

# Chapter 1

## Introduction

### 1.1 Flexible Manipulator: An Introduction

Utilization of robotic manipulators in various applications in recent years has increased with the demands of industrial automation. Robot manipulator consists of a sequence of links connected by means of joints. Links and joints are usually made as rigid as possible so as to achieve high precision in robot positioning. These robots are called as rigid link manipulators (RLM). However RLMs possess limited load-carrying capacity of only 5-10 percent of their own weight therefore are found to be inefficient with respect to power consumption and speed for manipulating payload. With the increase in the demand of the operational speed and high payload capacity the design called for light weight manipulators. Manipulators designed using lightweight materials for the links are flexible link manipulators (FLM). Light weight flexible structures are required in several fields of engineering, such as large telescope mirrors, aeroplane wings, large satellite structures, etc.

Flexible manipulators offer various merits over rigid counterparts like fast motion, large payload to weight ratio, etc. This results into low energy consumption, requirement of relatively smaller actuators, increased productivity, enhanced payload capacity, reduced non-linearity due to elimination of gears, lower mass and thus reduced overall cost. FLMs are found in many fields, such as space exploration, handling hazardous materials, manufacturing automation,

mining, construction, medicine and many more. The flexibility in robotics can appear in the joints or in the links. The flexibility link problem offers more challenges from a control point of view as compared to the joint flexibility.

Despite of the several advantages, the FLM could not make its deserving space in applications, as it offers certain research challenges right from system design, structural optimization to modeling, sensing and control. Various challenges of FLM are

- Light members generate undesirable deflections and vibrations due to the inertia and external forces.
- FLM imposes several constraints on the design of controller due to highly nonlinear and coupled dynamics evolved because of the flexibility.
- Flexible system is underactuated as there is no direct control available for the control of flexible modes.
- FLM exhibits nonminimum phase characteristic due to noncollocation. Noncollocation means the actuators are located at the joints, while the sensor gives the measurement of the tip displacement occurring at the end of the link. Nonminimum phase feature results in instability of the system and prevents the direct use of many conventional and effective control algorithms.
- The flexibility of FLM leads to undesirable oscillations at the tip of the link, which in turn demands complex closed-loop control.
- FLM is described by a highly non linear, coupled dynamics resulting in infinite degree of freedom (DOF) model, which is difficult to solve.
- Also the system becomes more complex when plant changes from single link to multiple links due to more number of DOF and increased interaction between the rigid and deformation motion.

Above mentioned issues make control design for FLM very challenging.

## 1.2 Modeling of FLM

Mathematical representation of any physical system is modeling. The first step towards designing an efficient control strategy is to develop an accurate dynamic model for any physical plant which can represent the actual dynamics of the system. Usually the advanced control laws are model based laws. Since better model can lead to better control, it is necessary to get an explicit and complete model of the system.

From the literature, it is seen that dynamic modeling of flexible robots is the most studied problem and considerable amount of research has been carried out on the development of dynamic models of FLM. Numerous papers are devoted to mathematical modeling and simulations of flexible link manipulators [1], [2].

For modeling of FLM, two strategies are adopted namely lumped parameter model and distributed parameter model.

In lumped parameter model, the flexibility is modeled by discrete, localized masses and rotary springs [3]. In this approach, a link can be divided into a number of rigid bodies connected by non-actuated joints. However this generates an approximate model where the flexibility is not captured accurately.

In distributed modeling, flexible manipulator is treated as a continuous dynamic system, where the dynamics of every point along the length of the link is different. This generates an infinite DOF model and is governed by nonlinear, coupled and partial differential equations (PDE). Exact solution of these equations is not feasible hence the equations are discretized using numerical techniques.

Commonly used numerical techniques are finite element method (FEM) and assumed mode method (AMM). In finite element method, a continuous flexible link is approximated as a system of discrete parts (finite elements) [4], [5]. Each of these parts is described by a finite number of generalized coordinates. A major disadvantage of FE method is the computational complexity and consequent software coding. However, this method allows irregularities in the structure and mixed boundary conditions hence found suitable in applications involving irregular structures.

