CHAPTER 5

SYNTHESIS OF RADIATION PATTERNS OF ARRAYS WITH DEFINED SIDELOBE STRUCTURE USING ACCELERATED PARTICLE SWARM OPTIMIZATION

5.1 Introduction

In antenna design pattern synthesis is one of the important problems to solve. Each applications starting from communications, radars, and radiation therapy systems, demand a specific type of radiation beam shape. For example, point to point communication requires pencil beams, ground mapping and airborne surveillance radars require cosecant patterns, search radars require flat and sector beams, IFF radars require sum and difference patterns, and Marine radars require asymmetric patterns. Some users require low first sidelobe levels. Some other requires low sidelobes in mid-sidelobe region. It is also often required to generate patterns with low sidelobes at the far end of the visible region. Interestingly some applications of communications require multiple beams with the equal height of the main beam (Skolnik 1980 & 1990).

To meet the above requirements and applications, several methods are reported in the literature. Some of them are Dolph-Chebyshev method, standard distributions like Taylor’s method, Fourier Transform, Laplace Transform, Woodward method, Modified Taylor’s method, Bayliss method etc (Rudge 1982).

The above methods are applied to produce patterns of different shapes. (Taylor 1955) reported a method of generation of sum patterns from continuous line sources. It has been possible to maintain required number of sidelobes of equal height by the selection of \( \bar{n} \). This variable provides \( (\bar{n} - 1) \) number of equal sidelobes close to the main beam. The remaining sidelobes taper exponentially. Taylor’s modified method is used to produce symmetric sum patterns (Elliott 2003). These patterns are useful in marine radars to overcome roll and pitch due to turbulent sea.

Sometime, perturbation techniques (Milligan 1985) are also used to synthesize a few typical shapes. It is difficult to use this technique to generate complex shaped patterns as it leads to convergence problems.

However, no work is reported in open literature, to produce low sidelobes in the middle of sidelobe region. In view of this, an attempt is made to produce beam using Accelerated Particle Swarm Optimization. The patterns of present interest are of symmetric nature. Accelerated Particle Swarm Optimization algorithm is simultaneously applied to generate the desired pattern accurately. The data presented in this paper is entirely useful for the array designers.

5.2 Optimization Techniques

Optimization of a specific parameter is very important in every field of human life. These include agricultural, product design, product sales, aircraft design, all industrial products, satellite design, radar and antenna design etc. Specific parameter is optimized within the given constraints, the number of constraints vary from parameter to parameter. Over the last several decades techniques and methods of optimization are reported in the open literature. Some of the useful techniques that are frequently used are Gradient, Random, Perturbation, Iterative, Simplex, Systematic Search, Genetic Algorithm, Simulated Annealing, Particle Swarm Optimization, Accelerated Particle Swarm Optimization, etc (Pierre 1969).
The Gradient technique is a very quick local optimization technique and it provides local optima rather than global. It is often used in the applications, where the point is required. In order to calculate gradient, first step specifies numerical approximation of the gradient of the cost function due to the dynamic range of the variable to be optimized it lies between 0 and 1.

Random optimization technique is relatively laborious and generates random points in the space given for optimization. Usually it is used as the first stage in this the points are randomly distributed and hence it is approximated very often.

Perturbation technique is used when the initial value is well defined. It involves incremental values till the parameter is optimized. The time consumed for optimization depends on incremental value and convergence. In Iterative technique the time consumed depends on number of iterations required for the convergence of the parameter to be optimized.

The Simplex algorithm provides robust local optimized solution if the initial optimization parameter values exist in the vicinity of the solution, it provides the best option. It has initial step between 0 and 1. It is possible to prescribe in this technique the tolerance for the coordinate (optimization variable) and tolerance for the cost function (Nelder and Mead 1965).

The Systematic Search approach as the name describes generates points systematically at the nodes of the grid in the given space. Even 3 to 4 variables can be optimized with this method. The variables arrangements in this technique are error for saved local minima and user defined iterations.

Genetic Algorithm (GA) is frequently used to optimize the parameters using Darwinian evolution principle (Johnson and Rahmat-Samii 1997). It provides the survival of the fittest. It is one of the most robust universal algorithms for optimization. It takes care of many optimization variables and huge optimization spaces. It is not preferred local optimization due to its slow convergence properties (Haupt 1995). It consists of the following parameters, they are algorithm type (continuous or binary), number of bits, pareto genetic algorithm, population size, number of surviving, crossover probability, mutation probability, total number of generations, keep from previous generation, probability of keeping, end calculations, specified tolerance for function, total number of populations, swap entities, and probability of swapping.

Simulated Annealing (SA) is used to optimize any quantity by simulating the annealing process (Kirkpatrick, Gelatt and Vecchi 1983). It is also a robust universal algorithm, unlike Genetic Algorithm, which can be used for local optimization also. It has following parameters, they are starting temperature, cooling scheme, and number of iterations per generation.

Particle Swarm Optimization (PSO) optimizes any quantity by simulating the movement of a bird flock, fish school. It is a very useful technique to address optimization problems with about 4 to 5 variables (Robinson and Rahmat-Samii 2004). It has the following parameters, they are number of particles in the swarm, inertia, cognitive coefficient, social rate, maximal velocity, end calculations if swarm converged, and relative tolerance with absolute tolerance for the cost-function (Rahmat-Sami and Jin 2007).

Accelerated Particle Swarm Optimization (APSO) which was developed by (Yang et al.) In accelerated particle swarm optimization, it is possible to accelerate the convergence of the algorithm using the global best only (Sunny Dayal, Raju, Mishra, and Varma Gottumukkala 2016). Virtual mass is introduced to stabilize the motion of the particles, and hence convergence occurs quickly.
5.3 Accelerated Particle Swarm Optimization

If \( \mathbf{X}_i \) is a position vector and \( \mathbf{V}_i \) is velocity vector of the \( i^{th} \) particle, new velocity vector is obtained from the following

\[
\mathbf{V}_i^{k+1} = \mathbf{V}_i^k + \alpha \epsilon_1 (\mathbf{g}^* - \mathbf{X}_i^k) + b \epsilon_2 (\mathbf{X}_i^* - \mathbf{X}_i^k)
\]

Here, 
\( \epsilon_1 \) and \( \epsilon_2 \) are random vectors each entry takes the values between 0 and 1. 
\( \alpha \) and \( \beta \) are acceleration constants, they are approximately \( \approx 2 \).

In order to extend particle swarm optimization algorithm, inertia function \( y \) is used. Here \( \mathbf{V}_i^k \) can be replaced by \( y \mathbf{V}_i^k \). That is

\[
\mathbf{V}_i^{k+1} = y \mathbf{V}_i^k + \alpha \epsilon_1 (\mathbf{g}^* - \mathbf{X}_i^k) + \beta \epsilon_2 (\mathbf{X}_i^* - \mathbf{X}_i^k)
\]

Here, 
\( y \in (0,1) \).

However typically \( y \) varies between 0.5 and 0.9. This is equivalent to introducing a virtual mass and stabilizes the motion of the particles to have fast convergence.

The current global best, \( \mathbf{g}^* \) and the individual best, \( \mathbf{X}_i^* \) are used in particle swarm optimization. Individual best increases the diversity in the best solution, where the diversity is simulated with randomness. The individual best is used only when the parameter to be optimized is non-linear and multimodal.

However, in accelerated particle swarm optimization, the global best is only used. Keeping this fact in view, the velocity vector is generated from

\[
\mathbf{V}_i^{k+1} = \mathbf{V}_i^k + \alpha \epsilon_n + \beta (\mathbf{g}^* - \mathbf{X}_i^k)
\]

Here, 
\( \epsilon_n \) is taken from \( \mathcal{M}(0,1) \). As a result, we have

\[
\mathbf{X}_i^{k+1} = \mathbf{X}_i^k + \mathbf{V}_i^{k+1}
\]

Updating the location to improve the convergence, we have

\[
\mathbf{X}_i^{k+1} = (1 - \beta) \mathbf{X}_i^k + \alpha \epsilon_n + \beta \mathbf{g}^*
\]

In most of the application, \( \alpha = 0.1 \) and \( \beta = 0.1\sim0.7 \). It is evident that accelerated particle swarm is the simplest and it has only two parameters instead of having more parameters like in particle swarm optimization. The accelerated particle swarm optimization reduces the randomness when the iterations are taken place. Hence it is possible to use a monotonically decreasing function like
\[ \alpha = \alpha_0 e^{-\gamma^k} \quad (5.6) \]

or

\[ \alpha = \alpha_0 \gamma^k \quad (5.7) \]

Here \( \gamma \) is in range 0 and 1. In the above expression \( k \) represents number of iterations. As \( \gamma \) is the controlling parameter \( \alpha \) can be written as \( \alpha = 0.7^k \), where \( k \in [0, k_{\text{max}}] \) and \( k_{\text{max}} \) is the maximum of iterations.

### 5.4 Array Design Using Accelerated Particle Swarm Optimization Algorithm

A typical uniform linear array is shown in Fig. 5.1.

