# Chapter 4

# Proposal of Fractional Approach for Control of a Flexible Link Manipulator

## 4.1 Introduction

Robust control technique namely the sliding mode control and higher order SMC, is investigated for the control of flexible manipulator. This chapter proposes the use of fractional calculus along with SMC to combine the advantages of both the theories. Fractional calculus is very well established mathematical concept but its applications from control perspective have been investigated recently. It has been proved that it allows more flexibility and freedom to existing mathematical tools. The field of automatic control systems has influenced from this new perspective of operators and significant number of contributions are reported on the fractional order (FO) variants of most popularly practised PID controllers [104]. Control fraternity has taken a serious note of fractional calculus by publishing a special issue of Asian Journal of Control on 'Advances in Fractional Order Control and Estimation' in May 2013.

SMC has also extended its applicability using fractional calculus to improve the performance of the system in fractional framework [105]- [108]. Factional order sliding mode control (FSMC) is the use of SMC either with fractional plants or with a fractional controller or both [109].

Motivation of this study is to investigate the useability of the fractional calculus for the control of flexible link manipulator (FLM) using sliding modes for enhancing the performance.

## 4.2 Introduction to Fractional Calculus

Integer order derivative and integration of a function are given as $\frac{df}{dt}$, $\frac{d^2f}{dt^2}$, $\int f dt$, etc but fractional calculus allows derivatives and integrals of arbitrary orders i.e. may be real or complex, for example $\frac{d^{0.5}f}{dt}$ or $\frac{d^{\pi}f}{dt}$. Therefore fractional calculus is defined as the generalization of classical calculus having arbitrary real orders of integration and differentiation [110].
Importance of fractional calculus (FC) can be realized from the following.

- In an aerial view of a city, all roads are seen exactly geometric; which are not in reality, but the exact shape is seen only from near distance. Similarly fractional calculus gives a microscopic view, while integer calculus gives a macroscopic one.
- FO derivative and IO derivative can be compared with a practical example of ramp and staircase respectively. Stairs give a certain level difference like an integer order derivatives as $1^{st}$, $2^{nd}$ or $3^{rd}$ order but the ramp gives a continuous slope between the two levels of a step, which can be analogous to any fractional order derivative lying between two integers. The comfort of ramp is indeed more with respect to steps.

FC can be used to describe system more accurately. However physical significance of fractional derivative of a function is not yet known. Therefore in spite of the long history of around 350 years, fractional calculus was not considered eligible for any applications due to its complexity and lack of physical and geometric interpretation. This limitation kept fractional calculus away from its applicability. By the end of 19th century, due to primary contributions of Liouville, Grunwald, Letnikov and Riemann, theoretical developments in this field took almost finished form.

Applications of fractional calculus to real-world problems are very recent. The work has essentially been motivated by the enormous numbers of very interesting and novel applications of fractional differential equations in various applied fields such as signal processing, system modeling and identification, economics, probability and statistics, astrophysics, chemical engineering, signal processing, dynamics of earthquakes, optics, geology, bio-sciences, medicines, control engineering and many more. Specific applications of fractional calculus is found in mathematical modeling of systems and control design in control theory.

## 4.3 Applications of FC for Modeling and Control: A Literature Review

Though the use of non-integer orders in systems theory is not a new concept, the last two decades have witnessed many successful applications of fractional order differentiation and integration. Potential advantages of fractional calculus have been successfully utilised by many of the researchers in various application fields. Especially it has proved to be very useful in modeling and control domain [111], [112]. Study in the field showed that FO models are more adequate than the classical IO model of any real plant [113]. Therefore FC is used for modeling the systems in the field of mechanics (theory of viscoelasticity and viscoplasticity), biochemistry (polymers and proteins), electrical engineering (transmission of ultrasound waves), medicine (modeling of human tissue under mechanical loads), etc [114]. Fractional derivative (FD) has proved to be an excellent tool for description of memory and hereditary properties of various materials and processes which are neglected in IO case. Therefore it is used in modeling mechanical and electrical properties of real materials, as well as in the description of rheological properties of rocks and in many other fields [115]. FD are also used in modeling dynamical processes in self similar and porous structures [116].

Last two decades have witnessed the use of FC in the development of the controllers. Many FO controllers are proposed in the literature. Stabilization of a fractional order system with a fractional order PID using Ziegler-Nichols approach is presented in [117]. Axtell, Bise illustrated fractional calculus applications in control systems [112]. Matignon, Manabe, Petras developed fractional order feedback control [118] - [120]. Controller based on FO quantitative feedback theory is given in [111]. Fractional order controller for multivariable system is proposed in [121]. Stabilization of fractional order differential equations are attempted by many researchers [122] - [124]. FO optimal control has been given by Agarwal in [125]. Estimators for FO systems are proposed in [126], [127].

Fractional order sliding mode control has been investigated by many researchers e.g [128], [129], etc. Valrio in his survey paper on fractional SMC, developed a systematic approach to design and implement FSMC to control physical plants [130]. Application of FC to robotic system was investigated by Vinagre et.al. [131] and in particular to SLFM by Monje et.al [132] using three nested loops.

To design fractional controller, some theorems and definitions of FC are required. Important and frequently used definitions of fractional calculus are covered in the following section.

## 4.4 Fractional Calculus Preliminaries and Definitions

Many mathematicians have defined fractional order integration and derivative. Therefore several definitions of fractional derivatives are reported in the literature, but the commonly used are Riemann-Liouville fractional Derivative (RLFD) and Caputo fractional Derivative (CFD). Here, J represents integration operator and D represents derivative operator.

