CHAPTER-V
STABILITY OF VISCOUS FLOW OVER A FLAT DEFORMABLE SHEET

5.1 INTRODUCTION
Boundary layer flow over solid surfaces was studied by Sakiadis[1], Erickson, Fan and Fox[2] under different velocity for the moving solid surfaces. These problems find their application in polymer industry where flow over stretching plastic sheet is required. Fansler and Danberg[3] gave a non-similar solution for two-dimensional boundary layer flow over a stretching sheet with a stream velocity proportional to the distance along the sheet extending the similarity solution of Mc Cormack and Crane[4]. Bhattacharya and Gupta [5] studied the linear stability of the boundary layer flow according as [4] considering three dimensional disturbances. In this paper a more generalization is made for the presence of two parameters \( k_1 \) and \( r \) for the equations of motion and the similarity solution. Their various effects on the stability analysis is discussed here.

5.2 FORMULATION OF THE PROBLEM
Let us consider a stretching sheet on the plane \( y=0 \) over which an incompressible viscous fluid is flowing. Using boundary layer
approximations, the equations of continuity and momentum (in the usual notations) are

\[ \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \]  \hspace{1cm} (1)

and

\[ u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} = \gamma \frac{\partial^2 u_0}{\partial y^2} \]  \hspace{1cm} (3)

The boundary conditions are as follows:

\[ u_0 = cx, \quad v_0 = 0 \quad \text{at} \quad y = 0 \]

And \( u_0 \to 0 \) as \( y \to \infty \) \hspace{1cm} (3)

Where \( c \) is a positive constant.

The above system of equations satisfying the boundary conditions has solution

\[ u_0 = cx \ F'(\eta) \ , \ v_0 = -(c \ \gamma)^{1/2} \ F(\eta) \]

\[ F(\eta) = \frac{1}{(1-r^m)(1-e^{-m}r^{m})}, \quad \eta = \frac{c}{\gamma}^{1/2} \ y \]  \hspace{1cm} (4)

We have to study the stability of (4) with respect to disturbances periodic in a normal direction to the plane of the basic flow. Hammerlin’s [6] analysis of differential equations obtained by Görtler [7] pointed out that instability can occur in the form of Taylor-Görtler vortices.

Writing the perturbed state as

\[ u = u_0 + \tilde{u} = u_0 + cx \ f_1 (\eta , z, t) \]
\[ v = v_0 + \tilde{v} = v_0 - \sqrt{c \gamma} f_2(\eta, z, t) \]

\[ w = \tilde{w} = \gamma f_3(\eta, z, t) \quad \text{----------------------------------}(5) \]

\[ p = p_0 + \tilde{p} = p_0 + \rho \gamma c f_4(\eta, z, t) \]

Where \( p_0 \) stands for the basic pressure distribution, \( w \) for the perturbation velocity component perpendicular to the \( xy \)-plane and \( u_0, v_0 \) are given in (4)

Assuming the perturbations are periodic in \( z \) as analysed by Gortler we can take as in [5]

\[ \tilde{u} = c_x u_1(\eta) \cos(\alpha z) e^{\beta t}, \quad \tilde{v} = -\sqrt{c \gamma} v_1(\eta) \cos(\alpha z) e^{\beta t} \]

\[ \tilde{w} = \gamma \alpha w_1(\eta) \sin(\alpha z) e^{\beta t}, \quad \tilde{p} = \rho \gamma c p_1(\eta) \cos(\alpha z) e^{\beta t} \quad \text{------(6)} \]

We have the equations of momentum in usual tensor notation as

\[ \rho \left[ \frac{d v_i}{dt} + v_k \frac{\partial v_i}{\partial x_k} \right] = -\frac{\partial p}{\partial x_i} + \eta_0 \frac{\partial^2 v_i}{\partial t^2} \frac{\partial}{\partial x_m} x_k \frac{\partial}{\partial x_k} - k_0 \left[ \frac{\partial}{\partial t} \left( \partial^2 v_i / \partial x_m \partial x_k \right) + \right. \]

\[ v_m \partial^3 v_i / \partial x_m \partial x_k \partial x_k - \partial v_i / \partial x_m \partial^2 v_m / \partial x_k \partial x_m - 2(\partial^2 v_i / \partial x_m \partial x_k) \partial v_m / \partial x_k \] \quad \text{-----}(7) \]

where \( k=1,2,3 \) and \((v_1, v_2, v_3) = (u, v, w)\)