![Geometry of Linear Array with equal spacing](image)

Fig. 5.1 Geometry of Linear Array with equal spacing

Considering a linear array of \( N \) isotropic antennas, antenna elements are equally spaced at distance \( d \) apart from each other along the \( x \) axis. The free space far-field pattern \( E(u) \) is given by (Ma 1974):\(^8\)

\[ E(u) = 2 \sum_{n=1}^{N} A(n) \cos[k(n - 0.5)du] \quad (5.8) \]

Here,

- \( k = \frac{2\pi}{\lambda} \), Wave number
- \( \lambda \) = Wavelength
- \( \theta \) = Angle of observer
Excitation of the $n^{th}$ element on either side of the array, array being symmetric

d = spacing between elements and

$u = \sin \theta$

Normalized radiation in $dB$ is given by:

$$E(u) = 20 \log_{10} \left[ \frac{|E(u)|}{|E(u)|_{\text{max}}} \right]$$  \hspace{1cm} (5.9)

In the design of array, amplitude distribution is considered to be optimized keeping phase and space parameters constant, for a specified sidelobe level, $A(n)$ is computed for $d = \frac{\lambda}{2}$ and excitation phase = 0.

The fitness function is expressed as

$$\text{Fitness function} = PSSL_o - PSSL_d$$  \hspace{1cm} (5.10)

Case 1 $PSSL_d = \begin{cases} -30 \text{ dB} & 0.4 \leq u \leq 0.6 \\ -45 \text{ dB} & \text{elsewhere} \end{cases}$

Case 2 $PSSL_d = \begin{cases} -35 \text{ dB} & 0.4 \leq u \leq 0.6 \\ -50 \text{ dB} & \text{elsewhere} \end{cases}$

Case 3 $PSSL_d = \begin{cases} -50 \text{ dB} & 0.4 \leq u \leq 0.6 \\ -55 \text{ dB} & \text{elsewhere} \end{cases}$

Here,

$PSSL_o$ = Peak Sidelobe level obtained

$PSSL_d$ = Peak Sidelobe level desired

**TABLE 5.1 – APSO Control Parameters**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swarm size</td>
<td>50</td>
</tr>
<tr>
<td>Social component</td>
<td>0.5</td>
</tr>
<tr>
<td>Randomness control component</td>
<td>0.97</td>
</tr>
<tr>
<td>Maximum Number of Iterations</td>
<td>7000</td>
</tr>
</tbody>
</table>

5.5 Results and Discussion

Using the expression Eq. 5.3-5.10, amplitude distributions are computed for desired sidelobe ranging -30 dB to -55 dB. The results are presented in Figs. 5.2, 5.4, 5.6, 5.8, 5.10, 5.12, 5.14, 5.16, 5.18, and 5.20. The excitation levels so obtained are introduced for the array element and their radiation patterns are computed. The elements are considered to be isotropic and the element pattern is uniform. The data obtained from the patterns, the resultant first sidelobe level, null to null beam width for different arrays are presented in Tables 5.12 – 5.14.
TABLE 5.2 – Optimized Element Amplitude Weights Obtained by APSO Method for $N = 10$
with SLL=30/-45dB, SLL=35/-50dB, and SLL=40/-55dB

<table>
<thead>
<tr>
<th>Element Number</th>
<th>$A(n)$ for SLL=30/-45dB using APSO</th>
<th>$A(n)$ for SLL=35/-50dB using APSO</th>
<th>$A(n)$ for SLL=40/-55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1620</td>
<td>0.1298</td>
<td>0.0981</td>
</tr>
<tr>
<td>2</td>
<td>0.3061</td>
<td>0.2871</td>
<td>0.2601</td>
</tr>
<tr>
<td>3</td>
<td>0.5996</td>
<td>0.5672</td>
<td>0.5348</td>
</tr>
<tr>
<td>4</td>
<td>0.8468</td>
<td>0.8397</td>
<td>0.8221</td>
</tr>
<tr>
<td>5</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>6</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>7</td>
<td>0.8468</td>
<td>0.8397</td>
<td>0.8221</td>
</tr>
<tr>
<td>8</td>
<td>0.5996</td>
<td>0.5672</td>
<td>0.5348</td>
</tr>
<tr>
<td>9</td>
<td>0.3061</td>
<td>0.2871</td>
<td>0.2601</td>
</tr>
<tr>
<td>10</td>
<td>0.1620</td>
<td>0.1298</td>
<td>0.0981</td>
</tr>
</tbody>
</table>
TABLE 5.3 – Optimized Element Amplitude Weights Obtained by APSO Method for $N = 20$
with $\text{SLL}=\text{-30/45dB}$, $\text{SLL}=\text{-35/50dB}$, and $\text{SLL}=\text{-40/55dB}$

<table>
<thead>
<tr>
<th>Element Number</th>
<th>$A(n)$ for $\text{SLL}=\text{-30/45dB using APSO}$</th>
<th>$A(n)$ for $\text{SLL}=\text{-35/50dB using APSO}$</th>
<th>$A(n)$ for $\text{SLL}=\text{-40/55dB using APSO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2014</td>
<td>0.1173</td>
<td>0.0682</td>
</tr>
<tr>
<td>2</td>
<td>0.2025</td>
<td>0.1496</td>
<td>0.1077</td>
</tr>
<tr>
<td>3</td>
<td>0.4252</td>
<td>0.3115</td>
<td>0.2294</td>
</tr>
<tr>
<td>4</td>
<td>0.4700</td>
<td>0.4061</td>
<td>0.3356</td>
</tr>
<tr>
<td>5</td>
<td>0.5805</td>
<td>0.5251</td>
<td>0.4651</td>
</tr>
<tr>
<td>6</td>
<td>0.7270</td>
<td>0.6646</td>
<td>0.6131</td>
</tr>
<tr>
<td>7</td>
<td>0.8243</td>
<td>0.7794</td>
<td>0.7423</td>
</tr>
<tr>
<td>8</td>
<td>0.9262</td>
<td>0.8766</td>
<td>0.8620</td>
</tr>
<tr>
<td>9</td>
<td>0.9991</td>
<td>0.9776</td>
<td>0.9581</td>
</tr>
<tr>
<td>10</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>11</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>12</td>
<td>0.9991</td>
<td>0.9776</td>
<td>0.9581</td>
</tr>
<tr>
<td>13</td>
<td>0.9262</td>
<td>0.8766</td>
<td>0.8620</td>
</tr>
<tr>
<td>14</td>
<td>0.8243</td>
<td>0.7794</td>
<td>0.7423</td>
</tr>
<tr>
<td>15</td>
<td>0.7270</td>
<td>0.6646</td>
<td>0.6131</td>
</tr>
<tr>
<td>16</td>
<td>0.5805</td>
<td>0.5251</td>
<td>0.4651</td>
</tr>
<tr>
<td>17</td>
<td>0.4700</td>
<td>0.4061</td>
<td>0.3356</td>
</tr>
<tr>
<td>18</td>
<td>0.4252</td>
<td>0.3115</td>
<td>0.2294</td>
</tr>
<tr>
<td>19</td>
<td>0.2025</td>
<td>0.1496</td>
<td>0.1077</td>
</tr>
<tr>
<td>20</td>
<td>0.2014</td>
<td>0.1173</td>
<td>0.0682</td>
</tr>
</tbody>
</table>
TABLE 5.4 – Optimized Element Amplitude Weights Obtained by APSO Method for \( N = 30 \)
with SLL=30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB

<table>
<thead>
<tr>
<th>Element Number</th>
<th>( A(n) ) for SLL=-30/-45dB using APSO</th>
<th>( A(n) ) for SLL=-35/-50dB using APSO</th>
<th>( A(n) ) for SLL=-40/-55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2225</td>
<td>0.1291</td>
<td>0.0824</td>
</tr>
<tr>
<td>2</td>
<td>0.1537</td>
<td>0.1290</td>
<td>0.0880</td>
</tr>
<tr>
<td>3</td>
<td>0.3363</td>
<td>0.2505</td>
<td>0.1902</td>
</tr>
<tr>
<td>4</td>
<td>0.3588</td>
<td>0.2975</td>
<td>0.2288</td>
</tr>
<tr>
<td>5</td>
<td>0.3748</td>
<td>0.3412</td>
<td>0.2972</td>
</tr>
<tr>
<td>6</td>
<td>0.5070</td>
<td>0.4301</td>
<td>0.3800</td>
</tr>
<tr>
<td>7</td>
<td>0.5677</td>
<td>0.5079</td>
<td>0.4600</td>
</tr>
<tr>
<td>8</td>
<td>0.6359</td>
<td>0.5929</td>
<td>0.5587</td>
</tr>
<tr>
<td>9</td>
<td>0.7231</td>
<td>0.6996</td>
<td>0.6574</td>
</tr>
<tr>
<td>10</td>
<td>0.7885</td>
<td>0.7696</td>
<td>0.7383</td>
</tr>
<tr>
<td>11</td>
<td>0.8077</td>
<td>0.8302</td>
<td>0.8193</td>
</tr>
<tr>
<td>12</td>
<td>0.8979</td>
<td>0.8938</td>
<td>0.8856</td>
</tr>
<tr>
<td>13</td>
<td>0.9589</td>
<td>0.9365</td>
<td>0.9357</td>
</tr>
<tr>
<td>14</td>
<td>0.9424</td>
<td>0.9757</td>
<td>0.9838</td>
</tr>
<tr>
<td>15</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>16</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>17</td>
<td>0.9424</td>
<td>0.9757</td>
<td>0.9838</td>
</tr>
<tr>
<td>18</td>
<td>0.9589</td>
<td>0.9365</td>
<td>0.9357</td>
</tr>
<tr>
<td>19</td>
<td>0.8979</td>
<td>0.8938</td>
<td>0.8856</td>
</tr>
<tr>
<td>20</td>
<td>0.8077</td>
<td>0.8302</td>
<td>0.8193</td>
</tr>
<tr>
<td>21</td>
<td>0.7885</td>
<td>0.7696</td>
<td>0.7383</td>
</tr>
<tr>
<td>22</td>
<td>0.7231</td>
<td>0.6996</td>
<td>0.6574</td>
</tr>
<tr>
<td>23</td>
<td>0.6359</td>
<td>0.5929</td>
<td>0.5587</td>
</tr>
<tr>
<td>24</td>
<td>0.5677</td>
<td>0.5079</td>
<td>0.4600</td>
</tr>
<tr>
<td>25</td>
<td>0.5070</td>
<td>0.4301</td>
<td>0.3800</td>
</tr>
<tr>
<td>26</td>
<td>0.3748</td>
<td>0.3412</td>
<td>0.2972</td>
</tr>
<tr>
<td>27</td>
<td>0.3588</td>
<td>0.2975</td>
<td>0.2288</td>
</tr>
<tr>
<td>28</td>
<td>0.3363</td>
<td>0.2505</td>
<td>0.1902</td>
</tr>
<tr>
<td>29</td>
<td>0.1537</td>
<td>0.1290</td>
<td>0.0880</td>
</tr>
<tr>
<td>30</td>
<td>0.2225</td>
<td>0.1291</td>
<td>0.0824</td>
</tr>
<tr>
<td>Element Number</td>
<td>( A(n) ) for SLL=-30/-45dB using APSO</td>
<td>( A(n) ) for SLL=-35/-50dB using APSO</td>
<td>( A(n) ) for SLL=-40/-55dB using APSO</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------------------------------</td>
<td>----------------------------------------</td>
<td>----------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>0.2448</td>
<td>0.1684</td>
<td>0.1129</td>
</tr>
<tr>
<td>2</td>
<td>0.2165</td>
<td>0.1131</td>
<td>0.0907</td>
</tr>
<tr>
<td>3</td>
<td>0.3572</td>
<td>0.2551</td>
<td>0.1870</td>
</tr>
<tr>
<td>4</td>
<td>0.3478</td>
<td>0.2383</td>
<td>0.1947</td>
</tr>
<tr>
<td>5</td>
<td>0.2908</td>
<td>0.2584</td>
<td>0.2167</td>
</tr>
<tr>
<td>6</td>
<td>0.4730</td>
<td>0.3288</td>
<td>0.2883</td>
</tr>
<tr>
<td>7</td>
<td>0.3831</td>
<td>0.3942</td>
<td>0.3542</td>
</tr>
<tr>
<td>8</td>
<td>0.5647</td>
<td>0.4488</td>
<td>0.4099</td>
</tr>
<tr>
<td>9</td>
<td>0.5572</td>
<td>0.5303</td>
<td>0.4889</td>
</tr>
<tr>
<td>10</td>
<td>0.6246</td>
<td>0.5928</td>
<td>0.5427</td>
</tr>
<tr>
<td>11</td>
<td>0.6763</td>
<td>0.6255</td>
<td>0.6020</td>
</tr>
<tr>
<td>12</td>
<td>0.8196</td>
<td>0.7165</td>
<td>0.6818</td>
</tr>
<tr>
<td>13</td>
<td>0.7631</td>
<td>0.7754</td>
<td>0.7536</td>
</tr>
<tr>
<td>14</td>
<td>0.8762</td>
<td>0.8121</td>
<td>0.7998</td>
</tr>
<tr>
<td>15</td>
<td>0.8494</td>
<td>0.8753</td>
<td>0.8518</td>
</tr>
<tr>
<td>16</td>
<td>0.8898</td>
<td>0.9147</td>
<td>0.9079</td>
</tr>
<tr>
<td>17</td>
<td>0.9940</td>
<td>0.9453</td>
<td>0.9381</td>
</tr>
<tr>
<td>18</td>
<td>1.0000</td>
<td>0.9837</td>
<td>0.9829</td>
</tr>
<tr>
<td>19</td>
<td>0.9937</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>20</td>
<td>0.9732</td>
<td>0.9999</td>
<td>1.0000</td>
</tr>
<tr>
<td>21</td>
<td>0.9732</td>
<td>0.9999</td>
<td>1.0000</td>
</tr>
<tr>
<td>22</td>
<td>0.9937</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>23</td>
<td>1.0000</td>
<td>0.9837</td>
<td>0.9829</td>
</tr>
<tr>
<td>24</td>
<td>0.9940</td>
<td>0.9453</td>
<td>0.9381</td>
</tr>
<tr>
<td>25</td>
<td>0.8898</td>
<td>0.9147</td>
<td>0.9079</td>
</tr>
<tr>
<td>26</td>
<td>0.8494</td>
<td>0.8753</td>
<td>0.8518</td>
</tr>
<tr>
<td>27</td>
<td>0.8762</td>
<td>0.8121</td>
<td>0.7998</td>
</tr>
<tr>
<td>28</td>
<td>0.7631</td>
<td>0.7754</td>
<td>0.7536</td>
</tr>
<tr>
<td>29</td>
<td>0.8196</td>
<td>0.7165</td>
<td>0.6818</td>
</tr>
<tr>
<td>30</td>
<td>0.6763</td>
<td>0.6255</td>
<td>0.6020</td>
</tr>
<tr>
<td>31</td>
<td>0.6246</td>
<td>0.5928</td>
<td>0.5427</td>
</tr>
<tr>
<td>32</td>
<td>0.5572</td>
<td>0.5303</td>
<td>0.4889</td>
</tr>
<tr>
<td>33</td>
<td>0.5647</td>
<td>0.4488</td>
<td>0.4099</td>
</tr>
<tr>
<td>34</td>
<td>0.3831</td>
<td>0.3942</td>
<td>0.3542</td>
</tr>
<tr>
<td>35</td>
<td>0.4730</td>
<td>0.3288</td>
<td>0.2883</td>
</tr>
<tr>
<td>36</td>
<td>0.2908</td>
<td>0.2584</td>
<td>0.2167</td>
</tr>
<tr>
<td>37</td>
<td>0.3478</td>
<td>0.2383</td>
<td>0.1947</td>
</tr>
<tr>
<td>38</td>
<td>0.3572</td>
<td>0.2551</td>
<td>0.1870</td>
</tr>
<tr>
<td>39</td>
<td>0.2165</td>
<td>0.1131</td>
<td>0.0907</td>
</tr>
<tr>
<td>40</td>
<td>0.2448</td>
<td>0.1684</td>
<td>0.1129</td>
</tr>
</tbody>
</table>
TABLE 5.6 – Optimized Element Amplitude Weights Obtained by APSO Method for $N = 50$
with SLL=$-30/-45$dB, SLL=$-35/-50$dB, and SLL=$-40/-55$dB

