6.1. Introduction

An application of electric field $E$ to a semiconductor sample produces a current density $J$ exhibiting the Ohmic behavior $J \sim E$, in the low field region. Above certain value of $E$, behavior of $J$ becomes non-Ohmic (sublinear in $E$). For still higher fields, $J$ saturates (or decreases) with the increasing $E$. In high electric field, electrons gain energy and thermalise among themselves rapidly leading to the establishment of hot electron temperature $T_e$ which is greater than the lattice temperature $T$. In the steady state, these electrons lose their energy by emission of phonons. Emission of optical phonons leads to the saturation of drift velocity and hence the current density in the high field region. Momentum relaxation of the electrons takes place through the scattering by the impurities and phonons. In the high field region, generally studied transport properties are field dependent carrier drift velocity $V_d$, and hence $J$, and the hot electron energy loss rate $P$. These properties govern the performance of the electronic devices operating in the high field region and are extensively studied, theoretically and experimentally, in bulk and low-dimensional semiconductor structures including hot phonon effect [6.1-6.7]. It would be interesting to investigate these properties in graphene and its structures. Low field transport properties of the graphene structures are under rigorous investigation [6.8, 6.9].

Graphene, that exhibits very high room temperature mobility ($\sim 2 \times 10^5$ cm$^2$/Vs), highest saturation drift velocity ($\sim 10^7$ cm/s), large current density ($\sim 10^9$ A/cm$^2$) is an attractive material for its applications in ultra-high speed radio frequency and analog devices. Graphene field effect transistors (GFETs) with large transconductance and saturation drain current are desirable for radio frequency (RF) and power gain performance. Voltage gain is another important figure of merit for analog high frequency applications. Hot electron energy relaxation is also central problem in graphene as it is an important issue in designing graphene devices operating in the high filed region. Energy loss rate studies provide insight into the thermal link between electrons and phonons. Electron heating by photons finds potential applications in bolometry and calorimetry. Since graphene sustains very high current density, it holds promise for an interconnect.

The feasibility of graphene applications in analog and RF devices depends mainly upon the non-linear response of drift velocity and current density to the external electric field. It is determined by the interaction of electrons with the disorders (ionized and neutral impurities), intrinsic phonons (acoustic and optical
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phonons) of graphene and surface polar phonons (for a graphene on substrate). The Ohmic region is generally believed to be limited by elastic scattering by disorders. At high fields electron-phonon interactions are important. In this field region electrons can lose their energy through only phonon modes as the dissipative channels. Moreover, the increasing drift velocity and electron temperature with the increasing field enhances electron-phonon scattering and thus momentum relaxation becomes more efficient.

There exist experimental and theoretical investigations of high field transport in monolayer graphene with a view to achieve the ultimate saturation velocity and performance in RF applications [6.10-6.22] and it is currently one of the hot topics in the electronic devices community. Hot electron energy relaxation, another current topic of high field transport, is also investigated both experimentally and theoretically in monolayer graphene [6.23-6.33].

First measurements of saturation velocity and current in monolayer graphene field effect transistor on SiO₂ substrate are reported by Meric et al [6.10] and saturation velocity is attributed to the scattering by interfacial phonons in the substrate. These results demonstrated that applications of 2D graphene for analog and radio frequency circuits are feasible. Barreiro et al [6.11], in monolayer graphene, have shown that the high field transport is very sensitive to the elastic scattering and current tends to saturate but never reaches complete saturation. Their model, based on Boltzmann equation, includes electron scattering due to charged and neutral impurities (disorder) and intrinsic optical phonons. The saturation is shown to be incomplete because of the competition between disorder and optical phonon scattering. Bai et al [6.12] have obtained a high performance graphene transistor by CVD grown with high dielectric constant material HfO₂ as the top gate material. A large transconductance and drain current saturation are reported with the channel length down to 300 nm.

Szafranek et al [6.34] have studied current voltage characteristics in bilayer GFETs and demonstrated that these are more suitable to achieve excellent current saturation and voltage gain. A poor voltage gain of monolayer GFETs, which could be a road block for the exploration of graphene in RF electronics, is shown to be significantly improved (six times more than that in MLG) by using bilayer graphene, where the band gap is tuned by an external electric field.
A theoretical nonlinear transport study is carried out by different groups in MLG [6.16-6.22]. Bistritzer and MacDonald [6.16] employ hydrodynamic approach with drifted Fermi-Dirac (F-D) distribution. At high fields, in clean samples, a nearly saturation drift velocity $V_{\text{sat}} \sim 10^7$ cm/s, and hence the saturation current density, is predicted. With the increasing $E$, the momentum loss rate $Q$ shows a sharp rise and energy loss rate $P$ shows almost a constant value at $V_{\text{sat}}$, indicating the enhanced electron-phonon scattering at high fields. Moreover, effect of disorder, which increases $Q$, is shown to reduce the drift velocity and saturation is not reached even at high fields, agreeing with the predictions of Barreiro et al [6.11].

DaSilva et al [6.17] have made detailed theoretical and experimental high field transport study in MLG. Using the energy and momentum balance equations and drifted Fermi-Dirac distribution, it is unambiguously demonstrated that, in high electric fields, the transport and the energy loss rate is predominantly by emitting surface polar phonons of the substrate resulting in current saturation. Cleaner samples are necessary to achieve full saturation at accessible bias fields $\approx 10^6$ V/cm. Perebeinos and Avouris [6.18] obtain the $V_{\text{sat}} = 8 \times 10^7$ cm/s by solving the Boltzmann transport equation in presence of intrinsic phonon, surface polar phonons and charged impurities. This value is much higher than the experimentally observed. Effect of self heating of graphene is shown to give full saturation current but lowers its value. Substrate dependence of velocity field characteristics are studied by Monte Carlo simulations and the electron scattering due to SPPs is shown to be crucial to describe saturation of drift velocity correctly [6.19]. Besides, there are high field transport studies by Bao et al [6.20], and Fang et al [6.22] using the drifted F-D distribution function for electrons.