So the linearised three dimensional perturbed equation of continuity and the equations of momentum are

\[ \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} = 0 \quad \text{----------------------------------}(8) \]

\[ \frac{\partial \tilde{u}}{\partial t} + u_0 \frac{\partial \tilde{u}}{\partial x} + \tilde{u} \frac{\partial u_0}{\partial x} + v_0 \frac{\partial \tilde{v}}{\partial y} + \tilde{v} \frac{\partial u_0}{\partial y} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x} + \nu \nabla^2 \tilde{u} \]
\[ -k^* \left[ \frac{\partial}{\partial t} \nabla^2 u + \left( u_0 \frac{\partial}{\partial x} + v_0 \frac{\partial}{\partial y} \right) \nabla^2 \tilde{u} + \left( \tilde{u} + \tilde{v} \frac{\partial}{\partial y} \right) \nabla^2 \tilde{u} \right] \]

\[ - \frac{\partial u_0}{\partial x} \nabla^2 \tilde{u} - \frac{\partial \tilde{u}}{\partial x} \nabla^2 u_0 - \frac{\partial \tilde{u}}{\partial y} \nabla^2 \tilde{v} - \frac{\partial \tilde{u}}{\partial \tilde{y}} \nabla^2 v_0 - 2 \left\{ \frac{\partial^2 u_0}{\partial y^2} \frac{\partial \tilde{v}}{\partial y} + \frac{\partial^2 \tilde{u}}{\partial x \partial \tilde{y}} \frac{\partial u_0}{\partial \tilde{y}} \right\} \]-----------------------------(9)

\[ \frac{\partial \tilde{v}}{\partial t} + u_0 \frac{\partial \tilde{v}}{\partial x} + \tilde{u} \frac{\partial \tilde{v}}{\partial x} + v_0 \frac{\partial \tilde{v}}{\partial y} + \tilde{v} \frac{\partial \tilde{v}}{\partial y} = - \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \tilde{y}} + \nu \nabla^2 \tilde{v} \]

\[ -k^* \left[ \frac{\partial}{\partial t} \nabla^2 \tilde{v} + \tilde{v} \right] \nabla^2 v_0 + v_0 \frac{\partial \tilde{v}}{\partial y} - \frac{\partial \tilde{v}}{\partial y} \nabla^2 \tilde{v} \]

\[ - \frac{\partial v_0}{\partial y} \nabla^2 \tilde{v} - \frac{\partial \tilde{v}}{\partial y} \nabla^2 v_0 - 2 \left[ \frac{\partial^2 v_0}{\partial y^2} \frac{\partial \tilde{v}}{\partial y} + \frac{\partial^2 \tilde{v}}{\partial y^2} \frac{\partial v_0}{\partial y} \right] \]-----------------------------(10)

\[ \frac{\partial \tilde{w}}{\partial t} + u_0 \frac{\partial \tilde{w}}{\partial x} + v_0 \frac{\partial \tilde{w}}{\partial y} = - \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \tilde{z}} + \nu \nabla^2 \tilde{w} - k^* \left[ \frac{\partial}{\partial t} \nabla^2 \tilde{w} - \frac{\partial \tilde{w}}{\partial \tilde{y}} \right] \nabla^2 v_0 \]

\[ -2 \frac{\partial^2 \tilde{w}}{\partial y^2} \]-----------------------------(11)