<table>
<thead>
<tr>
<th>$n$ Element Number</th>
<th>$A(n)$ for SLL= -30/-45dB using APSO</th>
<th>$A(n)$ for SLL= -35/-50dB using APSO</th>
<th>$A(n)$ for SLL= -40/-55dB using APSO</th>
<th>$n$ Element Number</th>
<th>$A(n)$ for SLL= -30/-45dB using APSO</th>
<th>$A(n)$ for SLL= -35/-50dB using APSO</th>
<th>$A(n)$ for SLL= -40/-55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2479</td>
<td>0.1861</td>
<td>0.1126</td>
<td>26</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.2570</td>
<td>0.1222</td>
<td>0.0778</td>
<td>27</td>
<td>0.9984</td>
<td>0.9973</td>
<td>0.9857</td>
</tr>
<tr>
<td>3</td>
<td>0.3050</td>
<td>0.2678</td>
<td>0.1657</td>
<td>28</td>
<td>0.9971</td>
<td>0.9978</td>
<td>0.9865</td>
</tr>
<tr>
<td>4</td>
<td>0.3730</td>
<td>0.2226</td>
<td>0.1756</td>
<td>29</td>
<td>0.9931</td>
<td>0.9889</td>
<td>0.9634</td>
</tr>
<tr>
<td>5</td>
<td>0.2746</td>
<td>0.2438</td>
<td>0.1675</td>
<td>30</td>
<td>0.9591</td>
<td>0.9398</td>
<td>0.9362</td>
</tr>
<tr>
<td>6</td>
<td>0.3347</td>
<td>0.2745</td>
<td>0.2347</td>
<td>31</td>
<td>0.9882</td>
<td>0.9284</td>
<td>0.8966</td>
</tr>
<tr>
<td>7</td>
<td>0.4352</td>
<td>0.3203</td>
<td>0.2647</td>
<td>32</td>
<td>0.8944</td>
<td>0.8830</td>
<td>0.8751</td>
</tr>
<tr>
<td>8</td>
<td>0.3574</td>
<td>0.3687</td>
<td>0.3092</td>
<td>33</td>
<td>0.8798</td>
<td>0.8493</td>
<td>0.8127</td>
</tr>
<tr>
<td>9</td>
<td>0.6081</td>
<td>0.4456</td>
<td>0.3787</td>
<td>34</td>
<td>0.8563</td>
<td>0.8265</td>
<td>0.7836</td>
</tr>
<tr>
<td>10</td>
<td>0.4341</td>
<td>0.4749</td>
<td>0.4143</td>
<td>35</td>
<td>0.7529</td>
<td>0.7661</td>
<td>0.7294</td>
</tr>
<tr>
<td>11</td>
<td>0.6405</td>
<td>0.5327</td>
<td>0.4650</td>
<td>36</td>
<td>0.7938</td>
<td>0.7310</td>
<td>0.6878</td>
</tr>
<tr>
<td>12</td>
<td>0.5935</td>
<td>0.5682</td>
<td>0.5126</td>
<td>37</td>
<td>0.7180</td>
<td>0.6692</td>
<td>0.6201</td>
</tr>
<tr>
<td>13</td>
<td>0.6935</td>
<td>0.6266</td>
<td>0.5834</td>
<td>38</td>
<td>0.6935</td>
<td>0.6266</td>
<td>0.5834</td>
</tr>
<tr>
<td>14</td>
<td>0.7180</td>
<td>0.6692</td>
<td>0.6201</td>
<td>39</td>
<td>0.5935</td>
<td>0.5682</td>
<td>0.5126</td>
</tr>
<tr>
<td>15</td>
<td>0.7938</td>
<td>0.7310</td>
<td>0.6878</td>
<td>40</td>
<td>0.6405</td>
<td>0.5327</td>
<td>0.4650</td>
</tr>
<tr>
<td>16</td>
<td>0.7529</td>
<td>0.7661</td>
<td>0.7294</td>
<td>41</td>
<td>0.4341</td>
<td>0.4749</td>
<td>0.4143</td>
</tr>
<tr>
<td>17</td>
<td>0.8563</td>
<td>0.8265</td>
<td>0.7836</td>
<td>42</td>
<td>0.6081</td>
<td>0.4456</td>
<td>0.3787</td>
</tr>
<tr>
<td>18</td>
<td>0.8798</td>
<td>0.8493</td>
<td>0.8127</td>
<td>43</td>
<td>0.3574</td>
<td>0.3687</td>
<td>0.3092</td>
</tr>
<tr>
<td>19</td>
<td>0.8944</td>
<td>0.8830</td>
<td>0.8751</td>
<td>44</td>
<td>0.4352</td>
<td>0.3203</td>
<td>0.2647</td>
</tr>
<tr>
<td>20</td>
<td>0.9882</td>
<td>0.9284</td>
<td>0.8966</td>
<td>45</td>
<td>0.3347</td>
<td>0.2745</td>
<td>0.2347</td>
</tr>
<tr>
<td>21</td>
<td>0.9591</td>
<td>0.9398</td>
<td>0.9362</td>
<td>46</td>
<td>0.2746</td>
<td>0.2438</td>
<td>0.1675</td>
</tr>
<tr>
<td>22</td>
<td>0.9931</td>
<td>0.9889</td>
<td>0.9634</td>
<td>47</td>
<td>0.3730</td>
<td>0.2226</td>
<td>0.1756</td>
</tr>
<tr>
<td>23</td>
<td>0.9971</td>
<td>0.9978</td>
<td>0.9865</td>
<td>48</td>
<td>0.3050</td>
<td>0.2678</td>
<td>0.1657</td>
</tr>
<tr>
<td>24</td>
<td>0.9984</td>
<td>0.9973</td>
<td>0.9857</td>
<td>49</td>
<td>0.2570</td>
<td>0.1222</td>
<td>0.0778</td>
</tr>
<tr>
<td>25</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>50</td>
<td>0.2479</td>
<td>0.1861</td>
<td>0.1126</td>
</tr>
</tbody>
</table>
TABLE 5.7 – Optimized Element Amplitude Weights Obtained by APSO Method for $N = 60$
with SLL=$-30/-45$dB, SLL=$-35/-50$dB, and SLL=$-40/-55$dB

<table>
<thead>
<tr>
<th>n</th>
<th>Element Number</th>
<th>$A(n)$ for SLL=$-30/-45$dB using APSO</th>
<th>$A(n)$ for SLL=$-35/-50$dB using APSO</th>
<th>$A(n)$ for SLL=$-40/-55$dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2601</td>
<td>0.1382</td>
<td>0.1421</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.2819</td>
<td>0.1626</td>
<td>0.0895</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.2880</td>
<td>0.1628</td>
<td>0.1806</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.3963</td>
<td>0.2797</td>
<td>0.1541</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.1551</td>
<td>0.1676</td>
<td>0.1447</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.2756</td>
<td>0.2246</td>
<td>0.1878</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.3092</td>
<td>0.2854</td>
<td>0.2495</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.4037</td>
<td>0.2478</td>
<td>0.2632</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.3914</td>
<td>0.3829</td>
<td>0.3185</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.5704</td>
<td>0.4085</td>
<td>0.3481</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.3841</td>
<td>0.4192</td>
<td>0.3633</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.4666</td>
<td>0.6264</td>
<td>0.4288</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.6600</td>
<td>0.5019</td>
<td>0.4889</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.4922</td>
<td>0.5166</td>
<td>0.5040</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.7304</td>
<td>0.5916</td>
<td>0.5594</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.6389</td>
<td>0.6646</td>
<td>0.5951</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.6601</td>
<td>0.6603</td>
<td>0.6449</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.7194</td>
<td>0.7366</td>
<td>0.6886</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.7902</td>
<td>0.7095</td>
<td>0.7356</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.8149</td>
<td>0.8208</td>
<td>0.7669</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.8576</td>
<td>0.7840</td>
<td>0.7899</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0.7992</td>
<td>0.9207</td>
<td>0.8518</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.9665</td>
<td>0.8545</td>
<td>0.8718</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.8154</td>
<td>0.9152</td>
<td>0.9061</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.0000</td>
<td>0.9036</td>
<td>0.9203</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0.9663</td>
<td>0.9582</td>
<td>0.9482</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>0.8915</td>
<td>0.9833</td>
<td>0.9667</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.9873</td>
<td>0.9950</td>
<td>0.9784</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0.9484</td>
<td>1.0000</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.9829</td>
<td>0.9684</td>
<td>0.9844</td>
<td></td>
</tr>
<tr>
<td></td>
<td>31</td>
<td>0.9829</td>
<td>0.9684</td>
<td>0.9844</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>0.9484</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>0.9873</td>
<td>0.9950</td>
<td>0.9784</td>
</tr>
<tr>
<td></td>
<td>34</td>
<td>0.8915</td>
<td>0.9833</td>
<td>0.9667</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>0.9663</td>
<td>0.9582</td>
<td>0.9482</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>1.0000</td>
<td>0.9036</td>
<td>0.9203</td>
</tr>
<tr>
<td></td>
<td>37</td>
<td>0.8154</td>
<td>0.9152</td>
<td>0.9061</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>0.9665</td>
<td>0.8545</td>
<td>0.8718</td>
</tr>
<tr>
<td></td>
<td>39</td>
<td>0.7992</td>
<td>0.9207</td>
<td>0.8518</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.8576</td>
<td>0.7840</td>
<td>0.7899</td>
</tr>
<tr>
<td></td>
<td>41</td>
<td>0.8149</td>
<td>0.8208</td>
<td>0.7669</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>0.7902</td>
<td>0.7095</td>
<td>0.7356</td>
</tr>
<tr>
<td></td>
<td>43</td>
<td>0.7194</td>
<td>0.7366</td>
<td>0.6886</td>
</tr>
<tr>
<td></td>
<td>44</td>
<td>0.6601</td>
<td>0.6603</td>
<td>0.6449</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>0.6389</td>
<td>0.6646</td>
<td>0.5951</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>0.7304</td>
<td>0.5916</td>
<td>0.5594</td>
</tr>
<tr>
<td></td>
<td>47</td>
<td>0.4922</td>
<td>0.5166</td>
<td>0.5040</td>
</tr>
<tr>
<td></td>
<td>48</td>
<td>0.6600</td>
<td>0.5019</td>
<td>0.4889</td>
</tr>
<tr>
<td></td>
<td>49</td>
<td>0.4666</td>
<td>0.4624</td>
<td>0.4288</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.3841</td>
<td>0.4192</td>
<td>0.3633</td>
</tr>
<tr>
<td></td>
<td>51</td>
<td>0.5704</td>
<td>0.4085</td>
<td>0.3481</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>0.3914</td>
<td>0.3829</td>
<td>0.3185</td>
</tr>
<tr>
<td></td>
<td>53</td>
<td>0.4037</td>
<td>0.2797</td>
<td>0.1541</td>
</tr>
<tr>
<td></td>
<td>54</td>
<td>0.3963</td>
<td>0.2246</td>
<td>0.1878</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>0.3092</td>
<td>0.2854</td>
<td>0.2495</td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>0.2756</td>
<td>0.2478</td>
<td>0.2632</td>
</tr>
<tr>
<td></td>
<td>57</td>
<td>0.3092</td>
<td>0.2854</td>
<td>0.2495</td>
</tr>
<tr>
<td></td>
<td>58</td>
<td>0.2756</td>
<td>0.2246</td>
<td>0.1878</td>
</tr>
<tr>
<td></td>
<td>59</td>
<td>0.2819</td>
<td>0.1626</td>
<td>0.0895</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.2601</td>
<td>0.1382</td>
<td>0.1421</td>
</tr>
</tbody>
</table>
TABLE 5.8 – Optimized Element Amplitude Weights Obtained by APSO Method for N = 70
with SLL=30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB

