CHAPTER – 8
SOME PROBLEMS IN MICROPOLAR
THEORY OF ELASTICITY
1 INTRODUCTION

The propagation of plane waves in a medium, consisting of two semi-infinite solid elastic mediums, separated by a plane interface are frequently studied in the book of elasto-dynamics. In classical theory of elasticity, Sommerfeld [133], Jeffreys [63], Muskat [80] and others have discussed wave propagation when the distance of a point source from the plane interface is finite. Their results are directly related to an important practical problem, that of refraction arrival from a source to a receiver in seismology of near earthquakes and in seismic refraction investigations.

The discrepancy between the classical theory of elasticity and experiments is particularly striking when the medium is granular or multi-molecular. To remove the deficiencies of classical theory, one may consider micropolar theory of elasticity. In micropolar theory of elasticity, the action across any infinitesimal surface element within a solid produces displacement $u(x, t)$ and rotation $\omega(x, t)$ in general. Two asymmetric tensors, $\gamma_{ij}$, called the strain tensor and $\chi_{ij}$, the curvature twist tensor describe the state of deformation of the elastic solid body. The state of stress is characterized by two asymmetric tensors, namely, the force stress tensor $\sigma_{ij}$ and the couple stress tensor $\mu_{ij}$, respectively. Following Nowacki [90] and Eringen [52], the disturbances created in a solid-solid elastic semi-spaces, have been studied due to the presence of a compressional wave source. It is assumed here that the boundaries are parallel planes and the problem is axisymmetric in cylindrical coordinate system ($r, \theta, z$). Further, it is assumed that $\omega_\theta = \omega_r = \omega_z = 0$ for the propagation of longitudinal waves and the displacement vector $u(u_r, 0, u_z)$ and the rotation vector $\omega(0, r, \omega_\theta, 0)$ depend only on $r, z, t$. It is mentioned that the other system deals with $u(0, u_\theta, 0)$ and $\omega(\omega_r, 0, \omega_z)$.

2. FUNDAMENTAL EQUATIONS AND RELATIONS

In the following, the tensorial index notation in rectangular coordinate system is used. The relations between state of stress and state of strain are linear and are given by [90].
\[
\sigma_{ij} = (\mu + \alpha)\gamma_{ij} + (\mu - \alpha)\chi_{ij} + \lambda\gamma_{kk}\delta_{ij},
\]
\[
\mu_{ij} = (\gamma + \varepsilon)\chi_{ij} + (\gamma - \varepsilon)\chi_{ij} + \beta\chi_{kk}\delta_{ij},
\]
where
\[
[i, j = 1, 2, 3]
\]
\[
\gamma_{ij} = u_{i,j} - \varepsilon_{kj}\omega_k \quad \text{and} \quad \chi_{ij} = \omega_{i,j},
\]
\[\varepsilon_{kj}\] denotes the unit anti-symmetric tensor, \(\gamma_{ij}\), known as the deformation tensor and \(\chi_{ij}\) is called the curvature twist tensor, \(\lambda\), \(\mu\) are Lame's elastic constants and \(\alpha, \beta, \gamma, \varepsilon\) are other material constants characterizing the micropolar character of the solid.

Inserting (2.1) into the equations of motion in absence of body forces and body couples
\[
\sigma_{ij,j} = \rho\ddot{u}_i,
\]
\[
\varepsilon_{ki}\sigma_{kj} + \mu_{ij,j} = J\ddot{\omega}_i
\]
(2.3).

And expressing \(\gamma_{ij}\) and \(\chi_{ij}\) in terms of displacement \(u_i\) and the rotation \(\omega_i\), in accordance with (2.2), we arrive at a system of equations expressed in vectorial form as
\[
(\mu + \alpha)\nabla^2 \overline{u} + (\lambda + \mu - \alpha)\text{graddiv}\overline{u} + 2\alpha\text{rot}\overline{\omega} = \rho\ddot{u}
\]
\[
(\gamma + \varepsilon)\nabla^2 \overline{\omega} + (\beta + \gamma - \varepsilon)\text{graddiv}\overline{\omega} - 4\alpha\overline{\omega} + 2\alpha\text{rot}\overline{u} = J\ddot{\omega}
\]
(2.4).

where \(\rho\) is material density, \(J\) is rotational inertia, dots denote the derivatives with respect to time \(t\) and comma denotes the partial differentiation with respect to space co-ordinates.