Using (6) in (8)-(11) and also using (4) we get

\[ U_i - v_i' + \alpha^2 w_1 = 0 \]-----------------------------(12)

\[ U_i'' + Fu_i' - (\beta + \tilde{\alpha}^2 + 2F')u_i + k_1 [ F u_i''' - u_i'' (2F' + \beta) + (2F' - \tilde{\alpha}^2 F + F' u_i' + \tilde{\alpha}^2 \beta u_1 ] = k_1 [ F v_i''' + 2 F'' v_i' - (F'' + \alpha^2 F') v_i ] \]

\[ -F'' v_i \]-----------------------------(13)

\[ v_i'' + F v_i' - (\tilde{\beta} + \tilde{\alpha}^2 + F') v_i + k_1 [ F v_i''' - (\tilde{\beta} + 3F) v_i'' - (3F'' + F' \tilde{\alpha}^2 v_i' + (\alpha^2 \tilde{\beta} + F'' + \tilde{\alpha}^2 F') v_i ] = -p_i' \]-----------------------------(14)
\[ w_1'' + F w_1' - (\bar{\beta} + \bar{\alpha}^2) w_1 + k_1 [F w_1''' - (\bar{\beta} + 2F) w_1'' - (\bar{\alpha}^2 + F + F') w_1'] + \bar{\alpha}^2 \bar{\beta} w_1 ] = -p_1' \] \hspace{1cm} \text{(15)}

Where \( \bar{\beta} \) stands for the derivative w.r.t. \( \eta \) and \( \bar{\alpha}^2 = \nu \alpha^2 /c \), \( \bar{\beta} = \beta /c \) \hspace{1cm} \text{(16)}

The boundary conditions are \( u_1 = v_1 = w_1 = 0 \) at \( \eta = 0 \)

And \( u_1 = v_1 = w_1 = 0 \) as \( \eta \to \infty \) \hspace{1cm} \text{(17)}

Using (17), we get from (12),

\[ \nabla_1 \cdot = 0 \text{ at } \eta = 0 \text{ and } \eta = \infty \]

So \( u_1 = v_1 = 0 \) at \( \eta = 0 \) and \( \eta = \infty \) \hspace{1cm} \text{(18)}

Now \( w_1 \) and \( p_1 \) eliminant from (12), (14) and (15) gives

\[ v_1'''' + F v_1''' + (F' - \bar{\beta} - 2 \bar{\alpha}^2) v_1'' + \bar{\alpha}^2 (\bar{\beta} + \bar{\alpha}^2 + F) v_1' + k_1 [F v_1'''' - (\bar{\beta} + F') v_1'''] - (2 \bar{\alpha}^2 F + 3F') v_1] \]

\[ = u_1'''' + F u_1''' + (F' - \bar{\beta} - \bar{\alpha}^2) u_1'' + k_1 [F u_1'''' - (\bar{\beta} + F') u_1'''] - (\bar{\alpha}^2 F + 3F') u_1'' \]

\[ - (\bar{\alpha}^2 F + F''') - \bar{\alpha}^2 F \] \hspace{1cm} \text{(19)}

Now differentiating (13) w.r.to \( \eta \) and combining with (19) we obtain,

\[ v_1'''' + F v_1''' + (F' - \bar{\beta} - 2 \bar{\alpha}^2) v_1'' + (F'' - \bar{\alpha}^2 F) v_1' + [\bar{\alpha}^2 (\bar{\beta} + \bar{\alpha}^2 - F') + F'''] v_1] \]

\[ + k_1 [F v_1'''' - (\bar{\beta} + F') v_1'''] - 2(\bar{\alpha}^2 F + 2F''') v_1'''' \]

\[ = 2(F' - \bar{\alpha}^2 F') v_1''' + \{ \bar{\alpha}^2 (4 F'' - \bar{\alpha}^2 F) - F'''} v_1' + \{F'' + \bar{\alpha}^4 (\bar{\beta} + F') v_1} \]
Let \( T = e^{\eta} \). So \( L = \frac{d}{d\eta} = -\tau T \frac{d}{dt} \) \hspace{1cm} \text{(21)}

