Chapter IV

Employment, Wages and Productivity Nexus in Indian Manufacturing Sector
4.1 Introduction:

This chapter discusses the relationship between employment, real wages, and labour productivity of Indian manufacturing sector. The relationship between real wages, employment and productivity are considered as the key variables in labour economics and have received substantial amount of attention in the economic literature. Moreover, we should also acknowledge the significance of the relationship between these three variables from a policy perspective to realize the state of full employment. Economic theories have been put forward to explain the interrelationships among these variables. The Classical, Neoclassical and Keynesian theories of employment postulates a close relationship between real wages and employment level where it is hypothesized that there exists a long-run inverse relationship between real wages and employment levels in the economy. However, the theories differ in terms of direction of causal flow. Classical and neoclassical models assume that the causal mechanism runs from wages to employment, while in the Keynesian version, the causality runs towards wages from employment (Mazumdar, 2003).2

The efficiency wage theory postulates the relationship between productivity and real wages, but presumes that the causal relationship runs from wages to productivity (Akerlof and Yellen, 1986). Moreover the performance based pay scheme, on the other hand, predicts higher real wages for higher productivity. Besides, changes in productivity may affect employment in two opposite ways. First, although the impact of an increase in productivity is to reduce the demand for labour as workers are more efficient. Secondly, the increase in productivity generates more employment, though the expansion of the market. The level of labourers efficiency is directly related to the wage received from a higher pay allows workers to improve their physical ability to work through improved nutrition and health. In addition, an increase in real wages likely to stimulate greater work effort and higher morals among employees that further enhances productivity. Nevertheless, the increase in real wages may have a negative effect on

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1 Say’s law of market is the core of Classical theory of employment. Classical economists believed in the existence of full employment in the economy. In case of unemployment, a general wage cut in money wages would take the economy to the full employment situation.
employment through a higher cost of labour that in turn may result in capital is being substituted for labour.

It has been widely accepted that one of the major determinant of the wage rate is productivity. From the previous chapter, we have confirmed that the high growth of manufacturing output along with a sluggish employment growth have been contributing to the labour productivity growth in the Indian manufacturing sector. The growth of the number of workers and total person engaged is as low as 1.31% and 1.29% respectively in comparison to the productivity growth of 5.73% (gross value added per workers) and 6.16% (gross value added per person engaged) per annum during 1973-74 to 2007-08. The slow rate of employment growth in the post liberalization period is a matter of concern that raises the questions about the presence of jobless growth in the Indian economy. This may be because of increasing use of capital deepening technology. Thus, the present chapter is intended to investigate the nexus between real wages, employment and productivity of Indian manufacturing sector.

This chapter is organized as follows. The next section highlights the theoretical arguments that have emerged from the classical school of thought. Third section discusses the methodology followed by the description of variables and data sources in the section Four. In the fifth section, we have presented the growth and trend analysis of employment, wage and productivity of Indian manufacturing sector. Finally, the sixth section presents the empirical findings followed by the conclusions.

4.2 Theoretical Underpinnings:
Wages and employment are two important variables, which affect the standard of living of the people to a larger extent. The importance attached to these leads, so many economists propose different underlying theories and laws in the economic history. Though, the present analysis deals with the establishment of the empirical relationship between employment, wage and labour productivity, it's worth to review some of the major school of economic thoughts on this doctrine before proceeding towards examining the empirical relationship among these variables.
To start with, the relationship between wage and employment has been discussed first by the classical economist. It is Adam Smith (1776)\(^3\) and Ricardo (1815, 1817)\(^4\), who believed the existence of substantial levels of wages. According to them, this substantial level of wage in the long-term remains fixed. However, the term subsistence level of wage is not lucidly defined in their analysis. During the latter period, it was Ricardo, who believes, wages are viewed as a part of interaction between the long-term dynamics of growth of the economy and distribution of total product among different classes such as labour, capital and landowners. With the expansion of the economy, the total wage expands in proportion to the growing absorption of labour in the production process. At the same time, Ricardo has also emphasized that the rate of profit, which is the important constituent of the national product has squished as the competition among the capitalist and landlord goes on and the cultivation is expanded from more fertile lands to lands of lower fertility, with the expansion of population. From this, it is clear that the share of profit is declining as a part of the national product. However, having said that, the share of wages keeps on rising only in proportion to the labour employed. Wages can thus be viewed as being determined essentially by the needs of human existence, with some temporary variations around it.

The wages under the classical thought is quite positive, which used as an instrument to arrive at the full employment level. They assume the existence of full employment without inflation, which is based on wage-price flexibility. In the classical thoughts, there are automatic forces\(^5\) in the economic system that tends to maintain full

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\(^3\)The Wealth of Nation, first published in 1776, touches upon broad topics such as division of labour, productivity, wages of labour and free market

\(^4\)In his Essay on the Influence of a Low Price of Corn on the Profits of Stock (1815), Ricardo articulated what came to be known as the law of diminishing returns. In 1817 David Ricardo published Principles of Political Economy and Taxation. In this text Ricardo integrated a theory of value into his theory of distribution. David Ricardo’s attempts to answer important economic issues took economics to an unprecedented degree of theoretical sophistication. He outlined the Classical system more clearly and consistently than anyone before had done. His ideas became known as the “Classical” or “Ricardian” School. Economists consider that Ricardo is the source of these concepts behind the so called “Iron Law of Wages”, according to this, wages naturally tend to a subsistence level. In his Theory of Profit, Ricardo stated that as real wages increases, real profits decreases because the revenue from the sale of manufactured goods is split between profits and wages.

\(^5\)Know as ‘Invisible hand’, a term coined by economist Adam Smith in his 1776 book to describe the self-regulating behavior of the marketplace.
employment. In the classical model of employment, changes in money wages and real wages are directly and proportionally relate to each other. The relationship between the wage and employment is based on the argument that, in a competitive economy a reduction in the money wage reduces the cost of production so the price of the products, thereby raising their demand. In order to meet the increasing demand for the products, more workers are employed to produce them.

In the employment front, classical economist believes the existence of full employment in the economy as a normal situation. The famous Say’s Law of Market, is the core of the classical theory, which postulate that “supply creates its own demand”. In Say’s words:

“It is production which creates markets for goods. A product is no sooner created than it, from that instant affords a market for other products to the full extent of its own value. Nothing is more favorable to the demand of one product, than the supply of another.”

Through this mechanism, where supply creates its own demand, would result in automatic achievement of full employment barring minor deviations on account of aberrations like frictional unemployment. James Mill (1808) supported Say’s in this front and he argues that, consumption is co-extensive with production and production is the cause, and the sole cause of demand it never furnishes supply without furnishing demand, both at the same time and both to an equal extent. Whatever the amount annually produced, can never exceed the amount of annual demand. Thus, the argument

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6 According to classical economist, full employment is a normal situation and any deviation from this level is something abnormal which automatically tends towards full employment.
7 The classical model of employment postulates direct and proportional relationship between money and real wage. When there is a cut in the money wage, the real wage is also reduced to the same extent which reduced unemployment and ultimately brings full employment in the economic system.
8 The 19th century French Economist, Jean Baptiste Say postulates this proposition.
9 In its original form, the law is applicable to a barter economy where goods are ultimately exchange for goods. According to Say, work being unpleasant, no person will work to make a product unless he wants to exchange it for some other product which he desires. Therefore the very act of supplying gods implies a demand for them.
10 James Mill, basing himself largely on what he found in the Wealth of Nations, by 1808 had presented a full and balanced discussion of the Law of Markets. Mill stated that income received from the production process would indeed be spent on commodities.
put forward by James Mill, support that, supply creates its own demand and there
cannot be general overproduction and hence general unemployment. However, Say’s
formulation of supply creates its own demand does not appear to be in exact conformity
with Ricardo’s conclusion of the long-term path of employment expansion would be
governed by surplus generated in the hands of the capitalist, who are believed to be the
only class which undertakes capital accumulation.

