Chapter 3

Control of SLFM using HOSMC

3.1 Introduction

In the previous Chapter, mathematical model of SLFM and 2LFM has been described. All those models are approximate representations of actual systems. In fact there is always a mismatch between the actual plant dynamics and its mathematical model used for the controller design. This difference mostly comes from external disturbances, unknown plant parameters and unmodeled dynamics. A controller therefore needs to assure the desired closed loop system performance in presence of these disturbances/ uncertainties.

This chapter is intended to illustrate the development of robust control algorithms using Higher Order Sliding Mode (HOSM) approach for control of FLM. Initially philosophy of sliding mode control is described along with its advantages and limitations. Then the notion of HOSM for the development of the control algorithms is presented. Further second order sliding mode (SOSM) using TA and STA controllers are proposed for control of SLFM.

3.2 Sliding Mode Control: A Robust control

SMC is a special variable structure control (VSC). It is essentially a nonlinear control technique. In VSC, two or more subsystems having fixed control structures are combined to exploit the useful properties of the individual subsystems to possess new properties that may not be present in any of the subsystems. The VSC that yields the closed-loop response totally insensitive to a particular class of uncertainties is called a sliding mode control (SMC) [86]. SMC utilizes a high speed switching law to drive the system trajectory on a user defined surface, called as a sliding surface and to maintain it there for all subsequent time. The sliding surface is designed to be inherently stable to render this robustness property to the system. The control ensures the system trajectories to reach the sliding surface and remain on it thereafter. SMC inherits the following advantages.

The controller is robust against parameter variation, unmodelled dynamics and external disturbances and uncertainties if it satisfies the matching condition. Load forecast is not required, accurate knowledge of plant parameters is not necessary. The design procedure is simple, cost of implementing the control is low and no parameter estimation is necessary. Therefore it is preferred against the other well known robust control techniques such as adaptive control, fuzzy control, $H\infty$ control, backstepping technique, etc.

In SMC, a stable sliding variable has to be selected such that the control input appears in the first time derivative of the sliding variable i.e. $\dot{s}$ [87]. This relates to relative degree one of the surface and thus classical SMC is also called as first order SMC. Here s is made zero in finite time while system states may converge to zero asymptotically.

For ideal sliding motion infinitely fast switching of the control is required which results in discontinuous control. However practically, switching is possible only at a finite frequency causing the trajectories to oscillate within a neighbourhood of the sliding surface. This high frequency motion is termed as chattering. This is a major drawback of SMC. Shortcomings of SMC are listed below.

- High frequency switching control action called ”chattering effect”, which limits the practical implementation of the control.
- It is robust to matched disturbances only.

- It requires complete state vector for implementing the control.
- Discontinuous control limits its practical application.

Chattering phenomenon makes sliding mode control unacceptable for mechanical systems where it may excite the unmodelled high frequency dynamics and cause increased wear and tear of the actuator. Many researchers attempted to deal with chattering issue [88]- [90]. Use of sigmoid or saturation function is one of the earliest reported approach [91] [92]. These functions in place of signum function provide a boundary layer for the switching to take place. Thus the control has attenuated chatter, but with reduced robustness. To overcome all the mentioned difficulties due to chattering, a new research paradigm called higher order sliding mode control (HOSMC) was proposed by Emelyanov, Levant et. al. [93]- [95]. The following section describes the notion of HOSM control.

3.3 Higher Order Sliding Modes: Preliminaries

Higher order sliding mode control is a generalized sliding mode control. It allows all restrictions of SMC to be removed, while preserving the main sliding mode features with improved sliding accuracy. Emelyanov initially presented the idea of acting on the higher derivatives of the sliding variable and provided second order sliding algorithms. Since then, a number of such controllers have been developed in the literature and are currently finding useful applications [96]- [99]. Theoretical concepts and definitions and its relation with higher order sliding mode is presented, in the following subsections.

3.3.1 Filippov's theory and sliding motion

Discontinuous control is inherent in SMC. It appears in the governing differential equation of the closed loop dynamics because of the 'signum' function. A solution of this differential equation containing discontinuity is required to analyze system. Such equations can be analyzed using Filippov's theory. Filippov's differential inclusion theory is used to understand and suitably interpret the differential equation with discontinuous right hand side [100].

Filippovs differential inclusion theory gives values of a vector function not at the discontinuity point but in its immediate neighborhood. Then, the function at the discontinuity point is replaced with an average function, taken from a set generated by the convex combination of the values at each side of the discontinuity point. Filippov solutions are absolutely continuous curves. Consider a general differential equation with discontinuous right hand side

$$\dot{x} = f(x) \tag{3.1}$$

where $x \in \mathbb{R}^1$ and $f(x)$ is a locally bounded measurable vector function. A continuous function $x(t)$ is said to be a solution of (3.1) in the Filippov sense if it is the solution of the differential inclusion

$$\dot{x} \in F(x) \tag{3.2}$$

where, $F(x)$ is the closed convex hull of all limits of $f(x)$.

Equation (3.2) is called a Filippov differential inclusion if the set $F(x)$ is non-empty, closed, convex, locally bounded and upper-semicontinuous.

Therefore, any solution of (3.1) is defined as an absolutely continuous function $x(t)$, satisfying the differential inclusion (3.2) almost everywhere. In the sequel, the solution of differential equations with discontinuous right hand side will be assumed in Flippovs sense.

