2xn flow shop scheduling problem including transportation time and break down interval with arbitrary lags.

• 4.1 Introduction
• 4.2 Mathematical Analysis
• 4.3 Numerical illustration
• 4.4 Remark/conclusion

4.1 - Introduction

In this present chapter we combine the idea of transportation time and break down interval with arbitrary lags and also deals with scheduling problem through heuristic decomposition in which machines break down interval and Arbitrary lags are given.

The effect of machines on the completion of job in an optimal sequence was firstly introduced by Johnson (1954). Milton and Johnson (1959) separately discussed n x 2 flow shop scheduling problem in which despite of processing time of job some addition arbitrary lags i.e. start lag and stop lag. Maggu, Das and kumar (1981) have studied “On equivalent job for job block in 2 x n sequencing problem with transportation time of jobs “Heuristic Algorithm for a weighted job n x 2 flow shop including break down interval” has presented by T.P. Singh (1993). “Minimize the weighted sum of completion times in a permutation flow shop problem has given by S.P. Bansal and P.K. Yadav (1995).

The objective of this chapter is to given heuristic decomposition principle which can obtain optimal or near optimal schedule of n-job two machine flow shop scheduling problem, wherein break down time of machine and arbitrary
lags is taken into account. “Start Lags and Stop Lags” for job i are defined as follows.

**Start Lags:** The start lags ($D_i \geq 0$) in the minimum time which must elapse between starting job i on the first machine and starting it on the second machine.

**Stop Lags:** The “Stop Lag ($E_i \geq 0$)” for the job i is the minimum time which elapses between completing job i on the first machine and completing it on the second machine.

It may be that “start Lag” and “stop Lag” are smaller than the respective processing time of jobs.

This chapter also concerns the breakdown problem of machine; it has been assumed that no machine fails and no disturbance occurs in the processing. The jobs get sudden breakdown due to failure of one or more component of machines for a certain internal of time.

Or

The machines are required to stop for certain interval of time due to in component of labor or machine (exclusive healing etc) or some other external cause.

### 4.2 - Mathematical analysis

Now the algorithm of N-jobs 2-machines flow shop scheduling problem with Arbitrary lags decomposed into the following steps.

**Step 1** - Let $t_i$ denote the effective transportation times defined by

$$t_i' = \text{Max} ( D_i - A_i, \ E_i - B_i, \ t_i)$$

**Step 2** - Let G and H be two fictitious machines having respective processing times for job i as $G_i$ and $H_i$ where $G_i$ and $H_i$ are defined as

$$G_i = A_i + t_i$$
\[ H_i = B_i + t_i \]

**Step 3** - Find optimal sequence by Johnson’s (1954) procedure for two machines n- jobs problem on reduced problem in step –2.

**Step 4** - The optimal sequence obtained in step III gives the optimal sequence for the original problem.
Transportation time \( t_i \) has been also defined by Maggu and Das (1980) is the minimum time for job \( i \) on the first machine and starting it on the second machine.

Now proof of this algorithm:
Let \( U_{ix} \) and \( T_{ix} \) denote starting and completion time at any job \( i \) on machine \( (X = A,B, i = 1,2, - - - - n) \) respectively in a sequences from the definition of start lag \( D_i \).

\[
D_i \leq U_{iB} - U_{iA}
\]

Now
\[
T_{iA} = U_{iA} + A_i
\]
\[
U_{iA} = T_{ia} - A_i
\]
\[
D_i \leq U_{iB} - (T_{iA} - A_i)
\]
\[
D_{iA} \leq U_{iB} - T_{iA}
\]

---------- (i)

\[
E_i \leq T_{iB} - T_{iA}
\]
\[
T_{iB} = U_{iB} + B_i
\]

Hence we have
\[
U_{iB} + B_i - U_{iA} \leq E_i
\]
\[
U_{iB} - T_{iA} \leq E_i - B
\]

---------- (ii)

Also from the definition of transportation time \( t_i \), we have
\[
U_{iB} - T_{iA} \geq t_i
\]

---------- (iii)
Let \( t_i = \max (D_i - A_i, E_i - B_i, t_i) \)  

\[ \text{-------- (iv)} \]

From (1) (2) and (3)

\[ U_iB - T_iA \geq t_i \]

Now we can reduce original problem after replacing these time (start lag, stop lag and Transportation).