Some of the uniqueness of the classical school of thoughts are also replicated in Marx
School of thoughts that incorporate the features like wages paid to labour and profits
goes to the hands of the capitalist class. Marx considers labour as the basic source of all
value, but when it engages in the production process, it produces more than what it
needed for its own subsistence does. Therefore, there remains a surplus, which is called
as “Surplus Value”. The capitalist appropriates this surplus value to enable capital
accumulation. Hence, there is an incentive to generate more surplus value by
incorporating capital-using technologies. Since this type of technological change needs
less labour than before to complete the work, so there emerges an ‘industrial reserve
army of labour’. The existence of this reserve army enables wages to be driven down to
the minimum subsistence level. Thus, like classical thought, wages are driven down
here as well, but through a very different mechanism. Here technological change is
viewed as forcing wages to remain low while simultaneously bring about a decline in
the rate of profit through the rise in the organic composition of capital. Once the
technology start-replacing labourers in the production process, the surplus value is
consistently declining, which in turn affect profit negatively. Thus, according to Marx,
the decay of capitalism is followed by the declining rate of profit. In one hand the
mechanism achieves the full employment through the emergence of the ‘reserve army’,
on the other hand, it pegged the wages to the subsistence level. Although, the Marxian
system results in bringing the wages to the subsistence level as in the Classical theory,
but in respect of the level of employment the two system deviate from each other.
Unlike Classical economist, Marxian does not believe in the permanent existence of the
full employment, rather Marxian dynamics keeps full employment permanently in
abeyance.
The neoclassical approach to wages is very different from that of the Classical and Marxian theory. As we have discussed above wage is determined by some factors, which are exogenous and related to sustenance of labour. Under neoclassical production framework, labour is considered to be a commodity, whose price (wage) is determined by the interaction between the demand for and supply of labour in the product market. Where the supply of labour is dependent upon disutility of labour which is to be overcome by the payment of wages as an incentive. On the other hand, demand for labourers dependent upon, the marginal productivity of labour, which declines with application of every successive unit of labour. This interaction brings out the equilibrium wage rate, which clears the labour market. Although, the interaction between the demand and supply schedule has taken place in the micro-level, but in the neoclassical theory, it has been generalized for the whole economy and the market demand schedule and market supply schedule determined the market clearing or equilibrium wage rate where, aggregate supply of labour is supposed to get fully absorbed. The equilibrium level of employment is taken to be a point at which there is automatic realization of full employment.

John Maynard Keynes in his General Theory of Employment, Interest and Money (1936), made a frontal attack on the classical and neoclassical postulates on wage and employment. The General Theory is written against the backdrop of Classical thoughts. This shows the disagreements by Keynes to that of the view propounded by the other two schools. He developed a new economic thought and policy that stand apart from the view developed over a century and which dominated economic thought and policy before and during the Great Depression. The major deviation of Keynesian economics from that of Classical and Neoclassical School is that, unlike the latter two schools of thought who believe in the principle of automatic adjustment in the system that brings the full employment, Keynes does not believe on account of ‘frictions’, within the economy, which occurs in respect of employment determination at the macro-level. The Keynesian analytical framework goes along with the Say’s Law to some extent, since the latter can be valid only if the marginal propensity to consume is

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11 By the ‘Classicalist’ Keynes meant the followers of Ricardo and all those, who adopted and preferred the theory of Ricardian Economics. They include in particular, J.S. Mill, Marshall, and Pigou.
12 Keynes vehemently criticized the classical theory of employment for its unrealistic assumptions. He argued that, the characteristics of the special case assumed by the Classical theory happen not to be those of the economic society in which we actually live, with the result that its teaching is misleading and disastrous if we try to apply it to the facts of experience.
taken to be one and whatever is saved is automatically invested. However, practically both of these assumptions are neither viable nor valid. Keynes rejects the fundamental classical assumption of full employment as a normal situation, rather he mentioned that the existence of involuntary unemployment (or underemployment) in the capitalist economy is a normal situation and full employment equilibrium is abnormal or a special situation. He also disagrees with the Classical postulates of wage rate as a mechanism to determine the employment, rather level of employment is depend upon aggregate demand, which can generate full employment only if the gap between the full employment level of income and consumption is filled up by an equivalent level of investment.

Moreover, the Keynesian are also disagreed with the viewpoint of the neoclassical formulation that states that the automatic achievement of full employment through wage adjustment. In Keynesian arguments, the full employment is automatically achieved through wage adjustment only if the demand for labour schedule is given. This condition is possible in the case of micro-level, but while dealing with the macroeconomic level, demand for labour scheduled cannot assume the level of income as a satirist previous condition. If we pursue, the neoclassical mechanism to arrive at the full employment by fixing the wage rate little lower, than at the macroeconomic level, the aggregate level of income and demand scheduled for the labour in the economy is shifting downwards. This in turn brings down the equilibrium point, which means a new equilibrium wage rate and employment level below the earlier one is achieved. This new equilibrium cannot in any meaningful sense be considered as the level of full employment.

In the Keynesian analysis, the importance given to the determination of wage rate is not as much as the importance attached for employment. However, the wage rate does occupy an important part in his formulation. He totally disagrees with the Classical

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13 Keynes refuted Say’s Law of Market, that supply always creates its own demand. He rather believe that all income earned by the labour class would not spent in buying products which they helped to produced. He maintained that part of the earning is saved and is not automatically invested because; saving and investment are two different functions, which are decided by two different groups. Therefore, the disequilibrium between saving and investment leads to the deficiency in the aggregate demand. This in turn leads to general overproduction because all that is produced is not sold and as a result, general unemployment.
school that money wage and real wages are directly related and proportion, he rather maintained that money wages are, by and large remains constant up to full employment. The classical notion of the subsistence wage, which they believe to remain fixed in the long-term, is refuted by Keynes and the money wage is taken to be at a reasonable level consistent with a decent level of living. Keynes mentioned that the money wage is supposed to rise beyond the full employment, where the labour market is becoming tight. Keynes is not fully agreed with the determination of wages by the marginal productivity of labour, but at the same time he is not disassociated with the marginal productivity of labour. He maintained that after the attainment of the full employment level, there persists a tendency for money wages to rise, the real wage level is supposed to be brought back into equality with marginal productivity through a rise in price level generated by the pressure of a rise in wages.

The behaviour of wage and price mentioned above in the Keynesian framework, where we come across a situation of persistent increase in the money wage rate beyond the full employment. On the other hand, with the increasing money wage, the real wage is about to stabilize at a level of the neutralizing effect of rising prices. The behaviour of wages and prices in the near full employment zone led to the development of the famous Post-Keynesian concept of the Phillips Curve. A.W. Phillips (1958)\textsuperscript{14} tries to empirically establish the relationship between the nominal wage rate and the level of employment. He found that the nominal wage rate started rising as the level of employment increased substantially to a high level, by analyzing a century of data pertaining to the British Economy. Unemployment rate as a counterpart of the level of employment, Phillips observed a negative relationship between the level of unemployment and the rate of increase of wage level.

The above review vindicates the fact that various schools of thought in the economic literatures differently represent the relationship between wage-employment and productivity. The empirical evidence as reviewed in the second chapter on the wage-

\textsuperscript{14} Phillips, A.W. wrote a paper in 1958 titled The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957, which was published in the quarterly journal Economica. In the paper Phillips describes how he observed an inverse relationship between money wage changes and unemployment in the British economy over the period examined. Similar patterns were found in other countries and in 1960 Paul Samuelson and Robert Solow took Phillips' work and made explicit the link between inflation and unemployment: when inflation was high, unemployment was low, and vice-versa.
employment–productivity relationship has been mixed, inconclusive and contradicting.
This chapter is particularly interested in finding out whether Indian manufacturing data
support any of the economic theories discussed previously. However, it is important to
mention over here that our intention here is not to establish or estimate the relationship
in the line of the different school of thoughts, rather we are interested to find out the
empirical relationship between labour productivity, employment and real wages by
employing various sophisticated time series econometric models. The findings of this
relationship between labour productivity, employment and real wages may be useful in
providing some observations to Indian policy makers in their implementations and
evaluation of labour policies.

4.3 Methodology:
Most of the empirical studies on the nexus of labour productivity, employment and real
wages focus on the existence of long-run equilibrium relationship between them. There
are different approaches available in the literature to empirically examine this behaviour.
One simple way of addressing this issue is using the correlation coefficient. However,
the use of correlation coefficient does not explain the existence of any long run and
dynamic relationship between the variables in view of non-stationary nature of the time
series variables. In this direction the next section presents the various time series
techniques i.e., Unit Root test (test of stationarity), Granger’s Non-Causality test in
Vector Auto Regression Block Exogeneity form, Johansen Maximum Likelihood
Procedure for co-integration test, Impulse Response Function, and forecasting Variance
Decomposition test that are employed to address the nexus between labour productivity,
employment and real wages.