Sliding Order:

Consider a non linear uncertain system

$$\dot{\mathbf{x}} = f(\mathbf{x}) = f(\mathbf{x}) + g(\mathbf{x})u \tag{3.3}$$

where $\mathbf{x} \in \mathbb{R}^{\mathbf{n}\times\mathbf{1}}$, $u \in \mathbb{R}^1$ is the control input.Consider a smooth output function

$$s = \mathbf{c}^T\mathbf{x} \tag{3.4}$$

where $\mathbf{c}^T \in \mathbb{R}^{1\times n}$ If time derivatives of output function s upto order $(r-1)$ exist i.e.

$$s, \dot{s}, ..., s^{(r-1)}$$

are continuous functions of $\mathbf{x}$ and are single valued and the next derivative s^r does not exist as a single-valued function of $\mathbf{x}$, then s is said to have relative degree r.

If the system in equation (3.3) has a relative degree r with respect to the sliding variable s defined in equation (3.4), then the sliding surface dynamics can be expressed as

$$\begin{aligned} s^r &= \mathbf{c}^T L_f^r \mathbf{x} + \mathbf{c}^T L_g L_f^{r-1} u \\ &= \varphi(\mathbf{x}) + \gamma(\mathbf{x}) u \end{aligned} \tag{3.5}$$

where L_g, L_f are Lie derivatives and $L_g L_f^{r-1} \mathbf{c}^T(x) \neq 0$ The above nonlinear uncertain dynamics of sliding variable (3.5) can be stabilised with the help of a discontinuous control u, such that sliding exists for order $r-1$. It implies that

$$s = \dot{s} = ... = s^{r-1} = 0. \tag{3.6}$$

Then the set (3.6) is called as r^{th} order sliding set and r is called as sliding order. The motion on the set (3.6) is said to exist in an r-sliding mode. This set constitutes an r-dimensional condition on the system states [94].

If u appears in $\dot{s}$, then $s = 0$, is called as first order sliding mode. The control that forces trajectories in the set

$$\{\mathbf{x} | s = 0\}$$

in finite time is classical first order SMC. In case of second order sliding modes a sliding set is

$$\{\mathbf{x} | s, \dot{s} = 0\}$$

The control appears explicitly in second derivative of s. The control that constraints trajectory to this 2-sliding set is called SOSMC.

3.3.2 SOSM Algorithms

The algorithm that assures second order sliding modes are SOSM algorithm. The controller which establishes second order sliding modes is called SOSMC.

Twisting Algorithm (TA)

The twisting algorithm (TA) has been proposed by [101]. For the dynamics

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -k_{11}sign(x_1) - k_{12}sign(\dot{x}_1)\end{aligned}$$

x_1 and x_2 converge to zero if $k_{11} > k_{12} > 0$. The control that assures above is called Twisting Control (TC). If the sliding variable is defined as $s = x_1$, then the TC needs to constrain system dynamics to

$$\ddot{s} = -k_{11}sign(s) - k_{12}sign(\dot{s}) \tag{3.7}$$

k_{11} and k_{12} are tuning parameters. Now for any system of relative degree 2 such as

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})u \tag{3.8}$$

and a sliding variable

$$s = \mathbf{c}^T\mathbf{x}$$

$\ddot{s}$ can take form

$$\ddot{s} = \phi(x) + \gamma(x)u \tag{3.9}$$

where $\phi(x) = \mathbf{c}^T L_f^2 x$ and $\gamma(x) = \mathbf{c}^T L_g L_f x$
With this assumption of $-C < \phi(x) < C$ and $k_m < \gamma(x) < k_M$ where C, K_m, K_M are known constants parameterizing the uncertainty of the original system. It is required to get u such that s and $\dot{s}$ are made zero in finite time. (3.7) can be written using differential inclusion

$$\ddot{s} \in [-C, C] + [K_m, K_M]u \tag{3.10}$$

From 3.7 and 3.10 the twisting controller takes the form,

$$u = -k_{11}sign(s) - k_{12}sign(\dot{s}) \tag{3.11}$$

The above controller ensures finite time convergence of s, $\dot{s}$ if

$$\begin{aligned}K_m(k_{11} + k_{12}) - C &> K_M(k_{11} - k_{12}) + C\\ K_m(k_{11} - k_{12}) &> C\end{aligned}$$

The detail discussion about the choice of k_{11} and k_{12} is given in [102].

3.3.3 Convergence of the Twisting Algorithm

The TA using variables is also written as

$$\ddot{s} = \frac{d\dot{s}}{dt} = \frac{d\dot{s}}{ds}\dot{s} = -k_M sgns - k_m sgn\dot{s}, \tag{3.12}$$

$$\begin{aligned} \ddot{s} &= -K_M sgns, \qquad s\dot{s} > 0 \\ &= -K_m sgns, \qquad s\dot{s} < 0 \end{aligned} \tag{3.13}$$

which implies

$$\begin{aligned} \frac{d\dot{s}}{ds} &= \frac{-K_M}{\dot{s}}, s > 0, \dot{s} > 0 \\ &= \frac{-K_m}{\dot{s}}, s > 0, \dot{s} < 0 \\ &= \frac{K_M}{\dot{s}}, s < 0, \dot{s} > 0 \\ &= \frac{K_m}{\dot{s}}, s < 0, \dot{s} < 0 \end{aligned}$$

The vector field takes the shape shown in Figure 3.1.