If we assume that displacement \(\overline{u}\) and rotation \(\overline{\omega}\) do not depend on \(x_2\), then for the system
\[
\overline{u} = (u_1, 0, u_3) \quad ; \quad \overline{\omega} = (0, \omega_2, 0),
\]
equation (2.4) may be written as
\[
(\mu + \alpha)\nabla^2 u_1 + (\lambda + \mu - \alpha)\Delta_{11} + 2\alpha\omega_{2,3} = \rho\ddot{u}_1
\]
\[
(\mu + \alpha)\nabla^2 u_3 + (\lambda + \mu - \alpha)\Delta_{33} - 2\alpha\omega_{2,1} = \rho\ddot{u}_3
\]
\[
(\gamma + \varepsilon)\nabla^2 \omega_2 - 4\alpha\omega_2 + 2\alpha(u_{1,3} - u_{3,1}) = J\ddot{\omega}_2
\]
(2.5) where
\[
\nabla^2 = (\ Partial differential operator)_{11} + (\ Partial differential operator)_{33}; \Delta = u_{1,1} + u_{3,3}
Expressing (2.5) in cylindrical co-ordinate system [93] we obtain

\[
(\mu + \alpha)(\nabla^2 u_r - \frac{u_r}{r^2}) + (\lambda + \mu - \alpha) \frac{\partial e}{\partial r} - 2\alpha \frac{\partial \omega_\theta}{\partial z} = \rho \frac{\partial^2 u_r}{\partial t^2}
\]

\[
(\mu + \alpha)\nabla^2 u_z + (\lambda + \mu - \alpha) \frac{\partial e}{\partial z} + 2\alpha \frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) = \rho \frac{\partial^2 u_z}{\partial t^2}
\]

\[
(\gamma + \varepsilon)\nabla^2 \omega_\theta - 4\alpha \omega_\theta + 2\alpha \left( \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) = J \frac{\partial^2 \omega_\theta}{\partial t^2}
\]

(2.6)

where

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial \theta^2} ; \quad e = \frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) + \frac{\partial u_z}{\partial z}
\]

The displacement components \( u_r \) and \( u_z \) can be written in terms of potentials \( \Phi \) and \( W \) as

\[
\begin{align*}
  u_r &= \frac{\partial \Phi}{\partial r} - \frac{\partial W}{\partial z} \\
  u_z &= \frac{\partial \Phi}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (rW)
\end{align*}
\]

(2.7)

and we introduce \( \Psi \) and \( \Gamma \) defined by

\[
W = -\frac{\partial \Psi}{\partial r} ; \quad \omega_\theta = -\frac{\partial \Gamma}{\partial r}
\]

(2.8)

Substituting (2.7) and (2.8) into (2.6) we obtain

\[
\begin{align*}
  \left[ \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right] \Phi &= 0 \quad (2.9) \\
  \left[ \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right] \Psi + p\Gamma &= 0 \quad (2.10) \\
  \left[ \nabla^2 - u_2^2 \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right] \Gamma - q\nabla^2 \Psi &= 0 \quad (2.11)
\end{align*}
\]

where

\[
\begin{align*}
  c_1^2 &= (\lambda + 2\mu)/\rho ; \quad c_2^2 = (\mu + \alpha)/\rho ; \quad u_2^2 = 4\alpha/(\gamma + \varepsilon) \\
  c_4^2 &= (\gamma + \varepsilon)/J ; \quad p = 2\alpha/(\mu + \alpha) ; \quad q = 2\alpha/(\gamma + \varepsilon)
\end{align*}
\]
3. GENERAL THEORY AND BOUNDARY CONDITIONS

We assume that \( z > 0 \) and \( z < 0 \) be two micropolar solid semi-spaces are in contact at \( z = 0 \). Here we shall use the subscript 1 for the upper solid and subscript 2 for the lower solid respectively. The compressional wave source being present in the upper solid substratum at \( (0, 0, h) \).

