Chapter – 3

Linewidth enhancement factor measurement of a Fabry-Perot laser diode through narrowband optical FM generation

3.1. Introduction:

The term linewidth enhancement factor (LEF) [3.1-3.16] owes its origin to the enhancement of linewidth of a semiconductor laser from its value as prescribed by Schawlow-Townes’ formula. It is also known as Henry factor or H-factor after the name of its proposer. It is also known as the chirp parameter of a semiconductor laser. The LEF originates from two effects, viz., the change in gain of the active region due to a change in carrier concentration on one hand, and the change in refractive index due to the change in carrier concentration in the active region of the laser diode (LD). The change in refractive index leads to a change in phase of the lightwave and the change in gain results in a change in amplitude. For this reason, the LEF is also known as phase-amplitude coupling factor.

It is a common fact that the adiabatic chirp makes a significant contribution to the value of LEF in a Fabry-Perot laser diode (FPLD) in the low modulation frequency region typically below 50 MHz. But, an experiment on LEF measurement of a FPLD by Nguyen et al. published in 2009 [3.9] shows that at low modulation frequencies the adiabatic chirp contribution to LEF is very small. The LEF vs. modulation frequency plot [3.9] is fairly flat within the limits of experimental error extending up to lower frequencies. So, the author has neglected the adiabatic chirp contribution in our analysis.

The LEF has a profound effect on the locking characteristics of an injection locked semiconductor laser introducing asymmetry in the lockband, on the linewidth and chirp of a directly modulated laser, behavior of the laser under optical feedback, stability of lasing modes and nonlinear dynamics of the laser diode (LD). There are a number of methods for the measurement of LEF of a laser diode such as Hakki-Paoli method [3.17], injection locking method [3.18], RF-modulation method [3.19], interferometric method [3.20] and amplified spontaneous emission method [3.21].

The Hakki-Paoli method [3.21] depends on the measurement of refractive index change and the differential gain with the variation in carrier density of the active region of the semiconductor laser which, in turn, requires a knowledge of laser parameters. This is difficult because all parameters of a commercial laser are not supplied by the manufacturer. This
method is applicable below threshold of the laser and does not give the value of LEF under actual lasing condition.

The injection locking method [3.18] measures the asymmetry of the lockband lying above and below the free-running slave laser frequency since this asymmetry depends on the LEF of the slave laser. In this case, care has to be taken to distinguish between lockband and capture range which are different due to the nonlinearity of the slave laser.

In the RF modulation method [3.19], the FP laser is directly modulated by high frequency modulating signal which produces both amplitude modulation (AM) and frequency modulation (FM). The existence of a relaxation frequency of the laser poses a problem in the measurement of LEF.

In self-mixing interferometric method [3.20], the LEF is determined from the resultant interferometric waveform. This requires accurate measurement of specific parameters of the waveform.

In amplified spontaneous emission (ASE) method [3.21], the laser is biased below threshold. So, the LEF is not determined under lasing condition.

A survey of relevant literature indicates that the effect of LEF on the characteristics of the semiconductor laser is so important that the determination of LEF [3.1-3.9] for the semiconductor lasers are being pursued till now. Most of the LEF determination techniques require a costly equipment like an optical spectrum analyzer or RF/microwave spectrum analyzer or both. It is thus important and useful to devise an experiment for the determination of LEF of a semiconductor laser which does not require high cost equipment. The author has proposed the concept of narrowband optical FM generation through direct modulation of LDs and measurement of the detected signal amplitude for the determination of the LEF.

Apart from a low cost design, the circuit is very simple in structure which requires a temperature-controlled FPLD, a modulating RF signal source, a photodiode (PD) and a CRO. This circuit simplicity is an advantage of our system. When an experimental setup requires an optical spectrum analyzer or an RF/microwave spectrum analyzer the system cost goes up. Our technique does not require this high cost component for LEF measurement. Only one cathode ray oscilloscope (CRO) is sufficient.