<table>
<thead>
<tr>
<th>n</th>
<th>A(n) for SLL=30/-45dB</th>
<th>A(n) for SLL=-35/-50dB</th>
<th>A(n) for SLL=-40/-55dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>using APSO</td>
<td>using APSO</td>
<td>using APSO</td>
</tr>
<tr>
<td>1</td>
<td>0.3195</td>
<td>0.2519</td>
<td>0.1411</td>
</tr>
<tr>
<td>2</td>
<td>0.2917</td>
<td>0.1491</td>
<td>0.0891</td>
</tr>
<tr>
<td>3</td>
<td>0.3349</td>
<td>0.2505</td>
<td>0.1688</td>
</tr>
<tr>
<td>4</td>
<td>0.3632</td>
<td>0.2537</td>
<td>0.1610</td>
</tr>
<tr>
<td>5</td>
<td>0.1807</td>
<td>0.1088</td>
<td>0.1198</td>
</tr>
<tr>
<td>6</td>
<td>0.3418</td>
<td>0.2595</td>
<td>0.1702</td>
</tr>
<tr>
<td>7</td>
<td>0.2378</td>
<td>0.2447</td>
<td>0.2024</td>
</tr>
<tr>
<td>8</td>
<td>0.4284</td>
<td>0.3022</td>
<td>0.2268</td>
</tr>
<tr>
<td>9</td>
<td>0.3678</td>
<td>0.3606</td>
<td>0.2780</td>
</tr>
<tr>
<td>10</td>
<td>0.4081</td>
<td>0.3544</td>
<td>0.2943</td>
</tr>
<tr>
<td>11</td>
<td>0.4446</td>
<td>0.3623</td>
<td>0.3168</td>
</tr>
<tr>
<td>12</td>
<td>0.4317</td>
<td>0.4058</td>
<td>0.3372</td>
</tr>
<tr>
<td>13</td>
<td>0.5678</td>
<td>0.4847</td>
<td>0.4128</td>
</tr>
<tr>
<td>14</td>
<td>0.5147</td>
<td>0.4949</td>
<td>0.4282</td>
</tr>
<tr>
<td>15</td>
<td>0.6264</td>
<td>0.5440</td>
<td>0.4666</td>
</tr>
<tr>
<td>16</td>
<td>0.5138</td>
<td>0.5671</td>
<td>0.5131</td>
</tr>
<tr>
<td>17</td>
<td>0.6501</td>
<td>0.5866</td>
<td>0.5288</td>
</tr>
<tr>
<td>18</td>
<td>0.6207</td>
<td>0.6100</td>
<td>0.5735</td>
</tr>
<tr>
<td>19</td>
<td>0.7742</td>
<td>0.7147</td>
<td>0.6273</td>
</tr>
<tr>
<td>20</td>
<td>0.7208</td>
<td>0.6612</td>
<td>0.6528</td>
</tr>
<tr>
<td>21</td>
<td>0.8704</td>
<td>0.7751</td>
<td>0.6904</td>
</tr>
<tr>
<td>22</td>
<td>0.7258</td>
<td>0.7327</td>
<td>0.7212</td>
</tr>
<tr>
<td>23</td>
<td>0.8467</td>
<td>0.8407</td>
<td>0.7611</td>
</tr>
<tr>
<td>24</td>
<td>0.7945</td>
<td>0.7926</td>
<td>0.7983</td>
</tr>
<tr>
<td>25</td>
<td>0.8689</td>
<td>0.9223</td>
<td>0.8203</td>
</tr>
<tr>
<td>26</td>
<td>0.8632</td>
<td>0.8356</td>
<td>0.8645</td>
</tr>
<tr>
<td>27</td>
<td>0.9554</td>
<td>0.9354</td>
<td>0.8687</td>
</tr>
<tr>
<td>28</td>
<td>0.8953</td>
<td>0.9103</td>
<td>0.9018</td>
</tr>
<tr>
<td>29</td>
<td>0.9909</td>
<td>0.9639</td>
<td>0.9249</td>
</tr>
<tr>
<td>30</td>
<td>0.9154</td>
<td>0.9826</td>
<td>0.9638</td>
</tr>
<tr>
<td>31</td>
<td>0.9995</td>
<td>1.0000</td>
<td>0.9435</td>
</tr>
<tr>
<td>32</td>
<td>0.9484</td>
<td>1.0000</td>
<td>0.9945</td>
</tr>
<tr>
<td>33</td>
<td>0.9955</td>
<td>1.0000</td>
<td>0.9995</td>
</tr>
<tr>
<td>34</td>
<td>0.9979</td>
<td>0.9999</td>
<td>0.9889</td>
</tr>
<tr>
<td>35</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n</th>
<th>A(n) for SLL=30/-45dB</th>
<th>A(n) for SLL=-35/-50dB</th>
<th>A(n) for SLL=-40/-55dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>using APSO</td>
<td>using APSO</td>
<td>using APSO</td>
</tr>
<tr>
<td>36</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>37</td>
<td>0.9979</td>
<td>0.9999</td>
<td>0.9889</td>
</tr>
<tr>
<td>38</td>
<td>0.9955</td>
<td>1.0000</td>
<td>0.9737</td>
</tr>
<tr>
<td>39</td>
<td>0.9484</td>
<td>1.0000</td>
<td>0.9945</td>
</tr>
<tr>
<td>40</td>
<td>0.9995</td>
<td>1.0000</td>
<td>0.9435</td>
</tr>
<tr>
<td>41</td>
<td>0.9154</td>
<td>0.9826</td>
<td>0.9638</td>
</tr>
<tr>
<td>42</td>
<td>0.9909</td>
<td>0.9639</td>
<td>0.9249</td>
</tr>
<tr>
<td>43</td>
<td>0.8953</td>
<td>0.9103</td>
<td>0.9018</td>
</tr>
<tr>
<td>44</td>
<td>0.9554</td>
<td>0.9354</td>
<td>0.8687</td>
</tr>
<tr>
<td>45</td>
<td>0.8632</td>
<td>0.8356</td>
<td>0.8645</td>
</tr>
<tr>
<td>46</td>
<td>0.8689</td>
<td>0.9223</td>
<td>0.8203</td>
</tr>
<tr>
<td>47</td>
<td>0.7945</td>
<td>0.7926</td>
<td>0.7983</td>
</tr>
<tr>
<td>48</td>
<td>0.8467</td>
<td>0.8407</td>
<td>0.7611</td>
</tr>
<tr>
<td>49</td>
<td>0.7258</td>
<td>0.7327</td>
<td>0.7212</td>
</tr>
<tr>
<td>50</td>
<td>0.8704</td>
<td>0.7751</td>
<td>0.6904</td>
</tr>
<tr>
<td>51</td>
<td>0.7208</td>
<td>0.6612</td>
<td>0.6528</td>
</tr>
<tr>
<td>52</td>
<td>0.7742</td>
<td>0.7147</td>
<td>0.6273</td>
</tr>
<tr>
<td>53</td>
<td>0.6207</td>
<td>0.6100</td>
<td>0.5735</td>
</tr>
<tr>
<td>54</td>
<td>0.6501</td>
<td>0.5866</td>
<td>0.5288</td>
</tr>
<tr>
<td>55</td>
<td>0.5138</td>
<td>0.5671</td>
<td>0.5131</td>
</tr>
<tr>
<td>56</td>
<td>0.6264</td>
<td>0.5440</td>
<td>0.4666</td>
</tr>
<tr>
<td>57</td>
<td>0.5147</td>
<td>0.4949</td>
<td>0.4282</td>
</tr>
<tr>
<td>58</td>
<td>0.5678</td>
<td>0.4847</td>
<td>0.4128</td>
</tr>
<tr>
<td>59</td>
<td>0.4317</td>
<td>0.4058</td>
<td>0.3372</td>
</tr>
<tr>
<td>60</td>
<td>0.4446</td>
<td>0.3623</td>
<td>0.3168</td>
</tr>
<tr>
<td>61</td>
<td>0.4081</td>
<td>0.3544</td>
<td>0.2943</td>
</tr>
<tr>
<td>62</td>
<td>0.3678</td>
<td>0.3606</td>
<td>0.2780</td>
</tr>
<tr>
<td>63</td>
<td>0.4284</td>
<td>0.3022</td>
<td>0.2268</td>
</tr>
<tr>
<td>64</td>
<td>0.2378</td>
<td>0.2447</td>
<td>0.2024</td>
</tr>
<tr>
<td>65</td>
<td>0.3418</td>
<td>0.2595</td>
<td>0.1702</td>
</tr>
<tr>
<td>66</td>
<td>0.1807</td>
<td>0.1088</td>
<td>0.1198</td>
</tr>
<tr>
<td>67</td>
<td>0.3632</td>
<td>0.2537</td>
<td>0.1610</td>
</tr>
<tr>
<td>68</td>
<td>0.3349</td>
<td>0.2505</td>
<td>0.1688</td>
</tr>
<tr>
<td>69</td>
<td>0.9979</td>
<td>0.9999</td>
<td>0.9889</td>
</tr>
<tr>
<td>70</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>(n)</td>
<td>(A(n)) for SLL=30/-45dB using APSO</td>
<td>(A(n)) for SLL=35/-50dB using APSO</td>
<td>(A(n)) for SLL=40/-55dB using APSO</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------</td>
<td>----------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>0.2897</td>
<td>0.2157</td>
<td>0.1725</td>
</tr>
<tr>
<td>2</td>
<td>0.3257</td>
<td>0.1764</td>
<td>0.0888</td>
</tr>
<tr>
<td>3</td>
<td>0.3645</td>
<td>0.2061</td>
<td>0.1906</td>
</tr>
<tr>
<td>4</td>
<td>0.3312</td>
<td>0.2487</td>
<td>0.1369</td>
</tr>
<tr>
<td>5</td>
<td>0.2553</td>
<td>0.1074</td>
<td>0.1110</td>
</tr>
<tr>
<td>6</td>
<td>0.1866</td>
<td>0.2028</td>
<td>0.1588</td>
</tr>
<tr>
<td>7</td>
<td>0.2967</td>
<td>0.2291</td>
<td>0.1916</td>
</tr>
<tr>
<td>8</td>
<td>0.2416</td>
<td>0.2784</td>
<td>0.2248</td>
</tr>
<tr>
<td>9</td>
<td>0.4095</td>
<td>0.2807</td>
<td>0.2452</td>
</tr>
<tr>
<td>10</td>
<td>0.3866</td>
<td>0.3248</td>
<td>0.2619</td>
</tr>
<tr>
<td>11</td>
<td>0.3147</td>
<td>0.3496</td>
<td>0.2779</td>
</tr>
<tr>
<td>12</td>
<td>0.4801</td>
<td>0.2890</td>
<td>0.3022</td>
</tr>
<tr>
<td>13</td>
<td>0.3508</td>
<td>0.4363</td>
<td>0.3751</td>
</tr>
<tr>
<td>14</td>
<td>0.4316</td>
<td>0.4163</td>
<td>0.3728</td>
</tr>
<tr>
<td>15</td>
<td>0.5587</td>
<td>0.4272</td>
<td>0.4090</td>
</tr>
<tr>
<td>16</td>
<td>0.4861</td>
<td>0.4974</td>
<td>0.4318</td>
</tr>
<tr>
<td>17</td>
<td>0.5136</td>
<td>0.5472</td>
<td>0.4783</td>
</tr>
<tr>
<td>18</td>
<td>0.6644</td>
<td>0.5217</td>
<td>0.4980</td>
</tr>
<tr>
<td>19</td>
<td>0.5142</td>
<td>0.6156</td>
<td>0.5544</td>
</tr>
<tr>
<td>20</td>
<td>0.5869</td>
<td>0.5792</td>
<td>0.5739</td>
</tr>
<tr>
<td>21</td>
<td>0.7272</td>
<td>0.6497</td>
<td>0.5943</td>
</tr>
<tr>
<td>22</td>
<td>0.5887</td>
<td>0.6552</td>
<td>0.6404</td>
</tr>
<tr>
<td>23</td>
<td>0.7242</td>
<td>0.7225</td>
<td>0.6806</td>
</tr>
<tr>
<td>24</td>
<td>0.7711</td>
<td>0.7372</td>
<td>0.7032</td>
</tr>
<tr>
<td>25</td>
<td>0.6671</td>
<td>0.7836</td>
<td>0.7286</td>
</tr>
<tr>
<td>26</td>
<td>0.8085</td>
<td>0.7526</td>
<td>0.7723</td>
</tr>
<tr>
<td>27</td>
<td>0.7486</td>
<td>0.8592</td>
<td>0.7841</td>
</tr>
<tr>
<td>28</td>
<td>0.8928</td>
<td>0.8170</td>
<td>0.8178</td>
</tr>
<tr>
<td>29</td>
<td>0.7618</td>
<td>0.8852</td>
<td>0.8660</td>
</tr>
<tr>
<td>30</td>
<td>0.8869</td>
<td>0.8765</td>
<td>0.8599</td>
</tr>
<tr>
<td>31</td>
<td>0.8556</td>
<td>0.8970</td>
<td>0.8891</td>
</tr>
<tr>
<td>32</td>
<td>0.7443</td>
<td>0.9170</td>
<td>0.9286</td>
</tr>
<tr>
<td>33</td>
<td>1.0000</td>
<td>0.9539</td>
<td>0.9234</td>
</tr>
<tr>
<td>34</td>
<td>0.8320</td>
<td>0.9707</td>
<td>0.9590</td>
</tr>
<tr>
<td>35</td>
<td>0.9674</td>
<td>1.0000</td>
<td>0.9758</td>
</tr>
<tr>
<td>36</td>
<td>0.9415</td>
<td>0.9935</td>
<td>0.9737</td>
</tr>
<tr>
<td>37</td>
<td>0.8938</td>
<td>0.9997</td>
<td>1.0000</td>
</tr>
<tr>
<td>38</td>
<td>0.9255</td>
<td>0.9999</td>
<td>1.0000</td>
</tr>
<tr>
<td>39</td>
<td>0.9257</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>40</td>
<td>0.9398</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
TABLE 5.10 – Optimized Element Amplitude Weights Obtained by APSO Method for \( N = 90 \)
with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB

<table>
<thead>
<tr>
<th>( n )</th>
<th>Element Number</th>
<th>( A(n) ) for SLL=-30/-45dB using APSO</th>
<th>( A(n) ) for SLL=-35/-50dB using APSO</th>
<th>( A(n) ) for SLL=-40/-55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3078</td>
<td>0.1975</td>
<td>0.1598</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.3331</td>
<td>0.2379</td>
<td>0.1093</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.2905</td>
<td>0.2178</td>
<td>0.1640</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.2949</td>
<td>0.2569</td>
<td>0.1708</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.3300</td>
<td>0.1479</td>
<td>0.0897</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.1480</td>
<td>0.1888</td>
<td>0.1406</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.5134</td>
<td>0.1844</td>
<td>0.1678</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.2615</td>
<td>0.2971</td>
<td>0.1908</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.3546</td>
<td>0.2453</td>
<td>0.2185</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.3182</td>
<td>0.2799</td>
<td>0.2476</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.1901</td>
<td>0.3045</td>
<td>0.2447</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.4230</td>
<td>0.2791</td>
<td>0.2526</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.4083</td>
<td>0.3544</td>
<td>0.3296</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.5411</td>
<td>0.4456</td>
<td>0.3311</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.5269</td>
<td>0.3735</td>
<td>0.3450</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.3889</td>
<td>0.4521</td>
<td>0.4015</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>0.4683</td>
<td>0.4550</td>
<td>0.3917</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>0.4428</td>
<td>0.4632</td>
<td>0.4347</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.5382</td>
<td>0.5084</td>
<td>0.4727</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.6975</td>
<td>0.5717</td>
<td>0.5201</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.6753</td>
<td>0.5378</td>
<td>0.4916</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>0.7031</td>
<td>0.5893</td>
<td>0.5917</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.6467</td>
<td>0.6370</td>
<td>0.5503</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>0.6733</td>
<td>0.6374</td>
<td>0.6337</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.5865</td>
<td>0.6832</td>
<td>0.6328</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>0.7045</td>
<td>0.7050</td>
<td>0.6774</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>0.7354</td>
<td>0.7104</td>
<td>0.6990</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>0.7573</td>
<td>0.7273</td>
<td>0.7269</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>0.8924</td>
<td>0.8036</td>
<td>0.7526</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.8231</td>
<td>0.7538</td>
<td>0.7943</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>0.9359</td>
<td>0.8707</td>
<td>0.7878</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.7680</td>
<td>0.7925</td>
<td>0.8326</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.9080</td>
<td>0.8694</td>
<td>0.8569</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>0.7979</td>
<td>0.9179</td>
<td>0.8627</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>0.8554</td>
<td>0.8226</td>
<td>0.9071</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.9461</td>
<td>0.9950</td>
<td>0.9126</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.9274</td>
<td>0.8502</td>
<td>0.9243</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>0.9675</td>
<td>0.9306</td>
<td>0.9377</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>0.9605</td>
<td>0.9671</td>
<td>0.9659</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.9861</td>
<td>0.9393</td>
<td>0.9668</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>0.9059</td>
<td>0.9958</td>
<td>0.9816</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>0.9844</td>
<td>1.0000</td>
<td>0.9947</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>0.9920</td>
<td>0.9599</td>
<td>0.9861</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>0.9291</td>
<td>0.9901</td>
<td>0.9989</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>1.0000</td>
<td>0.9628</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( n )</th>
<th>Element Number</th>
<th>( A(n) ) for SLL=-30/-45dB using APSO</th>
<th>( A(n) ) for SLL=-35/-50dB using APSO</th>
<th>( A(n) ) for SLL=-40/-55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>1.0000</td>
<td>0.9628</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>0.9291</td>
<td>0.9901</td>
<td>0.9989</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>0.9920</td>
<td>0.9599</td>
<td>0.9861</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>0.9844</td>
<td>1.0000</td>
<td>0.9947</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.9059</td>
<td>0.9958</td>
<td>0.9816</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>0.9861</td>
<td>0.9393</td>
<td>0.9668</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>0.9605</td>
<td>0.9671</td>
<td>0.9659</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>0.9675</td>
<td>0.9306</td>
<td>0.9377</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>0.9274</td>
<td>0.8502</td>
<td>0.9243</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>0.9461</td>
<td>0.9950</td>
<td>0.9126</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0.8554</td>
<td>0.8226</td>
<td>0.7943</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0.7979</td>
<td>0.7273</td>
<td>0.7269</td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>0.9080</td>
<td>0.8694</td>
<td>0.8569</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>0.7680</td>
<td>0.7925</td>
<td>0.8326</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.9359</td>
<td>0.8707</td>
<td>0.7878</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>0.8231</td>
<td>0.7538</td>
<td>0.7943</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>0.8924</td>
<td>0.8036</td>
<td>0.7526</td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>0.7573</td>
<td>0.7273</td>
<td>0.7269</td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>0.7354</td>
<td>0.7104</td>
<td>0.6990</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>0.7045</td>
<td>0.7050</td>
<td>0.6774</td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>0.5865</td>
<td>0.6832</td>
<td>0.6328</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>0.6733</td>
<td>0.6374</td>
<td>0.6337</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>0.6467</td>
<td>0.6370</td>
<td>0.5503</td>
<td></td>
</tr>
<tr>
<td>69</td>
<td>0.7031</td>
<td>0.5893</td>
<td>0.5917</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>0.6753</td>
<td>0.5378</td>
<td>0.4916</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>0.6975</td>
<td>0.5717</td>
<td>0.5201</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>0.5382</td>
<td>0.5084</td>
<td>0.4727</td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>0.4428</td>
<td>0.4632</td>
<td>0.4347</td>
<td></td>
</tr>
<tr>
<td>74</td>
<td>0.4683</td>
<td>0.4550</td>
<td>0.3917</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.3889</td>
<td>0.4521</td>
<td>0.4015</td>
<td></td>
</tr>
<tr>
<td>76</td>
<td>0.5269</td>
<td>0.3735</td>
<td>0.3450</td>
<td></td>
</tr>
<tr>
<td>77</td>
<td>0.5411</td>
<td>0.4456</td>
<td>0.3311</td>
<td></td>
</tr>
<tr>
<td>78</td>
<td>0.4083</td>
<td>0.3544</td>
<td>0.3296</td>
<td></td>
</tr>
<tr>
<td>79</td>
<td>0.4230</td>
<td>0.2791</td>
<td>0.2526</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>0.1901</td>
<td>0.3045</td>
<td>0.2447</td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>0.3182</td>
<td>0.2791</td>
<td>0.2526</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>0.3546</td>
<td>0.2791</td>
<td>0.2526</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>0.2615</td>
<td>0.2971</td>
<td>0.1908</td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>0.5134</td>
<td>0.1844</td>
<td>0.1678</td>
<td></td>
</tr>
<tr>
<td>85</td>
<td>0.1480</td>
<td>0.1888</td>
<td>0.1406</td>
<td></td>
</tr>
<tr>
<td>86</td>
<td>0.3300</td>
<td>0.1479</td>
<td>0.0897</td>
<td></td>
</tr>
<tr>
<td>87</td>
<td>0.2944</td>
<td>0.2569</td>
<td>0.1708</td>
<td></td>
</tr>
<tr>
<td>88</td>
<td>0.2905</td>
<td>0.2178</td>
<td>0.1640</td>
<td></td>
</tr>
<tr>
<td>89</td>
<td>0.3311</td>
<td>0.2379</td>
<td>0.1093</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.3078</td>
<td>0.1975</td>
<td>0.1598</td>
<td></td>
</tr>
</tbody>
</table>