**Boundary Conditions:** The boundary conditions are, at \( z = 0 \)

\[
\begin{align*}
(u_r)_1 &= (u_r)_2 ; & (u_z)_1 &= (u_z)_2 ; & (\omega_\theta)_1 &= (\omega_\theta)_2 \\
(\sigma_{zz})_1 &= (\sigma_{zz})_2 ; & (\sigma_{rr})_1 &= (\sigma_{rr})_2 ; & (\mu_{z\theta})_1 &= (\mu_{z\theta})_2
\end{align*}
\]

where

\[
\sigma_{zz}, \sigma_{rr}, \mu_{z\theta} \ \text{in terms of} \ \varphi, \psi \ \text{are}[90]
\]

\[
\begin{align*}
\sigma_{zz} &= \lambda \nabla^2 \varphi + 2\mu \frac{\partial}{\partial z} \left( \frac{\partial \varphi}{\partial z} - \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} \right) \\
\sigma_{rr} &= 2\mu \frac{\partial^2 \varphi}{\partial r \partial z} - (\mu + \alpha) \frac{\partial}{\partial t} \left( \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} \right) + (\mu - \alpha) \frac{\partial^3 \psi}{\partial r^2 \partial z} + 2\alpha \frac{\partial \Gamma}{\partial r} \\
\mu_{z\theta} &= -(\gamma + \varepsilon) \frac{\partial^2 \Gamma}{\partial r \partial z}
\end{align*}
\]

4. SOLUTION OF THE PROBLEM

Eliminating \( \psi \) and \( \Gamma \) from (2.10) and (2.11) we get

\[
\begin{bmatrix}
\left( \nabla^2 - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} \right) \nabla^2 - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} + \psi^2 \nabla^2 \end{bmatrix} (\psi, \Gamma) = 0
\]

where \( \psi^2 = \rho \eta \)

As a source in solid can produce both compressional and distortional waves and that an expression for spherical waves emitted by the point source at \( (0,0,h) \) may be written as [55]

\[
\varphi_0 = \frac{e^{-\sigma_1 R} e^{i\omega t}}{R} = \int_{-\infty}^{\infty} \frac{k}{\bar{\lambda}_1} e^{-k |z-h| + i\omega t} J_0 (kr) dk
\]

where

\[
\bar{\lambda}_1 = k^2 - \sigma_1^2 ; \quad \sigma_1^2 = \omega^2 / c_s^2 ; \quad R = \left( r^2 + (z - h)^2 \right)^{1/2}
\]
and hence the expressions for \( \varphi, \psi \) and \( \Gamma \) in two mediums are 

at \( z > 0 \)
\[
\varphi_1 = \varphi_0 + \int_0^\infty B_1(k)e^{-\lambda_1(z-h)}e^{ikt}J_0(\nu r)dk
\]
\[
\psi_1 = \int_0^\infty [C_1(k)e^{-\lambda_2(z-h)} + D_1(k)e^{-\lambda_3(z-h)}]e^{ikt}J_0(\nu r)dk
\]
\[
\Gamma_1 = \int_0^\infty [a_2C_1(k)e^{-\lambda_2(z-h)} + a_3D_1(k)e^{-\lambda_3(z-h)}]e^{ikt}J_0(\nu r)dk
\]

(4.4)

at \( z < 0 \)
\[
\varphi_2 = \int_0^\infty B_2(k)e^{\lambda_1(z-h)}e^{ikt}J_0(\nu r)dk
\]
\[
\psi_2 = \int_0^\infty [C_2(k)e^{\lambda_2(z-h)} + D_2(k)e^{\lambda_3(z-h)}]e^{ikt}J_0(\nu r)dk
\]
\[
\Gamma_2 = \int_0^\infty [a'_2C_2(k)e^{\lambda_2(z-h)} + a'_3D_2(k)e^{\lambda_3(z-h)}]e^{ikt}J_0(\nu r)dk
\]