1. The Riemann Liouville fractional order integration (RLFI) of a function $f(t)$ of order $\alpha$ is given as

$$J^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t-\tau)^{(\alpha-1)} f(\tau) d\tau \tag{4.1}$$

   where $\alpha$ is real number and $\Gamma(\cdot)$ is the Eulers gamma function given as $\Gamma(x) = \int_{0}^{t} e^{-t} t^{x-1} dt$. This definition is generalized by Gamma function and is determined using Cauchy's closed form for successive integration.

2. $\alpha^{th}$ order Riemann-Liouville fractional Derivative denoted as $D_t^{\alpha}$ of a function $f(t)$ is determined as

$$_{RL}\mathrm{D}_t^{\alpha} f(t) = \frac{d^m (J^{m-\alpha} f(t))}{dt^m} \tag{4.2}$$

   with $(m-1) < \alpha < m$

where $m$ is an integer and $\alpha$ is a real number. In order to find the $\alpha^{th}$ RLFD, first the function is integrated for order $(m-\alpha)$ and then it is differentiated with $m^{th}$ order where $m$ is the nearest integer greater than $\alpha$. By use of (4.1) and (4.2) RLFD can be expressed as

$$_{RL}\mathrm{D}_t^{\alpha} f(t) = \frac{d^m}{dt^m}\frac{1}{\Gamma(m-\alpha)}\int\limits_0^t (t-\tau)^{m-\alpha-1} f(\tau)d\tau \tag{4.3}$$

3. $\alpha^{th}$ order Caputo fractional Derivative denoted as $D_t^{\alpha}$ of a function $f(t)$ is determined as

$$_{c}\mathrm{D}_t^{\alpha} f(t) = J^{m-\alpha}\frac{d^m f(t)}{dt^m} \tag{4.4}$$

In order to find the $\alpha^{th}$ CFD, first the function is differentiated by $m^{th}$ order and then it is $(m-\alpha)$ fold integrated. By use of (4.1) and (4.4), CFD can be expressed as

$$_{c}\mathrm{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)}\int\limits_0^t (t-\tau)^{m-\alpha-1} \frac{d^m}{dt^m} f(\tau)d\tau \tag{4.5}$$

From the above definitions, it is observed that for calculating FO derivative, FC makes use of integer order differentiation and fractional order integration. Differentiation of a function $f$ of order say 2.2 (i.e. $\frac{d^{2.2}f}{dt}$) is found out in two ways. As $2 < 2.2 < 3$, first differentiate the function with order 3 and then integrate this result by order 0.8 by RL integration method, or first integrate the function with order 0.8 by RL integration method and then differentiate the result 3 times.

Applied problems demand usage of fractional derivatives, with physically interpretable initial conditions. From (4.3), it is seen that the initial condition arising in case of RLFD is unphysical (value of fractional integration at t=0) so RLFD definition has limitations in terms of its applications in modeling. Moreover from (4.5), it is seen that in case of CFD, the initial conditions have physical meanings since the value of the function and its derivatives at time $t = 0$ can be determined e.g. $f(0), \dot{f}(0)$ etc. Therefore CFD definition is more popular in physicists and engineers. However, if the initial condition of the function and therefore its derivatives are 0 at time $t = 0$, then RLFD definition can be used.

Important facts evolved from the definitions of the FDs are listed below.

- Fractional differintegration (i.e. differentiation and/or integration) interpolates the operation between the two integer order operations. When this order becomes one, it gives normal integer order operations.

- Definition of fractional derivative involves an integration, which is a non local property. Therefore fractional derivative is a non-local operator.
- Calculating fractional derivative of a function $f(t)$ at time $t = t_1$ requires all the past history, i.e. all $f(t)$ from $t = 0$ to $t = t_1$, hence fractional derivatives can be used for modeling systems with memory.
- Calculating space-fractional derivative of a function $f(x)$ at $x = x_1$ requires all non-local $f(x)$ values, therefore fractional derivatives can be used for modeling distributed parameter systems.

4. Laplace transform of the fractional derivatives and integrals are required while solving the fractional differential equations. These are given as

$$L[J^{\alpha} f(t)] = s^{-\alpha} F(s)$$

$$L[{}_{RL}D_t^{\alpha} f(t)] = s^{\alpha} F(s) - \sum_{k=1}^{m-1} s^k [{}_{R}D_t^{\alpha-k-1} f(t)]_{t=0}$$

$$L[{}_{c}D_t^{\alpha} f(t)] = s^{\alpha} F(s) - \sum_{k=1}^{m-1} s^{\alpha-k-1} f^{(k)}(0)$$

5. In FC, Mittag-Leffler function is frequently used. It is defined as

$$E_{\alpha}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)}$$

Similarly two-parameter Mittag-Leffler function is defined by

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + \beta)}$$

where Re $(\alpha) > 0$ and $\beta \in \mathbb{C}, t \in \mathbb{C}$.

6. Stability of linear fractional differential equation (FDE):
Existence and uniqueness of solutions for these FDEs are well established. Matignon [120] was pioneer in proposing a well-known stability result by an algebraic approach, where the necessary and sufficient conditions have been derived. The result is illustrated

below.

Consider the linear fractional system as $D^{\alpha}x(t) = Ax(t)$. The autonomous system with Caputo derivative and initial value $x_0 = x(0)$, where $0 < \alpha \leq 1$, is asymptotically stable if and only if $|arg(spec(A))| > \alpha\pi/2$. In this case the components of the state decay towards 0 like $t^{-\alpha}$.

With this basic understanding of the fractional calculus, fractional differintegrations can be used in the development of the controller to implement in real time application. This thesis investigates fractional PID controller and fractional SMC for SLFM.

## 4.5 PID Controller

PID controller is the most popular controller, used in the industries. To obtain the desired behavior, controller compares the actual operation of the system with the desired one, computes corrective actions based on reference model, and actuates the system to obtain the desired change. Since the usual tools to model dynamic systems are integrals and derivatives, PID algorithm uses the integration and derivative of the error between the desired and the actual output of the system. The control structure of PID controller is

$$u_{IOPID} = K_p e + K_i \int e + K_d \dot{e} \tag{4.6}$$

where $K_p, K_i, K_d$ are proportional, integral and derivative constants and $e$ is the error between the actual and desired value of the controlled variable.

