CHAPTER –I

INTRODUCTION

The Chapter-I is Introductory one, in which some of basic definitions and fundamental formulae of Riemannian spaces and complex spaces. (i.e Hermitian, Kaehlerian ,Tachibana spaces ,Kaehlerian and Sasakian manifold ) have been studied, which are necessary prerequisites for the following chapters.
CHAPTER – II

SOME PROPERTIES IN KAEBLERIAN , TACHIBANA AND EINSTEIN KAEBLERIAN SPACE

In the present Chapter , we have discussed the recurrence, bi-recurrence, symmetric and bi-symmetric properties of Kaehlerian , Tachibana and Einsteian Kaehlerian spaces , and several theorems have been investigated on it , also we have obtain following :

THEOREM (2.1) : If a Kaehlerian space satisfies any two of the following properties
(i) the space is Kaehlerian Ricci-recurrent ,
(ii) the space is Kaehlerian projective –recurrent ,
(iii) the space is Tachibana H-concircular recurrent , it must also satisfy the third .

THEOREM (2.2) : If a Kaehlerian space satisfies any two of the following properties
(i) the space is Kaehlerian Ricci-recurrent ,
(ii) the space is Kaehlerian space with recurrent Weyl-conformal curvature tensor ,
(iii) the space is Weyl-concircular recurrent , it must also satisfy the third .

THEOREM (3.1) : The necessary and sufficient condition for a K-C space to be Kaehlerian bi-recurrent is that
\[
(L_{ik,ab} - \lambda_{ab} L_{ik} ) \delta_{j}^{h} - (L_{jk,ab} - \lambda_{ab} L_{jk} ) \delta_{i}^{h} + (L_{j,ab} - \lambda_{ab} L_{j} ) g_{ik} - (L_{i,ab} - \lambda_{ab} L_{i} ) g_{jk} = 0
\]

THEOREM (3.2) : The necessary and sufficient condition for a K-K space to be Kaehlerian bi-recurrent is that
\[
(L_{ij,ab} - \lambda_{ab} L_{ij} ) g_{hk} + (L_{hi,ab} - \lambda_{ab} L_{hi} ) g_{jk} + (L_{hk,ab} - \lambda_{ab} L_{hk} ) g_{ij} + (L_{jk,ab} - \lambda_{ab} L_{jk} ) g_{hi} = 0
\]

THEOREM (3.3) : If a Keahler space satisfies any two of the following properties
(i) the space is Keahlerian Ricci bi-recurrent ,
(ii) the space is Keahler space with bi-recurrent Bochner curvature tensor ,
(iii) the space is Tachibana conharmomic bi-recurrent , it must also satisfy the third .
THEOREM (3.4) : If a Kaehler space satisfies any two of the following properties:
(i) the space is Kaehlerian Ricci bi-recurrent ,
(ii) the space is Kaehlerian projective bi-recurrent ,
(iii) the space is Tachibana Conharmonic* bi-recurrent, it must satisfy the third .

THEOREM (3.5) : Every Kaehlerian bi-recurrent space is Tachibana Conharmonic* bi-recurrent space .

THEOREM(3.6) : A necessary and sufficient condition for a Tachibana Conharmonic* bi-recurrent space to be Kaehlerian bi-recurrent is that the space be Kaehlerian Ricci bi-recurrent.

THEOREM (4.1) : If a Kaehler space satisfies any two of the following properties:
(i) the space is Kaehlerian Ricci -symmetric ,
(ii) the space is projective symmetric ,
(iii) the space is Tachibana H-concircular symmetric , it must also satisfy the third .

THEOREM (4.2) : If a Kaehler space satisfies any two of the following properties:
(i) the space is Kaehlerian Ricci symmetric ,
(ii) the space is Kaehlerian space with symmetric Weyl-conformal curvature tensor ,
(iii) the space is Weyl-concircular symmetric, it must also satisfy the third .