In this chapter, the author has proposed a novel method of measurement of LEF of a semiconductor laser through narrowband optical FM generation in direct modulation of a FPLD where the modulating signal is fed through a thermo-electric current (TEC) controller.
For narrowband optical FM generation, the author has restricted the optical IM index typically within 10%. A narrowband optical FM has a spectrum like AM (or IM), the only difference is that the two first order sidebands differ in phase by $\pi$ radian. The higher order FM sidebands are very small in amplitude and hence neglected in the analysis. The generated FM is said to be narrowband if the FM index is $\leq 0.5$. The LEF is estimated by detecting the sinusoidally modulated lightwave in a photodiode (PD) and measuring the amplitude of the detected sinusoidal signal.

### 3.2 Analysis:

The linewidth enhancement factor (LEF) is conventionally defined as

$$\alpha = \frac{4\pi}{\lambda_0} \frac{\partial n/\partial N}{\partial G/\partial N}$$  \hspace{1cm} (3.1)

where $\lambda_0$ is the vacuum wavelength of light, $n$ is the real part of the refractive index, $N$ stands for carrier concentration in the active region and $G$ is the gain of the active region of the LD. The linewidth broadening in a semiconductor laser was first explained by Henry [3.22] and described by the relation

$$\Delta \nu = \Delta \nu_{ST} \left(1 + \alpha^2\right)$$ \hspace{1cm} (3.2)

where $\Delta \nu_{ST}$ is the Schawlow-Townes’ formula for linewidth, $\Delta \nu$ is the broadened linewidth and $\alpha$ is the LEF of the LD.

Direct bias current modulation of the laser diode generates a lightwave which is modulated simultaneously in intensity and in frequency by the modulating signal. The output lightwave is thus IM-FM.

Let

$$I(t) = I_0 + I_m \sin \omega_m t$$ \hspace{1cm} (3.3)

be the bias current of the modulated LD. $I_0$ is the dc bias current and $I_m$ is the amplitude of the sinusoidal current modulation at angular frequency $\omega_m$. The modulated power of the output lightwave is expressed as

$$P(t) = P_0 \left(1 + m \sin \omega_m t\right)$$ \hspace{1cm} (3.4)

where $P_0$ is the average output power and $m$ is the power modulation index. According to Koch-Bower’s formula [3.23], the frequency deviation produced by bias current modulation in absence of adiabatic chirp is given by
\[ \Delta \nu(t) = -\frac{\alpha}{4\pi} \frac{\partial \ln P(t)}{\partial t} \]  

(3.5)

where \( \alpha \) is the linewidth enhancement factor of the LD fed with the bias current \( I_0 \). The change in phase due to this frequency modulation is expressed as

\[ \varphi(t) = 2\pi \int \Delta \nu(t) dt \]

\[ = -\varphi_0 - m_f \sin \omega_m t \]  

(3.6)

where \( \varphi_0 = \frac{\alpha}{2} \ln P_0 \) and \( m_f = \frac{\alpha}{2} m \). Here, we have assumed IM index, \( m \ll 1 \) such that

\[ \ln(1 + m \sin \omega_m t) \approx -m \sin \omega_m t \]  

(3.7)

The IM-FM lightwave can be represented as

\[ E(t) = \sqrt{P(t)} \cos(\omega_c t - \varphi_0 - m_f \sin \omega_m t) \]  

(3.8)

where \( E(t) \) is the electric field of the modulated lightwave with carrier frequency \( \omega_c \) in radian and \( m_f \) is the FM index. The field \( E(t) \) has been normalized so that \( |E(t)|^2 \) gives the power of the lightwave.

Equation (3.8) in the case of narrowband FM can be expanded as

\[ E(t) = \sqrt{P(t)} \left[ J_0(m_f) \cos(\omega_c t - \varphi_0) - J_1(m_f) \cos((\omega_c - \omega_m) t - \varphi_0) + J_1(m_f) \cos((\omega_c + \omega_m) t - \varphi_0) \right] \]  

(3.9)

where \( J_k(m_f) \) is Bessel function of order \( k \) and argument \( m_f \). Since \( m \ll 1 \) (which is 7% in our case) the generated FM index \( m_f \ll 1 \). Higher order Bessel functions being small in values have been neglected from the analysis.