However, in bilayer graphene, velocity and current saturation are rarely addressed [6.34, 6.35]. Li et al [6.35] have calculated velocity-field characteristics, employing Monte Carlo simulations, by considering electron scattering by surface polar phonons and charged impurities. The mobility and saturation velocity are found to be significantly reduced than those in monolayer graphene.

The hot electron energy loss rate has been studied theoretically in monolayer graphene [6.23-6.33] and bilayer graphene [6.26, 6.36]. At very low $T_e$, in MLG, $P \sim T_e^4$ and $n_e^{-1}$ ($n_e$ electron concentration) power laws, due to emission of acoustic phonons, are predicted [6.23, 6.26] and experimentally confirmed [6.27, 6.31-6.33]. On the contrary, $P \sim T_e^3$ is predicted and observed in disorder assisted power loss
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[6.30, 6.31] which is attributed to the ‘supercollisions’ - the impurity mediated electron-acoustic phonon interaction. At higher $T_e$, $P$ is shown to be due to intrinsic optical and surface polar phonons [6.24-6.26, 6.28].

From the above discussion, we see that only few studies deal with the high field transport in bilayer graphene [6.26, 6.34-6.36] although it has many advantages over MLG. Viljas and Heikkilä [6.26] have calculated power loss due to acoustic, optical and surface polar phonons. Li et al [6.35] have studied velocity-field characteristics employing the Monte Carlo simulations. Hot phonon effect is not taken in to account in these studies. In a recent work, Katti and Kubakaddi [6.36] have investigated the hot phonon effect on the electron power loss due to surface polar phonons.

It has been shown comprehensively in MLG that SPPs are the principal scattering mechanisms for the high field transport [6.17, 6.22]. $V_d$ and $T_e$ obtained from the theory including all the phonons are shown to be extremely well approximated in a model considering only SPPs and more than 95% of the power loss is shown to be due to SPPs [6.17]. The experimental work in case of MLG on a SiO$_2$ substrate also confirms that the high field transport is limited by SPP scattering [6.17]. Also, in the study of $V_d$ dependence on $E$ in BLG, it is shown that SPPs play a very significant role and that the intrinsic optical phonons are relatively weak source of interaction [6.35]. Hence, in the present work SPPs are considered to be the main source for power loss and scattering due to intrinsic optical phonons is ignored.

Two approaches have been used in the theoretical study of high field transport properties: (i) momentum and energy balance technique [6.16, 6.17, 6.20, 6.22] and (ii) Monte Carlo simulation [6.18, 6.22, 6.35]. The former one is an analytical model using Boltzmann theory and it is shown to provide an excellent account of high field transport in MLG [6.17].

In the present work, we investigate the high field transport properties namely, hot electron energy loss rate $P$, momentum loss rate $Q$, hot electron temperature $T_e$, and electron drift velocity $V_d$ in BLG. The analytical model of momentum and energy balance technique based on Boltzmann theory, including the hot phonon effect, is followed in the present work and some new features are brought out. BLG is taken to be on a substrate, a commonly used configuration. The energy relaxation is considered to be due to (i) acoustic phonons (APs) and (ii) surface polar phonons (SPPs) [6.26, 6.36]. The momentum relaxation is considered to be due to (i) charged
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6.2 Theory

We consider an n-type gapless BLG, on a substrate, with Fermi energy \( E_f \) away from the charge neutrality point, so that electrons are the only carriers. As discussed in Chapter 1, the eigen function of a 2D electron is given by

\[
\psi_{k,s}^K = e^{-ikr} \phi_{k,s} / \sqrt{A},
\]

where \( \phi_{k,s} = (e^{-i\theta_k}, s)/\sqrt{2} \) is the chiral wave function and \( A \) is the area of the graphene [6.9]. The corresponding energy eigen value is given by \( E_k = \hbar^2 k^2 / 2m \). It is assumed that electrons are equally distributed in the two layers of BLG.

Under the influence of electric field \( E \), the electron distribution is approximated to be drifted Fermi-Dirac distribution [6.16, 6.17, 6.22]

\[
f(E_k) = \left[ \exp \left( \frac{(E_k - k_0 - E_f)}{k_B T_e} \right) + 1 \right]^{-1}
\]

for electron temperature \( T_e > T \) with \( \hbar k_o = mV_d \) being the drift momentum of the electrons. It is believed that the use of drifted Fermi-Dirac distribution in the electron temperature model, used in the technique of energy and momentum balance equations [6.1, 6.7, 6.16, 6.17, 6.22, 6.40], has the following technical advantages over the Monte Carlo method [6.41]: (i) it is simple and does not require large computations, (ii) free from statistical fluctuations inherent in numerical Monte Carlo method and (iii) it takes care of incorporation of the effects of degeneracy and electron-electron interaction.

The energy and momentum balance equations, respectively, are given by

\[
eEV_d = P = \sum_j \langle dE_k / dt \rangle_j \quad \text{and} \quad eE = Q = \sum_j \langle dhk_E / dt \rangle_j,
\]

where \( \langle dE_k / dt \rangle_j \) and \( \langle dhk_E / dt \rangle_j \) are the average energy and momentum loss rates, respectively, \( j \) stands for different scattering mechanisms and \( \hbar k_E \) is the component of electron momentum in the direction of electric field \( E \).