PID controllers have the following effects

1. proportional (P) action increases the speed of the response, and decreases the steady-state error and relative stability;
2. Derivative (D) action increases the relative stability and the sensitivity to noise;
3. Integral (I) action eliminates the steady-state error and decreases the relative stability.

It is observed that by introducing more general control actions, more satisfactory compromises between the effects of P, I, D actions can be achieved. Combining these actions, more powerful and flexible design methods can be developed to satisfy the specifications of the system to be controlled.

### 4.5.1 Fractional Order PID Controllers

The fractional-order $PI^{\lambda}D^{\delta}$ controller has been proposed as a generalization of classical PID controller with integrator of real order $\lambda$ and differentiator of real order $\delta$ [104]. The fractional PID controller takes a form

$$u_{FOPID} = K_p e + K_i J^{\lambda}(e) + K_d D^{\delta}(e) \tag{4.7}$$

$K_p$, $K_i$, $K_d$ are the proportional, integration and differentiation constant and $\lambda$, $\delta$ are the fractional orders of the integral and derivative parts of the controller, respectively. Therefore this controller has five parameters to tune. The conventional PID controller can be considered as a case of the fractional PID (FOPID) controller with $\lambda = \delta = 1$. The objective is to design a fractional-order controller so that the system fulfils different performance specifications.

## 4.6 Fractional Order PID Control for SLFM

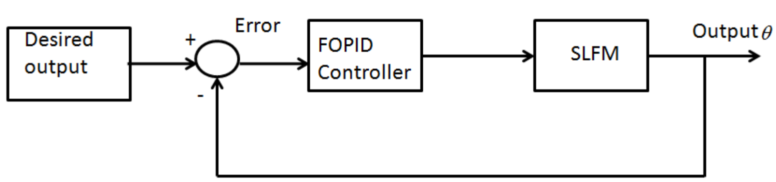


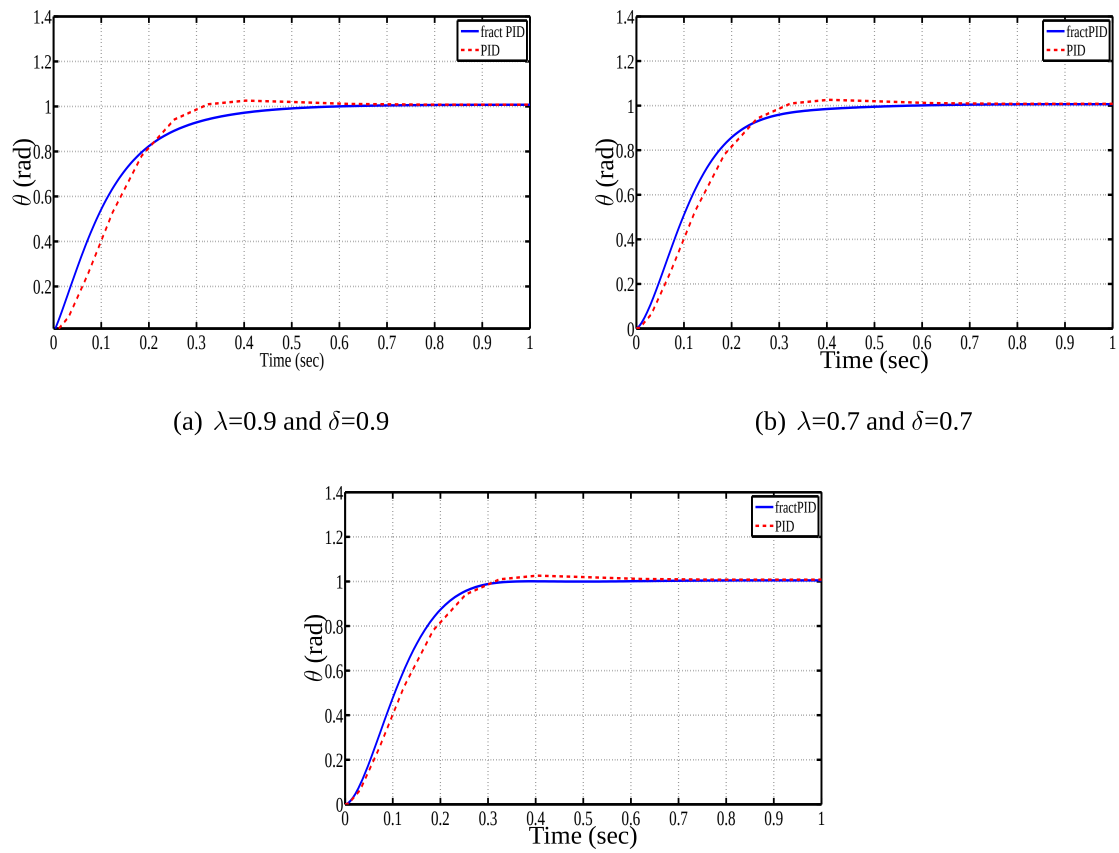

Figure 4.1: Block diagram of FOPID controller for SLFM plant

Figure 4.1 shows a simplified block diagram of a fractional order PID controller for a single link flexible manipulator plant. Dynamic model of this plant has been developed in Chapter 2. Fractional PID control is developed for this plant and is compared with integer order PID control. The desired performance specifications for $\theta$ are taken as settling time $T_s = 0.9$ second and % overshoot $< 10\%$ with minimum vibrations of the tip.