Let the machines A and B cease working in the interval (a, b) for the length of time \((b - a)\) due to break down of machines consideration in to account. Then find an optimal schedule minimizing the total elapsed time in which break down effective time interval is involved.

**Step 5** - Read the effect of break down time interval \((a, b)\) on all jobs of the optimal schedule in step (iii) for this, find total elapsed for the optimal schedule, say \(s : \alpha_1, \alpha_2 - - - \alpha_n\) in step(iii) in the following table A and B.

**Tableau A**

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>In-out</td>
<td>In-out</td>
</tr>
<tr>
<td>X_2</td>
<td>U_1A - T_1A</td>
<td>U_1B - T_1B</td>
</tr>
<tr>
<td>X_2</td>
<td>U_2A - T_2A</td>
<td>U_2B - T_2B</td>
</tr>
<tr>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>xn</td>
<td>UnA - TnA</td>
<td>UnB - TnB</td>
</tr>
</tbody>
</table>
Tableau B

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>(U_{iA}, T_{iA})</td>
</tr>
<tr>
<td>x2</td>
<td>(U_{2A}, T_{2A})</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>xn</td>
<td>(U_{nA}, T_{nA})</td>
</tr>
</tbody>
</table>

Where \((U_{ix}, T_{ix}), (i = \alpha_1, \alpha_2, \ldots, \alpha_n, X = A, B)\) denote interval of processing time of jobs \(\alpha_1, \alpha_2, \ldots, \alpha_n\) on machine \(X\).

**Step 6** Find \((U_{ix}, T_{ix}), (b_i, b_j) \cap (a, b)\) for \(i \neq j\)

**Case (i)** if \((U_{ix}, T_{ix}), \cap (a, b) = \emptyset\)

Then the optimal schedule \(S\) in step (iii) is still optimal and the given break down interval \((a, b)\) is not effective on the optimal schedule \(S\).

**Case (ii)** if \((U_{ix}, T_{ix}) \cap (a, b) \neq \emptyset\)

Then defined new processing time \(A_i^'\) and \(B_i^'\)

\[
A_i^' = A_i + (b - a)
\]

\[
B_i^' = B_i + (b - a)
\]

**Step 7** Formulate a new problem with processing times \(A_i^'\) and \(B_i^'\) for job \(j\) on machines \(A\) and \(B\) respective and find optimal schedule by Johnson’s method (1954) for the case (2).

**Step 8** The optimal schedule in step (vii) is now optimal or near optimal for the original problem.
4.2.1. **Notations**

N = number of jobs
X = machines A, B
A_i = Processing time of ith job on machine A
B_i = Processing time of ith job on machine B
T = Total elapsed time
t_i' = effective transportation time
t_i = transportation time
U_i = starting time of any job i
T_i = completion time of any job i
D_i = start lag
E_i = stop lag

4.3. **Numerical illustration**

4.3.1. **Example 1**

Consider the problem with 5 jobs \( \alpha_1, \alpha_2, \ldots, \alpha_n \) to be processed on two machines A and B in the order AB with their processing times given in the following table –

<table>
<thead>
<tr>
<th>JOB</th>
<th>MACH A</th>
<th>MACH B</th>
<th>TRANSPORTATION TIME</th>
<th>START LAG</th>
<th>STOP LAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(A_i)</td>
<td>(B_i)</td>
<td>(t_i)</td>
<td>(D_i)</td>
<td>(E_i)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>3</td>
<td>9</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

*Table 4.1*
Define effective Transportation Time $P_i$ as per step 1 of the Algorithm as:

\[ t_1' = \text{Max} (D_1 - A_1, E_1 - B_1, t_1) \]
\[ = \text{Max} (5 - 6, 4 - 2, 7) \]
\[ = \text{Max} (-1, 2, 7) \]
\[ = 7 \]

\[ t_2' = \text{Max} (D_2 - A_2, E_2 - B_2, t_2) \]
\[ = \text{Max} (6 - 3, 8 - 9, 2) \]
\[ = \text{Max} (3, 1, 2) \]
\[ = 3 \]

\[ t_3' = \text{Max}(D_3 - A_3, E_3 - B_3, t_3) \]
\[ = \text{Max} (4 - 5, 5 - 7, 5) \]
\[ = \text{Max} (-1, -2, 5) \]
\[ = 5 \]

\[ t_4' = \text{Max} (D_4 - A_4, E_4 - B_4, t_4) \]
\[ = \text{Max} (1 - 2, 4 - 5, 3) \]
\[ = \text{Max} (-1, -1, -3) = 3 \]

\[ t_5' = \text{Max} (D_5 - A_5, E_5 - B_5, t_5) \]
\[ = \text{Max} (3 - 4, 7 - 6, 4) \]
\[ = \text{Max} (-1, + 1, 4) \]
\[ = 4 \]