The electron energy loss rate \( P \) and momentum loss rate \( Q \) due to phonons are obtained, respectively, by finding the energy and momentum gained by the phonons from the carriers and dividing by the total number of carriers \( N_e \). Assuming that 2D
electrons interact with 2D phonons of energy $\hbar\omega_q$ and wave vector $\mathbf{q}$, the respective loss rates due to each of the phonon mechanisms are given by [6.1]

\[
P = -\frac{1}{N_e} \sum_{\mathbf{q}} \hbar \omega_q \left( \frac{\partial N_q}{\partial t} \right)_{el-ph}
\quad \text{and} \quad
Q = -\frac{1}{N_e} \sum_{\mathbf{q}} \hbar q_E \left( \frac{\partial N_q}{\partial t} \right)_{el-ph}.
\quad (6.2.3)
\]

Here $(\partial N_q/\partial t)_{el-ph}$ is the rate of change of phonon occupation number $N_q$, and $q_E$ is the component of phonon wave vector in the direction of electric field $E$. The rate of change of $N_q$ is given by

\[
\left( \frac{\partial N_q}{\partial t} \right)_{el-ph} = \frac{2\pi}{\hbar} g_v g_s \sum_k \left| C(q) \right|^2 g(\theta_{kk'}) \left[ (N_q + 1) f(E_{k+q}) \left( 1 - f(E_k) \right) \right]

- N_q f(E_k) \left( 1 - f(E_{k+q}) \right) \delta(E_{k+q} - E_k - \hbar\omega_q). \quad (6.2.4)
\]

In evaluating Eqn. (6.2.4), $f(E_k)$ is being approximated to the first order in the expansion

\[
f(E_k) = f^0(E_k) + (\hbar k . V_d/k_B T_e) f^0(E_k), \quad (6.2.5)
\]

where $f^0(E_k)$ is the F-D distribution at $T_e$. To simplify Eqns. (6.2.3) and (6.2.4) further, we convert the summations over $q$ and $k$ to integration, as given in Eqn. (4.2.6),

\[
\sum_q \rightarrow \frac{A}{4\pi^2} \int_0^{2\pi} q dq d\varphi \quad \text{and} \quad \sum_k \rightarrow \frac{A}{4\pi^2} \int_0^{2\pi} k dk d\psi,
\quad (4.2.6)
\]

where $\varphi$ is the angle between $\mathbf{k}$ and $\mathbf{q}$ and $\psi$ is the angle between $\mathbf{k}$ and $\mathbf{V_d} (\parallel \mathbf{E})$. Then taking $E$ along $y$-axis, $q_E = \mathbf{q} . \mathbf{E}/E = q \cos(\psi + \varphi)$. It can be shown that chiral overlap integral $g(\theta_{kk'}) = \cos^2 \theta_{kk'} = g(k,q) = [(k\pm q\cos\psi)/(k\pm q)]^2$, where $+$(-) corresponds to the phonon absorption (emission).

### 6.2.1 Energy and momentum loss rates due to acoustic phonons

Electrons are assumed to interact with only the longitudinal acoustic phonons via unscreened deformation potential coupling. Also, acoustic phonons are taken to be at thermal equilibrium at lattice temperature $T$, hence $N_q = N_q^0$ is taken in the following. Using Eqns. (6.2.4) and (6.2.5) in equation for $P$, we get,

\[
P_{el-ph} = -\frac{8\pi^2}{\hbar} \frac{A^2}{N_e} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} q dq d\varphi d\psi \left| C(q) \right|^2 g(\theta_{kk'}) N_q^0 \exp \left[ \frac{\hbar\omega_q}{k_B \left( \frac{1}{T} - \frac{1}{T_e} \right)} - 1 \right]

\times f(E_k) \left[ 1 - f(E_k + \hbar\omega_q) \right] \delta(E_k - E_k - \hbar\omega_q), \quad (6.2.6)
\]
We substitute for matrix element from Eqn. (3.2.14) and \( f(E_k) \) from Eqn. (6.2.5). The integration with respect to \( \psi \) is carried out using Dirac delta function (as done in section 4.2). This is followed by integration with respect to \( \varphi \). Angular integration gives contribution from only first term of Eqn. (6.2.5). Then, we obtain

\[
P_{el-ap} = -\frac{D^2 m^{3/2}}{n_s \sqrt{2 \hbar^2 \rho \pi^2}} \int_0^\infty dq \int_0^{\gamma_{a(q)}(q)} dE_k \frac{g(q,k)N^0_q}{\sqrt{E_k}} \times \left\{ \exp \left[ \frac{\hbar \omega_q}{k_B} \left( \frac{1}{T} - \frac{1}{T_c} \right) \right] - 1 \right\} f^0(E_k)[1 - f^0(E_k + \hbar \omega_q)]. \quad (6.2.7)
\]

To find the momentum loss rate, using phonon rate equation in equation for \( Q \), we write

\[
Q_{el-ap} = -\frac{g g_s A^2 m}{8 \pi^3 \hbar^2} \int_0^{\gamma_{a(q)}(q)} dE_k \int_0^{2\pi} dq \frac{d\varphi}{2\pi} \int_0^{2\pi} d\psi \hbar q \cos(\varphi + \psi) \left| C(q) \right|^2 \frac{g(\theta_{kk'})}{\sqrt{E_k}} \times \left\{ (N^0_q + 1) f(E_k)[1 - f(E_k - \hbar \omega_q)] \delta(E_k - E_k + \hbar \omega_q) \right. \left. - N^0_q f(E_k)[1 - f(E_k + \hbar \omega_q)] \delta(E_k + \hbar \omega_q - E_k + \hbar \omega_q) \right\}, \quad (6.2.8)
\]

