Chapter 3

Exponential function based Fuzzy Sliding Mode Control for Nonlinear Systems

In previous Chapter, the basic design method of FSMC for uncertain nonlinear systems was discussed. The FSMC method was successful in minimizing the chattering once the system dynamics is on sliding surface. In this Chapter, ExFSMC design method is discussed to achieve reduced reaching time to the sliding surface without chattering.

3.1 Introduction

Sliding mode control, based on the theory of variable structure systems, has attracted a lot of research on control systems for the last two decades. The salient advantage of sliding mode control is robustness against structured and unstructured uncertainties. In path tracking systems, however, the system invariance properties are observed only during the sliding phase. In the reaching phase, tracking may be hindered by disturbances or parameter variations. The straightforward way to reduce tracking error and reaching time is to increase the control gain of the controller. This may cause severe chattering in some dynamics of the system. Moreover, problems relating to simplification of design procedures, control performance enhancement in the reaching mode, and chattering alleviation remain to be explored fully.

However, SMC suffers from two major short comings during its performance. Like system susceptible to parameter perturbation and external disturbances during the reaching phase of the system [4] and high frequency chattering in control signal. The chattering can be minimized by number of methods, either using signum function with a high gain saturation function or by designing the observer [48, 58, 70–72], and by
design of higher order sliding mode control [59, 60].
The reduced chattering can be achieved by FSMC without sacrificing its robust performance.

FSMC was successful in chattering free performance and preserves invariance property once the system states are on sliding surface [18, 19, 66, 73–75]. However, the system is still susceptible to parameter perturbation and external disturbances, when state trajectories are off the sliding surface during the reaching phase of SMC [4]. To overcome this problem, various methods have been suggested either by eliminating or minimizing the reaching phase [70]. The reaching time minimization was done by shifting or rotating the sliding surface in phase space by fuzzy tuning approach to SMC [71, 76]. In these methods the reaching time was minimized by moving the sliding surface either shifting or rotating by fuzzy tuning or by adapting to arbitrary initial conditions. The movement of sliding surface makes the dynamics of the surface to make it sensitive to disturbances and parameter uncertainties [27].

In [28], exponential reaching law was proposed for MIMO nonlinear systems. The exponential function used to design reaching law, reduces the reaching time of the system dynamics to the sliding surface with minimum chattering for a small gain value.

Motivated from [28], in this study, an ExFSMC for nonlinear system is proposed. In the proposed method, FSMC is designed by defining the output membership functions on an exponential function based system gain. The exponential function will adapt dynamically the variation of the controlled system, which reduces the reaching time to the sliding surface. The proposed controller guarantees some properties, such as, system stability and robustness, along with fast reaching time.

The performance of the proposed ExFSMC is demonstrated by simulation results using a nonlinear system as compared to FSMC and modified SMC. The experimentation on position control of DC motor to validate the effectiveness of the proposed method.

### 3.2 Exponential Function Based Sliding Mode Control

In this section an exponential function based sliding mode control design method is discussed. Consider a nonlinear system (2.1) and sliding surface as (2.2). To satisfy the reaching condition (2.4), \( \dot{s} \) is chosen as,

\[
\dot{s} = -K \text{sign}(s) \quad \forall \ t \ K > 0 \tag{3.1}
\]

The reaching time can be obtained by integrating (3.1) with reference to time as follows,

\[
t_r = \frac{|s(0)|}{K}.
\]
where $t_e$ is the time required for the state vector to reach $s = 0$. The SMC has invariance property once the system dynamics are on the switching manifold (or in sliding phase). However the system is still vulnerable to the system perturbation and external disturbances during the reaching phase. This problem can be minimized by reducing the reaching time by choosing a high value of $K$ in (3.1). As value of $K$ increases, reaching time decrease, but this will lead to increase in chattering. The large value of $K$ may cause saturation problem and high frequency chattering in control input. To address reaching phase problem an exponential function is defined, so that, reduced reaching time with minimum chattering for a small value of $K$ can be achieved. Such a controller is called as exponential SMC, which is given as,

$$u = u_{eq} + \left(\frac{K}{N(s)}\right)\text{sign}(s)$$

where, $u_{eq}$ is obtained as in (2.5). The exponential function parameters are selected as given in [28]