Since the modulation frequency 220 kHz corresponds to the 3dB down point of the TEC controller, the actual modulation voltage (and also the current) applied to the LD is \( \frac{1}{\sqrt{2}} \) times the voltage (and also the current) applied to the modulation input of the TEC controller at a modulation frequency of 220 KHz. In experiment, the bandwidth of the TEC controller is 220 kHz. The modulation frequency taken is 220 kHz so that it just falls at the extreme of the bandwidth of the TEC controller. Since the modulation index is taken as 7.07% (i.e. \( m = 0.0707 \)), the generated optical FM is a narrowband one. Its spectrum contains the carrier and two side bands as seen in AM. The second and higher order sidebands in the generated FM are insignificant in amplitude and make no contribution in the power calculation. The FPLD used in our experiment has been designed by the manufacturer with feedback so as to
lase in a principal central mode at 1557.1 nm wavelength. So, we can neglect the effect of residual side modes of the FPLD in the modulation process.

The photodiode (PD) output current \( I_{PD} \) is given by

\[
I_{PD} = \eta P_{in}
\]

where \( \eta \) is the responsivity of the PD and \( P_{in} \) is the optical input power to the PD.

Now,

\[
P_{in} = P(t) \left[ J_0^2(m_f) + 2J_1^2(m_f) \right]
\]

The detected signal voltage output of the InGaAs photodiode is given by

\[
v_D = I_{PD} R_L
\]

\[
v_D = \eta P_{in} R_L = \eta P_0 R_L m \left[ J_0^2(m_f) + 2J_1^2(m_f) \right] \sin \omega_m t
\]

\[
v_D = v_{DO} \sin \omega_m t
\]

where

\[
v_{DO} = \eta P_0 R_L m \left[ J_0^2(m_f) + 2J_1^2(m_f) \right]
\]

and \( R_L \) is the load resistance at the output of PD and \( \eta \) is the responsivity of the InGaAs PD which has bandwidth of 1.2 GHz.

From (3.14) we have

\[
\left[ J_0^2(m_f) + 2J_1^2(m_f) \right] = \frac{v_{DO}}{\eta P_0 R_L m}
\]

\[
v_{DO} \text{ is measured from the CRO by observing the amplitude of detected signal. In experiment, } R_L = 50 \Omega \text{ and } \eta = 0.88 mA/mW. P_0 \text{ varies with the change in LD bias current } (I_{LD}). \text{ Typical value of } P_0 \text{ is } 4.83 mW \text{ for } I_{LD} = 80 mA. \text{ The value of the FM index } m_f \text{ is determined from (3.15) since the quantities of the right hand side of (3.15) are known. Now, } m_f = (m \alpha / 2), \text{ so that we can find}
\]

\[
\alpha = \frac{2m_f}{m}
\]

Since IM index, \( m \), is calculated from the relation

\[
m = \frac{I_{mod}}{I_{LD} - I_{th}}
\]
where $I_{\text{mod}}$ = modulation current amplitude, $I_{LD}$ is the LD bias current and $I_{th}$ is the LD threshold current.

Now, $m_f$ is obtained from (3.15) and $\alpha$ can be computed from (3.16).

3.3. Experimental results:

In numerical computation, we have used the following parameter values:

The author has used modulation current amplitude in the range, $I_{\text{mod}} = 0.59 mA - 3.77 mA$, LD bias current, $I_{LD} = 50 mA - 95 mA$, modulation voltage amplitude $v_{\text{mod}}$ in the range $5.9 mV - 37.7 mV$ applied to the TEC controller input, threshold current of LD, $I_{th} = 41.6 mA$, operating wavelength of FPLD, $\lambda_0 = 1557.1 nm$.

The IM index has been kept constant at 7.07%, i.e., $m = 0.0707$. The measurement has been done above the threshold. This has been carried out by suitably choosing the bias current, $I_{LD}$, and the current modulation amplitude, $I_{\text{mod}}$ such that the operating point of the LD is never driven down to threshold.

A schematic circuit diagram of the experimental set up is shown in figure 3.1.

