CHAPTER IV

TERNARY AND CASCADE FISSION CONTRIBUTIONS IN QUASI-FISSION REACTIONS

1. Introduction

Ternary fission contribution to fission products in high energy heavy ion collisions has been unambiguously established in recent experiments\(^1\)-\(^3\). There are no calculations for the ternary fission contributions in quasi-fission reactions. Here the statistical model is used to calculate ternary fission yields for the compound \(^{84}\text{Kr} + ^{238}\text{U}\). The calculation explains well the enhancement of yields at the lower mass region and the suppression of higher mass components of the yield.

Ever since Fleischer et al.\(^1\) detected ternary fission, there has been a lot of experimental and theoretical work to study the nature of the fissioning process. Becker et al.\(^4\) observed distinct three-pronged events in the quasi-fission reaction \(^{56}\text{Fe} + ^{238}\text{U}\). Theoretical investigations by Strutinsky et al.\(^5\) brought out the fact that such shapes existed only for low charge values (\(X < 0.7\)). Muzychka et al.\(^6\) attempted to explain the formation of three fragments by
cascade fission which is different from the direct ternary fission process. Diehl et al. 7) have made a LDM calculation for the saddlepoint energies for prolate and oblate ternary fission as a function of the fissility parameter X. They found that the saddlepoint energies for prolate shapes fall off more steeply than that for oblate shapes. In Ref. 8 the kinetic energy distribution of three fragments at the scission point is calculated using a statistical theory.

The object here is to calculate the ternary fission contribution to the quasi-fission reaction yield in the reaction $^{84}$Kr + $^{238}$U. The ratio $2^2/A \approx 50$ is quite large and if we extrapolate the experimental values for ternary to binary fission ratio $^4$, it could be greater than 5%. This is enough motivation to calculate the mass distribution of ternary fragments. The yield curve reported by Kratz 9 shows distinct signs of shell effects even at that high energy of excitation. Our theoretical calculations 10,11 indicate that the two humps around mass numbers $A \approx 132$ and 190 are the binary components whereas the one near $A \approx 100$ seems to result from a ternary fission.

The statistical theory of fission was first used by Pong 12 and later modified by Ignatyuk 13 to explain binary
fission yields. The two works mentioned above did not explain even the binary yields properly and it was modified slightly by us \(^{11}\) to give satisfactory results. Here we extend the previous calculation for binary to the case of ternary fission for it is straightforward and requires only the level densities of the three fragment combination; the product of the level densities gives the fragmentation probability in that particular mode characterised by \((A_1, A_2, A_3)\), the masses of the fragments. If one is interested only in the probability of observing one of the fragments as is usually the case in heavy-ion collisions, then the probability is summed over the other two fragment masses subject to the constraint that the total number of particles is conserved. It is interesting that the enhancement in the yield at low mass numbers \((A_1 \approx 110)\) in the reaction \(^{84}\text{Kr}(600 \text{ MeV}) + ^{238}\text{U}\) is well brought out in the present calculation. This calculation deals only with the direct ternary fission and no cascade fission probabilities are included. Attempts have already been made by others \(^{14}\) to take into account the latter process.

2. Method

The method of calculation is similar to that of the binary fission calculation done by us \(^{10},^{11}\). The probability
of observing a particular ternary combination is given by the product of the level densities $\rho_1$, $\rho_2$ and $\rho_3$ of the individual fragments

$$P(A_1, A_2, A_3) \propto \rho(A_1, Z_1) \rho(A_2, Z_2) \rho(A_3, Z_3). \quad (1)$$

The individual nuclear level densities are calculated by the method outlined in Ref. 11, as a function of temperature $T$. In brief, the level density formula of Bethe\textsuperscript{15} is used. The single particle level density parameters are extracted from the single particle spectrum generated by a Nilsson Hamiltonian\textsuperscript{16}. The parameters for the $\ell_s$ and $\ell^2$ terms are the ones given by Seeger\textsuperscript{17}. This ensures that the single particle spectrum could be used over a wide range of nuclei. The Hamiltonian is diagonalised using cylindrical basis\textsuperscript{16}. The excitation energies are obtained by subtracting the ground state energies of the system from the energy of the system at a temperature $T$. The energy of the system at a temperature $T$ is obtained by summing single particle energies with corresponding occupational probabilities (Fermi occupation numbers).

In quasi-fission reactions if one observes only one of the fragment masses as in Ref. 9, the probability as given in eq. (1), should be integrated over the other two fragment
masses subject to the constraint that the total number of nucleons are conserved. Here we do not include prompt neutron emission. The probability $P$ has to be normalised to the percentage of ternary fission in quasi-fission reactions given by experiments. In the case of $^{84}\text{Kr} + ^{238}\text{U}$ the percentage could be easily greater than 5% and this prompts us to look for the mass distribution in ternary fission as well.

3. Cascade fission contribution

As mentioned earlier the heavier component of the binary fragment in heavy ion reactions can fission again provided it has sufficient excitation energy. If $P_B(A)$ is the binary probability of observing the heavier fragment with mass $A$, then the total probability for a cascade fission is

$$P_{\text{cas}} = \sum_A P_B(A) \sum_{A_1'} \rho(A_1') \rho(A_2')$$

(2)

where the second summation gives the total probability of fission for the heavier fragment into smaller fragments with masses $A_1'$ and $A_2'$ subject to the condition $A_1' + A_2' = A$.

Since in heavy ion reactions adequate excitation energy is available for cascade or sequential fission $P_{\text{cas}}$ is
quite appreciable. This process while it decreases the probability of observing a heavier fragment in heavy ion collisions, it enhances the yield towards the lighter fragment side considerably. Observations made by Kratz et al.\(^9\) in the reaction \(^{84}\text{Kr} + ^{238}\text{U}\) is an example which illustrates this fact.

Here however we have not calculated the entire sequential fission yield contribution. A typical calculation for the most probable binary heavy fragment is shown in Fig.1.

In Fig.2, the experimentally observed ternary to binary fission ratio for various reactions characterised by N/Z is shown.

4. Results

The ternary fission contribution enhances the yield at the lower mass region. In the case of \(^{84}\text{Kr} + ^{238}\text{U}\), the lower mass ternary peak is around \(A \approx 95\) and a smaller peak at \(A \approx 135\). The binary fission peaks at \(A \approx 125\) and 197 and sequential fission (SF) yield of the heavier binary fission product are given in Fig.1. If the sequential fission of the entire heavy mass region of yield is included the experimental yield curve could be easily realised. In the present work the stress is on the ternary fission contribution to the mass yield in the quasi-fission reaction of heavy ions.
References

12. P. Fong, Phys. Rev. 82, 332 (1953); ibid. P. Fong, Phys. Rev. 102, 434 (1956).


Figure Captions

Fig. 1. The mass yield curve for the quasifission reaction $^{34}_{84}$Kr + $^{238}_{92}$U at a temperature $T = 2$ MeV. The sequential fission (SF) curve for the mass number $A = 198$ is also shown. The experimental data correspond to projectile energy $E_{\text{lab}} = 600$ MeV and are taken from Ref. 20. The dashed line represents the ternary fission contribution.

Fig. 2. The experimentally observed ternary to binary ratio taken from Ref. 3 and 4.
Fig. 1

$^{84}\text{Kr} + ^{238}\text{U}$

$T = 2 \text{ MeV}$

Yield %

Mass number