THEOREM (5.1) : A necessary and sufficient condition for a K-C*space to be Kaehlerian bi-symmetric is that

\[ (L_{ik,ab} \delta^h_j - L_{jk,ab} \delta^h_i ) + ( L^h_{j,ab} g_{ik} - L^h_{i,ab} g_{jk} ) = 0 \]

THEOREM(5.2) : A necessary and sufficient condition for a K-K* space to be Kaehlerian bi-symmetric is that

\[ (L_{ij,ab} g_{hk} + L_{hi,ab} g_{jk} + L_{hk,ab} g_{ij} + L_{jk,ab} g_{hi} ) = 0 \]

THEOREM (5.3) : If a Kaehler space satisfies any two of the following properties:
(i) the space is Kaehlerian Ricci bi-symmetric ,
(ii) the space is a Kaehlerian space with bi-symmetric parallel (or vanishing ) Bochner curvature tensor ,
(iii) the space is Tachibana Conharmonic*bi- symmetric , it must also satisfy the third.

**THEOREM (5.4)**: A necessary and sufficient condition for a Tachibana Conharmonic*bi-symmetric space to be Kaehlerian bi-symmetric in the sense of cartan is that the scalar curvature be constant.

**THEOREM (5.5)**: Every Kaehlerian bi-symmetric space is a Tachibana Conharmonic* bi-symmetric space.

**THEOREM (5.6)**: A necessary and sufficient condition for a Tachibana Conharmonic* bi-symmetric space to be Kaehlerian Ricci bi-symmetric is that

\[
R_{ijk,ab}^h + \lambda_{ab} \left[ T_{ijk}^h - R_{ijk}^h - \frac{1}{n-2} \left( g_{ik} R_j^h - g_{jk} R_i^h \right) \right] = 0 .
\]

**THEOREM (6.1)**: A necessary and sufficient condition for an E-K* space to be E-K* space with bi-symmetric parallel Bochner curvature is that the space be Kaehlerian bi-symmetric.

**THEOREM (6.2)**: A necessary and sufficient condition for an E-K* space to be an E-K* conharmonic bi-symmetric is that the space be Kaehlerian bi-symmetric.

**THEOREM (6.3)**: For every Kaehlerian bi-symmetric space , which is an E-K* space with bi-symmetric parallel Bochner curvature , the relation

\[
U_{ijk}^h = R_{ijk}^h + \frac{R}{n(n+2)} \left( g_{ik} \delta_j^h - g_{jk} \delta_i^h + F_{ik} F_j^h - F_{jk} F_i^h + 2 F_{ij} F_k^h \right),
\]

is satisfied.

**THEOREM (6.4)**: For every Kaehlerian bi-symmetric space, which is an E-K* space conharmonic bi-symmetric space ,the relation

\[
E_{ijk}^h = R_{ijk}^h + \frac{2R}{n(n+4)} \left( g_{ik} \delta_j^h - g_{jk} \delta_i^h + F_{ik} F_j^h - F_{jk} F_i^h + 2 F_{ij} F_k^h \right)
\]

is satisfied.
**THEOREM (6.5)**: A necessary and sufficient condition for an E-K* space to be E-K* conharmonic bi-symmetric is that the space be an E-K* space with bi-symmetric parallel Bochner curvature.

**THEOREM (6.6)**: For every E-K* conharmonic bi-symmetric space, which is an E-K* space with bi-symmetric parallel Bochner curvature tensor, the relation

\[
E_{ijk}^h - U_{ijk}^h - \frac{R}{(n+2)(n+4)} \left( g_{ik} \delta_j^h - g_{jk} \delta_i^h + F_{ik} F_j^h - F_{jk} F_i^h + 2F_{ij} F_k^h \right) = 0
\]

is satisfied.

**THEOREM (6.7)**: A flat Kaehler space is Kaehlerian bi-symmetric as well as an E-K* space with bi-symmetric parallel Bochner curvature.