## 4.7 Simulation Results

The SLFM plant in (2.21) has been simulated to follow a unit step input command. IOPID (4.6) and FOPID (4.7) were tested in simulation. A conventional PID controller is designed and tuned to get the desired performance indices as $T_s$= 0.9 second, % overshoot $<$ 10%. Controller gains were tuned to obtain the desired performance. Gains for IOPID were $K_p = 3.5, K_d = 1, K_i = .001$.

Then a fractional order PID controller was designed for the same plant and same values of tuning parameters $K_p, K_d, K_i$. It was further tuned for its fractional orders of integration and differentiation namely $I^{\lambda}$ and $D^{\delta}$ to get approximately same performance as that of PID controller. The following Figure 4.2 depicts some of the results obtained for different values of $\lambda$ and $\delta$ for FOPID controller.




(a) $\lambda$=0.9 and $\delta$=0.9

(b) $\lambda$=0.7 and $\delta$=0.7

(c) $\lambda$=0.5 and $\delta$=0.5

Figure 4.2: Output response of FOPID and IOPID

Figure 4.3 shows a comparative output response for variations of $\lambda$ and $\delta$.

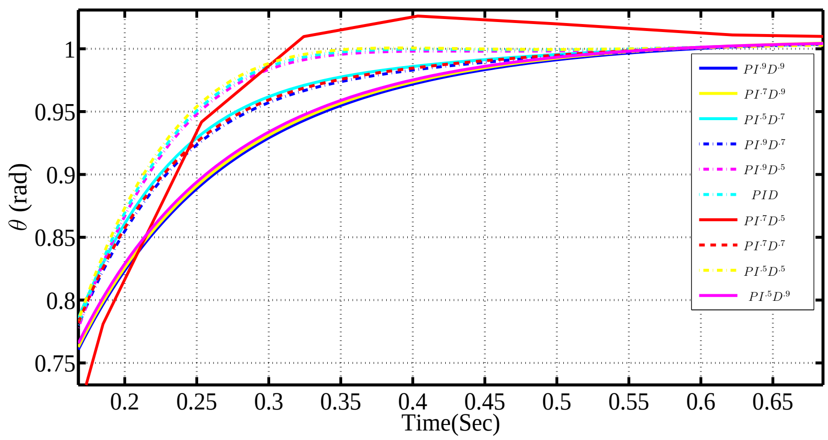



Figure 4.3: Fractional order PID

The change in the performance indices e.g. rise time $T_r$ and delay time $T_d$ due to change in the fractional variables $\delta$ and $\lambda$ of the fractional PID controller are tabulated in the following Table 4.1.

| $\delta$ | $\lambda$ | $T_r(sec)$ | $T_d(sec)$ |
|---|---|---|---|
| 0.9 | 0.9 | 0.36 | 0.2 |
| | 0.7 | 0.35 | |
| | 0.5 | 0.34 | |
| 0.7 | 0.9 | 0.29 | 0.17 |
| | 0.7 | 0.28 | |
| | 0.5 | 0.27 | |
| 0.5 | 0.9 | 0.26 | 0.1 |
| | 0.7 | 0.25 | |
| | 0.5 | 0.24 | |

Table 4.1: Comparison of control performance for fractional PID

From the Table 4.1, it is observed that as the order of fractional derivative and integration $\delta$ and $\lambda$ are reduced, rise time and delay time are also reduced increasing the corresponding speed of response of the system. Fractional PID controller thus can be tuned exactly according to the accuracy of the performance indices for a system.

## 4.8 Fractional Order Sliding Mode Control

Convincing results of FOPID motivated us to investigate fractional SMC. Fractional order SMC (FSMC) can be realized with fractional model or fractional controller or combination of both. Initially the fractional controller is investigated for the implementation of FSMC for control of SLFM. Fractional sliding surface is proposed wherein fractional order differentiation are used in the governing differential equation of the sliding surface.

### 4.8.1 Design of Sliding Surface

Sliding surface is proposed as a linear combination of states identified with fractional derivatives. The state vector is modified as,

$$\mathbf{x}_f = \begin{bmatrix} x_1 & x_2 & D^{\mu}x_1 & D^{\mu}x_2 \end{bmatrix} \tag{4.8}$$

This preposition redefines the system model as follows;

$$\dot{\mathbf{x}}_f = A\mathbf{x}_f + \mathbf{b}u \tag{4.9}$$

The surface proposed is

$$s_{fr} = \mathbf{c}^T\mathbf{x}_f \tag{4.10}$$

where $\mathbf{c}^T$ is a sliding surface matrix designed in Chapter 3 and given by (3.16).

$$\mathbf{c}^{\mathbf{T}} = [c_1 \ c_2 \ c_3 \ c_4] = [1.7321 \ \ -19.0164 \ 1.0784 \ \ -0.3358]$$

From (4.10) it is observed that the fractional-order differentiation of $x_1$ and $x_2$ is used to construct the sliding surface. Therefore this surface is a fractional order sliding surface.

### 4.8.2 Synthesis of Control Law

The control is devised using Gao's Proportional and constant rate reaching law

$$\dot{s}_{fr} = -k\text{sgn}(s_{fr}) - q(s_{fr}) \tag{4.11}$$

where $k$, $q > 0$ are tuning parameters chosen to ensure existence of sliding. This law ensures finite time convergence to the sliding surface.
Differentiating (4.10),

$$\dot{s}_{fr} = \mathbf{c}^T \dot{\mathbf{x}}_f$$

$$\dot{s}_{fr} = \mathbf{c}^T (A\mathbf{x}_f + \mathbf{b}u)$$

Equating it with the reaching law (4.11),

$$-k\text{sgn}(s_{fr}) - q(s_{fr}) = \mathbf{c}^T (A\mathbf{x}_f + \mathbf{b}u)$$

Control can be found as

$$u = (\mathbf{c}^T\mathbf{b})^{-1}(-\mathbf{c}^T A\mathbf{x}_f - \text{sgn}(s_{fr})) - qs_{fr}) \tag{4.12}$$

This control ensures existence of sliding.
Stability of the proposed fractional order sliding surface is analyzed subsequently.