We substitute again for \( \left| C(q) \right|^2 \) (from Eqn. (3.2.14)) and \( f(E_k) \) (from Eqn. (6.2.5)) and carry out integration with respect to \( \psi \) using Dirac delta function followed by integration with respect to \( \varphi \). Angular integration gives contribution from only second term of Eqn. (6.2.5), then we obtain

\[
Q_{el-ap} = \frac{D^2 m v_d}{2 \pi^2 \rho \sqrt{2 \hbar^2 k_B T_n s}} \int_0^{q_{max}} dq \int_0^{q_{min}} dq \left\{ q^2 dq \frac{g(q,k)(N^0_q + 1)\sqrt{\gamma_e(q)}}{\sqrt{E_k}} \times \left\{ \left| \left| f^0(E_k + \gamma_e(q))[1 - f^0(E_k + \gamma_e(q))][1 - f^0(E_k + \gamma_a(q) - \hbar \omega_q)] \right| \right\} \right. \left. + \int_{q_{min}}^{q_{max}} q^2 dq f^0(E_k + \gamma_a(q))[1 - f^0(E_k + \gamma_a(q))] \right. \left. \times \left\{ \frac{g(q,k)N^0_q \sqrt{\gamma_a(q)}}{\sqrt{E_k}} [1 - f^0(E_k + \gamma_a(q) + \hbar \omega_q)] \right\} \right\}. \quad (6.2.9)
\]

where \( q_{max} = 2 \sqrt{2 m E_k / \hbar^2} [1 \pm (m v_L / \sqrt{2 m E_k})] \), \((-)\) is for absorption (emission), \( q_{min} = 0 \) for both emission and absorption and \( g(q,k) = (1 - 2(q/2k)^3)^2 \) for quasi-elastic scattering. \( N^0_q \) is the Bose distribution function of acoustic phonons at lattice temperature \( T \) and \( \gamma_{e(a)}(q) = [(E_q \pm \hbar \omega_q)^2 / 4 E_q] \).
At low temperature, in the Bloch-Grüneisen (BG) regime, \(q \ll 2k_f\) (\(k_f\) being the Fermi wave vector) and \(\hbar \omega_q \approx k_B T\), following the approximations made in Eqn. (4.2.14)-(4.2.16), we obtain

\[
P_{el-ap} = \frac{D^2 m^{3/2}}{\sqrt{2} n_s \hbar^2 \rho \pi^2 (hv_s)^3 E_f^{1/2}} \int_0^{\infty} \frac{d(h \omega_q)(\hbar \omega_q)^3 [N_q(T_e) - N_q(T)]}{0}.
\]  

(6.2.10)

Then, using the integral given in Eqn. (4.2.18), we obtain energy loss rate in the form of power law, which is given by

\[
P_{el-ap} = F(T_e) - F(T) ,
\]

(6.2.11)

with

\[
F(T) = \frac{m^2 D^2 (k_B)^4 \zeta(4)}{\pi^{5/2} n_s^{3/2} \rho \hbar^3 (hv_s)^3 T^4}.
\]

(6.2.12)

Here \(\zeta(4)\) is the Riemann zeta function. Hence, in BG regime, we predict

\[
F(T_e) \sim T_e^4 \text{ and } n_s^{-3/2}.
\]

(6.2.13)

In the high temperature approximation \(\hbar \omega_q \ll k_B T, k_B T_e\), phonon distribution is taken in the equipartition regime \(N_q^0 = k_B T / \hbar \omega_q\). In this approximation, it can be shown that

\[
P_{el-ap} = G(T_e, E_f) (T_e - T),
\]

(6.2.14)

where \(G(T_e, E_f)\) is a function of \(T_e\) and \(E_f\) with a weak dependence on \(T_e\).

### 6.2.2 Energy and momentum loss rates due to surface polar phonons

As discussed in section 1.5.3, we assume 2D electrons to interact with 2D surface polar phonons of energy \(\hbar \omega_q = \hbar \omega_s\), where \(\omega_s\) is assumed to be independent of wave vector \(q\). The interaction of electrons with surface phonons is polar in nature and its matrix element is given by Eqn. (1.5.16) i.e. \(|M(k,k')|^2 = |C(q)|^2 g(\theta_{kk'})\), with \([C(q)]^2 = [e^2 F^2 / |\varphi(q)|^2] [e^{-2q \delta} + e^{-2q(d+c)}] / 2q\) [6.35]. In order to evaluate \(P\), we first simplify phonon rate equation \((\partial N_q / \partial t)_{el-ap}\) for surface polar phonons. We note that in Eqn. (6.2.5) for \(f(E_k)\), only first term \(f^0(E_k)\) contributes for the power loss due to angular integration. In this rate equation, we use

\[
(N_q + 1) f^0(E_k + \hbar \omega_s)[1 - f^0(E_k)] - N_q f^0(E_k)[1 - f^0(E_k + \hbar \omega_s)]
= [N_q + 1] e^{-2q / T_e} - N_q f^0(E_k)[1 - f^0(E_k + \hbar \omega_s)] .
\]

(6.2.15)

This leads Eqn. (6.2.4) for SPP to
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\[
\left( \frac{\partial N_q}{\partial t} \right)_{el-spp} = \Gamma(q)[(N_q + 1)e^{-\theta/T_e} - N_q],
\]
(6.2.16)

where

\[
\Gamma(q) = g_v g_s \frac{2\pi}{\hbar} \sum_k |C(q)|^2 g(\theta_{kk'}) f^0(E_k) [1 - f^0(E_k + \hbar\omega_s)] \\
\times \delta(E_k + \hbar\omega_s - E_{k'}) \delta_{k',k+q}.
\]
(6.2.17)

In Eqn. (6.2.17) summation over \( k \) is converted to an integration and angular integration is carried out using Dirac delta function. Then after substituting for respective \( |C(q)|^2 \), we get

\[
\Gamma(q) = \frac{m^{3/2} \epsilon^2 \beta_{rem} \hbar \omega_s (e^{-2qd} + e^{-2q(d+c)})}{2\sqrt{2\pi} \hbar^4 q^2 e_0 |e(q)|^2} \int \frac{dE_k}{\gamma_e(q) \sqrt{E_k - \gamma_e(q)}} \\
\times g_e(E_k,E_q) f^0(E_k) [1 - f^0(E_k + \hbar\omega_s)].
\]
(6.2.18)