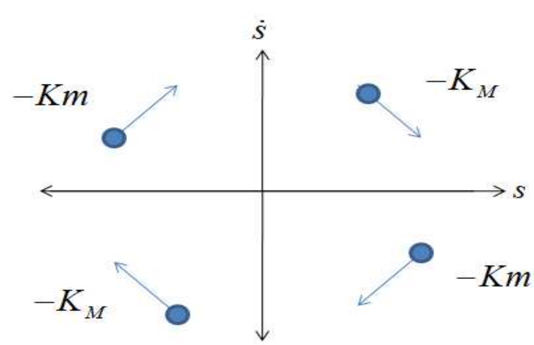

Figure 3.1: Vector field of Twisting Controller

It is obvious that this trajectory converges if $K_M > K_m > 0$. From (3.11) and (3.12) if

$$\begin{aligned} k_{11} + k_{12} &= K_M \\ k_{11} - k_{12} &= K_m \end{aligned}$$

convergence of $s, \dot{s}$ is assured.

The phase trajectory can be sketched starting from any initial condition, say $P_1 = (0, \dot{s}_0)$. It shows twisting nature as shown in Figure 3.2. The solution of (3.13) for the first quadrant can be found as follows

$$\begin{aligned} \int_0^{\dot{s}} \dot{s} d\dot{s} &= \int_{s_1}^{s} -K_M ds \\ \frac{1}{2}\dot{s}^2 &= K_M(s_1 - s) \\ s &= s_1 - \frac{\dot{s}^2}{2K_M} \end{aligned}$$

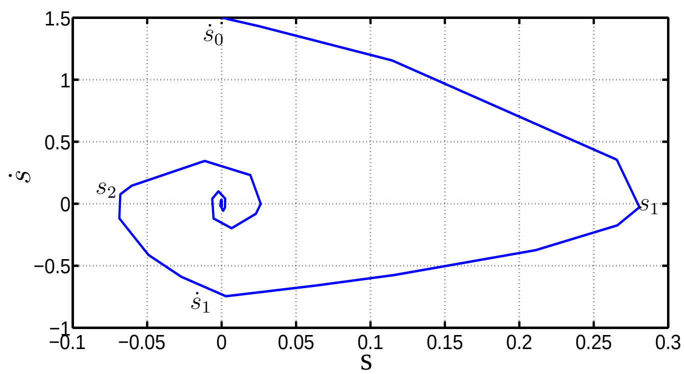

Figure 3.2: Twisting Trajectory

similarly an expression for the second quadrant, can be written as

$$s = s_1 - \frac{\dot{s}^2}{2K_m}$$

Taking the ratio of the absolute values of $\dot{s}_0$ and $\dot{s}_1$, we get

$$\frac{|\dot{s}_1|}{|\dot{s}_0|} = \sqrt{\frac{K_m}{K_M}} = q < 1$$

Extending this for the complete trajectory, we can generalize $\frac{|s_{i+1}|}{|s_i|} = q < 1$

This shows that the trajectory converges to the origin in finite time. The time of convergence can be calculated as shown below.

3.3.4 Time of Convergence

Let's take t_1 as the time taken by the auxiliary system trajectories to go from the point $\dot{s}_0$ to s_1. Integrating (3.13) in the first quadrant,

$$\int_{\dot{s}_0}^{\dot{s}} d\dot{s} = \int_0^t K_M dt$$

gives

$$\dot{s}(t) = -K_M t + \dot{s}_0$$

but $\dot{s}_0(t_1) = 0$, therefore we get $t_1 = \frac{\dot{s}_0}{K_M}$.
If t_2 is the time taken to travel from s_1 to $\dot{s}_1$, it can be found as $t_2 = \dot{s}_0\sqrt{\frac{1}{K_M K_m}}$.
Recursively the total time of convergence of the trajectory can be found as
$T = \frac{\alpha \dot{s}_0}{1-q}$, where $\alpha = \frac{1}{K_M} + \sqrt{\frac{1}{K_M K_m}}$.
The main problem in implementation of TA is increasing information demand, i.e. requirement of sliding variable and its (r-1) derivatives. These can be generated through differentiators which has its own inherent issues. It is also evident from the TA that the signum function is present in the control law (for a relative degree 2 system), thus retaining the undesirable chattering issue. With these limitations in TA, the Super twisting algorithm was investigated by Levant et. al.

Super Twisting Algorithm (STA)

The dynamics given by

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -k_1|x_1|^{1/2} sign(x_1) - k_2 \int sign(x_1) \end{aligned}$$

is called as supertwisting algorithm (STA).If a sliding variable is defined as $s = x_1$, the STA needs to constrain sliding dynamics to

$$\dot{s} = -k_1|s|^{1/2} sign(s) - k_2 \int sign(s) \tag{3.14}$$

with $k_1 > k_2$, s and $\dot{s}$ then converges to zero in finite time. Now for a system with relative degree 1

$$\dot{s} = \bar{\phi}(\mathbf{x}) + \bar{\gamma}(\mathbf{x})u$$

which can be written as

$$\dot{s} = u + d(x,t)$$

where $d(x,t)$ is the system perturbation.