The AMM approach [6] - [8] consists of representing the deflection of the manipulator as a summation of finite number of modes. Each mode is assumed as a product of two functions; one is spatial coordinate dependent on the distance along the length of the manipulator, and the other is a generalized coordinate, dependent on time. In principle, the summation amounts to an infinite number of modes. However, for practical purposes, a finite number of modes are used. This modal approach gives models with the minimal number of state variables. If we adopt some positional kinematic variables, i.e. a set of generalised coordinates, the model is said Lagrangian. The basic advantage of the Lagrangian model is the clear separation of the geometric, kinematic and dynamic modeling which gives a physical insight from the beginning of the modeling [9].

The other popular modeling technique adopted for FLM is the singular perturbation technique [10]. In this, the system characteristic modes are separated into two distinct groups: a set of low frequency or slow modes and a set of high frequency or fast modes. In case of flexible manipulators, the rigid-body modes are the slow modes and the flexible modes are the fast modes. The dynamics of the system can then be divided into two subsystems namely slow and fast. The slow subsystem is of the same order as that of the equivalent rigid manipulator and fast subsystem is of the order of elastic modes.

All the above referred modeling techniques are in the time domain.

An alternative method of modeling the FLM is to use a method based on frequency domain analysis [11], [12]. This method develops a concise transfer matrix model using the Euler-Bernoulli beam equation for a uniform beam. The weakness of this method is that it makes no allowance for interaction between the gross motion and the flexible dynamics of the manipulator, nor can these effects be easily included in the model. As a result, the model can only be regarded as approximate.

Looking at the need of developing a closed form of a dynamic model of FLM, Prof. Book came up with a recursive, lagrangian, assumed modes formulation for modeling a flexible arm [13] that described an efficient technique to develop the kinematics and dynamics of the FLM. This work has become one of the most cited and well-known studies in flexible robotics. After the theoretical development, first experimental platform was developed by Cannon [14], which

showed essential concept of noncollocated system and nonminimum phase nature of FLM. Point-to-point motion of flexible manipulators was studied exhaustively, but tracking control was proposed first time by Prof. Bayo [15] and was properly deliberated by Prof. Siciliano and De Luca [6]. This work proved to be a milestone in modeling of FLM.

### **1.3 Control of FLM: Literature Reported**

The challenge of control in flexible manipulators stems from several sources. First, the dynamic equations are highly coupled and nonlinear. Second, these equations are stiff due to the time-scale separation of the slow rigid modes and the fast flexible modes. Third, the presence of right half-plane zeros in a non-collocated sensor-actuator configuration imposes limitations on the control problem. These factors create problems in control design, analysis and simulation. The control objectives of the FLM are categorized as

1. End-effector position regulation.
2. Rest to rest end-effector motion control in fixed time.
3. Tracking of a desired angular (joint) trajectory.
4. Tracking of a desired end-effector trajectory.

Most of the research is related to control of a single link flexible manipulator (SLFM). This is due to the fact that in SLFM the mass matrix is only a function of deflection variables resulting in quadratic type nonlinearities, which can be approximated as a linear system. In case of the multi-link manipulator, the mass matrix contains joint position variables, which introduce considerable nonlinearities in the dynamic equations making it highly nonlinear and more complex, hence very few efforts towards control of multi link manipulators is available in the literature. Still the experiments conducted with SLFM provide a basis for multi-link investigations.