**THEOREM (6.8)**: A flat Kaehlerian space is Kaehlerian bi-symmetric as well as an E-K* conharmonic bi-symmetric.
CHAPTER - III

DECOMPOSITION OF CURVATURE TENSOR FIELD IN A KAEHLERIAN RECURRENT SPACE

In the present Chapter, the decomposition of the recurrent curvature tensor field in four different ways have been considered and several theorems have been established also we have obtained the following:

**THEOREM (2.1):** Under the decomposition (2.1) the Bianchi identities for $R_{ijk}^h$ take the form

\[ p_i X_{j,k} + p_j X_{k,i} + p_k X_{i,j} = 0 \]

and

\[ \lambda_a X_{j,k} + \lambda_j X_{k,i} + \lambda_k X_{i,j} = 0 \]

**THEOREM (2.2):** Under the decomposition of (2.1), the tensor fields $R_{ijk}^h$, $R_{ij}$ and $X_j$, $Y_k$ satisfies the relations

\[ \lambda_a R_{ijk}^h = \lambda_i R_{jk}^h - \lambda_j R_{ik}^h = p_i X_{j,k} \]

**THEOREM (2.3):** Under the decomposition (2.1), the quantities $\lambda_a$ and $V_j^h$ behave like the recurrent vector and tensor field. The recurrent form of these quantities are given by

\[ \lambda_{a,m} = \mu_m \lambda_a \]
\[ V_{j,m}^h = V_m V_j^h \]

**THEOREM (2.4):** Under the decomposition (2.1), the vector fields $p_i$, $X_j$, $Y_k$ behave like recurrent vectors and their recurrent forms are given respectively by

\[ p_{i,m} = (\mu_m + V_m) p_i \]

and

\[ (\lambda_m - V_m) X_j Y_k = X_{j,m} Y_k + X_j Y_{k,m} \]

**THEOREM (2.5):** Under the decomposition (2.1), the curvature tensor and holomorphically projective curvature tensor are equal if
\[ X_j Y_j \left\{ \left( p_i \delta^h_j - p_j \delta^h_i \right) + p_a \left( F'^a_i F'^h_j - F'^h_i F'^a_j \right) \right\} + 2 p_a X_j Y_j F'^a_i F'^h_k = 0 \]

THEOREM (2.6) : Under the decomposition (2.1), the scalar curvature \( R \), satisfying the relation

\[ \lambda \, R = g^h p_i X_i Y_j \]

THEOREM (3.1) : Under the decomposition (3.1) the Bianchi identities for \( R^h_{ijk} \) take the form

\[ X_i Y_{j,k} + X_j Y_{k,i} + X_k Y_{i,j} = 0 \]

and

\[ \lambda_i Y_{j,k} + \lambda_j Y_{k,i} + \lambda_k Y_{i,j} = 0 \]

THEOREM (3.2) : Under the decomposition of (3.1), the tensor fields \( R^h_{ijk} \), \( R_{ij} \) and \( Y_{j,k} \) satisfies the relation

\[ \lambda_i R^h_{ijk} = \lambda_j R_{ijk} - \lambda_k R_{ij} = X_i Y_{j,k} \]

THEOREM (3.3) : Under the decomposition (3.1), the quantities \( \lambda^a \) and \( P^b \) behave like the recurrent vectors. The recurrent form of these quantities are given by

\[ \lambda_{m,a} = \mu_m \lambda^a \]
\[ P_{m,b} = -\mu_m P^b \]

THEOREM (3.4) : Under the decomposition (3.1), the curvature tensor and holomorphically projective curvature tensor are equal if

\[ Y_{i,m} \left\{ \left( X_j \delta^h_j - X_j \delta^h_i \right) + X_j \left( F'^h_i F'^i_j - F'^i_i F'^h_j \right) \right\} + 2 X_i Y_{j,m} F'^h_i F'^i_k = 0 \]

THEOREM (3.5) : Under the decomposition (3.1), the vector \( X_j \) and the tensor \( Y_{j,k} \) satisfy the relation

\[ X_{j,k} \left( \lambda_m + \mu_m \right) = X_j Y_{j,km} + Y_{j,k} X_{i,m} \]

THEOREM (3.6) : Under the decomposition (3.1), the scalar curvature \( R \), satisfy the relation
\[ \lambda_u R = g^{ij} X_{j,k} \]

**THEOREM (4.1):** Under the decomposition (4.1) the Bianchi identities for \( R_{ijk}^h \) takes the forms

\[ X_i \phi_k + X_j \phi_i + X_k \phi_j = 0 \]

and

\[ \lambda_i \phi_k + \lambda_j \phi_i + \lambda_k \phi_j = 0 \]