4.3.1 Test of Stationarity:
Before employing any time series model to examine the interlinkages between the
different variables under study, one has to test the stationarity properties of the time
series variables. This study applies unit root tests to examine the stationarity properties
of the variables. A stochastic process \( \{ y_t \} \) is said to be stationary if for all \( t \) and \( k \),
(i) \( E[y_t] = E[y_{t+k}] = \mu \) for all \( t \)
(ii) $\text{Var}(y_t) = \text{Var}(y_{t+k}) = \delta^2$

or, $E((y_t - \mu)^2) = E((y_{t+k} - \mu)^2) = \delta_y^2 = \gamma_0$

(iii) $\text{Cov}(y_t, y_{t+k}) = \text{Cov}(y_{t+j}, y_{t+j+k})$

or, $E((y_t - \mu)(y_{t+k} - \mu)) = E((y_{t+j} - \mu)(y_{t+j+k} - \mu)) = \gamma_k$

Where, $\mu, \delta_y^2$ and all $\gamma_k$ are constants. The covariance may depend on $k$, the lag length.

The above conditions are also referred as conditions of weak stationarity, second order stationarity or wide sense stationarity. A strongly stationary process need not have finite mean and variance (i.e. $\mu$ and/or $\gamma_0$ need not be finite).

A simple first order autoregressive process can be expressed by the following general equation:

$$y_t = \mu_0 + \mu_1 t + \alpha y_{t-1} + \varepsilon_t \quad (4.1)$$

Where, $\{y_t\}$ is the stochastic process, $\mu_0$, $\mu_1$ and $\alpha$ are parameters and $\varepsilon_t$ is a random disturbance term with white noise properties. $\mu_0$ is called drift or constant or intercept. The nature of the time series described by the equation (4.1) depends on the parameter values. If $\mu_1 \neq 0$ and $|\alpha| < 1$, then $y_t$ follows a deterministic trend. The presence of autoregressive component, $\alpha y_{t-1}$, will mean that there may be short-run deviations, but the series will return to trend eventually. A series of this sort is known as a trend stationary (TS) process, as the residuals from the regression of $y_t$ on a constant and a trend will be stationary. If $\mu_0 = 0$, $\mu_1 = 0$ and $\alpha = 1$, the series is said to follow a simple random walk, a unit root process. If $\mu_0 \neq 0$, $\mu_1 = 0$ and $\alpha = 1$, the series is said to follow a random walk with drift. Any stochastic process, which becomes stationary after differencing once, is called a difference stationary (DS) process, for e.g. a simple random walk process is a DS process. Likewise, any time series, which becomes stationary after de-trending is called a TS process.

In time series literature, there are both formal and informal tests of stationarity. The informal tests include time series plots and use of Correlogram. Statistical packages use
Box-Pierce Q-statistics and Ljung-Box (LB) Q-statistics for testing stationarity of a series. These two statistics are based on autocorrelation coefficients of several lag lengths. The formal tests of nonstationarity are also known as unit root tests or test of random walk series. These include the Dickey-Fuller (DF), Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests to check the presence of unit root in the data. These tests are necessitated because the usual Student’s t-test is inappropriate to test the null hypothesis.

4.3.1.1 Dickey-Fuller and Augmented Dickey-Fuller Tests:

The basic Dickey-Fuller (DF) test examines whether the value of the parameter \( \alpha = 1 \) in equation (4.1), in other words, whether the underlying first order difference equation has a unit root. Specifically, assuming the absence of trend term in equation (4.1) and rewriting it in a modified form as below:

\[
\Delta y_t = \mu_0 + \delta y_{t-1} + \epsilon_t
\]  

(4.2)

where, \( \Delta y_t = y_t - y_{t-1} \). The null hypothesis is that the \( \{ y_t \} \) process has a unit root, i.e. \( H_0: \delta = \alpha - 1 = 0 \). Since \(-1 \leq \alpha \leq 1\), it follows that \(-2 \leq \delta \leq 0\).

More generally, if the given time series follows a \( p \)th order autoregressive process [AR(p)] or even autoregressive moving average process [ARMA(p,q)], an extended Dickey-Fuller test called augmented Dickey-Fuller (ADF) test. Specifically, if the original time series follows AR(p), it can be represented as,

\[
y_t = \mu_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \epsilon_t
\]  

(4.3)

After suitable mathematical manipulation, Equation (4.3) can be rewritten as,

\[
\Delta y_t = \mu_0 + \delta y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta y_{t-i+1} + \epsilon_t
\]  

(4.4)

Where, \( \delta = -(1 - \sum_{i=1}^{p} \alpha_i) \) and \( \beta_i = \sum_{j=i}^{p} \alpha_j \).
Equation (4.4) is also recommended if the residuals sequence, \( \{e_t\} \) in equation (4.2), is not a white noise, for e.g. when \( e_t \)'s are autocorrelated. There are different forms of DF and ADF tests, which are possible by including trend terms in equations (4.2) and (4.4), and also excluding drift (intercept or constant) term, \( \mu_0 \), from these equations. The DF test is a special case of the ADF test when \( p = 1 \). To test the significance of \( \delta \) in equations (4.2) and (4.4), the usual Student’s t-statistic critical values cannot be used. Initially, Dickey-Fuller and later MacKinnon have developed the appropriate test statistic, known as \( \tau \)-statistic, and its critical values using Monte Carlo simulations. The critical values of \( \tau \)-statistic are made available under alternative assumptions of drift, trend, sample size and level of significance. They are abbreviated as \( \tau \) (no drift and no trend), \( \tau_\mu \) (only drift) and \( \tau_\tau \) (with both drift and trend). Dickey-Fuller have also provided the critical F-test values, known as \( \Phi_1 \), \( \Phi_2 \), and \( \Phi_3 \), for pair-wise joint tests of significance for \( \mu_0 \) and \( \mu_1 \). Thus, the null hypothesis that \( \delta = 0 \) can be rejected if the computed t-value for the coefficient \( \delta \) is greater than the critical \( \tau \)-value in absolute magnitude. It has been shown that the same DF test critical values are valid for the ADF test as well.

### 4.3.1.2 Phillips-Perron Test:

One of the important assumptions of DF test is that error terms are uncorrelated, homoscedastic as well as identically and independently distributed (iid). Phillips and Perron (1988) have modified the DF test, known as PP test, which can be applied to situations where the above assumptions may not be valid. Another advantage of PP test is that it can also be applied to frequency domain approach, which is more recent and an alternative to the usual time domain approach, to time series analysis. The derivation of the PP test statistic is quite involved and hence not given here. The PP test has been shown to follow the same critical values as that of DF test, but has greater power to reject the null hypothesis of unit root. However, the PP test seems to be biased towards rejecting the null hypothesis of a unit root, when the error series follows a negative moving average process. In such situations, it is recommended to use the ADF test, rather than the PP test.
4.3.2 Johansen Maximum Likelihood Procedure:

Engel-Granger co-integration procedure will have a problem if there is more than one co-integrating vector present in a vector of variables. If the number of variables exceeds two, we cannot rule out the possibility of more than one co-integrating vector. So in a multivariate system, the Engel-Granger approach will not be the appropriate method to use. Johansen (1988) Maximum Likelihood Procedure is the appropriate method to test co-integration among a vector of variables. This procedure estimates co-integrating relationship in a system of equations unlike single equation method of Engel-Granger. Thus, it makes use of all the available information in the long run and short run fluctuations of each variable and allows for testing of more than one co-integrating vector.

Like Engel-Granger technique, it requires that the variables should be integrated of same order. Although forms of the Johansen tests can detect differing orders of integration, it is wise not to mix variables with different orders of integration. The second important requirement is selection of lag length as Johansen procedure is quite sensitive to the lag length. One can use Likelihood Ratio test to select lag length. Alternatively, lag length can be selected using multivariate generalization of the AIC or SBC.

The Johansen test for co-integration begins by considering the unrestricted reduced form of a system of variables, which by assumption can be represented as a finite order Vector Auto Regression (VAR) model.

\[ X_t = A_0 + A_1 x_{t-1} + A_2 x_{t-2} + \ldots + A_p x_{t-p} + \epsilon_t \]  \hspace{1cm} (4.5)

where, \( x_t \) = the \((n \times 1)\) vector \((x_{t1}, x_{t2}, \ldots, x_{tn})\)

\( A0 \) = an \((n \times 1)\) vector of constants.

\( Ai \) = an \((n \times n)\) matrix of parameters.
\( \varepsilon_i = \) an independently and identically distributed n-dimensional vector with mean 0 and variance \( \Sigma_\varepsilon \).