Now equation (13) and (20) become,

\[
L^2 u_{i} + \left( \frac{1}{r} \right) (1-T) L u_{i} - \left( \beta + \alpha^2 + 2F \right) u_{i} + k_{1} \left[ \left( \frac{1}{1-T} \right) L^3 u_{i} + \left( 2T + \beta \right) L^2 u_{i} \right.
\]
\[
+ \left\{ 2T - \left( \alpha^2 / r \right) (1-T) \right\} L u_{i}
\]
\[
+ \alpha^2 \beta u_{i} \right] = \tau T v_{i} + k_{1} \left[ -r T L^2 v_{i} + 2r^2 T L v_{i} + r \left( r^2 + \alpha^2 \right) T v_{i} \right] \quad \text{(22)}
\]

\[
r L^4 v_{i} + \left( 1-T \right) L^3 v_{i} + r \left( T - \beta - 2 \alpha^2 \right) L^2 v_{i} - \left\{ r^2 T + \alpha^2 \left( 1-T \right) \right\} L v_{i}
\]
\[
+ r \left[ \alpha^2 \left( \beta + \alpha^2 - T \right) + r^2 T \right] v_{i} +
\]
\[
k_{1} \left[ \left( 1-T \right) L^5 v_{i} - r \left( \beta + T \right) L^4 v_{i} + 2 \alpha^2 \left( 1-T \right) - 2r^2 T \right] L^3 v_{i} - 2r \left( 2r^2 T - \alpha^2 \beta - \alpha^2 \right)
\]
\[
\times T L^2 v_{i} + \left\{ \alpha^2 \left(-4r^2 T + \alpha^2 \left(1-T\right)\right) + r^4 T \right\} L v_{i} + r \left\{ r^4 T - \alpha^2 \left( \beta + T \right) \right\} v_{i} \right]
\]
\[
= 2r T \left( L u_{i} - ru_{i} \right) + 2k_{1} r \left( T - T \right) + \text{(23)}
\]

The boundary condition (18) reduces to \( u_{i} = v_{i} = L v_{i} = 0 \) at \( T = 0 \) and

\( T = 1 \) \hspace{1cm} \text{(24)}

Equations (22) and (23) together with the boundary conditions in (24)

constitute an eigen value problem for stability.
5.3 DISCUSSION AND SOLUTION OF THE PROBLEM

To solve the above problem we expand \( u_j \) in a set of trial functions \( T_j^j (1-T) \)

\( j=1,2,------ \)

such that for all \( j \) it satisfies (24).

Similarly \( v_i \) can also be expanded by the set of trial functions \( T_l(1-T)^j \)

\( j=1,2,------ \) for all \( j \) it also satisfies (24)

Writing \( u_i \) as \( u_j \) for j-term approximation and \( v_i \) as \( v_j \) we get

\[ U_j = T_j^j (1-T) \quad \text{and} \quad v_j = T(1-T)^j \quad \text{--------------------------------------------(25)} \]

Now consider the integral

\[ I = \int_0^1 \left[ u_i M_j + u_i N_j + v_i P_j + v_i Q_j \right] dz = A_{ij} + B_{ij} + C_{ij} + D_{ij} \quad \text{---------(26)} \]

Where

\[ A_{ij} = \int_0^1 u_i M_j \ dz = \int_0^1 u_i L^2 + (1/r)(1-T)L \ - (\bar{\beta} + \bar{\alpha}^2 + 2F^1) + k_1 \{(1/r)(1-T)L^3 + (2T+ \bar{\beta})L^2 \}
+ \{(2T - (\bar{\alpha}^2)(1-T))L + (\bar{\alpha}^2 \bar{\beta})\} u_j \ dz = a_i a_j \bar{A}_{ij} \quad \text{-------------------------(27)} \]

\[ B_{ij} = \int_0^1 u_i [-rT - k_1 \{-rT L^2 + 2r^2 T L + r(r^2 + \bar{\alpha}^2)T\} v_j \ dz = a_i b_j \bar{B}_{ij} \quad \text{---------(28)} \]