### 4.8.3 Stability Analysis

Substituting the system dynamics $x_3 = D^\mu x_1$ and $x_4 = D^\mu x_2$, surface becomes

$$s_{fr} = c_1x_1 + c_2x_2 + c_3D^\mu x_1 + c_4D^\mu x_2$$

which can be written as

$$s_{fr} = c_1x_1 + c_2x_2 + c_3D^{\mu-1}\dot{x}_1 + c_4D^{\mu-1}\dot{x}_2 \tag{4.13}$$

During sliding $s_{fr} = 0$,

$$c_1x_1 + c_2x_2 + c_3(D^{\mu-1}\dot{x}_1) + c_4(D^{\mu-1}\dot{x}_2) = 0 \tag{4.14}$$

Therefore

$$c_3 D^{\mu-1}\dot{x}_1 + c_4(D^{\mu-1}\dot{x}_2) = -c_1 x_1 - c_2 x_2$$

Taking $(1-\mu)^{th}$ order derivative and assuming $\beta = 1-\mu$, gives

$$c_3\dot{x}_1 + c_4\dot{x}_2 = -c_1 D^{\beta} x_1 - c_2 D^{\beta} x_2 \tag{4.15}$$

Taking Laplace of (4.15) and assuming $x_2(0) = 0$, which is valid for no initial vibrations, we get

$$c_3(sX_1(s) - X_1(0)) + c_4(sX_2(s)) = -c_2 s^{\beta} X_2(s) - c_1(s^{\beta}X_1(s) - s^{\beta-1}X_1(0))$$

Rearranging,

$$X_2(s)(c_4 s + c_2 s^{\beta}) = c_3 X_1(0) + c_1 s^{\beta-1} X_1(0) - c_3 s X_1(s) - c_1 s^{\beta} X_1(s)$$

$X_2(s)$ is calculated as

$$X_2(s) = \frac{c_3 X_1(0)}{(c_4 s + c_2 s^{\beta})} + \frac{c_1 s^{\beta-1} X_1(0)}{(c_4 s + c_2 s^{\beta})} - \frac{c_3 s X_1(s)}{(c_4 s + c_2 s^{\beta})} - \frac{c_1 s^{\beta} X_1(s)}{(c_4 s + c_2 s^{\beta})}$$

$$X_2(s) = \frac{c_3 X_1(0)}{c_2 s^{\beta}(c_4/c_2 s^{1-\beta}+1)} + \frac{c_1 s^{\beta-1} X_1(0)}{c_2 s^{\beta}(c_4/c_2 s^{1-\beta}+1)} - \frac{c_3 s X_1(s)}{c_2 s^{\beta}(c_4/c_2 s^{1-\beta}+1)} - \frac{c_1 s^{\beta} X_1(s)}{c_2 s^{\beta}(c_4/c_2 s^{1-\beta}+1)}$$

which further implies,

$$\begin{aligned} X_2(s) = {} & \frac{c_3 X_1(0)}{c_2 s^{\beta}(1+s^{1-\beta}c_4/c_2)} + \frac{c_1 X_1(0)}{c_2 s(1+s^{1-\beta}c_4/c_2)} \\ & - \frac{s c_3 X_1(s)}{c_2 s^{\beta}(1+s^{1-\beta}c_4/c_2)} - \frac{c_1 X_1(s)}{c_2(1+s^{1-\beta}c_4/c_2)} \end{aligned}$$

Let $c_2/c_4 = a$ and taking the Laplace inverse, the solution takes the form as

$$x_2(t) = k_a E_{\mu}(-at^{\mu})x_1(0) + k_b(t^{\mu-1}E_{\mu}(-at^{\mu}))x_1(t) \tag{4.16}$$

where $k_a, k_b$ are constants and $E_{\mu}$ is a Mittag-Leffler (ML) function defined as

$$E_{\mu}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\mu k + 1)}$$

Decay of ML function is a power law type function near t=0 and inverse power law type asymptotically. It decays at a faster rate than $e^{\pm t}$ near t=0.

From (4.16) it is observed that $x_2(t)$ converges to $x_1(t)$ and $x_1(0)$ through a decaying ML function. Substituting $x_2(t)$ in (4.14), we get the equation only in terms of $x_1(t)$ and as $t \to \infty$ the fractional order equation becomes

$$c_1 x_1 + c_3 D^{\mu} x_1 = 0 \tag{4.17}$$

which implies

$$c_3 D^{\mu} x_1 = -c_1 x_1 \tag{4.18}$$

taking $(1-\mu)^{th}$ order derivative of (4.18),

$$\dot{x}_1 = -c_1/c_3 D^{1-\mu} x_1 \tag{4.19}$$

Taking Laplace with the initial conditions and simplifying this equation,

$$x_1(t) = k_c E_{\mu}(-bt^{\mu}) x_1(0) \tag{4.20}$$

where $k_c$ and $b$ are constants. Again this is a convergent ML function. From equations (4.16) and (4.20) it is observed that $x_1(t)$ and $x_2(t)$ converge to $0$ asymptotically. Thus the system is stable during sliding.

### 4.8.4 Chattering Attenuation

State $x(t)$ of fractional order system decays towards $0$ with power rate function $t^{-\mu}$ and integer order system decays towards $0$ with exponential function $e^{-At}$. It means that the energy transfer is slower with fractional order sliding surface than that with integer order counterpart during sliding [109]. Also, it can be seen that because of the integration effect of the operator $D^{\mu-1}$ in (4.13), the proposed FSMC method has smaller chattering amplitude than the conventional SMC method.