The equation (4.5) can be reformulated into a Vector Error Correction Model (VECM) form:

\[
\Delta x_t = A_0 + \sum_{i=1}^{p-1} \pi_i \Delta x_{t-i} + \pi x_{t-p} + \varepsilon_t
\]  

(4.6)

\[
\pi = - \left[ I - \sum_{i=1}^m A_i \right]^{-1}
\]

\[
\pi_j = - \left[ I - \sum_{i=1}^m A_i \right]^{-1}
\]

Where \( I \) = an \((n\times n)\) identity matrix.

The equation (4.6) contains information on both the short run and long run adjustment to change in \( x_t \), via the estimates of \( \pi_i \) and \( \pi \) respectively. As it is shown in Johansen (1988), \( \pi = \alpha \beta \), where \( \alpha \) represents the speed of adjustment to disequilibrium, while \( \beta \) is a matrix of long run coefficients such that the term \( \beta \Delta x_{t-1} \) embedded in (4.6) represents up to \( n-1 \) co-integrating relationship in the multivariate model which ensure that the \( x_t \) converge to their long run steady state solutions. Hence the matrix \( \beta \) is the matrix of co-integrating parameters and \( \alpha \) is the matrix of the speed of adjustment parameters.

The key feature to note in (4.6) is the rank of \( \pi \); the rank of \( \pi \) is equal to the number of independent co-integrating vectors. If rank (\( \pi \)) = 0, the matrix is null, so there is no co-integration among the set of \( n \) variables, that means, there is no linear combination of variables that is stationary. Hence the equation (4.6) will become a usual VAR model in first differences. Instead, if rank (\( \pi \)) = \( n \), the vector process is stationary, that means there are ‘\( n \)’ linear independent combinations of \( x_t \) that are stationary. So in this case all the variables are stationary. In the intermediate case, if rank (\( \pi \)) = 1, there is a single
co-integrating vector and the expression $\pi x_{t-p}$ is the error correction factor. For other cases in which $1 < \text{rank } (\pi) < n$, there are multiple co-integrating vectors.

The number of distinct co-integrating vectors can be obtained by checking the significance of the characteristic roots of $\pi$. The number of co-integrating vectors is equal to the rank of the matrix $\pi$ and the rank of the matrix is equal to the number of characteristic roots that differ from zero. The Johansen methodology allows determining the number of characteristic roots that are statistically different from zero. If the variables in $x_t$ are not co-integrated, the rank of $\pi$ is zero and all the characteristic roots ($\lambda_i$) will equal to zero. In practice, one can obtain only estimates of $\pi$ and the characteristic roots. The test for the number of characteristic roots that are insignificantly different from unity can be conducted using the following two test statistics:

\[
\hat{\lambda}_{\text{trace}}(r) = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \\
\hat{\lambda}_{\text{max}}(r, r+1) = -T \ln(1 - \hat{\lambda}_{r+1})
\]

where, $\hat{\lambda}_i$ is the estimated value of the characteristic roots (also called eigen values) obtained from the estimated $\pi$ matrix.

$T$ = the number of usable observation.

$r$ = the number of co-integrating vectors.

When the appropriate values of ‘r’ are clear, these statistics are simply referred to as ‘$\lambda_{\text{trace}}$’ and ‘$\lambda_{\text{max}}$’. The first statistic tests the null hypothesis that the number of distinct co-integrating vectors is less than or equal to ‘r’ against a general alternative. It is clear that $\lambda_{\text{trace}}$ equals zero when all $\lambda_i = 0$. The second statistic tests the null hypothesis that the number of co-integrating vectors is $r$ against the alternative of $r+1$ co-integrating vectors. Johansen and Juselius (1990) provide the critical values of the $\lambda_{\text{trace}}$ and $\lambda_{\text{max}}$ statistics. The estimated values of the above two statistics are compared with the Johansen and Juselius critical value to determine the number of co-integrating vectors exist among the variables. One of the most interesting aspects of the Johansen procedure is that it allows for testing restricted forms of the co-integrating
vector (s). One can impose restrictions on the co-integrating vectors or adjustment coefficients, and accordingly conclude whether restrictions are binding or not by using a statistic, which is proposed by Johansen.

4.3.3 Vector Auto Regression Model:

Both univariate single equation model and multivariate models such as intervention analysis and transfer function analysis shows the dynamic relationships among variables by allowing the time path of the ‘dependent’ variable to be influenced by the time path of an ‘ independent’ or ‘ exogenous’ variable. If it is known that there is no feedback, these analyses can be very effective tools for forecasting and hypothesis testing. But the major limitation of these analyses is that many economic systems do exhibit feedback. In Practice, it is not always known if the time path of a series designated to be ‘independent’ variable has been unaffected by the time path of the so-called ‘dependent’ variable. And if one is not confident that a variable is actually exogenous, the natural way is to treat each variable symmetrically. That is where the importance of VAR lies. By the very construction, a VAR system consists of a set of variables, each of which is related to lags of itself, and of all other variables in the system. In other words, a VAR system consists of a set of regression equations, each of which has an adjustment mechanism such that even small changes in one variable component in the system may be accounted for automatically by possible adjustments in the rest of the variables in the system. Thus VAR provide a fairly unrestricted approximation to a reduced form structural model without assuming beforehand any of the variables as exogenous. Thus, by avoiding the imposition of a priori restrictions on the model the VAR add significantly to the flexibility of the model. Furthermore, by incorporating the lagged terms of the variables, the VAR become useful in capturing the empirical regularities embedded in the data, which consequently enables one to obtain deeper insights into the channels through which the macro policy variables such as demand for money and interest rates percolate the system in establishing the relationship between stock and foreign exchange market.

Let the present wage rate \( \{ W_t \} \) sequence be affected by current and past realization of the employment rate i.e. \( \{ E_t \} \) sequence and let the time path of employment rate \( \{ E_t \} \)
sequence be affected by current and past realizations of the wage rate \( \{ W_t \} \). Now the unrestricted VAR system can be written as:

\[
W_t = b_{10} + b_{12} E_t + \gamma_{11} W_{t-1} + \gamma_{12} E_{t-1} + \varepsilon_{Wt} \tag{4.7}
\]

\[
E_t = b_{20} + b_{21} W_t + \gamma_{21} W_{t-1} + \gamma_{22} E_{t-1} + \varepsilon_{Et} \tag{4.8}
\]

Where, it is assumed that (i) \( \{ W_t \} \) and \( \{ E_t \} \) are stationary, (ii) \( \varepsilon_{Wt} \) and \( \varepsilon_{Et} \) are white-noise disturbances with standard deviations of \( \sigma_S \) and \( \sigma_E \) respectively, and (iii) \( \{ \varepsilon_{Wt} \} \) and \( \{ \varepsilon_{Et} \} \) are uncorrelated white-noise disturbances.

The structure of the system incorporates feedback since \( W_t \) and \( E_t \) are allowed to affect each other. For example, \( +b_{12} \) is the contemporaneous effect of a unit change of \( E_t \) on \( W_t \) and \( \gamma_{12} \) is the effect of a unit change in \( E_{t-1} \) on \( W_t \). The terms \( \varepsilon_{Wt} \) and \( \varepsilon_{Et} \) are pure innovations (or shocks) in \( W_t \) and \( E_t \) respectively. If \( b_{21} \) is not equal to zero, \( \varepsilon_{Wt} \) has an indirect contemporaneous effect on \( E_t \) and if \( b_{12} \) is not equal to zero, \( \varepsilon_{Et} \) has an indirect contemporaneous effect on \( W_t \).

Now equation (4.7) and (4.8) are not reduced form equations since \( W_t \) has a contemporaneous effect on \( E_t \) and \( E_t \) has a contemporaneous effect on \( W_t \). But using the matrix algebra, the system of equations can be transformed into more usable and compact form. Rewriting the system of equations in matrix form we get,

\[
Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t \tag{4.9}
\]

Where,

\[
B = \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} ; \quad x_t = \begin{bmatrix} W_t \\ E_t \end{bmatrix} ; \quad \Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} ; \quad \Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}
\]

and,
Equation (4.9) represents a VAR in primitive form. Premultiplication of $B^{-1}$ in Equation (4.9) give us the Vector Autoregressive (VAR) model in standard form:

$$x_t = A_0 + A_1 x_{t-1} + e_t \quad (4.10)$$

Where,

$$A_0 = B^{-1} \Gamma_0; \quad A_1 = B^{-1} \Gamma_1; \quad \text{and} \quad e_t = B^{-1} \varepsilon_t$$

The process in equation 4.10 looks like an auto regressive process but with a difference that $x_t$, $A_0$ and $e_t$ are now vectors i.e. all the variables and the disturbance terms are now in vector form.