This \( \Gamma(q) \) is substituted in Eqn. (6.2.16). In turn, resulting phonon rate equation is used to obtain energy loss rate \( P_{el-spp} \) due to surface polar phonons. It is found to be given by

\[
P_{el-spp} = (\hbar\omega_s / 2m_s) \int_0^\infty \frac{q \Gamma(q)}{(N_q + 1) \exp(-\hbar\omega_s / k_B T_e) - N_q) dq}.
\]
(6.2.19)

Following the steps given in case of acoustic phonons, substituting the matrix element corresponding to surface polar phonons, the momentum loss rate due to these phonons is given by

\[
Q_{el-spp} = \frac{m^2 \epsilon^2 \beta_{rem} (\hbar\omega_s) V_d}{2\pi^2 \omega_n \hbar^3} \int_0^\infty \frac{e^{-2qd} + e^{-2q(d+c)}}{|e(q)|^2} dq \\
\times \left\{ \frac{(N_q + 1)}{\gamma_e(q) + \hbar\omega_s} \right\} \\
\times \left( -\frac{\partial f^0(E_k)}{\partial E_k} \left[ 1 - f^0(E_k - \hbar\omega_s) \right] dE_k \right) \\
+ \left( \frac{(N_q \sqrt{\gamma_a(q)} E_k - \gamma_a(q))^{-1/2}}{\gamma_a} \right) \\
\times \left( -\frac{\partial f^0(E_k)}{\partial E_k} \left[ 1 - f^0(E_k + \hbar\omega_s) \right] dE_k \right),
\]
(6.2.20)
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where \( g_{a(e)}(E_k, E_q) = \frac{1}{4E_k} \left( \sqrt{E_k^2 + E_q^2} + E_k \pm E_q \right) \) and \( \gamma_{a(e)}(q) = \frac{1}{4E_q} \). The lower limit of the energy integration is with the addition of phonon energy, as electrons with energy lower than phonon energy cannot emit phonons.

6.2.3 Hot surface polar phonons

In the high field region, we believe that the SPP emission rate by electrons is much faster than the phonon decay rate leading to the increase in the phonon population. These phonons are called ‘hot phonons’. The distribution function \( N_q \) of these hot phonons would be different from its equilibrium Bose distribution \( N_q^0 = \{ \exp(\hbar\omega_q/k_BT)-1 \}^{-1} \). The hot phonon effect is shown to be important with respect to interface modes in narrow GaAs/AlAs QWs [6.42, 6.43]. Hot phonon effect on energy loss rate due to intrinsic optical phonons in MLG [6.25] and SPPs in BLG [6.36] has also been investigated. Besides, in a monolayer graphene device on a polar substrate, the experimentally reported smaller saturation drift velocity compared to the calculated one is attributed to the ‘self heating effect’ in which SPPs and the intrinsic optical phonons of graphene are assumed to be heated [6.18]. In view of this, we take account of the hot phonon effect on the energy and momentum loss rates due to SPP in BLG. The hot phonon distribution \( N_q \) for SPPs is obtained by solving Boltzmann equation for phonons in the following.

The phonon Boltzmann equation, describing the rate of change of phonon occupancy \( N_q \), in steady state, is given by

\[
\frac{\partial N_q}{\partial t} \bigg|_{\text{pp}\rightarrow \text{el}} + \frac{\partial N_q}{\partial t} \bigg|_{\text{el}\rightarrow \text{pp}} = 0. \tag{6.2.21}
\]

Here, the first term is due to phonon scattering and second term is due to scattering of phonons by electrons. In the relaxation time approximation,

\[
\frac{\partial N_q}{\partial t} \bigg|_{\text{pp}\rightarrow \text{el}} = - \frac{(N_q - N_q^0)}{\tau_p}, \tag{6.2.22}
\]

where \( \tau_p \) is the phonon relaxation time. We use Eqn. (6.2.16) and Eqn. (6.2.22) in Eqn. (6.2.21). Then, rearranging the terms, solution for \( N_q \) of SPP is obtained. It is given by
$$N_q = \frac{[N^0_q + \tau_p \Gamma(q) \exp(-\hbar \omega_q / k_B T_e)]}{[1 + \tau_p \Gamma(q) \{1 - \exp(-\hbar \omega_q / k_B T_e)\}]}.$$  \hspace{1em} (6.2.23)

Thus, $N_q$ depends upon $\tau_p$, $T_e$ and $n_s$ at a given lattice temperature.

**6.2.4 Momentum loss rate due to impurity scattering**

The momentum loss rates due to Coulomb impurity and short-range disorder scattering are obtained in terms of their respective average momentum relaxation time $\langle \tau \rangle$ i.e.

$$Q_{imp} = m V_d \langle \tau \rangle_{imp}.$$  \hspace{1em} (6.2.24)

The momentum relaxation time due to impurity is given by Eqn. (3.2.22) [6.37].

Using the respective average momentum relaxation times (Eqns. (3.2.23) and (3.2.24)) in Eqn. (6.2.24), we obtain momentum loss rate

$$Q_{CI} = m V_d \langle \tau \rangle_{CI},$$  \hspace{1em} (6.2.25)

due to charged impurity and

$$Q_{SD} = m V_d \langle \tau \rangle_{SD}.$$  \hspace{1em} (6.2.26)

due to short-range disorder.