## 4.9 Simulation Results

Simulation studies are carried out to demonstrate the effectiveness of the proposed FSMC. The plant in (2.21) was simulated with controller in (4.12). The aim was to stabilize the output i.e.

motor deflection angle $\theta$ in about 4 seconds from the initial condition of 0.6 rad. Controller tuning parameters were tuned to get the desired performance. The time response specifications have been approximately maintained same with SMC and FSMC. Both the controllers were designed to yield the settling time of 4 seconds. The results are then compared with respect to control efforts required. To reduce the chattering, sign function is replaced by a sigmoid function for both the schemes.

The simulation parameters are:

For SMC Controller : Gains: k=1.5, q=6

For FSMC Controller : Gains: k=0.3, q=1.8, $\mu = 0.45$

Figures 4.4 and 4.5 evolution of the actuation angle and angular velocity respectively whereas Figure 4.6 and 4.7 show the evolution of the tip deflection and its rate. Finite convergence of the sliding surface is seen from Figure 4.7 and the control efforts are depicted from Figure 4.15.

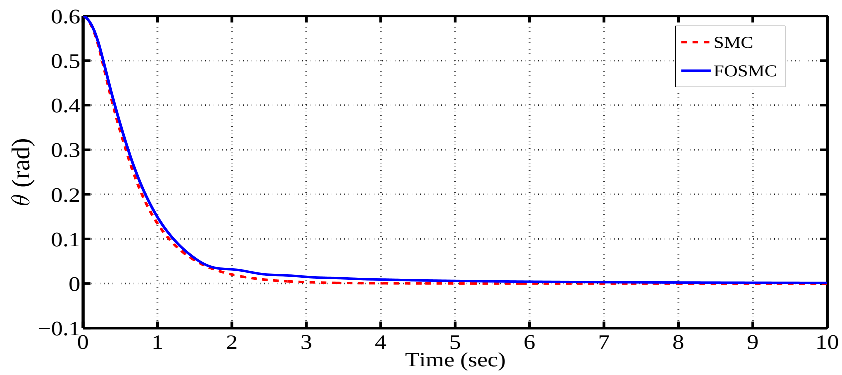



Figure 4.4: Plot of output angular displacement verses time

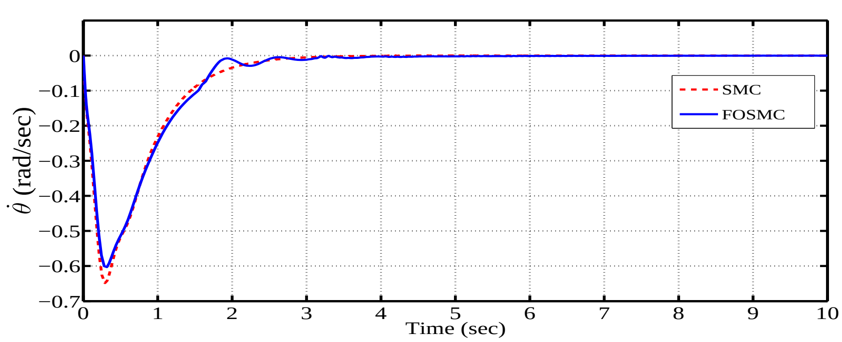



Figure 4.5: Plot of variation of angular velocity

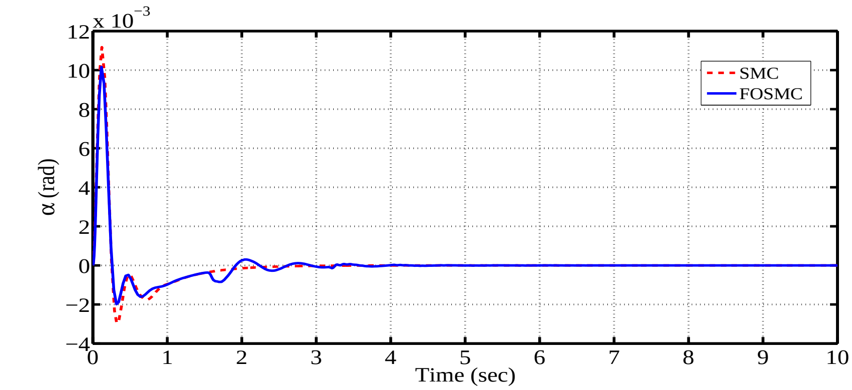



Figure 4.6: Plot of tip displacement verses time

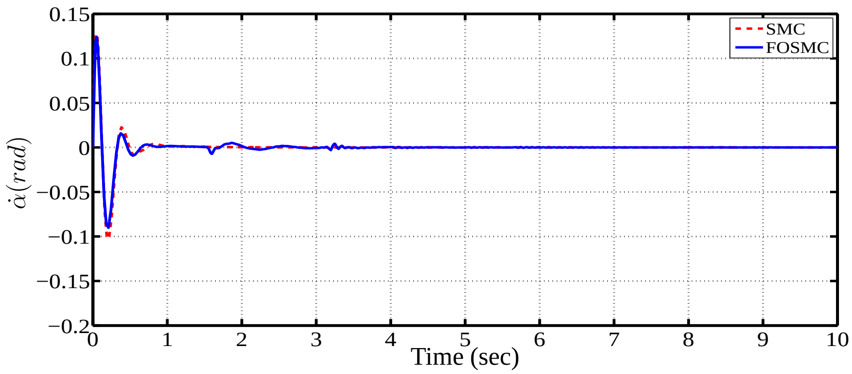



Figure 4.7: Rate of change of tip displacement

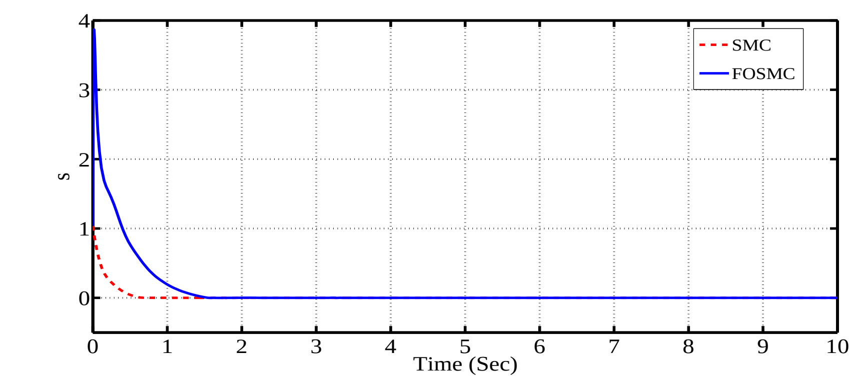



Figure 4.8: Evolution of sliding surface

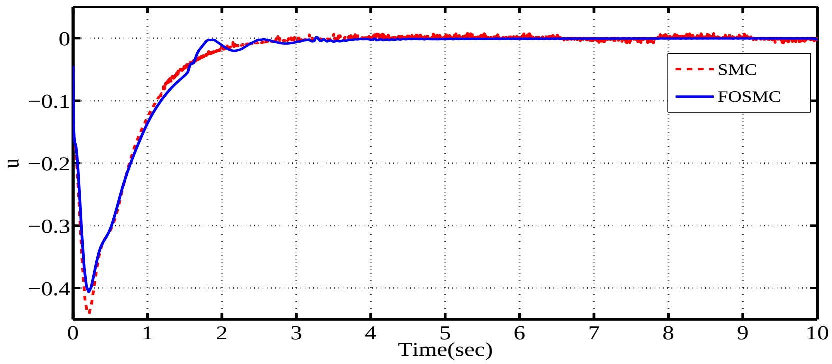



Figure 4.9: Control efforts

Performance of both the controllers namely SMC and FSMC are almost same, however reduced controller gains are used in FSMC compared to SMC. This is a measure of control efforts utilised for implementing the control. Also it is observed from Figure 4.15, that chattering is reduced in FSMC compared to SMC.

The robustness against external disturbance and parameter variations is also tested. Performance of the controllers is tested for $10\%$ uncertainty in plant parameters alongwith a matched disturbance of 0.01sin(t). The simulation parameters are:

SMC Controller : Gains: k=7, q=10

FSMC Controller : Gains: k=4, q=2, $\mu = 0.7$

The simulation results for robustness investigations are shown from Figure (4.10) to Figure (4.14). Figure (4.10) and Figure (4.11) show plots of stabilization of $\theta$ and $\dot{\theta}$, while Figure (4.12) and Figure (4.13) show that of $\alpha$ and $\dot{\alpha}$. Sliding surface is shown by Figure (4.14) and control efforts by Figure (4.15).