For notational purposes, we can define $a_{i0}$ as element $i$ of the vector $A_0$; $a_{ij}$ as the element in row $i$ and column $j$ of the matrix $A_1$; and $e_{it}$ as the element $i$ of the vector $e_t$. Using this notation, the equation 4.10 can be rewritten in the equivalent form:

$$W_t = a_{i0} + a_{i1} W_{t-1} + a_{i2} E_{t-1} + e_{it} \quad (4.11)$$

$$E_t = a_{20} + a_{21} W_{t-1} + a_{22} E_{t-1} + e_{2t} \quad (4.12)$$

It is important to note that the error terms (i.e. $e_{1t}$ and $e_{2t}$) are composites of the two shocks $\varepsilon_{Wt}$ and $\varepsilon_{Et}$. Since $e_t = B^{-1} \varepsilon_t$, $e_{1t}$ and $e_{2t}$ can be computed as:

$$e_{it} = (\varepsilon_{Wt} + b_{12} \varepsilon_{Et})/(1 - b_{12} b_{21}) \quad (4.13)$$

$$e_{2t} = (\varepsilon_{Et} + b_{21} \varepsilon_{Wt})/(1 - b_{12} b_{21}) \quad (4.14)$$

Since $\varepsilon_{Wt}$ and $\varepsilon_{Et}$ are white noise processes, it follows that both $e_{1t}$ and $e_{2t}$ have zero means, constant variances and are individually serially un-correlated. However, the critical point to be noted is that the covariance between $e_{1t}$ and $e_{2t}$ will not be zero so that two shocks will be correlated. In the special case where $b_{12} = b_{21} = 0$ (i.e. if there are no contemporaneous effects of $W_t$ on $E_t$ and $E_t$ on $W_t$) the shocks will be
uncorrelated. It is useful to determine the variance and covariance matrix of the $e_{1t}$ and $e_{2t}$ shocks as:

$$
\Sigma = \begin{bmatrix}
\text{var}(e_{1t}) & \text{cov}(e_{1t}, e_{2t}) \\
\text{cov}(e_{1t}, e_{2t}) & \text{var}(e_{2t})
\end{bmatrix}
$$

Since all elements of $\Sigma$ is time independent, we can use the more compact form:

$$
\Sigma = \begin{bmatrix}
\sigma_1^2 & \sigma_{12} \\
\sigma_{21} & \sigma_2^2
\end{bmatrix}
$$

Where $\text{var}(e_{it}) = \sigma_i^2$ and $\sigma_{12} = \sigma_{21} = \text{cov}(e_{1t}, e_{2t})$.

Now let me discuss the different steps that are associated with the computation of VAR models:

1. **Selection of Variables in the System**:

The variables to be included in the VAR are selected according to the relevant economic model. Otherwise, no explicit attempt is made to 'pare down' the number of parameter estimates. Suppose a multivariate VAR is given as follows:

$$
X_t = A_0 + A_1 X_{t-1} + A_2 X_{t-2} + \ldots + A_p X_{t-p} + e_t \quad (4.15)
$$

Where,

- $X_t$ = an $(n \times 1)$ vector containing each of the $n$ variables included in the VAR
- $A_0$ = an $(n \times 1)$ vector of intercept terms
- $A_i$ = $(n \times n)$ matrices of coefficient, and
- $e_t$ = an $(n \times 1)$ vector of error terms.

In the above example, the matrix $A_0$ contains $n$ intercept term and each matrix $A_i$ contains $n^2$ coefficients, hence $n+pn^2$ terms need to be estimated.

2. **Test of Stationarity**:

Before estimating the VAR, Unit Root tests will be applied to examine the stationary properties of the variables. Here in the study, two Unit Root tests \textit{viz.} Augmented
Dickey-Fuller (ADF) test and Phillips-Perron (PP) test have been conducted to examine the stationary properties of the variables. The detailed discussions regarding these tests have already been discussed in the previous section.

**4.3.3.3 Choice of Lag Length:**

To check lag length, first, a longest plausible length or the longest feasible length is chosen given degrees of freedom considerations. For example, using quarterly data, lag length of 12 is chosen. Second, the VAR is estimated and variance and covariance matrix of residuals is formed. Variance and covariance matrix of residuals from the 12-lag model can be called as $\Sigma_{12}$. Now suppose we want to determine if 8 lag is appropriate. After all restricting the model from 12 to 8 lags would reduce the number of estimated parameters by $4n$ in each equation.

Since the goal is to determine whether lag 8 is appropriate for all equations, an equation by equation F-test on lags 9 through 12 is not appropriate. Instead the proper test for this cross equation restriction is a likelihood ratio test. Now the VAR is re-estimated over the sample using eight lags and variance and covariance matrix of residuals $\Sigma_8$ is obtained. The likelihood ratio statistic is:

$$(T) \left( \log|\Sigma_8| - \log|\Sigma_{12}| \right)$$

Sims (1980) recommends using

$$(T - c) \left( \log|\Sigma_8| - \log|\Sigma_{12}| \right)$$

Where, $T$ = number of usable observations, $c$ = number of parameters estimated in each equation of the unrestricted system; and $\log|\Sigma_n|$ is the natural logarithm of the determinant of $\Sigma_n$. In the example given above $c = 12n+1$, since each of the equations of the unrestricted model has 12 lags for each variable plus an intercept term.

This statistic has the asymptotic $\chi^2$ distribution with degrees of freedom equal to the number of restrictions in the system. Clearly, if the restriction of a reduced number of lags is not binding, then $\log|\Sigma_8|$ will be equal to $\log|\Sigma_{12}|$. Large values of this sample statistic would mean the restriction of only lag 8 is binding, hence we can reject the null
hypothesis that lag length = 8. If the calculated value of the statistics less than \( \chi^2 \) at a specified significance level, then null of only 8 lags cannot be rejected. In this way lag length of a VAR model gets determined and once it is determined, the model can be estimated by applying the Ordinary Least Square (OLS) method.

### 4.3.3.4 Exogeneity in VAR Model:

A necessary condition for the exogeneity of \( W_t \) is that current and past values of \( E_t \) do not affect \( W_t \). The sequence \( \{W_t\} \) may not be exogenous to \( \{E_t\} \) even though \( \{E_t\} \) does not Granger cause \( \{W_t\} \). Because pure shocks to \( \{E_t\} \) i.e. \( \varepsilon_{Et} \) may affect the value of \( \{W_t\} \) though \( \{E_t\} \) sequence does not Granger cause the \( \{W_t\} \) sequence.

A block exogeneity test is useful for detecting whether to incorporate a variable into a VAR. Given the above distinction between causality and exogeneity, the multivariate generalization of the Granger causality test should be called a ‘block causality’ test. In any event the issue is to determine whether lags of one variable say \( W_t \) Granger cause any of the variables in the system. In the three variable case with \( W_t, E_t \) and \( L_t \), the test whether lags of \( W_t \) Granger cause either \( E_t \) or \( L_t \). In essence, the block exogeneity restricts all lags of \( W_t \) in the \( L_t \) and \( E_t \) equations to be equal to zero. This cross equation restriction is properly tested using the likelihood ratio test given as follows:

\[
(T - c) \left( \log |\Sigma_r| - \log |\Sigma_u| \right)
\]

Where, \( \Sigma_u \) and \( \Sigma_r \) are the variance and covariance matrixes of the unrestricted and restricted systems respectively.

### 4.3.3.5 Impulse Response Function (IRF):

An essential tool to analyze the dynamic interrelationships among variables in a VAR is the vector moving average (VMA) representation. Just as an auto regression has a moving average representation, a VAR can be written as a VMA.

Recalling the equation (4.9) we have,
\[ B x_t = \Gamma_0 + \Gamma_t x_{t-1} + \varepsilon_t, \] which represents a VAR in primitive form. The VMA representation of an equation (4.9) expresses the variables \( W_t \) and \( E_t \) in terms of current and past values of the two shocks \( \varepsilon_{St} \) and \( \varepsilon_{Et} \).