**6.2.5 Total energy and momentum loss rates**

Finally, using equations for $P$ and $Q$ due to all the mechanism, we can express Eqn. (6.2.2) in the form

$$e E V_d = P = \sum_j f_{ej}(T_e) \text{ and } e E = Q = V_d \sum_j f_{mj}(T_e).$$  \hspace{1em} (6.2.27)

where $f_{ej}(T_e)$ and $f_{mj}(T_e)$ are the $T_e$ dependent terms on the right hand side of energy and momentum loss rate equations, respectively, due to $j$ th scattering mechanism and the summation is over the different relevant scattering mechanisms. Then, the field dependent mobility can be expressed through the equation

$$\mu(T_e) = \frac{V_d}{E} = e / \sum_j f_{mj}(T_e).$$  \hspace{1em} (6.2.28)

The coupled energy and momentum loss rate Eqns. (6.2.27) are numerically solved to obtain $V_d$ and $T_e$ as a function of electric field $E$. Moreover, $V_d$ vs $E$ due each mechanism is obtained by solving the respective coupled energy and momentum balance equations.
6.3 Results and discussion

We present the numerical calculations for a BLG with SiO$_2$ substrate using $D=20$ eV [6.16, 6.23, 6.33]. The material parameters used are listed in Table 3.2.

Figure 6.1. (a) $P/T_e^3$ vs $T_e$, (b) $P/P_{12D}$ vs $T_e$ due to acoustic phonons for different $n_s$. 

(b) $P/P_{12D}$ vs $T_e$ due to acoustic phonons for different $n_s$. 

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First, we present the low temperature ($T_e < 50$ K) power loss study due to acoustic phonons at lattice temperature $T = 0.1$ K. The $P/T_e^3$ due to only acoustic phonons is shown in Figure 6.1(a) as a function of $T_e$ for electron concentrations $n_s = 0.5, 1.0$ and $1.5 \times 10^{12}$ cm$^{-2}$. Curves are also shown due to $T_e^4$ law obtained in BG regime (from Eqn. (6.2.11)) and due to setting $g(E_k) = 1$, a function arising from the chiral nature of the electrons in Eqn. (6.2.6). A new feature is that a kink is observed around $T_e = 8$ K in the curves calculated using Eqn. (6.2.6), which disappears when chiral property is ignored (i.e. when $g(E_k) = 1$). The position of this kink is $n_s$ dependent and is found to shift towards higher $T_e$ with the increasing $n_s$.

Denoting the $P = P_{12D}$ (ideal 2D) for the case with $g(E_k) = 1$, $P/T_{12D}$ vs $T_e$ is shown in Figure 6.1 (b). It shows a minimum and the position of the minimum shifts to higher $T_e$ for larger $n_s$. Effect of chiral character is found to reduce $P$ and maximum reduction is about 55%. Similar behavior is observed in the phonon-drag thermopower of BLG (Figure 4.3) [6.44]. This behavior is found to be missing in the energy loss rate calculations of Viljas and Heikkilä [6.26]. The predicted $P \sim T_e^4$ law in BLG (Eqn. 6.2.13) is same as found in MLG [6.23] and it is again a characteristic of 2D phonons. This is in contrast with $P \sim T_e^5$ in conventional 2DEG for unscreened deformation potential coupling with 3D phonons [6.45]. Additionally, we point out that, the range of $T_e$ for the validity of $T_e^4$ law is reduced due to the chiral character of the electrons.

We compare the heat resistance $R_h$ obtained using our calculated $P$ with the experimental results. The heat resistance $R_h = (dT_e/dP_T)$ is calculated using Eqn. (6.2.11) for $P$, where $P_T = N_e P$ ($= n_s A P$). For the sample of Yan et al [6.46] with $E_f = 33$ meV and $A = 100$ μ m$^2$, we obtain $R_h = 4.14 (T/5)^{-3}$ K/nW which is in reasonable agreement the experimentally observed value of $2 (T/5)^{3.45}$ K/nW [6.46].

The $n_s$ dependence of $P$ is predicted to be $P \sim n_s^{-3/2}$, in BLG, which is same as that in conventional 2DEG [6.45] and is in contrast with $P \sim n_s^{-1/2}$ in MLG [6.23]. We attribute the $n_s^{-3/2}$ dependence to the parabolic dispersion of electron energy. In MLG, the recent experimental observations of low temperature hot electron energy loss rate [6.32, 6.33] are in excellent agreement with the predicted $T_e^4$ law and $n_s^{-1/2}$ dependence [6.23]. Similar experimental observations in BLG may validate our predictions in the present work.
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Figure 6.2. $Q/T_e^2$ vs electron temperature $T_e$ due to acoustic phonons for different $n_s$.

Low temperature $Q/T_e^2$, due to APs, as a function of $T_e$ is shown in Figure 6.2 at $T=0.1$ K. It is found that $Q \sim T_e^{2.5}$ at very low $T_e$. The $Q$ behavior also shows a kink around $T_e = 8$ K which is again attributed to the chiral character of the electrons in BLG. This behavior is also reflected in very low temperature acoustic phonon limited mobility in BLG (Figure 3.2) [6.47]. $Q$ is found to decrease with the increasing $n_s$.