**THEOREM (4.2):** Under the decomposition of (2.1), the tensor field \( R_{ij}^h \), \( R_j \) and \( \phi_{jk} \) satisfies the relations

\[ \lambda_u R_{ijk} = \lambda R_{jk} - \lambda_j R_{ik} = X_j \phi_{jk} \]

**THEOREM (4.3):** Under the decomposition (4.1), the identities \( \lambda_a \) and \( V^h \) behave like the recurrent vectors. The recurrent form of these quantities are given by

\[ \lambda_{a,m} = \mu_m \lambda_a \]

\[ \lambda^h_m = -\mu_a V^h \]

**THEOREM (4.4):** Under the decomposition (4.1), the vector \( X_j \) and the tensor \( \phi_{jk} \) satisfy the equation

\[ X_j \phi_{jk} (\lambda_m + \mu_m) = X_j \phi_{jk,m} + X_{j,m} \phi_{jk} \]

Where \( \phi_{jk,m} \) is the covariant differentiation with respect to \( x^m \) of \( \phi_{jk} \)

**THEOREM (4.5):** Under the decomposition (4.1), the curvature tensor and holomorphically projective curvature tensor are equal if

\[ \phi_{im} \{ (X_j \delta_k^h - X_j \delta_k^h) + X_j (F^h_j F^i_j - F^h_j F^i_j) \} + 2 X_j \phi_{jm} F^h_j F^i_j = 0 \]

**THEOREM (5.1):** Under the decomposition (5.1) the Bianchi identities for \( R_{ijk}^h \) takes the forms

\[ Y_i \phi_j \psi_k + Y_j \phi_i \psi_i + Y_k \phi_i \psi_j = 0 \]

and
\[(5.4) \quad \lambda_\alpha \phi_j \psi_k + \lambda_j \phi_k \psi_\alpha + \lambda_k \phi_\alpha \psi_j = 0\]

**THEOREM (5.2):** Under the decomposition of (5.1), the tensor fields $R^h_{\alpha j k}$, $R^h_{\alpha j k}$ and vector $\phi_j$, $\psi_k$ satisfies the relations

\[(5.8) \quad \lambda_\alpha R^h_{\alpha j k} = \lambda_j R^h_{j k \alpha} - \lambda_k R^h_{j k \alpha} = Y_j \phi \psi_k\]

**THEOREM (5.3):** Under the decomposition (5.1), the quantities $\lambda_\alpha$ and $X^h$ behave like the recurrent vectors. The recurrent form of these quantities are given by

\[(5.11) \quad \lambda_{\alpha, m} = \mu_m \lambda_\alpha\]
\[(5.12) \quad X^h_{\alpha, m} = -\mu_m X^h\]

**THEOREM (5.4):** Under the decomposition (5.1), the curvature tensor and holomorphically projective curvature tensor are equal if

\[(5.19) \quad \phi \psi_m \{ (Y_j \delta^h - Y_j \delta^h) + Y_j \left( F^h_i F^h_j - F^h_j F^h_i \right) \} + 2Y_j \phi \psi_j F^h_j F^h_i = 0\]

**THEOREM (5.5):** Under the decomposition (5.1), the vector $Y_j$, $\phi_j$ and $\psi_k$ satisfy the relation

\[(5.26) \quad Y_j \phi \psi_k (\lambda_\alpha + \mu_m) = Y_j \phi \psi_{k, \alpha} + Y_{j, \alpha} \phi \psi_k + Y_j \phi_{j, \alpha} \psi_k\]

**THEOREM (5.6):** Under the decomposition (5.1), the scalar curvature $R$, satisfy the relation

\[(5.27) \quad \lambda R = g^m \phi \psi_k\]
CHAPTER – IV

SUB-SPACE OF A SUB-SPACE OF A TACHIBANA SPACE AND UNION CURVES IN AN ALMOST TACHIBANA HYPER SURFACE

In the present Chapter, we have studied the properties of subspace of subspaces of a Tachibana space, Union curves, Union curvature in an almost Tachibana Hyper surface and several theorems have been established.