Early control laws for FLM are mostly linear and designed for point-to-point motion [16]. However to accomplish desired motion of the end effector, tip-trajectory tracking control is demanded in many of the applications of FLM. It can be achieved by several control

approaches such as output redefinition, inversion technique, input output linearization, input shaping, pole placement, computed torque control etc. Out of these inversion control is commonly used. If the joint (rigid body) variables are taken as the system outputs, inversion can be easily applied for joint angular trajectory control. Therefore it is convenient to pursue a joint-based control strategy for trajectory tracking. It results in stable response even with nonlinear control laws, however this generates undesirable error in the end effector positioning [17]. For achieving tip-trajectory control, tip deflection has to be selected as an output. For this output, the link flexibility leads to unstable closed loop response due to nonminimum-phase nature of FLM [18], [19]. Therefore accurate end effector trajectory tracking can be investigated using output redefinition.

From the early experimental work in this area, the work of Cannon and Schmitz [20], Hastings and Book [13] aimed at the end-point regulation problem. It was shown that output redefinition can be used for achieving smaller tracking errors. The output is defined such that the non minimum phase system becomes minimum phase. Many choices of such outputs are suggested by the researchers. Wang and Vidyasagar [21] introduced the reflected-tip position as a new output for a SLFM. The work of Hashtrudi and Khorasani [22] based on an integral manifold approach may also be interpreted as a form of output redefinition. In this work, new outputs are defined and are categorized as fast and slow outputs. This transforms the original tracking problem into tracking of slow output and stabilizing of fast dynamics. De Luca and Lanari [23] studied the regions of sensor and actuator locations for achieving minimum phase property for SLFM. Moallem et al. [24] proposed an observer based inverse dynamics control strategy which resulted in small tip-position tracking errors while maintaining robust closed-loop performance for a class of FLM. Khalil [25] initiated the experiment to control the end-effector of a FLM by measuring the tip position and using that measurement for applying torque to the other end (joint) of the link.

For end effector trajectory tracking with input output linearization approach, Wang and Vidyasagar [26] have shown that the nonlinear flexible-link system is not in general input-state feedback linearizable, however the system is locally input-output linearizable. In [27], [28] nonlinear state feedback have been used to compensate coupled nonlinear terms and accurate trajectory tracking have been obtained by inversion of the input/output map of the given system.

Controllers for FLM have been developed through pole placement techniques. Based on the

concept of pole assignment in linear systems, transmission zero assignment was introduced by Patel and Misra [29] and applied to SLFM. Here the basic idea is to add a feedforward block to the plant so that the zeros of the new system are at required locations in the left half-plane. It is then possible to use output feedback and ensure that the closed-loop system poles are at desirable locations in the left half-plane.

Singer and Seering [30] developed an input shaping method which transforms each sample of desired input into a new set of impulses that do not excite the system resonances. The input is delayed by half the damped natural period of the system to cancel the vibration induced by the original input. Tzes et al. [31] used the same principle to damp out vibration using the vector diagram approach. Torfs et al. [32] combined feedback linearization with input shaping technique to control the vibration of a SLFM. A Backstepping approach is presented for SLFM in [33].

Recently, intelligent control methods have been applied to FLM [34]. Intelligent control is the discipline in which control algorithms are developed by emulating certain characteristics of intelligent biological systems. Moudgal et al. [35] have implemented a fuzzy supervisory control law for vibration damping of a flexible two-link manipulator. Neural network based controller with SMC for flexible multi-link manipulator has been proposed in [36].

Amongst the nonlinear approaches for the design of controllers for FLM, singular perturbation theory [37] proved to be attractive due to the two-time-scale nature of the system dynamics. The control strategy is based on stabilizing the fast dynamics and tracking the joint trajectories. Singular perturbation model has been developed in [38] and [39] for the case of multilink manipulators. Mostly the researchers attempting this strategy, have taken joint positions as outputs to avoid the problems due to the non-minimum phase nature of the plant. However, it suffers from the drawback of large tip position errors.

Despite of the complexity, controllers for multi link manipulators also have been developed by many researchers. Khorami et al. [40] developed rigid-body based controller with input preshaping for two-link flexible manipulator. Utkin [41] developed sliding mode control for a multi-link flexible manipulator based on the pole assignment approach. An adaptive energy based robust control scheme for multi-link flexible robots is proposed in [42], [43].