In what follows, we present numerical results of high field properties, at $T=4.2$, 77 and 300 K for $n_s=1 \times 10^{12}$ cm$^{-2}$, covering large $T_e$ range where electron scattering due to SPPs plays a significant role. Before giving these high field results, we briefly discuss the hot phonon distribution function $N_q$ of SPPs [6.36]. Hot phonon distribution (Eqn. (6.2.23)) is shown in Figure 6.3 (a) as a function of normalized phonon wave vector $q/q_0$, with phase matched wave vector $q_0 = \sqrt{2m\hbar \omega / \hbar^2}$. The values of the $\tau_p$ chosen are closer to the values used in the literature for intrinsic optical phonons in graphene [6.25], carbon nanotubes [6.48] and in GaAs heterostructures [6.5, 6.42, 6.43, 6.49]. The calculated $N_q$ is found to vary strongly with the wave vector $q$. The most probable hot phonons produced are in the region around $q_0$ because of the requirement of conservation of energy and momentum. Interestingly, a dip is observed in $N_q$ and it is a new feature in BLG. This feature is not found in hot phonon distribution of conventional 2D semiconductor systems [6.42, 6.49]. This dip is vanishing (see continuous curve) if the overlap integral due to
chiral wave function is set to one i.e. \( g(E_k) = 1 \). Hence, the appearance of dip may be attributed to the chiral nature of the electrons. \( N_q \) is larger for higher \( T_e \), because large number of phonons are emitted at higher \( T_e \). Also, for a given \( T_e \), \( N_q \) is found to be larger for larger \( \tau_p \) (Figure 6.3 (b)) as phonons will decay slowly for larger \( \tau_p \). Besides, the depth of the dip is found to be larger for smaller \( \tau_p \). \( N_q \) is found to be larger for higher lattice temperature (Figures 6.3(c) and (d)). Also, width of the maximum \( N_q \) region decreases with the increasing \( T \).

In Figure 6.4 (a) \( P \) vs \( T_e \) is shown for both APs and SPPs at \( T=4.2 \) K with (without) hot phonon effect choosing \( \tau_p = 5 \) (0) ps. In the high \( T_e \) region (>50 K), \( P \) due to APs shows a nearly linear \( T_e \) dependence. Consequently, in the predicted \( P_{el-ap} = G(T_e, E_j) (T_e - T) \) behavior, \( G(T_e,E_j) \) will be nearly independent of \( T_e \) and this is comparable to \( P \sim (T_e - T) \) dependence in conventional 2DEG [6.5]. We observe that the \( P \) due to SPPs is dominant for about \( T_e > 150 \) (120) K, with (without) hot phonon effect as compared to dominance of LO phonons above 50 K in GaAs heterojunction.

Figure 6.3. Hot phonon distribution \( N_q \) vs normalized phonon wave vector \( q/q_0 \). Solid (dashed) curve represents \( N_q \) with (without) chiral property. (a) For different \( T_e \). (b) For different \( \tau_p \). (c) for \( T=77 \) K and (d) for \( T=300 \) K.
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Figure 6.4. Energy loss rate $P$ vs electron temperature $T_e$ with and without hot phonon effect for lattice temperature (a) 4.2 K, (b) 77 K and (c) 300 K.
This can be attributed to the smaller LO phonon energy of 36 meV in GaAs compared to 59.98 meV energy of SPPs in graphene. Hot phonon effect is found to reduce $P$ due to SPPs significantly. For example, at $T_e = 300$ K, $P$ is reduced by about 2.5 times. However, the magnitude of the hot phonon effect is found to be relatively smaller in BLG compared to GaAs heterojunction [6.5, 6.50].

For $T = 77$ K, $P$ due to acoustic phonons is dominant for about $T_e < 120$ (100) K for $\tau_p = 5$ (0) ps (Figure 6.4 (b)). $P$ due to acoustic phonons is almost linear in $T_e - T$. The hot phonon effect reduces the power loss due to SPP by about 5 times (for $\tau_p = 5$ ps) at $T_e = 300$ K. Similarly, power loss calculations as function of $T_e$ (300 –1000 K), at $T = 300$ K are presented in Figure 6.4 (c). $P$ due to SPP is dominant over the entire $T_e$ range. It is 40 (50) times greater than that of the acoustic phonons at $T_e = 500$ (1000) K. Hot phonon effect on $P$ due to SPP is found reduce $P$ by two times at $T_e = 1000$ K.

The momentum loss rates $Q$ due to APs and SPPs are shown as function of $T_e$ in Figures 6.5 (a)-(c) for lattice temperatures 4.2, 77 and 300 K respectively. At $T=4.2$ K, $Q$ due to SPPs (Figure 6.5 (a)) is found to be dominant for about $T_e > 180$ (140) K with (without) hot phonon effect. It is observed that hot phonon effect significantly reduces $Q$ due to SPPs. For example, at $T_e=300$ K it is found to be reduced by a factor of about 2.5.
Figure 6.5. Momentum loss rate $Q$ vs electron temperature $T_e$ with and without hot phonon effect for (a) $T = 4.2$ K, (b) $T = 77$ K (c) $T = 300$ K.

For $T = 77$ K, $Q$ due to SPPs dominates for about $T_e > 220$ (150) K with (without) hot phonon effect. Hot phonon effect is found to reduce $Q$ by about 2.5 times at $T_e = 300$ K. For $T = 300$ K, $Q$ is dominated by SPPs over the entire temperature range. The reduction in magnitude of $Q$ due to hot phonon effect is by about 1.5 times at $T_e = 1000$ K.
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Figure 6.6. Electron temperature $T_e$ vs electric field $E$. It is shown with and without hot phonon effect. Curves are presented due to phonons (APs and SPPs combined) and phonons with impurity scattering.

While calculating field dependence of $T_e$ and $V_d$, we consider additionally elastic scattering due to impurity in the momentum loss rate. The representative values of $n_i = 1 \times 10^{11}$ cm$^{-2}$ and $n_d V_0^2 = 2$ (eVÅ)$^2$ are chosen [6.37].

In Figure 6.6, $T_e$ is shown as a function of $E$ for lattice temperature 4.2 K. Calculations are shown for $T_e$ due to (i) phonon (AP+SPP) scattering and (ii) with phonon and impurity scattering. $T_e$ is found to increase more rapidly in low field region than in the high field region. This may be attributed to the increased role of SPPs in energy dissipation in the high field region. For a given field, $T_e$ due to only phonons (APs and SPPs together) is larger compared to the case when impurity scattering is included. Hot phonon effect is found to enhance $T_e$ significantly. For example, at $E=1.0$ kV/cm, $T_e$ enhances by about a factor of 1.5. Hot phonon effect on $T_e$ increases with the increasing electric field. This may be attributed to the reduced power loss of the electrons due to the increased hot phonons. Similar behavior is observed in GaN heterojunction [6.51]. $T_e$ values obtained here may be compared with those in MLG [6.20].