Let G and H be two fictitious machines introduced as per step 2, with $G_i$ and $H_i$ given in the following table for job $i$

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine G</th>
<th>Machine H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>$G_i$</td>
<td>$H_i$</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>6 + 7 = 13</td>
<td>2 + 7 = 9</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>3 + 3 = 6</td>
<td>9 + 3 = 12</td>
</tr>
</tbody>
</table>
Now as per step 3, applying Johnson’s procedure for the above reduced problem, we have \((\alpha_4, \alpha_2, \alpha_5, \alpha_3, \alpha_1)\) as the optimal sequence/schedule.

The total elapsed time for the schedule \((\alpha_4, \alpha_2, \alpha_5, \alpha_3, \alpha_1)\) is given as below:

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine A</th>
<th>Transportation time</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In out</td>
<td>(t_i)</td>
<td>in-out</td>
</tr>
<tr>
<td>(\alpha_4)</td>
<td>0 - 2</td>
<td>7</td>
<td>9 – 13</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>2 - 5</td>
<td>2</td>
<td>13 - 22</td>
</tr>
<tr>
<td>(\alpha_5)</td>
<td>5 - 9</td>
<td>5</td>
<td>22 – 28</td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>9 - 14</td>
<td>3</td>
<td>28 – 35</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>14 - 20</td>
<td>4</td>
<td>35 – 37</td>
</tr>
</tbody>
</table>

Here total elapsed time for this optimal sequence is 37. Here it may be observed as follows:

\[
D_i \leq (U_i B - U_i A)
\]

\[
D_1 = 5 \leq (U_1 B - U_1 A) \\
\leq (35 - 14) = 21
\]

\[
D_2 = 6 \leq (13 - 2) = 11
\]

\[
D_3 = 4 \leq (28 - 9) = 19
\]

\[
D_4 = 1 \leq (9 - 0) = 9
\]

\[
D_5 = 3 \leq (22 - 5) = 17
\]
\[ t_i \leq U_i B - T_i A \]
\[ t_1 = 7 \leq (35 - 20) = 15 \]
\[ t_2 = 2 \leq (13 - 5) = 8 \]
\[ t_3 = 5 \leq (28 - 14) = 14 \]
\[ t_4 = 3 \leq (9 - 2) = 7 \]
\[ t_5 = 4 \leq (22 - 9) = 13 \]
\[ E_i \leq (T_i B - T_i A) \]
\[ E_1 = 4 \leq (37 - 20) = 17 \]
\[ E_2 = 8 \leq (22 - 5) = 17 \]
\[ E_3 = 5 \leq (35 - 14) = 21 \]
\[ E_4 = 4 \leq (13 - 2) = 11 \]
\[ E_5 = 7 \leq (22 - 9) = 13 \]

Let us suppose that break down interval is (7, 12) for 5 hours.
By step (1) of decomposition Principle, the optimal scheduling by Johnson’s
method, neglecting the break down interval (7, 12) is

\[ S = (\alpha_4, \alpha_2, \alpha_5, \alpha_3, \alpha_1) \]

Now to read the effect of the break down interval (7,12) on the jobs of the
schedule S, we compute total elapsed time for S in the following tableau.

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>in – out</td>
<td>in – out</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>0 – 2</td>
<td>2 – 7</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>2 – 5</td>
<td>7 – 16</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>5 – 9</td>
<td>16 – 22</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>9 – 14</td>
<td>22 – 29</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>14 – 20</td>
<td>29 – 31</td>
</tr>
</tbody>
</table>

_table 4.4_
Table 4.5

Now $\{2, 7\} \cap \{7, 12\} \neq \emptyset$ Therefore according to step (iii) the new processing time of job $(\alpha_4, \alpha_2, \alpha_5, \alpha_3, \alpha_1)$ are given in the following table as per step (iv)

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>(0, 2)</td>
<td>(2, 7)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>(2, 5)</td>
<td>(7, 16)</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>(5, 9)</td>
<td>(16, 22)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>(9, 14)</td>
<td>(22, 29)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>(14, 20)</td>
<td>(29, 31)</td>
</tr>
</tbody>
</table>

Table 4.6

The optional schedule for the reduced problem by Johnson's technique is $(\alpha_4, \alpha_2, \alpha_5, \alpha_3, \alpha_1)$ which is not same, when break down interval is not involved.