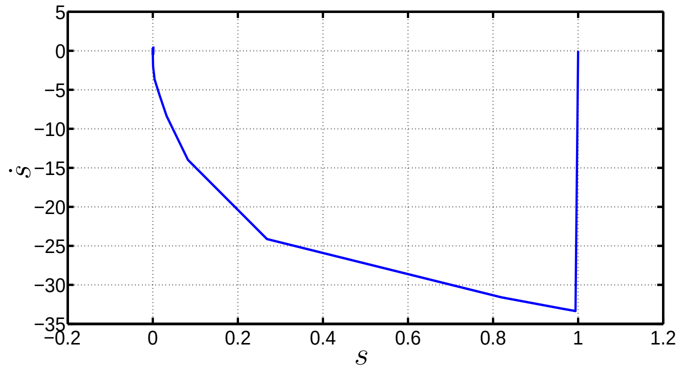

Figure 3.3: Super-twisting algorithm phase trajectory

It is proved in [103] that if the system perturbations are bounded as,

$$|d(x,t)| \leq \delta |s|^{1/2}$$

(where δ is a positive constant), the sliding exists in finite time if the gains are selected as;

$$k_1 > 2\delta,$$

$$k_2 > k_1 \frac{5\delta k_1 + 4\delta^2}{2(k_2 - 2\delta)}$$

It ensures exact finite time convergence, for any bounded uncertainty. For some positive constants k_1 and k_2 system converges to the second sliding mode set $s = \dot{s} = 0$. The controller is called as STA controller (STAC).

The features of STA are summarized below:

- STA ensures finite time convergence of s and $\dot{s}$.
- It is continuous.
- It can be implemented to relative degree 1 system w.r.t.s
- It does not require derivative of s.

3.4 HOSM Differentiator

Often differentiation of variable is required e.g for implementing TA, $\dot{s}$ is required. Supertwisting algorithm is used to get exact differentiation of a variable [101]. Let x_1 be a variable whose differentiation is required i.e. $\dot{x}_1$. It is assumed that $|\ddot{x}_1| \leq \chi$, with χ as a known constant. Considering $\dot{x}_1 = x_2$, the first order Levant differentiator [101] is

$$\left.\begin{aligned} \dot{\hat{x}}_1 &= -\eta_1|\hat{x}_1 - x_1|^{1/2}sign(\hat{x}_1 - x_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 &= -\eta_2 sign(\hat{x}_1 - x_1) \end{aligned}\right\} \tag{3.15}$$

$\hat{x}_1$, $\hat{x}_2$ are the representations of the estimates. Finite time, exact estimate $\dot{\hat{x}}_1$ is the required derivative obtained provided the gains are tuned as $\eta_1 > 1.5\chi$ and $\eta_2 > 1.1\chi$.

3.5 Control of SLFM using TA and STA

This section describes HOSMC design using TA and STA for the SLFM. The mathematical model of the SLFM developed using classical approach is described in Chapter 2, Section 2.2 (2.21). The design of SMC includes two stages. The first step is to design a stable sliding surface to assure asymptotic convergence of the system states during sliding. The second is concerned with the synthesis of the control law that drives the trajectory to this sliding surface in finite time and ensures to stay on it thereafter. This feature allows the system to have an enhanced performance, including insensitivity to parametric uncertainties and rejection to disturbances that satisfy the matching condition [95].

3.5.1 Surface Design

For the system in (2.21), the sliding surface is chosen to be a function of the states and is defined as

$$s = \mathbf{c}^T\mathbf{x} \tag{3.16}$$

where $\mathbf{x} \in \mathbb{R}^{(4\times1)}$ is a surface matrix to be designed. A stable surface can be designed using defined approaches like quadratic minimization, pole placement etc [73]. For $(\mathbf{b}) \in \mathbb{R}^1$, there exists an invertible matrix $T \in \mathbb{R}^{4\times4}$ such that

$T\mathbf{b} = [\mathbf{0}\ \mathbf{b_2}]^{\mathbf{T}}$, where $\mathbf{b_2} \in \mathbb{R}^{\mathbf{1\times1}}$ and is nonsingular. Using the coordinate transformation, $\mathbf{z}(t) \leftrightarrow T\mathbf{x}(t)$, where

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.7071 & 0 & 0.5000 & 0.5000 \\ -0.7071 & 0 & 0.5000 & 0.5000 \\ 0 & 0 & 0.7071 & -0.7071 \end{bmatrix}$$

The system dynamics in transformed co-ordinates are

$$\dot{\mathbf{z}}_1 = A_{11}\mathbf{z}_1 + A_{12}\mathbf{z}_2 \tag{3.17}$$

$$\dot{\mathbf{z}}_2 = A_{21}\mathbf{z}_1 + A_{22}\mathbf{z}_2 + b_2 u \tag{3.18}$$

where

$$\mathbf{z}_1 = [x_1\ \ x_2\ \ x_3]^T, \mathbf{z}_2 = [x_2]$$

and

$$A_{11} = \begin{bmatrix} 0 & 0.5000 & 0.5000 \\ -202.1000 & 0.3536 & 0.3536 \\ -202.1000 & -0.3536 & -0.3536 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.7071 \\ 0.5000 \\ -0.5000 \end{bmatrix}$$

$A_{21} = [586.6158\ \ -13.8946\ \ -13.8946]$, $A_{22} = [-19.6500]$

$b_2 = [48.9352]$

Partitioning s compatibly,

$$s = c_1\mathbf{z}_1 + c_2\mathbf{z}_2 \tag{3.19}$$

Here $c_1 \in \mathbb{R}^{1\times3}$ and $c_2 \in \mathbb{R}^{1\times1}$, then $det(c\mathbf{b}) = det(c_2\mathbf{b}_2)$.