In Figure 6.7, electron drift velocity $V_d$ is shown as a function of electric field $E$ at $T=4.2$ K. $V_d$ is shown separately by considering scattering, in the momentum relaxation, due to only phonons (clean system) and impurities with phonons. It is
found that, in the clean system, the saturation velocity ($\sim 10^7$ cm/s) is reached at relatively much smaller fields. We observe from the curves due to only phonons, that the hot phonon effect reduces $V_d$ by about 3 times at saturation. When impurity scattering is included, $V_d$ is reduced significantly in the low field region as it dominates the low field mobility. Moreover, the near saturation of $V_d$ begins at much higher $E$. We note that the complete saturation is not observed in the range of the field considered. This is more so when the sample is with impurities. Similar observations are made with Monte Carlo simulation in BLG [6.35] and in MLG [6.11, 6.17] for the range of $E$ up to 5 kV/cm. The reduced rate of increase of $V_d$ in the high field region is attributed to the emission of SPPs. The maximum $V_d$ that we predict at 5 kV/cm is $3\times10^7$ cm/s in clean sample and about $1\times10^7$ cm/s in the sample with impurities.

$V_d$ as a function of $E$ is shown in Figure 6.8 for two lattice temperatures $T=4.2$ and 300 K due to only phonons and phonons with impurities with hot phonon effect ($\tau_p=5$ ps). We note that the near saturation of $V_d$ is achieved at larger $E$ for larger $T$. Slope of $V_d$ vs $E$, in the low field region, is smaller for larger $T$, indicating the lower mobility at larger $T$. Such behavior is observed in MLG [6.16] and in the conventional 2DEG [6.7]. With the increasing $E$ the difference in $V_d$ for the two lattice
temperatures, slowly decreases and it is similar to the field dependent electrical conductivity behavior in MLG [6.20]. Inset of

Figure 6.8. Electron velocity $V_d$ vs electric field $E$, with hot phonon effect, due to phonons and impurity scattering. Inset is $V_d$ vs $E$, only due to phonons, at $T=4.2$ and 300 K with and without hot phonon effect.

Figure 6.8 shows the magnitude of the hot phonon effect on $V_d$ at two lattice temperatures 4.2 and 300 K.

It is to be noted that electron-acoustic phonon interaction via unscreened deformation (scalar) potential (DP) coupling, taken in the present calculations, is simplified one. However, there exists calculations of resistivity [6.52] and hot electron power loss [6.29] with a new electron-acoustic phonon coupling due to change in bond length and bond angle between carbon atoms, which is described by an effective gauge field in terms of vector potential (VP). This vector potential is expressed in terms of an unscreened coupling parameter $\beta \sim 2-3$ and with the respective strength $\hbar k_f \beta / ma \approx (8-12)\sqrt{n_s}$ eV ($n_s$ in $10^{12}$ cm$^{-2}$) in BLG [6.52]. At low $T$, resistivity [6.52] and hot electron power loss [6.29] due to VP coupling are predicted to be $\sim T^4$, same as that due to unscreened DP coupling, and its contribution is shown to dominate in the low $T$ regime. Consequently, a generalized in-plane electron-acoustic phonon coupling is introduced adding the scalar and vector potential couplings. It is argued that, in case of MLG, replacing $D$ by a fitting parameter $\tilde{D}$ (Eq.(48) of Ref. [6.52]), which is resultant of screened DP and VP couplings, will give an excellent agreement with expected results of resistivity [6.53] for $D \sim 3$ eV.
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and $\hat{D} \sim 10$-20 eV. Hence, we believe that, inclusion of vector potential coupling may lead to a choice of new $D$ whose value, in the literature, is in the range 3-30 eV [6.26, 6.52]. Besides, there is need for establishing screening of DP coupling with the detailed comparison of theoretical calculations with the different experimental data, particularly, of low $T$ hot electron power loss [6.32, 6.33] and phonon-drag thermopower [6.23], which depend only on electron-acoustic phonon coupling. In view of this, at this juncture, we have taken the simpler model of unscreened DP coupling for electron-acoustic phonon interaction for illustration.

6.4 Conclusions

High field transport properties are studied, in a supported BLG, as a function of electron temperature $T_e$ and electric field $E$, considering the electron scattering due to acoustic phonons, surface polar phonons and impurity. At very low temperature, $T_e^4$ power law and $n_e^{-3/2}$ dependence are predicted for hot electron energy loss rate due to acoustic phonons. The calculated heat resistance from the powerloss is in reasonable agreement with the experimental observations. A kink is observed in this low $T_e$ regime which is a new feature. The hot phonon distribution of SPPs shows a dip, and it may be attributed to the chiral nature of electrons. Hot phonon effect is found to enhance the electron temperature $T_e$ and reduce the drift velocity $V_d$ significantly. Velocity-field characteristics due to only phonons show a larger saturation velocity $V_d (\geq 10^7 \text{ cm/s})$ setting in at relatively lower fields. Inclusion of impurity scattering reduces $V_d$ significantly as these external disorders play an important role in limiting the mobility, besides, the near saturation of $V_d$ begins at higher fields. The non-linear behavior and saturation of $V_d$ may be attributed to the emission of SPPs at higher fields. When calculations are to be compared with experimental results, the screening of acoustic phonon deformation potential and vector potential coupling are required to be taken in to account to validate the screening of their effect.
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References


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