Writing (4.10) and (4.11) in matrix form; we get

\[
\begin{bmatrix}
W_t \\
E_t
\end{bmatrix} = \begin{bmatrix} a_{10} & a_{11} & a_{12} \\
a_{20} & a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix}
W_{t-1} \\
E_{t-1}
\end{bmatrix} + \begin{bmatrix} e_{1t} \\
e_{2t}
\end{bmatrix} \quad (4.16)
\]

Now, recalling the VAR model in standard form i.e. equation (4.10) we have,

\[ x_t = A_0 + A_1 x_{t-1} + e_t \]

If we iterate backwards and assume that stability condition is met, then the particular solution for \( x_t \) is:

\[ x_t = \mu + \sum_{i=0}^{\infty} A_i^t e_{t-i} \quad (4.17) \]

Where \( \mu = [\bar{W}, \bar{E}] \)

Using equation (4.16) we can rewrite equation (4.15) as

\[
\begin{bmatrix}
W_t \\
E_t
\end{bmatrix} = \begin{bmatrix} \bar{W} \\
\bar{E}
\end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix} e_{1t-i} \\
e_{2t-i}
\end{bmatrix} \quad (4.18)
\]

Equation (4.16) expresses \( W_t \) and \( E_t \) in terms of the \{\varepsilon_1\} and \{\varepsilon_2\} sequences. However it is insightful to rewrite (4.17) in terms of \{\varepsilon_{Wt}\} and \{\varepsilon_{Et}\} sequences. From equation (4.12) and (4.13) the vector of error terms can be written as:

\[
\begin{bmatrix}
e_{1t} \\
e_{2t}
\end{bmatrix} = (1/b_{12} - b_{12} b_{21}) \begin{bmatrix} 1 & b_{12} \\
b_{21} & 1
\end{bmatrix} \begin{bmatrix} \varepsilon_{Wt} \\
\varepsilon_{Et}
\end{bmatrix} \quad (4.19)
\]

So that (4.17) and (4.18) can be combined to form:

\[
\begin{bmatrix}
S_t \\
E_t
\end{bmatrix} = \begin{bmatrix} \bar{S} \\
\bar{E}
\end{bmatrix} + (1/b_{12} - b_{12} b_{21}) \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \begin{bmatrix} 1 & b_{12} \\
b_{21} & 1
\end{bmatrix} \begin{bmatrix} \varepsilon_{St} \\
\varepsilon_{Et}
\end{bmatrix} \quad (4.20)
\]
To simplify the above notation, now define the 2 x 2 matrix $\Phi_i$ with elements $\Phi_{jk}^{(i)}$ such that:

$$\phi_i = \begin{bmatrix} A_i' / (1 - b_{21} b_{12}) & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

Hence moving average representation of (4.18) and (4.19) can be written in terms of $\{\varepsilon_{W_t}\}$ and $\{\varepsilon_{E_t}\}$ sequences:

$$\begin{bmatrix} W_t \\ E_t \end{bmatrix} = \begin{bmatrix} \bar{W} \\ \bar{E} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}^{(i)} & \phi_{12}^{(i)} \\ \phi_{21}^{(i)} & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} \varepsilon_{W_{t-i}} \\ \varepsilon_{E_{t-i}} \end{bmatrix}$$

or more compactly:

$$X_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} \quad (4.21)$$

The moving average representation is an especially useful to examine the interaction between $\{W_t\}$ and $\{E_t\}$ sequences. The coefficients of $\phi_i$ can be used to generate the effects of $\varepsilon_{W_t}$ and $\varepsilon_{E_t}$ shocks on the entire time paths of the $\{W_t\}$ and $\{E_t\}$ sequences. The four elements $\phi_{jk}^{(0)}$ are called as impact multipliers. For example the coefficient $\phi_{12}^{(0)}$ is the instantaneous impact of a one unit change in $\varepsilon_{E_t}$ on $W_t$. Similarly, the elements $\phi_{11}^{(1)}$ and $\phi_{12}^{(1)}$ are the one period response of unit changes in $\varepsilon_{W_{t-1}}$ and $\varepsilon_{E_{t-1}}$ on $W_t$ respectively.

The four sets of coefficients $\phi_{11}^{(i)}$, $\phi_{12}^{(i)}$, $\phi_{21}^{(i)}$ and $\phi_{22}^{(i)}$ are called impulse response function. Plotting the impulse response functions [i.e. plotting the coefficients of $\phi_{jk}^{(i)}$ against $i$] is a practical way to visually represent the behaviour of the $\{W_t\}$ and $\{E_t\}$ series in response to the various shocks. With knowledge of knowing all the parameters of the primitive system (4.7) and (4.8) it is possible to trace out the time paths of the effects of pure $\varepsilon_{W_t}$ or $\varepsilon_{E_t}$ shocks. However, this methodology is not applicable if the estimated VAR is under or over identified. Here in this example the estimated VAR is under identified, because primitive VAR systems contains 10 parameters but VAR in standard form contains only 9 parameters. So an additional restriction on the VAR system must be imposed in order to identify the impulse responses. One possible identification restriction is to use the Choleski decomposition. For example, it is
possible to contain the system such that the contemporaneous value of $W_t$ does not have a contemporaneous effect on $E_t$. Finally, this restriction is represented by setting $b_{21} = 0$ in the primitive system. In terms of (4.19) the error terms can be decomposed as follows:

$$e_{1t} = e_{Wt} + b_{12}e_{Et}$$

$$e_{2t} = e_{Et}$$

(4.22)

(4.23)

Equation (4.22) shows all the observed errors from the $\{e_{2t}\}$ sequence are attributed to $e_{Et}$ shocks. Given the calculated $\{e_{Et}\}$ sequence, knowledge of the values of the $\{e_{1t}\}$ sequence and the correlation coefficient between $e_{1t}$ and $e_{2t}$ allows for the calculation of the $\{e_{Wt}\}$ sequences using (4.22). Although this decomposition contains the system such that an $e_{Wt}$ shock has no direct effect on $E_t$, there is an indirect effect in that lagged values of $W_t$ affect the contemporaneous value of $E_t$. The key point is that the decomposition forces potentially important asymmetry on the system since an $e_{Et}$ has contemporaneous effects on both $W_t$ and $E_t$. For this reason equation (4.22) and (4.22) imply an ordering of variables. An $e_E$ shock directly affects $e_{1t}$ and $e_{2t}$ but a $e_{Wt}$ shocks does not affect $e_{2t}$. Hence, $E_t$ is ‘prior’ to $W_t$. Alternatively by putting $b_{12} = 0$, the errors can be decomposed as:

$$e_{1t} = e_{Wt}$$

$$e_{2t} = b_{21}e_{Wt} + e_{Et}$$

It is crucial to note that the importance of the ordering depends on the magnitude of the correlation coefficient between $e_{1t}$ and $e_{2t}$. For example if the correlation coefficient is equal to zero, the ordering is immaterial. Finally, (4.22) and (4.23) can be replaced with $e_{1t} = e_{Wt}$ and $e_{2t} = e_{Et}$. On the other hand, if the correlation coefficient is unity (so that two shocks are equivalent), it is inappropriate to attribute the shock to a single source.

### 4.3.3.6 Variance Decomposition:

Variance decomposition is used to detect the causal relation among the variables. It explains the extent to which a variable is explained by the shocks in all the variables in the system. The forecast error variance decomposition explains the proportion of the
movements in a sequence due to its own shock versus shocks to the other variable. The VAR in standard form i.e. (4.14) is written as follows:

\[ X_t = A_0 + A_1 X_{t-1} + e_t \]

Now suppose the coefficient \( A_0 \) and \( A_1 \) is known and we want to forecast the various values of \( X_{t+i} \) conditional on the observed value of \( X_t \). Updating the above equation by one period (i.e. \( X_{t+1} = A_0 + A_1 X_t + e_{t+1} \)) the conditional expectation of \( X_{t+1} \) is:

\[ E_t X_{t+1} = A_0 + A_1 X_t \]

Here one step ahead forecast error is \( X_{t+1} - E_t X_{t+1} = e_{t+1} \). Similarly the two steps ahead forecast error of \( X_{t+2} \) are:

\[ E_t X_{t+2} = [1 + A_1] A_0 + A_1^2 X_t \]

The two steps ahead forecast error [i.e. the difference between the realization of \( X_{t+2} \) and the forecast] is \( e_{t+2} + A_1 e_{t+1} \). In general the n-step ahead forecast is:

\[ E_t X_{t+n} = [1+A_1+A_1^2+ \ldots +A_1^{n-1}]A_0 +A_1^n X_t \]

And that the associated forecast error is:

\[ e_{t+n} + A_1 e_{t+n-1} + A_1^2 e_{t+n-2} + \ldots + A_1^{n-1} e_{t+1} \]

It is possible to write the forecast errors in terms of the \( \varepsilon_{St} \) and \( \varepsilon_{Et} \) shocks. The forecast error variance decomposition tells the proportion of the movements in a sequence due to its own shock versus shocks to the other variable. If \( \varepsilon_{yt} \) shocks explain none of the forecast error variance of \( \varepsilon_{St} \) at all forecast horizons, it can be said that \{\( W_t \)\} sequence is exogenous. In such a circumstance, the \{\( W_t \)\} sequence would evolve independently of the \( \varepsilon_{Et} \) shocks and of \{\( E_t \)\} sequence. On the other hand if \( \varepsilon_{Et} \) shocks explain all of the forecast error variance in \{\( W_t \)\} sequence at all forecast horizons then \{\( W_t \)\} would be entirely endogenous.