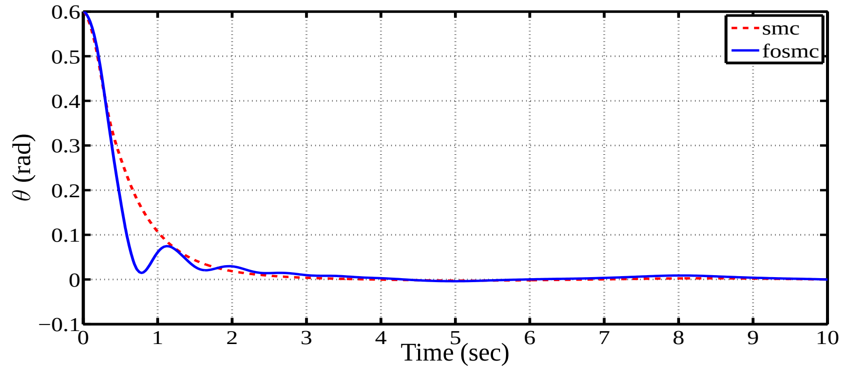



Figure 4.10: Output angular displacement verses time

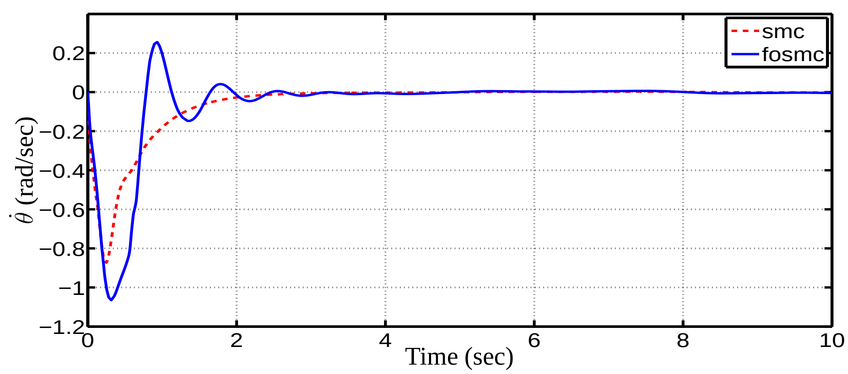



Figure 4.11: Plot of variation of angular velocity

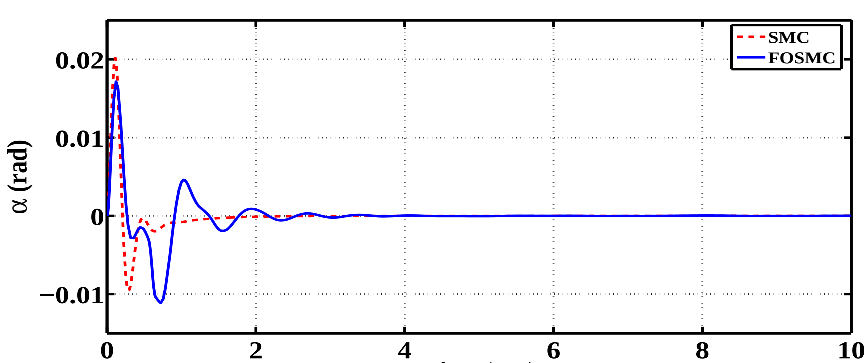



Figure 4.12: Plot of tip displacement verses time

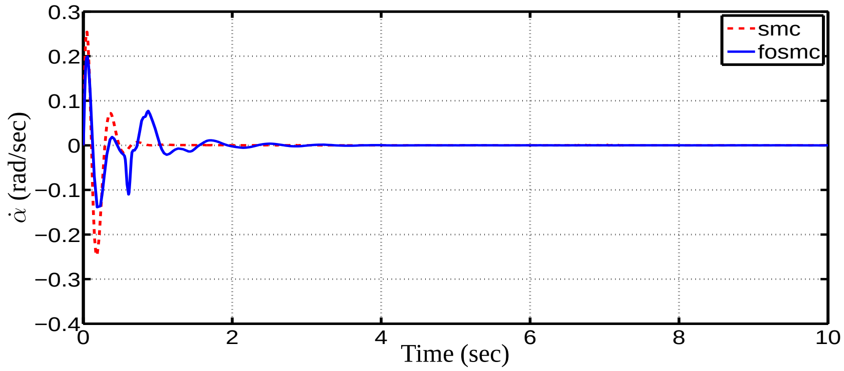



Figure 4.13: Rate of change of tip displacement

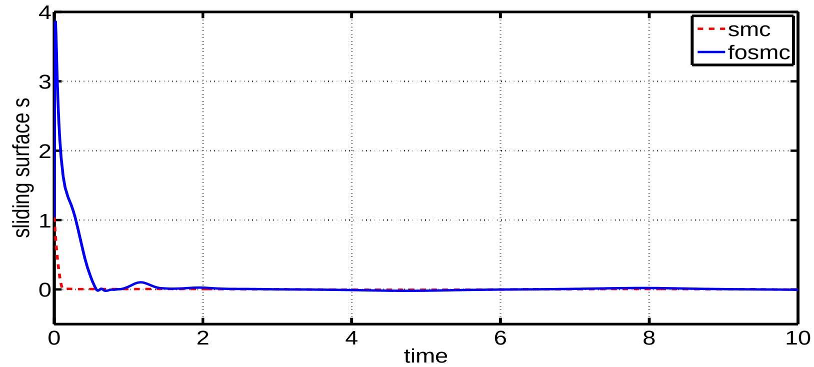



Figure 4.14: Evolution of sliding surface

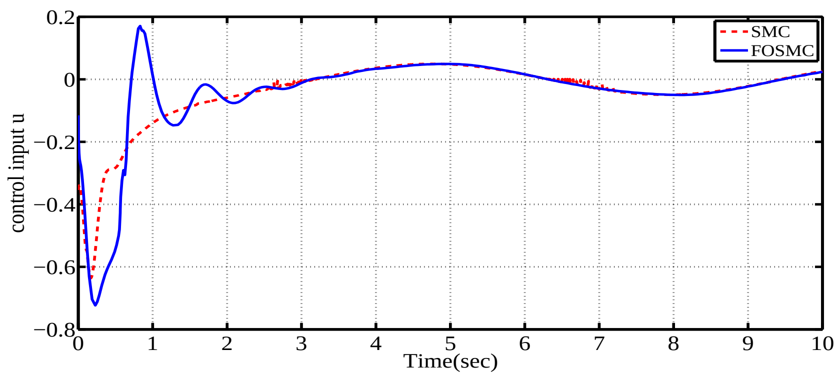



Figure 4.15: Control efforts

The angular displacement and the vibrating tip position stabilizes in approximately 4 seconds. Both the controllers ensure robustness against parametric variation and external disturbance. Qualitative comparison of both the controllers for stabilization of the output considering the uncertainty and external disturbance is tabulated in Table 4.2.

| Strategy | Parameter Var | | Parameter Var + Dist | |
|---|---|---|---|---|
| | $T_s(sec)$ | $\|u\|_2$ | $T_s(sec)$ | $\|u\|_2$ |
| SMC | 3 | 2.63 | 3 | 3.67 |
| FSMC | 3 | 2.06 | 3 | 3.2 |

Table 4.2: Comparison of control performance for SMC and FSMC

Table 4.2 shows the comparison of control performances with FSMC and SMC. It shows reduction in the control input with FSMC of about 20% for approximately same time domain specifications.

## 4.10 Summary

Fractional calculus (FC) can be considered as a super set of integer-order calculus. It has been used to devise control to accomplish desired performance with less control than what integer calculus promises. The introduction has given the thought that there is a wonderful universe of mathematics lying within the boundary of one complete differentiation and one complete integration.

In this Chapter two fractional controllers namely fractional PID and fractional SMC have been investigated for the control of SLFM system. Simulation results were presented to demonstrate the effectiveness of the FOPID, maintaining the advantages of integer order PID. The extra degrees of freedom using FOPID made it possible to further improve the performance compared with conventional integer order PID controller.

Fractional sliding surface was designed by incorporating fractional derivatives instead of integer derivatives. The same sliding surface matrix designed for integer SMC was considered while obtaining fractional sliding surface. Gao's reaching law has been considered to obtain fractional sliding mode control.

Simulation results have been presented to demonstrate the effectiveness of the fractional controllers. It has been found that FSMC consumes less control energy to yield the same performance as compared to that of SMC. Moreover FSMC exhibits less chattering.

With the encouraging results of fractional control, study in fractional calculus was continued to examine the efficacy of fractional model, which is followed in the coming chapter.