For the matrix $(s\mathbf{b})$ to be nonsingular, the necessary condition is $det(c_2) \neq 0$.

During sliding motion, s becomes 0. Therefore

$c_1\mathbf{z}_1 + c_2\mathbf{z}_2 = 0$, $\mathbf{z}_2 = -c_2^{-1}c_1\mathbf{z}_1 = -M\mathbf{z}_1$, where

$$M = c_2^{-1}c_1, M \in \mathbb{R}^{1\times3}$$

Using this, the null space dynamics become

$$\dot{\mathbf{z}}_1 = A_{11}\mathbf{z}_1 + A_{12}Mz_1$$

M is designed to yield desired poles of $(A_{11} - A_{12}M)$.

The desired poles are $[-8 \ \ -11 \ \ -15]$

The designed sliding surface matrix is $\mathbf{c}^T = [1.7321 \ \ -19.0164 \ 1.0784 \ \ -0.3358]$

and the surface is $s = \mathbf{c}^T\mathbf{x}$ giving

$$s = [1.7321 \ \ -19.0164 \ 1.0784 \ \ -0.3358]\mathbf{x}$$

for SLFM plant.

3.5.2 Controller synthesis using SMC for SLFM

The sliding mode control is synthesized using surface s designed in previous section (3.16). Differentiating the same

$$\dot{s} = \mathbf{c}^T\dot{\mathbf{x}}$$

Substituting for $\dot{\mathbf{x}}$ from system dynamics

$$\dot{s} = \mathbf{c}^T(Ax + \mathbf{b}u) \tag{3.20}$$

Gao's constant plus proportional reaching law is

$$\dot{s} = -ksgn(s) - qs, \tag{3.21}$$

where $k > 0$ and $q > 0$ are the tuning parameters. Using (5.20) in (5.19), the control law becomes

$$u_{(SMC)} = (\mathbf{c}^T\mathbf{b})^{-1}(-\mathbf{c}^T A\mathbf{x} - ksgn(s) - qs) \tag{3.22}$$

3.5.3 Controller synthesis using TA

The same surface in (3.16)is used for devising TAC. Differentiating the surface

$$\dot{s} = \mathbf{c}^T\dot{\mathbf{x}}$$

Substituting for $\dot{\mathbf{x}}$ from system dynamics,

$$\dot{s} = \mathbf{c}^T(A\mathbf{x} + \mathbf{b}u) \tag{3.23}$$

The designed surface of SLFM has relative degree one wrt control input. Differentiating (3.23),

$$\ddot{s} = \mathbf{c}^T(A\dot{\mathbf{x}} + \mathbf{b}\dot{u}) \tag{3.24}$$

Defining augmented control $v = \dot{u}$, and using twisting algorithm

$$\ddot{s} = -k_{11}sign(s) - k_{12}sign(\dot{s}), \tag{3.25}$$

the control for SLFM is derived as follows.

$$v_{(TA)} = (\mathbf{c}^T\mathbf{b})^{-1}(-\mathbf{c}A\dot{\mathbf{x}} - k_{11}sgn(s) - k_{11}sgn\dot{s}) \tag{3.26}$$

Therefore controller using TA bcomes

$$u_{(TA)} = \int(\mathbf{c}^T\mathbf{b})^{-1}(-\mathbf{c}A\dot{\mathbf{x}} - k_{11}sgn(s) - k_{11}sgn\dot{s}) \tag{3.27}$$

3.5.4 Controller Synthesis using STA

The surface designed in (3.16) is used to device STA control Differentiating the surface (3.16)

$$\dot{s} = \mathbf{c}^T\dot{\mathbf{x}}$$

Substituting for $\dot{\mathbf{x}}$ from system dynamics

$$\dot{s} = \mathbf{c}^T(A\mathbf{x} + \mathbf{b}u) \tag{3.28}$$

and equating with (**??**), the control for SLFM is derived as follows.

$$-k_1|s|^{1/2}sign(s) - k_2\int sign(s) = \mathbf{c}^T(A\mathbf{x} + \mathbf{b}u)$$

The control is thus found as,

$$u_{(STA)} = (\mathbf{c}^T\mathbf{b})^{-1}(-\mathbf{c}^TA\mathbf{x} - k_1|s|^{1/2}sgn(s) - \int k_2sgn(s)) \tag{3.29}$$

3.6 Simulation Results

This section illustrates the performance of the controllers through simulation results. The controllers designed in the previous section, were tested in simulation for the SLFM plant to demonstrate the effectiveness of the proposed HOSM controllers. The aim was to stabilize the output i.e. motor deflection angle θ to the required position with minimum vibrations. SLFM Model (2.21) and controllers developed using SMC (3.22) and HOSMC; namely TA based (3.27) and STA based (3.29) were simulated.