### 4.4 Description of Variables and Data Source:

The variables considered in the above model are labour productivity (real gross value added per workers: LP\(_t\)), real wages (real wages paid to workers: RW\(_t\)) and employment (number of workers: E\(_t\)). In order to measure the variables, we have used the data on
gross value added, number of workers, total wages to workers, WPI and CPI of industrial workers (CPI_{iw}). The labour productivity is defined as real gross value added per workers. Following Goldar (1986), we have preferred gross value added as an index of output in place of net value added because depreciation charges in the Indian industries are known to be highly arbitrary, fixed by income tax authorities and seldom represent actual consumption. The concepts those are published in the ASI on payments to labour are wages to a worker, total emoluments, and contribution to provident fund, other welfare expenses as labour compensation. The present chapter has taken total wages and extended emoluments (emoluments + provident funds and other benefits) as labour compensation corresponds to the number of workers and total person engaged. Similarly, the concepts on labour employments include workers, employees, total person engaged. The present study considers workers (E_1) and total person engaged (E_2) as labour variables. However, due to discontinuation of data since 1997-98, number of employees cannot be used as labour variable. As ASI provide value added and wage variables in nominal term, we have used the respective WPI and CPI_{iw} series to arrive at a real gross value added and real wages. For the empirical time series analysis, we have considered real value added per workers as labour productivity (LP_1), real wages (RW_1) as representative of real wage and total number of workers (E_1) as employment variables. This preference can have two justifications, first, to have a consistent time series data based on the uniformity of the approach of the variables, where the labour productivity, labour compensation is explained for the corresponding labour variables. Secondly, these series are consistently available over the entire study period over 1973-74 to 2007-08.

The present study uses annual data spanning the period from 1973-74 to 2007-08. The data are collected from Annual Survey of Industries (ASI) published by the Central Statistical Organization. Besides, the study has also relied on the Economic Political Weekly (EPW) Research Foundation, data base on Indian Manufacturing sector. The data on Wholesale Price Index (WPI) and Consumer Price Index of industrial workers (CPI_{iw}) have been collected from various issues of Handbook of Statistics on Indian Economy, published by the Reserve Bank of India (RBI). WPI deflector for respectively 2-digit industry groups with 1993-94 as the base is used to deflate output, while the CPI for industry workers at 1993-94 are used to adjust inflationary effect on the labour payment variable.
4.5 Growth and Trend Analysis:
In this section, we embark on growth and trend analysis of wage, employment and productivity. This facilitates to understand the broad trend of these variables. Further, this section tries to analyses the relationship between the wages and employment, wages with productivity and employment with productivity of Indian Manufacturing sector. The present analysis is carried on both at aggregate and disaggregated 2–digit industrial group level\(^\text{15}\). Before proceed to our empirical part of this chapter, we first present the trends of number of factories, number of workers, number of non-workers, total persons engaged, labour compensation per workers, labour compensation per person engaged, productivity per workers, productivity per person engaged, average number of workers per factory, average number of non-workers per factory and average number of person engaged per factory of Indian manufacturing sector over the period of 35 years of the study.

### Table-4.1: Trends of Wages, Employment and Productivity in Aggregate Manufacturing.


- refers to non-availability of data.

Note: Figures in the table represents compound growth rates. The growth rates are showing the antilogarithms of the relevant regression co-efficient minus one when the equations estimated are of the form $\ln Y = \alpha + \beta T$, where $\ln Y$ stands for Natural Logarithm of the dependent variable ‘Y’ and ‘T’ refers to time. Variables in row 9 -11, are in numbers.

\(^{15}\) The present study only consider 2-digit NIC industrial groups for the analysis not only for convenience rather, with frequent change in NIC and methodology related to collect and compilation of data, it is not possible to get a time series data on further disaggregated NIC levels for such a long period.
Table 4.1 represents the trends of the important variables, which provide information about the dynamics of labour productivity, employment and real wages in Indian manufacturing. The period of analysis spanning from 1973-74 to 2007-08, which is further divided into four sub-periods to better understand the trend of growth. The four sub-periods are (i) 1973-74 to 1979-80, which is considered to be the period of industrial recovery or pre-reform period, (ii) 1980-81 to 1989-90, represent by and large, the creeping liberalization phase, during this period most of the reforms have been initiated under Mr. Rajiv Gandhi’s Prime-Ministership, (iii) 1990-91 to 1999-2000, the decade when India embarks liberalization lastly, 2000-01 to 2007-08, a period which has seen significant implications and advantages of the LPG policies implemented during the nineties.

From Table 4.1, it is observed that the trends of most of the variables are following one way or another. Number of factories in Indian manufacturing have registered the least rate of growth (1.01%) during the eighties, followed by the highest growth (8.15%) registered during in seventies. However, the growth of a number of factories has recovered during the post reform periods. Employment variables, such as the number of workers, number of non-workers and total person engaged during the period of the study, revealed a rising trend. With a negative growth registered during 80s, employment variables recovered gradually in the reform period and the employment growth has accelerated significantly during the period 2000-01 to 2007-08, with number of workers and total person engaged grew at an annual average rate of 5.88% and 5.70% respectively. While looking at productivity growth, we observed that growth of productivity per workers (LP<sub>1</sub>) and productivity per person engaged (LP<sub>2</sub>) are rising constantly. However, we do not observe the same trend in labour compensation. Though labour productivity has risen consistently since eighties labour compensation to workers (real wage per workers: RWW) and labour compensation to total person engaged (Extended emoluments per person engaged: RWP) were showing weak trend over the period. Real wage per workers is growing at an annual average rate of 1.37% over the entire study period (1973-74 to 2007-08), while it has registered a significantly high
growth of 5.43% during the seventies, the period of industrial recovery when the highest numbers of factories were open up (the number of factories growth was 8.15% over the period). After which the growth of labour compensation per workers and per person engaged were eased to 3.56% and 3.87% respectively during eighties. The growth of labour compensation has decelerated since 80s and reached the all time low over the fourth sub-periods 2000-01 and 2007-08 at 0.08% and 1.12% for RWW and RWP respectively. Contrast to classical thought, rising labour productivity does not associated with rising labour compensation per labour in Indian registered manufacturing sector.

Similarly, if we compare the growth rate of labour compensation to that of their corresponding employment growth, we observe that growth rates of real wages to workers (RWW) and persons engaged (RWP) is consistently coming down over the four decades. In both the cases the growth rate has substantially decelerated from the creeping liberalization phase of the eighties to the full liberalisation period of the nineties and beyond that. Contrary to this, the growth rate of employment is substantially accelerated over the same period. Thus, we can conclude that, the employment growth has rebounded in the post reform period of negative growth during the eighties. This finding is contrary to the finding of Mathur and Mishra16 (2007) work, where they found consistently declining rate of growth of the number of workers and number of employees. While our finding of successive decline trend of the growth rates of wage per workers, is consistent with Mathur & Mishra findings. The major departure of our findings in wage and employment front to that of Mathur and Mishra’s because of the fact that, the difference in the data set as they have taken all India ASI data to represent organized manufacturing while in our case the aggregate manufacturing only representing the sum of 22 two-digit industrial groups. Secondly, it is because of the difference in time periods under consideration to define four sub-periods decade wise. Third, the use of different price base. Moreover, our findings on labour productivity also differ from that of Mathur and Mishra (2007). The growth of labour productivity (LP1 and LP2) has consistently risen since the eighties, which Mathur and Mishra observed an easing labour productivity during the eighties that of the seventies and that has further slowed down in the nineties.

4.5.1 Growth and Structure of Employment:
It is clear from the analysis that employment growth is significantly rising in Indian manufacturing. Moreover, from the analysis of the previous chapter, it was clear that the contribution of manufacturing (registered) sector for the employment generation has been very significant during the post liberalization period. In this section, we will elaborately discuss the employment growth and its composition in the organized manufacturing sector.

