Chapter 5

Topological properties of $M^A$

In this chapter, topological properties, namely, separation axioms, covering axioms, connectedness, etc. of $M^A$ have been worked out.

5.1 Separation Axioms

**Proposition 5.1.1:** Let $M^A$ be the n-dimensional Minkowski space with A-topology A. Then $M^A$ is Hausdorff space.

**Proof:** Since $M^E$ is Hausdorff, for any $x \neq y$ and $x, y \in M$, therefore there exist open sets $G_x$ and $G_y$ in $M^E$, containing $x$ and $y$ respectively, such that $G_x \cap G_y = \emptyset$. The A-topology being finer than the Euclidean topology, $G_x, G_y \in A$. Clearly $G_x \cap G_y = \emptyset$. Hence $M^A$ is Hausdorff.

**Proposition 5.1.2:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is $T_1$.

**Proof:** By Proposition 5.1.1, $M^A$ is Hausdorff and Hausdorffness implies $T_1$-ness [8]. This implies $M^A$ is $T_1$.

**Proposition 5.1.3:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is not regular.

**Proof:** Let $\{t_n\}$ be a sequence of time-like lines passing through a point $z$. Let $z_n \in t_n$ such that $d(z_n, z) \to 0$. Let $Z = \{ z_n : n \in N \}$ and $z \notin Z$. The set $Z$ is closed in $M^A$ [11]. Let $G_1$ and $G_2$ be open sets of $M^A$, containing $Z$ and $z$ respectively. Clearly, an open set $G_1$ intersects $G_2$. Hence $M^A$ is not regular.

**Proposition 5.1.4:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is not normal.

**Proof:** Let $M^A$ be normal. Also $M^A$ is $T_1$ by Proposition 5.1.2. Hence $M^A$ is $T_4$ which implies
that $M^A$ is regular a contradiction to Proposition 5.1.3. Hence $M^A$ is not normal.

5.2 Metrizability

**Proposition 5.2.1:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is not metrizable.

**Proof:** Let $M^A$ be a metrizable space. Since a metrizable space is regular [8]. Hence $M^A$ is regular a contradiction to Proposition 5.1.3. This implies that $M^A$ is not metrizable.

5.3 Covering Axioms

**Proposition 5.3.1:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is non-compact.

**Proof:** Since $M^E$ is non-compact, there exist an open cover $\mathcal{U}$ of $M^E$ which has no finite sub-cover. Since A-topology is finer than the Euclidean topology, therefore $\mathcal{U}$ is an open cover of $M^A$ as well. Since $\mathcal{U}$ has no finite sub-cover, $M^A$ is non-compact.

**Proposition 5.3.2:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is not paracompact.

**Proof:** Since a paracompact Hausdorff space is normal [8], and $M^A$ is Hausdorff and not normal by Proposition 5.1.1 and Proposition 5.1.4, hence $M^A$ is not paracompact.

**Proposition 5.3.3:** Let $M^A$ be the n-dimensional Minkowski space with A-topology. Then $M^A$ is not locally compact.

**Proof:** Since a Hausdorff locally compact space is regular [8], the result follows from Proposition 5.1.3.

5.4 Connectedness

**Lemma 5.4.1:** Let $M$ be the n-dimensional Minkowski space, $A$ and $E$ be the A-topology and
Euclidean topology on $M$ respectively and $\gamma : [0,1] \rightarrow M$ be the map defined by
$\gamma(t) = (1 - t)x + ty, \ t \in [0,1]$ for any $x, y \in M$. Then $\gamma([0,1])^A = \gamma([0,1])^E$.

**Proof:** Since for any $t \in [0,1]$, image of $\gamma([0,1])$ is the straight line in $M$ and $[0,1]^E = [0,1]^A$, hence $\gamma([0,1])^A = \gamma([0,1])^E$.

**Proposition 5.4.2:** Let $M^A$ be the n-dimensional Minkowski space with $A$-topology. Then $M^A$ is path-connected.

**Proof:** Let $x, y \in M$. The map $\gamma : [0,1] \rightarrow M^E$ defined by $\gamma(t) = (1 - t)x + ty, \ t \in [0,1]$ is continuous. This implies that $\gamma : [0,1] \rightarrow \gamma([0,1])^E$ is continuous, $\gamma : [0,1] \rightarrow \gamma([0,1])^A$ is continuous by Lemma 5.4.1. Hence $\gamma : [0,1] \rightarrow M^A$ is continuous. Thus $\gamma$ is a path from $x$ to $y$. This completes the proof.

**Proposition 5.4.3:** Let $M^A$ be the n-dimensional Minkowski space with $A$-topology. Then $M^A$ is connected.

**Proof:** Since path-connectedness implies connectedness [8] and $M^A$ is path-connected, hence the result follows from Proposition 5.4.2.

**Remark:** Since the space $M^A$ is connected, hence clopen sets of $M^A$ are $\emptyset$ and $M$ only.