The modulation response of the FPLD when the modulating signal is fed through the TEC controller for two modulation current values of 10 mA and 20 mA are shown in figure 3.2.

The low-frequency values of detected output voltages are 55 mV and 115 mV respectively for modulation current of 10 mA and 20 mA respectively. The 3 dB modulation bandwidth of the TEC controller measured from the response curves appears to be 220 kHz.

For a given LD, the threshold current, $I_{th}$, is fixed. The value of IM index has been kept constant at 7.07% level throughout the experiment. The corresponding modulation current, $I_{\text{mod}}$, is calculated from (3.17). The FM index is calculated from (3.15) and the variation of FM index $(m_f)$ with the bias current above threshold $(I_{LD} - I_{th})$ is shown in figure 3.3. The measured value of the LEF $(\alpha)$ as a function of the LD bias current above threshold, $(I_{LD} - I_{th})$, is shown in figure 3.4 Referring to figure 3.3 it is seen that the FM index $m_f$ exceeds 0.35 for $I_{LD} < 50 mA$. For narrowband FM concept to be valid, $m_f$ should kept below 0.5.
Figure 3.1: Schematic circuit diagram for the experimental set up for the measurement of LEF of the FPLD. **FPLD**: Fabry-Perot laser diode; **PD**: Photodiode; **CRO**: cathod ray oscilloscope; **TEC**: Thermoelectric current and temperature controller.
The resolution of our LEF measurement is 0.2/ mA. An important result of the present work is that the LEF of FPLD is found to decrease linearly with the bias current above threshold. In FPLD, as the bias current increases, more and more carriers are injected into the active region. As a result, the refractive index of the active region decreases. On the other hand, the linear gain of the active region of the LD increases with the increase in carrier density and hence with the increase in bias current. Thus, a reduction in $\frac{\partial n}{\partial N}$ together with an increase in $\frac{\partial G}{\partial N}$ leads to a reduction in LEF with increasing bias current. Hence, the plot in figure 3.4 is justified. The beauty of the present technique is that it has circuit simplicity as compared with other techniques, requiring no costly equipment like optical spectrum analyzer for the measurement of LEF. To the best of authors’ knowledge, the decrease in LEF with increase in bias current of the LD is being reported in literature for the first time.

The experimental circuit diagram is shown in figure 3.1 The Fabry-Perot laser diode (FPLD) operates at $1.1557 \text{ nm}$ and has a threshold current of $41.6 \text{ mA}$. The operating temperature is maintained at $23^\circ \text{C}$ by a thermoelectric current and temperature controller. The modulation frequency used in the experiment is $220 \text{ kHz}$ which falls just at the extreme of the bandwidth of the TEC controller. The InGaAs photodiode having a bandwidth of $1.2 \text{ GHz}$ has a responsivity of $0.88 \text{ mA/mW}$ and its output is connected with a load resistance of $50 \Omega$. The TEC controller modulation sensitivity, $S = 100 \text{ mA/volt}$. The value of modulation current, $I_{\text{mod}}$, is determined from a knowledge of LD bias current and the IM index ($m$) which is fixed. The required modulation voltage is then calculated from the modulation sensitivity of the TEC controller and the value of $I_{\text{mod}}$.

### 3.4. Conclusions:

A novel concept has been presented in this chapter for the measurement of LEF in a semiconductor laser which is based on the narrow-band optical FM generation in a directly modulated FPLD. The IM index has been kept constant at $7.07\%$ throughout the measurement, which validates the approximation made in (3.7). This small value of IM index, $m$, leads to small values of FM index $m_f$ which is essential for narrowband optical FM generation. The decrease in refractive index and an increase in gain of the active region of the LD produces a fall in LEF. The LEF decreases linearly with the increase in the bias current above threshold. The experimental results on LEF variation has been explained with physical reasoning.
Figure 3.2: Experimental modulation response of the Fabry-Perot laser diode driven through the TEC controller.
Figure 3.3: Variation of FM index with the LD bias current above threshold. Solid line shows an empirical fit.
Figure 3.4: Measured value of LEF as a function of the LD bias current above threshold. Solid line shows an empirical fit.
References:


