54] have defined the peakedness measure and Index of Dispersion measure for Counts (IDC) which take into account the temporal dependence in the network traffic. For a given time interval of length \( \tau \) the index of dispersion for counts is defined as the ratio of variance to mean, number of arrivals in the interval \([0, \tau]\), as given by equation 6.3:

\[ I_\tau = \frac{Var[N(\tau)]}{E[N(\tau)]]} \] (6.3)

Recent studies like [26] have used the Hurst Parameter as a measure of Burstiness via the concept of self-similarity.

6.5 Internet Traffic Models

6.5.1 Renewal Traffic Models

Important characteristics of renewal traffic process is that the \( A_n \) (Inter-arrival) times are Independent and Identically Distributed (IID), while their distribution is allowed to be general. Also the superposition of the independent renewal processes does not result into another renewal process. Even though it is analytically very simple, it suffers a modeling drawback in which the autocorrelation function of inter-arrival times is zero for all nonzero
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lags. Modern studies shows that it is important to capture the autocorrelations since the autocorrelations reveal the phenomenon like burstiness. Thus the models which capture the correlation structure of the traffic would be important to model the future traffic.

6.5.2 Poission Processes

In the telecommunication literature we find that Poissions traffic models are the simplest and oldest ones. Poission process is characterised as a renewal process with rate parameter \( \lambda \) and inter-arrival times \( \{A_n\} \), which comes from the exponential distribution as given in equation 6.1. Thus it is a counting process satisfying equation 6.4:

\[
p\{N(t) = n\} = \frac{\exp(-\lambda t) + (\lambda t)^n}{n!}
\]

(6.4)

Important properties of the Poission process are as follows:

- It is a memoryless process.

- Superposition of independent Poission processes results into another Poission process whose rate is the sum of the rates of individual processes.

6.5.3 Failure of Poission Models

As it is well known that most of the conventional work in queuing theory and communication network design is based on the assumption that the packet arrival process is Poisson. In simple terms, the Poisson arrival process means that events (e.g., traffic accidents, telephone calls, customer arrivals, packet arrivals) occur independently at random times, with a well defined average rate. More specifically, the inter-arrival times between events are exponentially distributed and independent, and no two events happen at exactly the same time.

Poisson models are attractive mathematically because of the memoryless property of the exponential distribution: even knowing the time that has elapsed since the last event
 provides no hint as to when the next event will occur. These types of models are elegant to mathematical analysis, leading to closed form expressions for the mean waiting time, variance in queuing network models.

Detailed studies of Internet network traffic show that the packet arrival process is not Poisson. That is to say, the inter-arrival times between packets are not exponentially distributed, nor are they independent. Rather, the packet arrival process is bursty: packets arrive in batches that make the traffic far more bursty than predicted by a Poisson process. As a result, the queuing behavior can be much more variable than predicted by a Poisson model.

This non-Poisson structure is due in part to the protocols used for data transmission. This observation creates doubt on the value of simple, Poisson network models used in network performance studies.

### 6.5.4 Bernoulli Processes

This is a slotted time equivalent of the Poission process. In this, if the probability of arrivals in a given slot is \( p \) then for the \( k^{th} \) slot the number of arrivals are Binomial defined by equation 6.5

\[
p \{ N_k = n \} = \binom{k}{n} p^n (1 - p)^{(k-n)}
\]

where \( n = 1, 2, \ldots, k \)

### 6.5.5 Markov Models

A Markov process states that a set of random variables forms a Markov chain if the probability that the value of the next possible random variable depends entirely upon the current random variable and not upon any other previous value for the random variable which preceded this most previous one. Thus, a Markov process forms a chain which effectively links
the value of the current random variable to the value of the most recent random variable. In other words, given the value of the most recent random variable, we may chain the entire random sequence merely by working backward one value at a time.

A random sequence of trials \((X_1, X_2, \cdots)\) must satisfy the following properties in order to be considered a stochastic process which forms a Markov chain:

1. Each outcome must be of finite set of outcomes \((a_1, a_2, \cdots, a_n)\) where this set is called the state space of the system. The state space is defined as the set of possible values or states that a random variable \(X\) may assume.

2. The outcome of any trial depends only upon the outcome of the most recent outcome i.e. the outcome preceding this most current one.

The transition probabilities \((p_{ij})\) of a finite Markov chain may be arranged in an \(m\) by \(n\) matrix called the transition matrix, that is,

\[
P = \begin{pmatrix}
p_{11} & p_{12} & \cdots & p_{1n} \\
p_{21} & p_{22} & \cdots & p_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
p_{m1} & p_{m2} & \cdots & p_{mn}
\end{pmatrix}
\]

If we now take the outcome \(a_i\), there will be a corresponding row \((p_{i1}, p_{i2}, \cdots, p_{im})\) of the appropriate transition matrix \(P\) which gives a row vector representing the probabilities of all possible values of the next possible outcome. This row vector is actually a probability vector if the components of the vector are non-negative and their sum is equal to one. Since the components of the vector \((p_{i1}, p_{i2}, \cdots, p_{im})\) are probabilities, their respective values must be, by definition, non-negative and their sum, again by definition, must be equal to one. A random variable \(X\) relies upon not only its state space and the dependency among its possible values but also another parameter which involves the allowable times at which changes or alterations in the state of the variable may occur. A discrete time Markov
process allows state changes to occur at some finite intervals. The analog of the discrete time Markov chain is the continuous time Markov process which allows state changes to occur anywhere within a finite set of intervals.

The Discrete time Markov chain is a process, \(X(t)\), where possible state changes occur only at the instants of time which are the set of positive integers \((0, 1, \ldots)\); for example a state changes may occur only if \(t = 0\) or \(t = 6\) and not if \(t = 9.1\) or \(t = 4.7\). Once a discrete time Markov process is in a given state, it may remain in that state for a period of time which must be geometrically distributed. The continuous time Markov process on the other hand must be in a given state for a period of time which is exponentially distributed.

Thus in limited sense we say that Markov process can capture traffic burstiness, because of nonzero autocorrelations in inter-arrival times.

**Markov Modulated Poisson Processes (MMPP)**

We encounter many point processes whose arrival rates vary in random fashion over the measuring time scale. Markov Modulated Poisson Process has been in use to model such processes because of its ability to model the random arrival rate point processes and also to capture the correlations between the inter-arrival times. Thus in \(i^{th}\) state of \(M\) arrivals, the arrival rate is \(\lambda_i\), which changes as transition into other state occurs. Alternately, arrivals obey Poisson process, however the Poisson parameter changes in accordance with the state of associated Markov chain. Thus MMPP is a doubly stochastic process which is considered in studies like [55],[56], [57],[58], [59],[60]. Markov Modulated Poisson Process may be discrete or continuous in nature.
As observed in [61], the models employed for simulations are not confirmed by empirical studies. Thus simulation results diverge from the reality in the Internet. This is because it is difficult to model entire Internet with accurate and simple, Global Internet Model. Therefore the models should be specific to the research questions being investigated. Also for understanding the dynamics of the behavior, model should facilitate the parameter setting.

In this study we have used time series approach to model given traffic traces. We have found that AR2 (Auto-regressive) and MA2 (Moving-average) models does perform reasonably well. Detailed time series analysis and modeling procedure is explained in the next chapter.