\[ C_{ij} = \int_0^1 v_i [-2rT(L - r) - 2k_1 \{(r-1)T L^2 + r(1-r)T L\} u_j \ dz = b_i a_j \bar{C}_{ij} \quad \text{---------(29)} \]

and

\[ D_{ij} = \int_0^1 v_i [rL^4 + (1-T)L^3 + r(T - \bar{\beta} - 2 \bar{\alpha}^2)T^2 - (r^2 T + \bar{\alpha}^2 (1-T))L \]

\[ + r[\bar{\alpha}^2 \ -T] + \bar{\alpha}^2 \quad \text{-----------------------------------------(30)} \]
\[ k_1 \{(1-T) L^5 - r(\bar{\beta} + T)L^4 - 2\{ \bar{\alpha}^2 (1-T) - 2r^2 T\} L^3 - 2r(2r^2 T - \bar{\alpha}^2 \bar{\beta} - \bar{\alpha}^2 T) L^2 + \{ \bar{\alpha}^2 (-4r^2 T + 16) + \bar{\alpha}^2 (1-T) \} L + r[r^4 T - \bar{\alpha}^4 (\bar{\beta} + T)] \} v_j dz \]

\[ = b_i b_j \tilde{D}_{ij} \]

For \( I \) to be stationary, \( \frac{\partial I}{\partial a_i} = 0 \), \( \frac{\partial I}{\partial b_i} = 0 \), \( \frac{\partial I}{\partial \omega_i} = 0 \),

\[ \text{i.e.,} \quad \tilde{A}_{ij} = \tilde{B}_{ij} = \tilde{C}_{ij} = \tilde{D}_{ij} = 0 \]

\[ \text{Now writing } \int T^i(1-T)^j \text{ dz} = F(i,j) \quad \text{----------------------------------------(33)} \]

\[ 0 \]

We get, \( \tilde{A}_{ij} = (r^2 j^2 - \bar{\alpha}^2) F(i+j,2) - r^2 (2j+1) F(i+j+1,1) - jF(\hat{l}+j,3) - F(\hat{l}+j+1,2) + \]

\[ k_1 \{(r^2 (3j^2 + 3j + 1) - 2r^2 j^2 - 2rj + r^2 j - \bar{\alpha}^2) F(i+j+1,2) + (\bar{\alpha}^2 j - r^2 j^3) F(i+j,3) + \}

\[ 2(r^2 (2j+1) + 2r - r^3) F(i+j+2,1) \} \quad \text{-----------------------------------------(34)} \]

\[ \tilde{B}_{ij} = -r[F(\hat{l}+j+1,3) + k_1 \{(r^2 (1-j^2 - 2j + \bar{\alpha}^2) F(i+j+1,3) + F(\hat{l}+j+2,2) 2r^2 (2j+3) - 2r^2 \}

\[ F(i+j+3,1) \}] \quad \text{-----------------------------------------(35)} \]

\[ \tilde{C}_{ij} = -2r^2 \{ F(i+j+2,2) - (j+1) F(i+j+1,3) + k_1 r(r-1) \{ j(j+1) F(i+j+1,3) - \}

\[ 2(j+1) F(i+j+2,2) \} \} \quad \text{-----------------------------------------(36)} \]

\[ \tilde{D}_{ij} = F(i+j,4) \{ r^5 j^4 - r^3 (\bar{\beta} + 2 \bar{\alpha}^2) j^2 + r^2 (\bar{\beta} + \bar{\alpha}^2) j \} + F(i+j+1,3) \{ -2r^5 \}

\[ (4j^3 + 6j^2 + 4j + 1) + 2r^3 (\bar{\beta} + 2 \bar{\alpha}^2) (2j+1) \} + F(i+j+2,2) \{ 2r^5 (6j^2 + 12j + 7) - 2r^3 \} \]
\[
(\beta + a^2) + \{ a^2 r - r^3 j^3 \} F(i+j, 5) + F(i+j+1, 4)(2r^3 (3j^2 + 3j + 1) + r^3 j - 2a^2 r + r(r^2 - a^2)) + F(i+j+2, 3)(-2r^3 (2j+1) - 2r^3 - 6r^3 (j+1)) + 2r^3
\]