95
<table>
<thead>
<tr>
<th>n</th>
<th>A(n) for</th>
<th>A(n) for</th>
<th>A(n) for</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SLL=30/-45dB using APSO</td>
<td>SLL=35/-50dB using APSO</td>
<td>SLL=40/-55dB using APSO</td>
</tr>
<tr>
<td>1</td>
<td>0.1719</td>
<td>0.2124</td>
<td>0.1600</td>
</tr>
<tr>
<td>2</td>
<td>0.3008</td>
<td>0.2021</td>
<td>0.1066</td>
</tr>
<tr>
<td>3</td>
<td>0.3673</td>
<td>0.2485</td>
<td>0.1590</td>
</tr>
<tr>
<td>4</td>
<td>0.3255</td>
<td>0.2407</td>
<td>0.1239</td>
</tr>
<tr>
<td>5</td>
<td>0.2953</td>
<td>0.1114</td>
<td>0.1101</td>
</tr>
<tr>
<td>6</td>
<td>0.2394</td>
<td>0.1527</td>
<td>0.1036</td>
</tr>
<tr>
<td>7</td>
<td>0.1513</td>
<td>0.1668</td>
<td>0.1937</td>
</tr>
<tr>
<td>8</td>
<td>0.2110</td>
<td>0.2762</td>
<td>0.1560</td>
</tr>
<tr>
<td>9</td>
<td>0.2937</td>
<td>0.2279</td>
<td>0.1913</td>
</tr>
<tr>
<td>10</td>
<td>0.2282</td>
<td>0.2738</td>
<td>0.1845</td>
</tr>
<tr>
<td>11</td>
<td>0.3517</td>
<td>0.2499</td>
<td>0.2169</td>
</tr>
<tr>
<td>12</td>
<td>0.3252</td>
<td>0.2380</td>
<td>0.2234</td>
</tr>
<tr>
<td>13</td>
<td>0.3260</td>
<td>0.3481</td>
<td>0.3175</td>
</tr>
<tr>
<td>14</td>
<td>0.3165</td>
<td>0.3556</td>
<td>0.2953</td>
</tr>
<tr>
<td>15</td>
<td>0.3998</td>
<td>0.3505</td>
<td>0.2824</td>
</tr>
<tr>
<td>16</td>
<td>0.3357</td>
<td>0.4140</td>
<td>0.3545</td>
</tr>
<tr>
<td>17</td>
<td>0.3705</td>
<td>0.3522</td>
<td>0.3107</td>
</tr>
<tr>
<td>18</td>
<td>0.4926</td>
<td>0.5049</td>
<td>0.3927</td>
</tr>
<tr>
<td>19</td>
<td>0.4961</td>
<td>0.3940</td>
<td>0.4229</td>
</tr>
<tr>
<td>20</td>
<td>0.4212</td>
<td>0.5171</td>
<td>0.4202</td>
</tr>
<tr>
<td>21</td>
<td>0.5114</td>
<td>0.4942</td>
<td>0.4742</td>
</tr>
<tr>
<td>22</td>
<td>0.3521</td>
<td>0.4802</td>
<td>0.4838</td>
</tr>
<tr>
<td>23</td>
<td>0.5588</td>
<td>0.5802</td>
<td>0.5053</td>
</tr>
<tr>
<td>24</td>
<td>0.5817</td>
<td>0.6163</td>
<td>0.5439</td>
</tr>
<tr>
<td>25</td>
<td>0.6656</td>
<td>0.5817</td>
<td>0.5312</td>
</tr>
<tr>
<td>26</td>
<td>0.5411</td>
<td>0.6211</td>
<td>0.5949</td>
</tr>
<tr>
<td>27</td>
<td>0.5996</td>
<td>0.6637</td>
<td>0.6181</td>
</tr>
<tr>
<td>28</td>
<td>0.5131</td>
<td>0.6674</td>
<td>0.6476</td>
</tr>
<tr>
<td>29</td>
<td>0.7171</td>
<td>0.6927</td>
<td>0.6949</td>
</tr>
<tr>
<td>30</td>
<td>0.6057</td>
<td>0.7291</td>
<td>0.6740</td>
</tr>
<tr>
<td>31</td>
<td>0.7921</td>
<td>0.7225</td>
<td>0.7135</td>
</tr>
<tr>
<td>32</td>
<td>0.6462</td>
<td>0.7695</td>
<td>0.7295</td>
</tr>
<tr>
<td>33</td>
<td>0.5497</td>
<td>0.7897</td>
<td>0.7656</td>
</tr>
<tr>
<td>34</td>
<td>0.8044</td>
<td>0.8338</td>
<td>0.7921</td>
</tr>
<tr>
<td>35</td>
<td>0.7368</td>
<td>0.7966</td>
<td>0.8194</td>
</tr>
<tr>
<td>36</td>
<td>0.7341</td>
<td>0.8093</td>
<td>0.8297</td>
</tr>
<tr>
<td>37</td>
<td>0.8228</td>
<td>0.8246</td>
<td>0.8553</td>
</tr>
<tr>
<td>38</td>
<td>0.7468</td>
<td>0.8929</td>
<td>0.8809</td>
</tr>
<tr>
<td>39</td>
<td>0.6157</td>
<td>0.8935</td>
<td>0.8612</td>
</tr>
<tr>
<td>40</td>
<td>1.0000</td>
<td>0.9023</td>
<td>0.9226</td>
</tr>
<tr>
<td>41</td>
<td>0.6590</td>
<td>0.9320</td>
<td>0.8885</td>
</tr>
<tr>
<td>42</td>
<td>0.9097</td>
<td>0.9563</td>
<td>0.9557</td>
</tr>
<tr>
<td>43</td>
<td>0.8010</td>
<td>0.9120</td>
<td>0.9723</td>
</tr>
<tr>
<td>44</td>
<td>0.7808</td>
<td>0.9883</td>
<td>0.9419</td>
</tr>
<tr>
<td>45</td>
<td>0.8372</td>
<td>0.9533</td>
<td>0.9666</td>
</tr>
<tr>
<td>46</td>
<td>0.8221</td>
<td>0.9679</td>
<td>0.9408</td>
</tr>
<tr>
<td>47</td>
<td>0.9543</td>
<td>0.9842</td>
<td>0.9870</td>
</tr>
<tr>
<td>48</td>
<td>0.7176</td>
<td>0.9623</td>
<td>0.9857</td>
</tr>
<tr>
<td>49</td>
<td>0.9448</td>
<td>1.0000</td>
<td>0.9951</td>
</tr>
<tr>
<td>50</td>
<td>0.7781</td>
<td>0.9822</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Fig. 5.2 Element amplitude weights obtained by APSO method for $N = 10$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.