In this Chapter, we have obtained the following:

THEOREM (3.1) : The mean curvature vector field of $T_n^c$ in $T_i^c$ is the sum of the mean curvature vector of $T_n^c$ in $T_m^c$ and the relative mean curvature vector of $T_n^c$ w.r.t. $T_m^c$ and $T_i^c$.

THEOREM (4.1) : Suppose that $\lambda^n$ is a unit vector field of $T_i^c$ defined along $T_n^c$ and normal to $T_i^c$ is umbilical with mean curvature vector $\eta$ w.r.to $\lambda^n$. If the mean curvature vector $\Gamma$ of $T_n^c$ in $T_i^c$ satisfies $\Gamma^2 \leq \eta^2$ then $T_n^c$ is minimal in $T_m^c$ and is minimal in $T_i^c$ iff $\eta = 0$.

THEOREM (4.2) : Suppose $T_m^c$ is totally umbilical in $T_i^c$, then the mean curvature vector $R^c(\lambda_m, T_i^c)$ of $T_m^c$ in $T_i^c$ coincides with the relative mean curvature vector of $T_n^c$ w.r.to $T_m^c$ and $T_i^c$.

THEOREM (7.1) : The necessary and sufficient condition that the geodesic curvature of a curve be equal to the union curvature relative to the congruence $\lambda$ is that either

(a) the curve is asymptotic or

(b) the congruence $\lambda$ is normal to the hypersurface or

(c) the component of $\lambda$ tangential to the hypersurface is tangential to the curve.

THEOREM (7.2) : The necessary and sufficient condition that the union curve relative to $\lambda$ be a geodesic of the hypersurface is that either

(a) it is an asymptotic line or

(b) the congruence $\lambda$ is normal to the hypersurface or

(c) the components of the vector $\lambda$ tangential to the hypersurface is tangential to the curve.
**THEOREM (7.3) :** The necessary and sufficient condition that the union curvature (or a curve of $T_n^a$) relative to $\lambda$ be expressed in the form

\[(7.6) \quad T_a = T_s - \frac{T_n}{C} \left( g_{a\bar{\beta}} t^a t^{\bar{\beta}} \right)^{\frac{1}{2}} \sin \phi \]

are that

\[(7.7a) \quad t^a = \nu \frac{du^a}{ds} + wp^a, \]

\[(7.7b) \quad \bar{t}^{\bar{a}} = \nu \frac{du^{\bar{a}}}{ds} + w p^{\bar{a}}. \]

Where $\nu$ and $w$ are real parameters.

**THEOREM (8.1) :** The geodesic curvature of the union curve relative to the congruence $\lambda$ can be expressed in the form

\[ T_s = \frac{\Omega_{a\bar{\beta}}}{C} \frac{du^a}{ds} \frac{du^{\bar{\beta}}}{ds} \left( g_{a\bar{\beta}} t^a t^{\bar{\beta}} \right)^{\frac{1}{2}} \sin \phi \]
CHAPTER – V

CURVATURE COLLINEATIONS AND PROJECTIVE MOTION IN TACHIBANA SPACES

In the present Chapter, we have studied curvature collineations in an almost Tachibana recurrent space and the necessary and sufficient condition for curvature collineations in such a space has been investigated, further, we have studied projective motion in an almost Tachibana space with recurrent curvature tensor.

In this Chapter, we have obtained the following:

**THEOREM (3.1):** A necessary and sufficient condition for an $AT^c_\mu\nu$ space to admit a CC is that there exists a transformation of the form (2.1) such that the vector $v$ satisfies.

\[(3.6) \quad (h_{lm,j} + h_{mj,i} - h_{ij,m})_j - (h_{lm,j} + h_{mj,h} - h_{ij,m})_j = 0\]

where $h_{ij} = v_{i,j} + v_{j,i}$

\[(3.7) \quad (v_{i,mj} + v_{m,ji} - v_{i,im})_j - (v_{i,mj} + h_{m,jh} - h_{ij,m})_j = 0\]

\[(3.8) \quad LR_{jh} = 0\]