\[
F(i+j+2, 2) + \kappa \{ F(i+j, 5) \} ^{-r} j^5 + 2a^2 r^3 j^3 - a^4 r + F(i+j+1, 4)(2r^5 (5j^4 + 10j^3 + 10j^2 + 5j + 1) - r^5 j^4 - 4a^2 r^3 (3j^2 + 3j + 1) - 4r^5 j^3 - 2r^3 (2r^2 - a^2) j^2 - r^3 (r^2 - 4a^2) j + 2d^4 r + r(r^4 - a^4)) + F(i+j+2, 3)(-10r^5 (2j^3 + 6j^2 + 7j + 3) + 2r^5 (4j^3 + 6j^2 + 4j + 1) + 12a^2 r^3 (j+1) + 8r^5 (3j^2 + 3j + 1) + 4r^3 (2r^2 - a^2)) + F(i+j, 4) \{-r^5 \beta + 2a^2 \beta r^3 j^2 - a^4 \beta r\} + F(i+j+1, 3)(2r^5 \beta (4j^3 + 6j^2 + 4j + 1) - 4a^2 \beta r^3 (2j+1)) + F(i+j+2, 2)(-2r^5 (6j^2 + 12j + 7) + 24r^5 (j+1) - 4r^3 (2r^2 - a^2)) \]

Now writing \( \mathbf{A}_{ij} = a_{ij} + \beta a_{ij}', \mathbf{B}_{ij} = b_{ij}, \mathbf{C}_{ij} = c_{ij} \) and \( \mathbf{D}_{ij} = d_{ij} + \beta d_{ij}' \)

Equation (32) becomes,

\[
\begin{vmatrix}
a_{ij} + \beta a_{ij}' & b_{ij} \\
c_{ij} & d_{ij} + \beta d_{ij}'
\end{vmatrix} = 0
\]

For 1-term approximation \( i=j=1 \). Equation (38) then reduces to

\[
(a_{11} + \beta a_{11}')(d_{11} + \beta d_{11}') - b_{11} c_{11} = 0
\]

Noting that

\[
\int T^i (1-T)^j \, dz = F(i,j) = (i! \, j!)/(i+j+1)! \]

0
We can easily verify that for small \( k, b, c \) are small compared to 
\( a_{11}, d_{11}, d_{11}', a_{11}' \)

Hence neglecting \( b_{11} c_{11} \) we get,

\[
(a_{11} + \bar{\beta} a_{11}')(d_{11} + \bar{\beta} d_{11}') = 0 \quad \text{---------------------------------------------(41)}
\]

\[
giving \quad \bar{\beta} = -\frac{a_{11}}{a_{11}'} \quad \text{and} \quad \beta = -\frac{d_{11}}{d_{11}'} \quad \text{---------------------------------(42)}
\]

When \( r=1, k =0, \ a^2 =10 \) we get \( (\bar{\beta}_1)_{1\text{-term}}=-14.5, (\bar{\beta}_2)_{1\text{-term}}=-13.3 \)-----(43)

As the two roots which are same as that obtained by Bhattacharya and Gupta [5]

For 2-term approximation, \( i=j=2 \). Equation (38) then becomes

\[
\begin{vmatrix}
 a_{11} + \bar{\beta} a_{11}' & a_{12} + \bar{\beta} a_{12}' & b_{11} & b_{12} \\
 a_{21} + \bar{\beta} a_{21}' & a_{22} + \bar{\beta} a_{22}' & b_{21} & b_{22} \\
 c_{11} & c_{12} & d_{11} + \bar{\beta} d_{11}' & d_{12} + \bar{\beta} d_{12}' \\
 c_{21} & c_{22} & d_{21} + \bar{\beta} d_{21}' & d_{22} + \bar{\beta} d_{22}'
\end{vmatrix} = 0 \quad \text{--------44)}
\]