Fig. 5.3 Optimized Sum Pattern by APSO method for $N = 10$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.
Fig. 5.4 Element amplitude weights obtained by APSO method for $N = 20$, with SLL=-30/-45 dB, SLL=-35/-50 dB, and SLL=-40/-55 dB.

Fig. 5.5 Optimized Sum Pattern by APSO method for $N = 20$, with SLL=-30/-45 dB, SLL=-35/-50 dB, and SLL=-40/-55 dB.
Fig. 5.6 Element amplitude weights obtained by APSO method for $N = 30$, with SLL=$-30/-45$ dB, SLL=$-35/-50$ dB, and SLL=$-40/-55$ dB.

Fig. 5.7 Optimized Sum Pattern by APSO method for $N = 30$, with SLL=$-30/-45$ dB, SLL=$-35/-50$ dB, and SLL=$-40/-55$ dB.
Fig. 5.8 Element amplitude weights obtained by APSO method for $N = 40$, with SLL=$-30/-45$ dB, SLL=$-35/-50$ dB, and SLL=$-40/-55$ dB.

Fig. 5.9 Optimized Sum Pattern by APSO method for $N = 40$, with SLL=$-30/-45$ dB, SLL=$-35/-50$ dB, and SLL=$-40/-55$ dB.
Fig. 5.10 Element amplitude weights obtained by APSO method for $N = 50$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.

Fig. 5.11 Optimized Sum Pattern by APSO method for $N = 50$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.
Fig. 5.12 Element amplitude weights obtained by APSO method for $N = 60$, with SLL=-30/-45 dB, SLL=-35/-50 dB, and SLL=-40/-55 dB.

Fig. 5.13 Optimized Sum Pattern by APSO method for $N = 60$, with SLL=-30/-45 dB, SLL=-35/-50 dB, and SLL=-40/-55 dB.
Fig. 5.14 Element amplitude weights obtained by APSO method for $N = 70$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.

Fig. 5.15 Optimized Sum Pattern by APSO method for $N = 70$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.
Fig. 5.16 Element amplitude weights obtained by APSO method for $N = 80$, with SLL=$-30/-45$ dB, SLL=$-35/-50$ dB, and SLL=$-40/-55$ dB.

Fig. 5.17 Optimized Sum Pattern by APSO method for $N = 80$, with SLL=$-30/-45$ dB, SLL=$-35/-50$ dB, and SLL=$-40/-55$ dB.
Fig. 5.18 Element amplitude weights obtained by APSO method for \( N = 90 \), with SLL=\(-30/-45\) dB, SLL=\(-35/-50\) dB, and SLL=\(-40/-55\) dB

Fig. 5.19 Optimized Sum Pattern by APSO method for \( N = 90 \), with SLL=\(-30/-45\) dB, SLL=\(-35/-50\) dB, and SLL=\(-40/-55\) dB.
Fig. 5.20 Element amplitude weights obtained by APSO method for $N = 100$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.

Fig. 5.21 Optimized Sum Pattern by APSO method for $N = 100$, with SLL=-30/-45dB, SLL=-35/-50dB, and SLL=-40/-55dB.
### TABLE 5.12 – Null to Null Beamwidth for Optimized Sum Pattern Using Accelerated Particle Swarm Optimization (APSO)

<table>
<thead>
<tr>
<th>Number of Elements</th>
<th>SLL=−30/−45dB using APSO</th>
<th>SLL=−35/−50dB using APSO</th>
<th>SLL=−40/−55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>42.48°</td>
<td>45.14°</td>
<td>48.22°</td>
</tr>
<tr>
<td>20</td>
<td>17.82°</td>
<td>19.89°</td>
<td>22.36°</td>
</tr>
<tr>
<td>30</td>
<td>11.79°</td>
<td>12.97°</td>
<td>14.29°</td>
</tr>
<tr>
<td>40</td>
<td>8.56°</td>
<td>9.67°</td>
<td>10.45°</td>
</tr>
<tr>
<td>50</td>
<td>6.82°</td>
<td>7.62°</td>
<td>8.38°</td>
</tr>
<tr>
<td>60</td>
<td>5.71°</td>
<td>6.38°</td>
<td>6.88°</td>
</tr>
<tr>
<td>70</td>
<td>4.85°</td>
<td>5.34°</td>
<td>5.98°</td>
</tr>
<tr>
<td>80</td>
<td>4.20°</td>
<td>4.75°</td>
<td>5.13°</td>
</tr>
<tr>
<td>90</td>
<td>3.44°</td>
<td>4.58°</td>
<td>4.58°</td>
</tr>
<tr>
<td>100</td>
<td>3.36°</td>
<td>3.78°</td>
<td>4.17°</td>
</tr>
</tbody>
</table>

### TABLE 5.13 – First Sidelobe Level for Optimized Sum Pattern Using Accelerated Particle Swarm Optimization (APSO)

<table>
<thead>
<tr>
<th>Number of Elements</th>
<th>SLL=−30/−45dB using APSO</th>
<th>SLL=−35/−50dB using APSO</th>
<th>SLL=−40/−55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-30.83 dB</td>
<td>-35.27 dB</td>
<td>-40.21 dB</td>
</tr>
<tr>
<td>20</td>
<td>-30.00 dB</td>
<td>-35.00 dB</td>
<td>-40.93 dB</td>
</tr>
<tr>
<td>30</td>
<td>-30.03 dB</td>
<td>-35.27 dB</td>
<td>-40.17 dB</td>
</tr>
<tr>
<td>40</td>
<td>-30.02 dB</td>
<td>-36.02 dB</td>
<td>-40.00 dB</td>
</tr>
<tr>
<td>50</td>
<td>-30.06 dB</td>
<td>-35.00 dB</td>
<td>-40.18 dB</td>
</tr>
<tr>
<td>60</td>
<td>-30.69 dB</td>
<td>-35.23 dB</td>
<td>-40.00 dB</td>
</tr>
<tr>
<td>70</td>
<td>-30.00 dB</td>
<td>-35.00 dB</td>
<td>-40.18 dB</td>
</tr>
<tr>
<td>80</td>
<td>-30.19 dB</td>
<td>-35.78 dB</td>
<td>-40.00 dB</td>
</tr>
<tr>
<td>90</td>
<td>-30.12 dB</td>
<td>-35.00 dB</td>
<td>40.19 dB</td>
</tr>
<tr>
<td>100</td>
<td>-30.16 dB</td>
<td>-35.82 dB</td>
<td>-40.88 dB</td>
</tr>
</tbody>
</table>

### TABLE 5.14 – Inner Closed Sidelobe Level for Optimized Sum Pattern Using Accelerated Particle Swarm Optimization (APSO)

<table>
<thead>
<tr>
<th>Number of Elements</th>
<th>SLL=−30/−45dB using APSO</th>
<th>SLL=−35/−50dB using APSO</th>
<th>SLL=−40/−55dB using APSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-45.02 dB</td>
<td>-50.01 dB</td>
<td>-55.07 dB</td>
</tr>
<tr>
<td>20</td>
<td>-45.00 dB</td>
<td>-50.00 dB</td>
<td>-55.10 dB</td>
</tr>
<tr>
<td>30</td>
<td>-45.17 dB</td>
<td>-50.12 dB</td>
<td>-55.27 dB</td>
</tr>
<tr>
<td>40</td>
<td>-45.00 dB</td>
<td>-50.84 dB</td>
<td>-55.00 dB</td>
</tr>
<tr>
<td>50</td>
<td>-45.15 dB</td>
<td>-50.09 dB</td>
<td>-55.36 dB</td>
</tr>
<tr>
<td>60</td>
<td>-45.93 dB</td>
<td>-50.18 dB</td>
<td>-55.01 dB</td>
</tr>
<tr>
<td>70</td>
<td>-45.19 dB</td>
<td>-50.13 dB</td>
<td>-55.34 dB</td>
</tr>
<tr>
<td>80</td>
<td>-45.00 dB</td>
<td>-50.78 dB</td>
<td>-55.03 dB</td>
</tr>
<tr>
<td>90</td>
<td>-45.15 dB</td>
<td>-50.17 dB</td>
<td>-55.41 dB</td>
</tr>
<tr>
<td>100</td>
<td>-45.00 dB</td>
<td>-50.88 dB</td>
<td>-55.17 dB</td>
</tr>
</tbody>
</table>
5.6 Conclusion

Accelerated Particle Swarm Optimization algorithm is found to be very useful for the optimization of desired patterns. The convergence has become simple compared to Particle Swarm Optimization. From the amplitude distribution presented, it is found that it exhibits a gradual taper from the centre to the end with symmetric behavior. As the number of elements is increased in the array the beamwidth is found to be decreased without much change in sidelobe levels. It has been possible with the present work to synthesize the required patterns using Accelerated Particle Swarm Optimization successfully.

The realized radiation patterns presented in Figs. 5.3, 5.5, etc., indicate good approximation to the specified sidelobe ratios. As the number of elements are increased in the array, the beamwidth is found to be reduced.