The tuning parameters were For SMC, $k = 7$ and $q = 10$,

For STA, $k_1 = 4$ and $k_2 = 3$,

For TA, $k_{11} = 1.5$ and $k_{12} = 1.4$.

Figure 3.4 shows evolution of the angular displacement of SLFM and Figure 3.5 shows its corresponding angular velocity for all the three controllers.

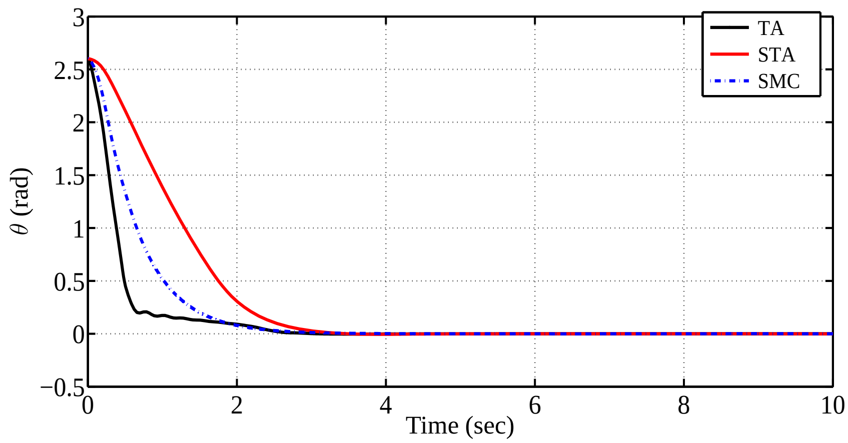

Figure 3.4: Evolution of Angular Displacement

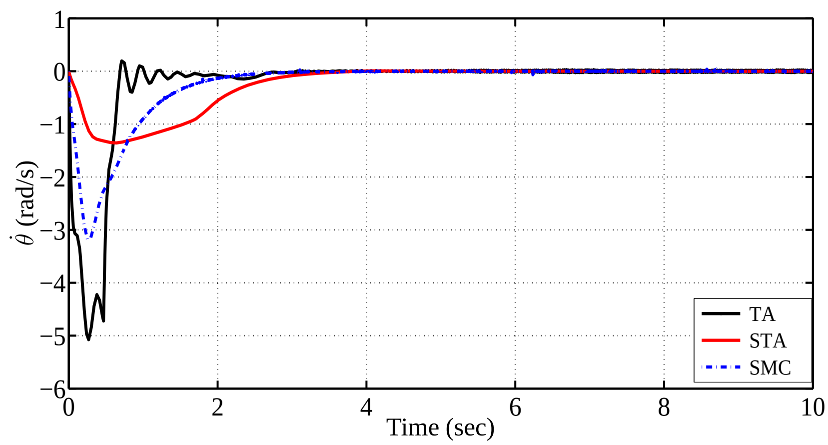

Figure 3.5: Evolution of Angular Velocity

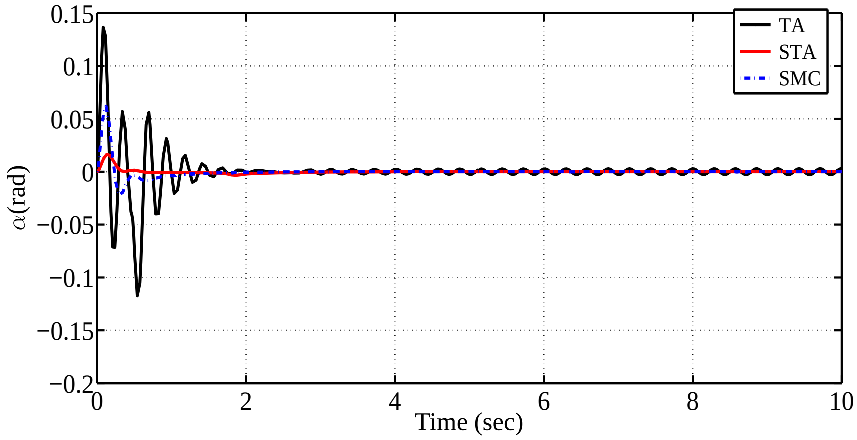

Figure 3.6: Evolution of link's tip displacement

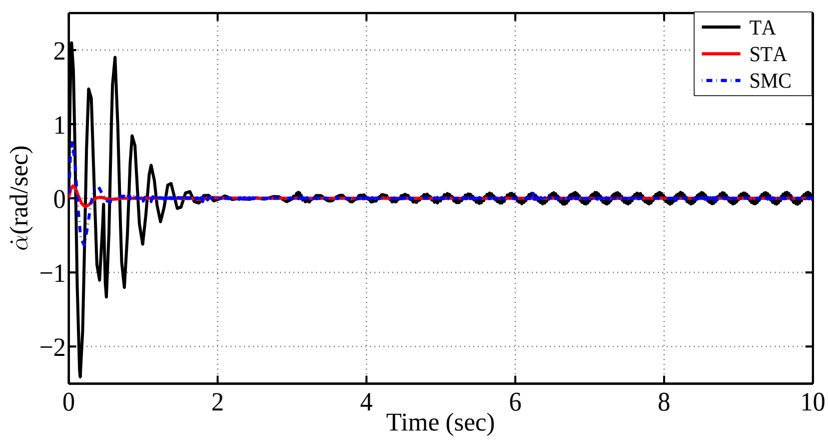

Figure 3.7: Rate of change of tip displacement vs time

Figure 3.6 and Figure 3.7 shows tip displacement and rate of change of tip displacement respectively. It is observed that STA shows less vibrations of the tip position compared to SMC.