Since \( b_{ij}, c_{ij} \) are small compared to \( a_{ij}, d_{ij} \) and their primes so we obtain

\[
\begin{vmatrix}
 a_{11} + \bar{\beta} a_{11}' & a_{12} + \bar{\beta} a_{12}' & =0..(45) & d_{11} + \bar{\beta} d_{11}' & d_{12} + \bar{\beta} d_{12}' & =0 \quad (46)
\end{vmatrix}
\]
\[
\begin{vmatrix}
 a_{21} + \bar{\beta} a_{21}' & a_{22} + \bar{\beta} a_{22}' & d_{11} + \bar{\beta} d_{11}' & d_{12} + \bar{\beta} d_{12}'
\end{vmatrix}
\]
Taking $r=1, k =0$, $\alpha^2 =10$ we get

\begin{align*}
(\beta_1)_{2\text{-term}} &= -12.33, \\
(\beta_3)_{2\text{-term}} &= -28.67 \text{ as solution of (45)}
\end{align*}

and \begin{align*}
(\beta_2)_{2\text{-term}} &= -11.58, \\
(\beta_4)_{2\text{-term}} &= -32.66 \text{ as solution of (46)}
\end{align*}

Which are same to the result investigated in\cite{5}

For the third approximation \hspace{1em} det($A_0$)$=0$ and det($D_0$) $= 0$

Has approximately the same solution for $\beta$ as in \cite{5}

**FIGURE CAPTION**

Fig5.1 plots $-\log_{10} k_1$ versus $-\beta_1$ and $-\beta_2$ where $r=1, \alpha^2 = 10$.

Fig5.2 plots $r$ versus $-\beta$ where $k_1 =.0001, \alpha^2 = 10$.

Fig5.3 plots $-\log_{10} k_1$ versus $-\beta$ for $\alpha^2 =10, r =2$.

Fig5.4 plots $r$ versus $-\beta$ for $k_1 =.0001, \alpha^2 =10$

Fig5.5 plots $-\log_{10} k_1$ versus $-\beta$ for $\alpha^2 =10, r =1$.

### 5.4. RESULTS AND EFFECTS OF NEWLY INTRODUCED PARAMETERS $k$ AND $r$

Fig5.1 states that for fixed values of $r$ and $\alpha^2$ ($r=1, \alpha^2 =10$) the magnitude of the smaller root $\beta_1$ increases with $k_1$ but reverse is the case for another root $\beta_2$. Fig5.2 signifies that for fixed $k_1 =.00001, \alpha^2 = 10$ with the increase of $r$, magnitude of $\beta_1$ increases but that of $\beta_2$ also increases. Also $\beta_1, \beta_2$ are both negative. So $r$ has stabilizing effect on the system. Fig5.3 and Fig5.4 give the same graphs for two-terms approximation. Also Fig5.5 gives the variation of $\beta_1, \beta_2$ with the variation of $k_1$ having same meaning for the 3-term approximation.
5.5. CONCLUSION

The flow discussed here is similar to the stagnation-point flow and asymptotic suction profile. The present study throws some light on the stability of a flow which has tremendous bearing in the polymer industry.

References

$K_1 = 0.0001, r = 1, \alpha^2 = 10$

$\bar{\rho}_1 = -13.32, \bar{\rho}_1 = -14.52$

$K_1 = 0.001 \quad \bar{\rho}_1 = -13.48 \quad \bar{\rho}_1 = -14.69$
$K_1 = 0.01 \quad \bar{\rho}_1 = -15.30 \quad \bar{\rho}_1 = -16.66$
$K_1 = 0.02 \quad \bar{\rho}_1 = -18.06 \quad \bar{\rho}_1 = -19.63$

Fig. 5.1
Fig. 5.2
$K_1 = 0.0001, \quad \alpha^2 = 10$
Fig. 5.3
\[ \alpha^2 = 10, r = 2 \]
Fig. 5.4

$\alpha^2 = 10, K_1 = 0.00001$
$a^2 = 10, r = 1$