$$N(s) = \rho_0 + (1 - \rho_0) \exp^{-\alpha|s|}. \quad (3.2)$$

an offset value $\rho_0 < 1$ and $\alpha$ are strictly positive values. The reaching time is

$$t_{er} = \frac{\rho_0}{k_e} |s(0)| + \frac{1 - \rho_0}{k_e \alpha}.$$ 

where $k_e$ can be obtained using an exponential function (3.2) as as given in [28]

$$k_e = \frac{K}{N(s)}, \quad k_e > 0 \quad (3.3)$$

### 3.3 Exponential Function Based Fuzzy Sliding Mode Controller

The ExFSMC design method is an extension of FSMC method. The design method is discussed by considering the system (2.1) and the crisp sliding surface (2.2) as input to the FLC. The triangular form and trapezoidal form of input membership functions are defined on the universe of discourse $s$. The output membership functions are singleton functions defined on the universe of discourse $u_c = u_{eq} \pm k_e$. The general fuzzy control rule for ExFSMC is,

$$R^i : \text{if } s \text{ is } F^n_s, \text{ then } u_c \text{ is } F^m_{u_c}$$

where, $u_c$ is control output. The sup-min compositional rule of inference is,

$$\mu_{seR^i}(u_c) = \sup_{z} \in s \left[ \min \left[ \mu_{F^n_z}(s), \min \left[ \mu_{F^n_3}(s), \mu_{u_c}(u_c) \right] \right] \right].$$
The (2.14) used to obtain the crisp control output $u_e$.

The output of ExFSMC is,

$$u_e = FSMC s = -k_e \text{sat}(s/\Phi)$$

The overall control output has the form,

$$u = u_{eq} - k_e \text{sat}(s/\Phi)$$

(3.4)

The exponential equation (3.2), will not affect the stability of the system, as the value of $N(s)$ remains strictly positive. The value of $k_e$ in (3.3) dynamically adapts to the variation of the sliding surface by changing the value of $K/N(s)$ between $k_e$ and $k_e/\rho_0$.

3.3.1 Design steps for exponential function based fuzzy sliding mode control

The design steps of ExFSMC and FSMC are similar.

1. Design a sliding surface $s$ for the systems.

2. Derive $u_{eq}$ using (2.5).

3. Design the input and output fuzzy membership functions on the universe of discourse of $s$ and $u_e = u_{eq} \pm k_e$.

4. Define the inference engine as (2.13).

5. Define the rule base as,

   (a) if $s$ is NB then $u_e$ is Bigger.
   
   (b) if $s$ is NM then $u_e$ is Big.
   
   (c) if $s$ is ZE then $u_e$ is Medium.
   
   (d) if $s$ is PM then $u_e$ is Small.
   
   (e) if $s$ is PB then $u_e$ is Smaller.

6. Obtain $u_e$ using (2.14) and total control law has the form as in (3.4).

The (3.4) will drive the state trajectories towards the sliding surface and satisfy the inequality (2.4). The stability of the system for proposed controller can be proved using Theorem 2.2.1.
3.4 Simulation studies

The performance of the ExFSMC is illustrated by considering two simulation examples and a real time experimentation.

3.4.1 Example 1

To test the performance of proposed control methodology, simulation test has been carried out on an uncertain nonlinear system example 2.16. The sliding surface and equivalent control and initial values are consider same as in section 2.3. The parameters of the exponential functions are selected as $\alpha = 2$, $p = 1$, $\delta_0 = 0.08$, $K = 5$, $\Phi = 3$ and $C_1 = 2$. The initial values are $x_1(0) = 1$, $x_2(0) = 2$.

The dynamic behavior of state variable $x_1(t)$, control input $u$ and sliding surface obtained using ExFSMC are shown in Figures (3.1) and (3.3).