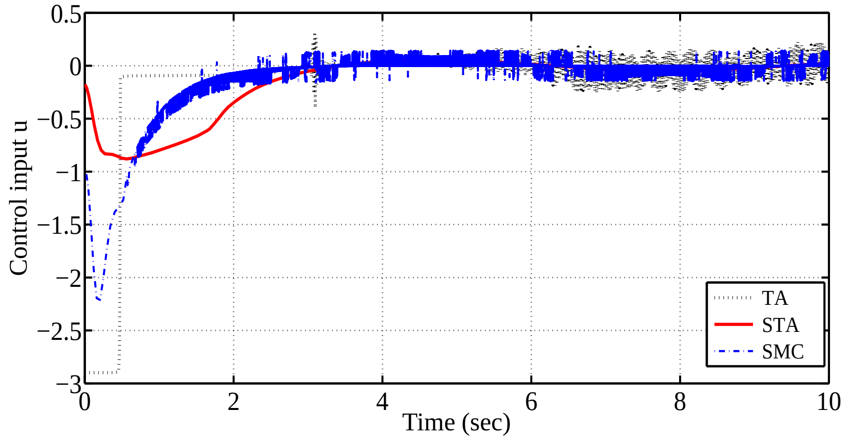

Figure 3.8: Control Input

It is evident that for same settling time of 3 sec, HOSM shows reduced chattering in the control input as compared to SMC. Also in HOSM strategies, STA shows less control efforts compared to TA. The error in stabilization of the actuation angle θ is calculated using norm one of its signal. It is seen to be minimum for STA approach. This analysis is tabulated in Table 3.1.

	SMC	TA	STA
$\|\|u\|\|_2$	40.44	15.49	3.93
$\|\|\theta\|\|_1$	11.32	9.11	6.45

Table 3.1: Comparison of control performance for SMC and HOSMC

Control efforts with STA are found to be minimum compared to that SMC and TA. It is almost reduced by 60% compared with SMC, whereas TA shows a reduction of about 33% w.r.t.efforts required by SMC. Thus both the HOSM controllers outperform SMC. Also the stabilization accuracy of TA controller is 18% better and that of STA controller is 34% better than that of SMC.

The SLFM is also simulated to track a step command of 2 rad, 0.1 Hz frequency at the angular joint. The corresponding evolution of states is plotted from Figure 3.9 to Figure 3.12. The system is also subjected to a disturbance of $0.05tsin(t)$ and parametric variation of 10% in system parameters. The controller parameters were found as

For SMC, $k = 7.4$ and $q = 10.3$,

For STA, $k_1 = 3.7$ and $k_2 = 5$,

For TA, $k_{11} = 2.5$ and $k_{12} = 2.1$.