To reduce the error of the tip deflection, vibration control technique is combined along with control approaches. Vibration control techniques for flexible structures are generally classified

into two categories: passive and active control [44]. Passive damping proposes selection of materials having good damping properties for manufacturing flexible links i.e. using composites. It has been explored by many researchers [45]- [48].

Active control utilizes the principle of wave interference. This is realized by generating an actuator torque to counteract with the unwanted disturbances and thus it results in reduction of the level of vibration. Active control of FLM can in general be divided into two categories: open-loop and closed-loop control. Closed-loop control uses measurements of the system states and change the actuator input accordingly to reduce the system oscillations.

In open loop control, independent modal space control (IMSC) method is used [49], [50]. When free motion of a system consists mainly of a limited number of clearly separable modes, then it is possible to control these modes directly using the IMSC, where the controller is designed for each mode independent of other modes.

All of these control methods have been implemented for the planar robotic case. However, the principal shortcoming of many of these controllers is the robustness. Therefore more research is being carried out in the field of robust control of FLM. Out of these, sliding mode control is investigated for control of FLM in the current study for SLFM as well as two link flexible manipulator (TLFM).

## **1.4 Sliding Mode Control of FLM**

Control systems subjected to external disturbances and model uncertainties has been the main challenge of modern control theory. Uncertainties due to flexibility and modeling approximations result in performance deterioration or even instability. In order to design an efficient and robust controller for FLM, a special control technique capable of coping with such challenges is required. Among the different existing robust control techniques, Sliding mode control (SMC) is recognized as one of the most promising robust control technique [51]. SMC was firstly proposed and developed by Prof. Emelyanov and Prof. Utkin in the early 1950s in the Soviet Union [52], [53]. The most important feature of the SMC is its ability to ensure robust performance which is completely insensitive to certain types of disturbances and uncertainties [54].

In SMC approach, the system states are forced to stay on some predefined constraint called as sliding surface. Any deviation from the constraint is steered back to it by a control input. Thus it becomes completely insensitive to certain types of disturbances and uncertainties. Theory of the SMC was well established in the decade of 1980s but its applications are quite recent. Now a days, SMC technique is very successfully used in engineering applications demanding very precise and complex control e.g. flight control, missile guidance control, robotic control etc.

Qian and Ma [55] studied the application of SMC to single link flexible manipulators (SLFM), where they controlled the end-point position in a non-collocated manner. There after the study in the field of controlling the FLM using SMC increased with rapid pace [56]- [58]. Most of the publications in the control of FLM using SMC are found using singularly perturbed model, where slow and fast subsystems are separated. The fast and slow dynamics once decoupled, can be controlled such that the rigid motion and elastic motion converge to zero asymptotically. However as complete states are required for implementing this control, it is invariably used only for slow dynamic states i.e. joint angles, since the flexible states are difficult to obtain. Then the fast subsystem is controlled by any other control method.

Two nonlinear controllers namely SMC and  $H_\infty$  control for a SLFM, are presented in [59] and [60] where the adaptive algorithm was used to estimate the perturbation part of system. Similar approach using SMC was shown in [61] with gain scheduling parameters and an optimal controller was designed to stabilize fast subsystem. Using inverse dynamics, nonsingular terminal SMC was designed to make the input-output subsystem converge to its equilibrium point in finite time [62]. In [63], performance of SMC has been compared with a LQR controller. Controller structure consisting of two nested feedback loops has been shown in [64] where the inner loop has been designed for the motor angle, controlled by a PD controller and the outer loop deals with the constraint force, which was controlled by SMC. Two controllers based on SMC for a SLFM are described in [65]. The first controller was a generalization of the computed torque and was used in conventional rigid robot control while in the second method, an adaptive version of SMC has been proposed.