4.5.1.1 Growth of Employment:
Annual Survey of Industries (ASI) provides data on various labour input such as the number of workers, number of employees and total person engaged. To analyse the employment growth in the Indian manufacturing sector, we consider number of workers \((E_1)\) and total person engaged \((E_2)\) as our labour variables. However, for better understanding of the composition and growth of employment trend we estimate ‘number of non-workers’, which we derived by deducting the number of workers from total person engaged. In absolute term both number of workers and total person engaged are rising in the post liberalisation period. The total person engaged in the organized manufacturing sector has been rising in the post liberalization period. The trend of the employment growth in the manufacturing as a whole has not been uniform throughout the study period. The employment in terms of the number of workers and the total persons engaged revealed an increasing level of employment in the manufacturing sector up to the year 1995-96. In the next six years i.e. up to 2002-03, the percentage of the manufacturing employment declined. However, the manufacturing sector has experienced a steady rise in employment generation post 2002-03.Goldar (2000) had tried to analyze the pattern of growth in employment in organized manufacturing in the 1990’s. The job security regulations were the main cause of the deceleration in the growth of employment. Another cause that came into view was the sharp hike of the real wage. The slowdown in employment growth was the result of capital deepening strategy. The composition of output of the manufacturing changed in favour of less labour intensive industries. There were faster growth in the low employment intensive industries and slow growth in the high employment intensive industries. The
distribution of employment by employment size had shown a marked change in favour of small size firms. The firms with 50-500 employees gained very much while the firms with 2000-5000 plus lost significantly.\(^{17}\)

Figure 4.1 and 4.2, represents the growth of the number of workers (E\(_1\)) and total person engaged (E\(_2\)) are showing the rising trend over the entire period of the study. However, only during the first half of the 90s, employment growth has decelerated. Employment growth of workers and total person engaged was -0.24% and -0.13% respectively, during the eighties, which has recovered to 1.58% and 1.67% during the nineties. However, the growth of employment has significantly increased during 2000-01 to 2007-08, with number of workers and total person engaged growth rebounded to 5.88% and 5.70%. Figure 4.3 shows the growth rate of the number of non-workers was also followed similar trend like Figure 4.1 and 4.2.

![Figure 4.1: Growth of Employment (Number of Workers: E\(_1\))](image)

**Ln (E\(_1\)) = \alpha + \beta T + \epsilon**

**Ln (E\(_1\)) = 15.30+0.013 T**

**R\(^2\) = 0.810**

Source: Author’s compilation from Annual Survey of Industries, CSO and EPW Research Foundation

\(^{17}\) Refer the trend analysis of labour productivity by ‘Size-of-Employment’ in Chapter-III
The growth of employment in the manufacturing for the post reform period from 1990-91 to 2007-08 was very much significant. The growth rate for the entire period was 1.31% and 1.29% per annum for number of workers and total person engaged respectively. However, the growth rate of the number of workers and the total person engaged showed a very augmenting growth of 5.88% and 5.70% per annum during the period 2000-01 to 2007-08. Goldar (2011) had tried to explain the discontinuity of unemployment growth in the last 15 years in the organized manufacturing sector. From 1995-96 to 2002-03, the employment had fallen at a rate of 1.5% per annum while in the next five years (2003-04 to 2007-08) it had accelerated at 7.5% per annum. Therefore, the argument of jobless growth is no longer a matter of concern. The recent repairs are due to the structural changes in favor of labour intensive technology. During this period the growth in employment was relatively faster among private companies. The labour intensive of private companies was higher than that of the aggregated level. The disappointing growth of employment is primarily due to labour market rigidities. The industrial disputes act caused nearly 3 million less jobs. However, the interest rate differentiates in the employment growth has proven the fact that the different degrees of relation given to the laws by the states contributing to the differentials.

**Figure 4.2: Growth of Employment (Total Person Engaged: E2)**

\[
\ln (E2) = \alpha + \beta T + \varepsilon
\]

\[
\ln (E2) = 15.654 + 0.013 T
\]

\[
R^2 = 0.7032
\]

Source: Author’s compilation from Annual Survey of Industries, CSO and EPW Research Foundation
Table 4.2, represents the compound growth rate of employment and labour productivity of disaggregate 2-digit industry groups. At disaggregated 2-digit group level, the growth of employment varies across 2-digit industrial groups. Wearing apparel (18), Rubber & Plastic products (25), Fabricated Metal products (28) and Furniture manufacturing (36) are the groups with the highest growth rate during the entire period of study, Table 4.2. However, industrial groups such as Woods products (20), Publishing and printing (22), Non-metallic minerals (26), Accounting and computing machines (30) and Other transport equipments (35) have experienced negative employment growth for the entire period of study. Tobacco products (16), Leather products (19), Paper products (21), Petroleum products (23), Chemical products (24), Non-metallic products (26), Electrical machinery (31), Radio, television and communication (32), Medical precision and optical (33) and Motor vehicles...
(34) have above average employment growth. These industries are capital-intensive in nature. Having said this, the high growth rate of employment in these 2-digit industry groups is recited either due to increased use of labour intensive technology or as a mere accumulation of capital. As a result of which, the demand for labour increases. The other industries have also registered growth, but the trend is very slow. The industries except electrical and transport industries, are supposed to be labour-intensive. These industries have a large employment base, which may be an important reason to report slow growth.

At 2-digit level the growth rates of employment in different industries in different sub-periods are entail a different story. During the seventies, the employment growth was above average for a majority of 2-digit groups, whereas during eighties, we observed a weak employment growth. More than half of the 22 two-digit groups have negative or zero employment growth. However, with the introduction of LPG policy in the early 90s, the employment growth in the aggregate manufacturing, recovered to 1.58% (workers) and 1.67% (total person engaged) per annum from the negative growth of -0.24% and -0.13% respectively in 80s. In the post liberalization period Wearing apparel (18), Leather products (19), Non-metallic mineral products (26), Basic metals (27), Fabricated Metal products (28), Machinery Equipments (29), Office accounting & computing machinery (30), Electrical Machinery (31), Motor vehicles (34), Other transport equipments (35) and Furniture & other manufacturing (36) registered high growth rates. The government provides various incentives as a policy measures to support highly capital intensive industries. Moreover the financial support to acquire more advance machineries through import also encouraged along with rising FDI investment are the factors that have enhanced employment growth.
### Table 4.2: Employment and Productivity Growth in Indian Manufacturing Sector  
(In percentage)

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Note: Figures in the table represents compound growth rates. The growth rates are showing the antilogarithms of the relevant regression co-efficient minus one when the equations estimated are of the form LnY = α + β T, where LnY stands for Natural Logarithm of the dependent variable ‘Y’ and ‘T’ refers to time.

Source: Author’s compilation from Annual Survey of Industries and EPW Research Foundation
From Table 4.2, it is observed that, during 2000-01 to 2007-08, Indian manufacturing experienced the highest employment growth. All the two-digit groups except Food products (15) and Tobacco products (16, registered a negative growth) have seen a sharp rebound in the employment growth. Amid open up of the world trade and rising domestic demand, employment growth has improved significantly to cater the rising market demand. Also the period from 2001-2004 during the NDA government, the employment growth in India was very high, which showed the glimpses of the positive effects of the new economic policy. As a result of which, the labour market rigidities were reduced. Thus, during this sub-period, the growth rate was highest among all the sub-periods and significantly higher than the average growth rate. Development policy favoring manufacturing rectifies an economic imbalance and gives a dynamic momentum. The manufacturing sector involved in the characteristics of increasing returns to scale and the economies of scale. These embodied characteristics of the manufacturing sector, have been proven operators in the case of India. The productivity of the Indian manufacturing sector is largely dependent upon the operation of economies of scale. It is found that in the large employment size factories the productivity is higher. This is only because of the use of the roundabout technology of production and greater division of labour. The greater division of labour raises the labour productivity. In the meantime, the large scale operation in the production and the use of specialized machinery invites more and more employment opportunity. Das (2007) examined the Kaldor’s hypothesis by linking the growth of output, employment and productivity for India in the pre and post reform phases. Kaldor argued for a highly significant relationship between capital intensity and rising productivity. However, he observed that output growth played an important role in determining productivity and employment growth. The faster the rate of output growth through capital intensity in manufacturing industry, the faster will be the rate of labour transference from outside manufacturing.

4.5.1.2 Structure of Manufacturing Employment:
After analyzing the employment growth and its trends in Indian manufacturing, now we examine the structure and composition of employment in the Indian manufacturing sector. As mentioned in the previous section, ASI provides many types