4.3.2 Example

Obtain optimal sequence for 3 job 2 machine problem given by the following table.
Table 4.7

\[ t'_1 = \text{Max} (-3,0,2) = 2 \]
\[ t'_2 = \text{Max}(-6,-9,5) = 5 \]
\[ t'_3 = \text{Max}(-1,-3,1) = 1 \]

Now according to step (ii) machines G and H are fictitious with \( G_i \) and \( H_i \) given in the following table

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine G</th>
<th>Machine H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( G_i )</td>
<td>( H_i )</td>
</tr>
<tr>
<td>1</td>
<td>10 + 2 = 12</td>
<td>9 + 2 = 11</td>
</tr>
<tr>
<td>2</td>
<td>8 + 5 = 13</td>
<td>12 + 5 = 17</td>
</tr>
<tr>
<td>3</td>
<td>6 + 1 = 7</td>
<td>10 + 1 = 11</td>
</tr>
</tbody>
</table>

Table - 4.8

Now as per step (iii) Applying Johnson’s processor \((3,2,1,)\) is the optional schedule

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine A</th>
<th>Transportation time</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In - out</td>
<td>( t_i )</td>
<td>in-out</td>
</tr>
<tr>
<td>3</td>
<td>0 - 6</td>
<td>1</td>
<td>7 – 17</td>
</tr>
<tr>
<td>2</td>
<td>6 - 8</td>
<td>5</td>
<td>19 – 31</td>
</tr>
<tr>
<td>1</td>
<td>14 - 24</td>
<td>2</td>
<td>31 - 40</td>
</tr>
</tbody>
</table>

Table - 4.9
Here total elapsed time –40 for the optimal sequence 3,2,1, here it may be observed that

\[
\begin{align*}
D_1 &= 7 \leq 31 - 14 = 17 \\
D_2 &= 2 \leq 19 - 16 = 13 \\
D_3 &= 5 \leq 7 - 0 = 7 \\
t_1 &= 2 \leq 33 - 24 = 9 \\
t_2 &= 5 \leq 19 - 14 = 5 \\
t_3 &= 1 \leq 7 - 6 = 1 \\
E_1 &= 9 \leq 40 - 24 = 16 \\
E_2 &= 3 \leq 31 - 14 = 17 \\
E_3 &= 7 \leq 17 - 6 = 11
\end{align*}
\]

To find the sequence minimizing Total elapsed time when the break down of machine occurs in the time internal (7,9) and (15,17) of 2 hours duration each.

Now ignoring break down effect the optimal sequence is \( S=(3,2,1,) \)

The total elapsed time for the sequence \( S \) is given the following table.

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>in-out</td>
<td>in-out</td>
</tr>
<tr>
<td>3</td>
<td>0 - 6</td>
<td>7 - 17</td>
</tr>
<tr>
<td>2</td>
<td>6 - 14</td>
<td>19 - 31</td>
</tr>
<tr>
<td>1</td>
<td>14 - 24</td>
<td>31 - 40</td>
</tr>
</tbody>
</table>

Table 4.9

<table>
<thead>
<tr>
<th>Job</th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(0, 6)</td>
<td>(7, 17)</td>
</tr>
<tr>
<td>2</td>
<td>(6, 14)</td>
<td>(19, 31)</td>
</tr>
<tr>
<td>1</td>
<td>(14, 24)</td>
<td>(31, 40)</td>
</tr>
</tbody>
</table>

Table 4.10
Now here \((0, 6) \cap (7, 9) = \phi\)
\[(6, 14) \cap (15, 17) = \phi\]

Here the break down time interval \((a,b)\) is not aficionado on the of final schedule \(S\).

**Remark** - The problem may be generalized by taking \(m\)-machines \(n\) jobs.