In [66], the SLFM system is divided in a slow subsystem and a fast one. Fuzzy SMC has been designed for slower one and LQR for faster one alongwith singular perturbation method.

Robust control using sliding modes has been developed in [57] and extended with state and disturbance estimation for SLFM [67]. Some researchers have proposed Terminal SMC using the output redefinition method [68]- [62]. In most of the works, rigid dynamics are controlled using HOSMC.

As FLM plant goes from single link to multiple links, complexity increases due to added coupled dynamics. Not much efforts on the application of SMC for controlling TLFM are found.

In [71], a hybrid sliding mode consisting of frequency shaped optimal sliding mode and terminal sliding mode is proposed and applied to TLFM. STA based HOSM controller with an adaptation methodology for TLFM is presented in [72].

In most of reported SMC for SLFM, chattering is alleviated using continuous approximation of discontinuous control. This is eventually a trade off in ideal sliding and robustness.

### **1.4.1 Motivation and Control Objectives**

With extensively carried out and continuously improving quality of research, the rigid manipulators have successfully made their way in all corners of relevant engineering applications. The FLM with their proven capabilities are now occupying larger space in the field of research. In spite of the fact that the design of their controllers is challenging, their utility is retaining the interest of researchers and end users in them. Control of FLM is the key competence for robot manufacturers, researchers and industries and the current development is aimed at increasing its performance and exploring new functionalities in modeling and control of these manipulators. The power consumption, weight and cost form important parameters of an efficient robotic system. Thus there is a continuous need to improve the models and control approaches for FLM.

FLM imposes various constraints on the design of controller due to its nonlinear and coupled, underactuated and non-minimum phase dynamics. Moreover it is subjected to external disturbances and uncertainties due to parameter variations. This demands a robust controller. This has motivated to examine SMC. Implementation of the conventional SMC technique faces certain drawbacks such as undesirable chattering in the control input, requirement of complete state vector for implementing the control and relative degree restriction of sliding surface with respect to control input [73]. Higher order sliding modes has been evolved to deal with issue of

chattering in conventional SMC. In HOSM chatter in the control input is reduced and zeroing of the sliding variable and its higher derivatives is ensured. Very few results of control of FLM using HOSM are available [74] - [79].

In [74], HOSMC is designed for the control of a one DOF flexible link space robotic arm with payload. [75]- [79] are also based on HOSMC applications.

With the motivation to capture better dynamics and yield robust control, fractional calculus together with SMC is investigated for SLFM. Following are control objectives set while investigating problem of robust control of SLFM and TLFM.

- Examination of robust control using SMC and HOSMC for stabilization and tracking of SLFM.
- State and disturbance estimation for implementing the proposed controllers.
- Development of fractional order controllers for SLFM.
- Investigation of fractional order model and fractional controller for SLFM.
- Control of TLFM using state and output feedback.
- Experimental validation.

## **1.5 Contributions to Thesis**

Control of a desired angular (joint) trajectory with minimum tip vibrations using sliding mode control has been chosen as a main control objective for the research work. The thesis aims to address two major issues regarding FLM namely plant modeling and control design. In this work, modeling and control of SLFM and a two link flexible manipulator (TLFM) is investigated. The study focuses on three main steps of engineering practice: Theoretical development, software simulation and practical implementation. The followings are major contributions:

### **1.5.1 Control of SLFM using Sliding Modes**

A robust performance of SLFM using first order SMC is examined. Stable sliding surface is designed. The resulting control is discontinuous. Therefore second order sliding mode controllers using twisting and supertwisting algorithms (TA and STA resp) are examined. For TA, STA based controllers, the surface is of relative degree one and is a function of complete state vector. This controller requires information of complete state vector for implementing the control law. Finite time convergence of the sliding variable and its derivative is achieved in both of the HOSM controllers.