**THEOREM (3.2):** An $AT^c_\mu\nu$ space every CC is an RC in (3.6), if we interchanging the indices j and m, add the resulting equation (3.6) we get

**THEOREM (3.3):** A necessary condition for a transformation of the form (2.1) to define a CC is that

\[(3.9) \quad h_{jm,ih} - h_{jm,hi} = 0\]

it is of interest to note that (3.9) could also be obtained by starting with

\[(3.10) \quad g_{ia}R_{jkm}^a + g_{ia}R_{skm} = 0\]

**THEOREM (4.1):** In an $AT^c_\mu\nu$ space every M is a CC similarly, from the condition (2.3) of an AC, it follows that we may state.
THEOREM (4.2): In an $AT^c_n$ - space every AC is a CC.

Also, it follows immediately from the definition of HM that from (2.11) satisfies (2.4) and hence as a consequence of theorem (4.2), we state

THEOREM (4.3): In an $AT^c_n$ - space, every HM is a CC.

From Yano [5] (p.167), it is known that if a transformation is both a conf. M and PC, then it is an HM. Hence we have the following as a consequence of the theorem (4.3).

THEOREM (4.4): In an $AT^c_n$ - space, if a transformation is both a conf. M and a PC, then it is a CC.

THEOREM (4.5): The necessary and sufficient condition for a PC to be a CC is that

\[(4.3) \quad \phi_{ijh} = 0\]

where $\phi_{ijh} = (n+1)^{-1} v_{ijh}$,
i.e a PC must be an SPC.

THEOREM (4.6): The necessary and sufficient condition for a conf. C to be a CC is that

\[(4.7) \quad \sigma_{ijh} = 0\]

where $\sigma_{ijh} = n^{-1} v_{ijh}$,
i.e the conf. C must be a S and C.

THEOREM (4.7): Every S conf. M is a S conf. C.

THEOREM (6.1): If an $AT^c_n$ space $n \geq 3$ admits an infinitesimal projective motion, the motion should be of the form

$$x^h = x^h + v^h (x) \, dt, \quad L \{ L^h \} = A^h F_j + A^h_j F_i, \quad F_j = \frac{1}{(n-2)} L \lambda_j.$$

THEOREM (6.2): If $L \lambda_i$ denotes a parallel vector, then an $AT^c_n$ space admits a general projective motion iff
\[ LR_{ijk}^h = 0 . \]

We know, in order the projective motion becomes affine motion, it is necessary and sufficient that we have \( F_i = 0 \), or \( L\lambda_i = 0 \), consequently, we get the following theorem.

**THEOREM (6.3):** A projective motion admitted in an \(*AT^c_n\)-space become an affine motion, in the same space it is necessary and sufficient that we assume

\[ L\lambda_i = 0 . \]
CHAPTER – VI

RECURRENCE AND SYMMETRIC PROPERTIES IN A KAHLERIAN MANIFOLD AND ITS SUBMANIFOLD.

In the present Chapter, we have defined and studied Kaehlerian manifold with recurrent and symmetric properties. Totally real submanifold of a Kaehlerian manifold, the almost product and almost decomposable complex manifold has been discussed and several theorem have been established.

In the present Chapter, we have obtained the following:

THEOREM (2.1) : Every Kaehlerian recurrent manifold is a Kaehlerian manifold with recurrent projective curvature tensor.

THEOREM (2.2) : Every Kaehlerian recurrent manifold is a Kaehlerian manifold with recurrent Bochner curvature tensor.

THEOREM (2.3): If a Kaehler manifold satisfies any two of the following properties
(i) the manifold is Kaehlerian Ricci –recurrent,
(ii) the manifold is Kaehlerian projective recurrent,
(iii) the manifold is Kaehlerian recurrent, it must be also satisfy the third.

THEOREM (2.4): If a Kaehler manifold satisfies any two of the following properties
(i) the manifold is Kaehlerian Ricci –recurrent,
(ii) the manifold is Kaehlerian manifold with recurrent Bochner curvature tensor,
(iii) the manifold is Kaehlerian recurrent, it must be also satisfy the third.