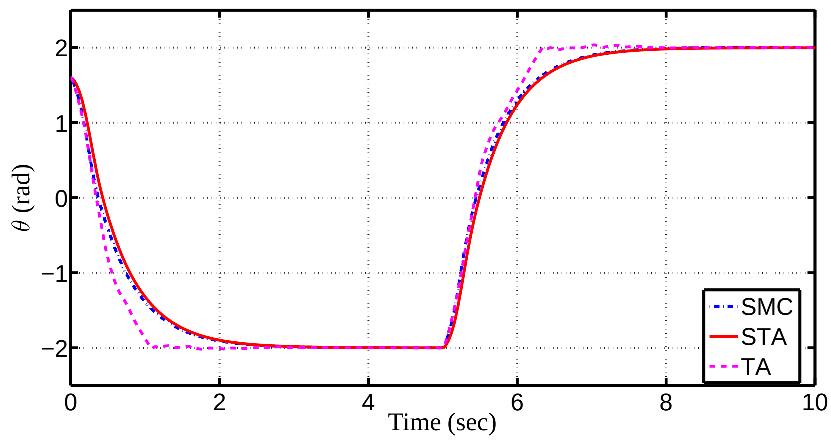

Figure 3.9: Evolution of Angular Displacement

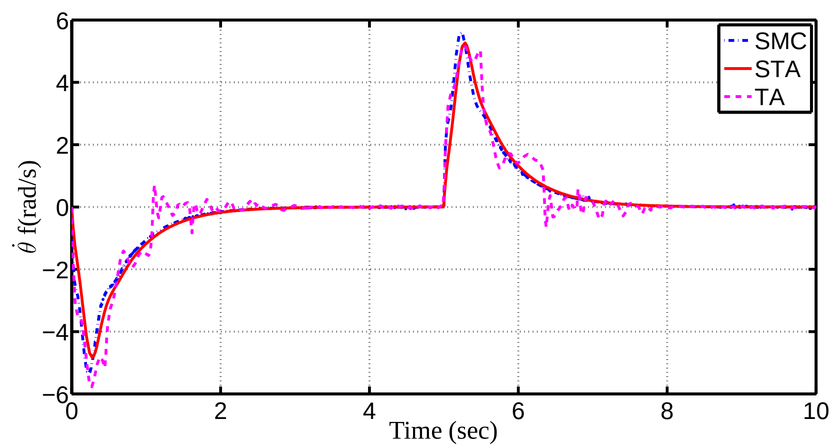

Figure 3.10: Evolution of Angular Velocity

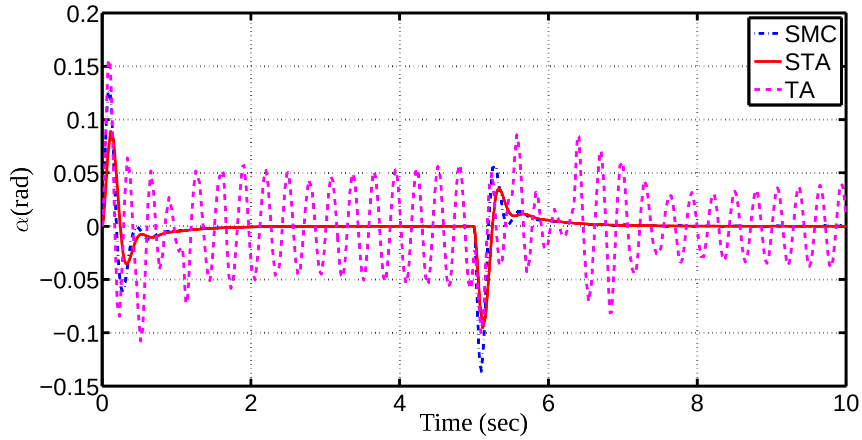

Figure 3.11: Evolution of link's tip displacement

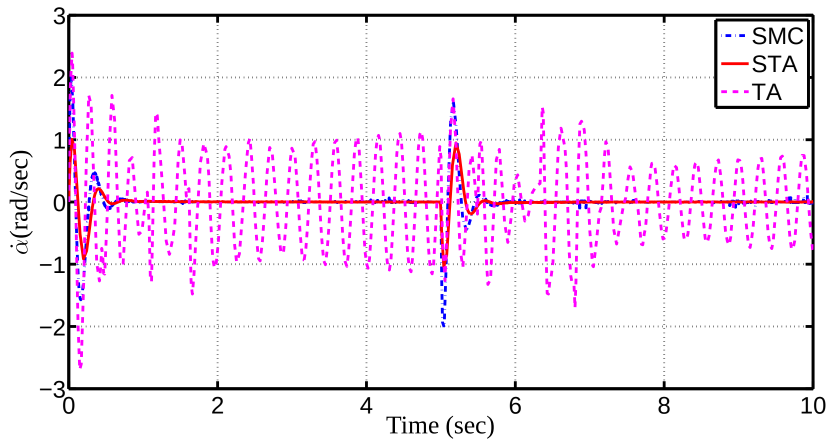

Figure 3.12: Rate of change of tip displacement vs time

It is observed that STA and SMC shows less vibrations of the tip position compared to TA. However, the control efforts show attenuated chattering due to STA as shown in Figure 3.13.

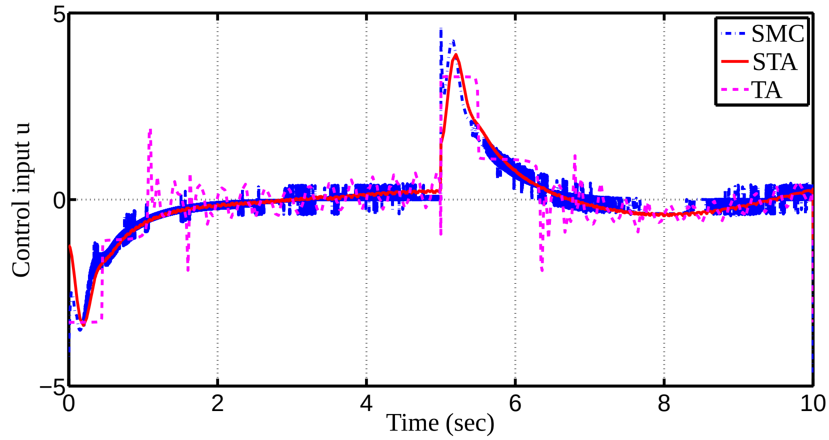

Figure 3.13: Control Input

These results ensure robustness property of all sliding mode schemes: SMC, STA and TA.

3.7 Summary

In this Chapter, classical sliding mode concept with its generalization in higher modes has been introduced. Stable sliding surface with relative degree one with respect to control input has been designed using pole placement method. This surface has been used to develop the first and higher order SMC. Three controllers were synthesized to achieve stabilization and tracking namely SMC, SOSMC using TA and STA. These designed controllers have been tested for nominal plant dynamics and also with parametric uncertainties and disturbances in the system. For implementation of TA controller, time derivative of the sliding variable s is also required. HOSM differentiator has been used to provide $\dot{s}$.

Simulation results have been presented to demonstrate the effectiveness of the TA and STA based HOSMC. The stability has been guaranteed even in the presence of parametric variations as well as the disturbance, ensuring the robustness of the proposed schemes. Proposed STA and TA based SOSMC outperforms the SMC in attenuating the chattering with less control efforts. All these control strategies were designed for a SLFM, which represents a class of underactuated system. The proposed technique can therefore be extended to such multi state systems.