### **1.5.2 Fractional Order SMC for SLFM**

Fractional calculus is examined for control of SLFM. Fractional PID controller is developed for controlling the FLM. Further fractional sliding mode control for SLFM is investigated. Fractional order sliding mode control (FSMC) is the use of SMC using fractional approach. FSMC is realized by using fractional order (FO) differentiation and integration in the governing differential equation of the sliding surface. Stability of the proposed fractional sliding surface is proved using Lyapunov stability criterion, utilizing the property of decaying Mittag-Leffler function of the fractional calculus. FSMC is compared with the integer order SMC. Control efforts, chattering and the performance of FSMC and SMC are analyzed.

### **1.5.3 Control of SLFM: Fractional Approach**

To capture better dynamics, fractional model of SLFM is proposed. In attempting an accurate fractional model of SLFM, a non-commensurate order model is evolved. The proposed fractional model is compared with the integer order model. This model is validated experimentally. Controller is developed using the proposed model. A novel fractional sliding surface is proposed for this non-commensurate fractional model. While designing control, fractional reaching law is used. For implementing the control of the fractional system, information of all the fractional states is needed. Therefore fractional SM observer is designed. Its stability

is proved using Lyapunov stability criterion. Effect of parameter variations and disturbance is also analyzed. The proposed method is validated in simulations and experiments both.

#### **1.5.4 SMC for TLFM**

For multiple links, the problem of tip control becomes more complex due to the coupled dynamics of the successive links. Model of TLFM is developed neglecting the link coupling of the serial mechanism. This decoupled model is used for control design. To assure accurate positioning of the tip of the TLFM with minimum vibrations, HOSMC is proposed along with the disturbance observer (DO). To reduce the control efforts, the technique of disturbance decoupling is used. An estimated signal representing the disturbance is compensated by augmenting control. This is to limit conservative control.

#### **1.5.5 Output Feedback Control for TLFM using Sliding Modes**

For model based control, an adequate model has to be developed which can characterize the dynamics of the actual system. Therefore to capture the flexibility, a model of TLFM is derived using assumed mode method (AMM) in conjunction with the Lagrangian approach. For this model, HOSMC is designed for position control of TLFM. All the states are not always measurable as the internal variables are inaccessible for measurement, like flexible modes. Therefore output feedback control is developed. The information of outputs i.e. joint angle positions is used to synthesize the control law. Second order sliding mode control using twisting and super twisting algorithm are developed. The resulting control is simple and implementable.

#### **1.5.6 SMC for TLFM using Continuous TA**

For systems with relative degree two, HOSMC using TA stabilizes sliding variable and its derivative to zero in finite time. However TA produces discontinuous control resulting in chattering. A new class of continuous homogeneous generalization of the Twisting Algorithm (CTA) is examined for a TLFM plant, which is a system with relative degree two. The sliding

surface is taken as a function of output, therefore the controller requires only the knowledge of the output making the control implementable.

## **1.6 Organization of Thesis**

The thesis is organized as follows:

Chapter 1 gives the introduction of the flexible link manipulator along with the challenges faced to control it. The chapter briefly reviews the work on modeling and control of FLM reported in the literature. Dynamic modeling of the single link and two link flexible manipulator is described in detail in Chapter 2. Mathematical models are developed for the plants used for simulation and experimentation. A dynamic model of a TLFM is derived using Euler Lagrangian approach in conjunction with the assumed mode approach. Chapter 3 reviews the evolutions of sliding mode controllers in the last two decades. It also illustrates the features and the design methods of higher order sliding mode control. Control synthesis using HOSM for FLM is demonstrated.

Chapter 4 gives the introduction of Fractional calculus and its application to control theory and to SMC theory in particular. Fractional order SMC (FSMC) is discussed in detail by designing FO sliding surface. Fractional order model of SLFM is proposed in Chapter 5. FSMC with FO observer is developed.

Chapter 6 illustrates the controllers developed for TLFM using state and output feedback control technique based on HOSM. The proposed controllers are validated through simulation and experimentation.

Chapter 7 concludes the research work. It highlights some future directives.