THEOREM (2.5) : The necessary and sufficient condition for a Kaehlerian projective recurrent manifold to be recurrent manifold is that

\[ \sum_{i,j,k} \lambda_{ij} K_{jk} - \lambda_{ik} K_{jk} = \sum_{i,j,k} \lambda_{ij} K_{jk} - \lambda_{ik} K_{jk} \]

\[ + \sum_{i,j,k} \lambda_{ij} K_{jk} - \lambda_{ik} K_{jk} = \sum_{i,j,k} \lambda_{ij} K_{jk} - \lambda_{ik} K_{jk} \]

\[ = 0 \]

THEOREM (3.1): Every Kaehlerian symmetric manifold is Kaehlerian manifold with symmetric H-projective curvature tensor.
THEOREM(3.2) : Every Kaehlerian symmetric manifold is a Kaehlerian manifold with symmetric Bochner curvature tensor.

THEOREM(3.3) : If a Kaehler manifold satisfies any two of the following properties
(i) the manifold is Kaehlerian Ricci-symmetric,
(ii) the manifold is Kaehlerian projective symmetric,
(iii) the manifold is Kaehlerian symmetric, it must also satisfy the third.

THEOREM(3.4) : If a Kaehler manifold satisfies any two of the following properties
(i) the manifold is Kaehlerian Ricci-symmetric,
(ii) the manifold is Kaehlerian manifold with symmetric Bochner curvature tensor,
(iii) the manifold is Kaehlerian symmetric, it must also satisfy the third.

THEOREM(5.1) : Every bi-recurrent complex manifold is a K*-complex manifold.

THEOREM(5.2) : If an almost product and almost decomposable complex manifold satisfies any two of the following properties
(i) it is bi-recurrent complex manifold,
(ii) it is Ricci-bi-recurrent complex manifold,
(iii) it is a K*-complex manifold, then it must also satisfy the third.

THEOREM(6.1) : Every bi-symmetric complex manifold is a Bochner bi-symmetric complex manifold.

THEOREM(6.2) : If an almost product and almost decomposable complex manifold satisfies any two of the following properties
(i) it is a bi-symmetric complex manifold,
(ii) it is a Ricci bi-symmetric complex manifold,
(iii) it is a Bochner bi-symmetric complex manifold. Then it must also satisfy the third.

THEOREM(7.1) : Let $M^n$, $n \geq 4$ be a totally umbilical, totally real submanifold of a Kaehlerian manifold $M^{2n}$ with vanishing Bochner curvature tensor, then $M^n$ is conformally flat.
THEOREM (8.1) : Let $M^3$ be a totally geodesic, totally real submanifold of a Kaehlerian manifold with vanishing Bochner curvature tensor. Then $M^3$ is conformally flat.
CHAPTER – VII

SASAKIAN MANIFOLD AND ITS SUBMANIFOLDS.

In the present Chapter, we have studied the Sasakian manifold, its submanifold some recurrent and symmetric properties has been discussed and several theorem have been established.

4-THEOREM: Prove that the contact Bochner curvature tensor \( B^b_{\mu \lambda} \) satisfies the identity

\[
\nabla B^b_{\mu \lambda} = -2m \left[ \nabla \xi \xi - \nabla \xi + \xi \left( F_{\mu \lambda} + M_{\mu \lambda} \right) - \xi \left( F_\mu + M_\mu \right) \right]
\]

(4.1)

THEOREM(5.1): ( [2] , Chapter-II Propert. 7.3 ). When the structure vector field \( f^h \) is normal to the submanifold \( M^a \) of a Sasakian manifold \( M^{2m+1} \), we have

\[
\xi f^a = 0, \quad h_{\xi \alpha} f^a = 0
\]

and consequently the submanifold is totally geodesic with respect to the structure vector field \( f^h \).

THEOREM(5.2): If a submanifold of a Sasakian manifold is invariant, then the structure vector field is tangent to the submanifold.

THEOREM (5.3): When the structure vector field \( f^h \) is normal to the submanifold, the submanifold is anti-invariant.

THEOREM(5.4): In order for a submanifold tangent to the structure vector field of a Sasakian manifold to be a contact CR submanifold, it, is necessary and sufficient that

\[
f_{\xi} f^a = 0
\]