From Figures (3.1)-(3.3) it can be seen that, the proposed control method was successful in stabilizing the system with smaller overshoot as compared to modified SMC and FSMC shows faster reaching time with similar control effort.
3.4.2 Example 2

To test the performance of proposed control methodology, simulation test has been carried out on a nonlinear inverted pendulum example 2.17. The sliding surface and equivalent control and initial values are consider same as in section 2.3. The parameters of the exponential functions are selected as $\alpha = 5$, $p = 0.5$, $\delta_0 = 0.01$, $k_e = 5$, $\Phi = 3$ and $C_1 = 2$. The initial values are $x_1(0) = -0.0524$, $x_1(0) = 0$.

The results are summarized as follows:

<table>
<thead>
<tr>
<th>Method</th>
<th>ISE</th>
<th>$|E|$</th>
<th>$t_r$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ExFSMC</td>
<td>0.0016</td>
<td>$1.4982 \times 10^{-17}$</td>
<td>0.25</td>
</tr>
<tr>
<td>FSMC</td>
<td>0.0359</td>
<td>$1.7538 \times 10^{-47}$</td>
<td>0.41</td>
</tr>
<tr>
<td>SMC</td>
<td>0.0369</td>
<td>$1.7625 \times 10^{-47}$</td>
<td>0.42</td>
</tr>
</tbody>
</table>

The $u_e$ term in (3.4) is obtained by following the FSMC design procedure discussed in...
section 3.3.1. The input and output membership functions are selected as in (2.3.1). The dynamic behavior of state variable, control input and sliding surface are obtained using ExFSMC, FSMC and modified SMC are shown in Figures (3.4) – (3.6). From the figures it is observed that, the ExFSMC method was able to stabilize the system faster than FSMC and modified SMC methods with almost similar control effort. The qualitative analysis of proposed method compared to FSMC and modified SMC based on $ISE$, second norm of error and reaching time is shown in Table 3.1. The ExFSMC results show faster reaching time to the sliding surface with reduction in chattering.

3.4.3 Real time experimentation

The real time experimentation is done on the Quanser SRV02 position control using DC servomotor. The experiment setup is shown in Figure (3.7). The proposed controller and other existing controllers are engaged on the experiment setup. The power module used is the Quanser UPM1503. The data acquisition board used is a Q8 controlPaQ-FW and the rotary servo plant is SRV02. This model is equipped with a Vishary
Spectral model 132 This model is equipped with potentiometer, encoder and tachometer. The potentiometer and encoder sensors measure the angular position of the load gear and tachometer can be used to measure its velocity. The tachometer prevents any latencies in the timing of the response and ensures that the speed of the motor is accurately measured. The electrical and mechanical system parameters used were $V_{nom} = 6.0 V$, $R_m = 2.6 \Omega$, $L_m = 0.18 mH$, $m_b = 0.038 kg$, $r_d = 0.005 m$, $m_{max} = 5 kg$, $f_{max} = 50 Hz$, $I_{max} = 1 A$, $\omega_{max} = 628.3 rad/s$, $J_{m, rotor} = 3.90 \times 10^{-7} kg.m^2$.

In this experiment the performance of the position control of DC motor is checked to have faster settling of system dynamics with minimum chattering. The exponential function is proposed to adjust the output membership functions for monitoring the system control performance. The DC motor state variables are position $x_1$, speed $x_2$ respectively. The control parameters are selected as $\alpha = 2$, $p = 1$, $\delta_0 = 0.02$, $K = 5$, $\Phi = 3$.

The real time experimentation results are shown in Figures (3.8)-(3.10). From the results it is observed that the real time and simulation results differ as the simulation model of the DC motor is based on approximation.

![Experimental setup of Quanser SRV02.](image-url)
Figure 3.8: Response of position of DC motor to ExFSMC

Figure 3.9: Response of velocity of DC motor to ExFSMC

Figure 3.10: Response of control input ExFSMC to DC motor
3.5 Conclusion

In this Chapter, design method of an ExFSMC is discussed. The controller design method has similar approach as of FSMC. The exponential function based gain is used as normalization factor. The reaching time to the sliding surface is reduced by making the normalization factor of output membership function of ExFSMC dynamically adaptable using an exponential function. The performance of the controller is illustrated by simulation using a nonlinear inverted pendulum system. The applicability of the controller is demonstrated by results of real time experimentation on position control of DC motor. From the results it is concluded that, the proposed methodology shows better results with reduced reaching time.