(4.5)

where

\[
\lambda_{2,3}^2 = k^2 + \frac{1}{2}\left[ v_2^2 - v_3^2 - \sigma_2^2 - \sigma_3^2 \pm \left( v_2^2 - v_3^2 + \sigma_2^2 + \sigma_3^2 \right)^2 - 4 \sigma_2^4 \left( \sigma_2^2 - v_2^2 \right)^2 \right]^{1/2}
\]

\[
\sigma_2^2 = \frac{\omega^2}{c_2^2} ; \quad \sigma_4^2 = \frac{\omega^2}{c_4^2} ; \quad a_i = -\left[ \lambda_i^2 + \sigma_i^2 - k^2 \right] / p \quad [i = 2,3]
\]

(4.6)

Now \( \lambda_i [i = 1,2,3] \) and \( a'_j [j = 2,3] \) are given by replacing the subscript 1 of the material constants by 2.

Substituting the expressions for \( \varphi, \psi \) and \( \Gamma \) from (4.4) and (4.5) in the boundary conditions (3.1) we obtain

\[
\hat{B}_1 - \lambda_2 \hat{C}_1 - \lambda_3 \hat{D}_1 - \hat{B}_2 - \lambda_2' \hat{C}_2 - \lambda_3' \hat{D}_2 = -(k/\lambda_1)e^{-\lambda_1 h}
\]
\[
\lambda_1 \hat{B}_1 - b_2 \hat{C}_1 - b_3 \hat{D}_1 + \lambda_1' \hat{B}_2 + b'_2 \hat{C}_2 + b'_3 \hat{D}_2 = -ke^{-\lambda_1 h}
\]
\[
-c_1 \hat{B}_1 + c_2 \hat{C}_1 + c_3 \hat{D}_1 + c'_1 \hat{B}_2 + c'_2 \hat{C}_2 + c'_3 \hat{D}_2 = (k/\lambda_1)[\rho_1 \omega^2 - 2\mu_1 k^2]e^{-\lambda_1 h}
\]
\[
2\lambda_1 \mu_1 \hat{B}_1 - d_2 \hat{C}_1 - d_3 \hat{D}_1 + 2\lambda_1' \mu_2 \hat{B}_2 + d'_2 \hat{C}_2 + d'_3 \hat{D}_2 = -2\mu_1 ke^{-\lambda_1 h}
\]
\[
(\gamma + \varepsilon)a_2 \lambda_2 \hat{C}_1 + (\gamma + \varepsilon)a_3 \lambda_3 \hat{D}_1 + (\gamma' + \varepsilon')a'_2 \lambda_2' \hat{C}_2 + (\gamma' + \varepsilon')a'_3 \lambda_3' \hat{D}_2 = 0
\]
\[
a_2 \hat{C}_1 + a_3 \hat{D}_1 - a'_2 \hat{C}_2 - a'_3 \hat{D}_2 = 0
\]

(4.7)

where
\[ b_2 = p a_2 + \lambda_2^2 + \sigma_2^2 ; \quad b_3 = p a_3 + \lambda_3^2 + \sigma_2^2 ; \quad c_1 = 2 \mu_1 \lambda_1^2 - \lambda_1 \sigma_1^2 = 2 \mu_1 k^2 - \rho_1 \omega^2 \]
\[ c_2 = 2 \mu_1 (p \lambda_2 a_2 + \lambda_2^3 + \lambda_2 \sigma_2^2) ; \quad c_3 = 2 \mu_1 (p \lambda_3 a_3 + \lambda_3^3 + \lambda_3 \sigma_2^2) \quad (4.7a) \]
\[ d_2 = 2 \mu_1 \lambda_2^2 + (2 \alpha + p[\mu + \alpha]) a_2 + \sigma_2^2 (\mu + \alpha) ; \quad d_3 = 2 \mu_1 \lambda_3^2 + (2 \alpha + p[\mu + \alpha]) a_3 + \sigma_2^2 (\mu + \alpha) \]
and
\[ \hat{B}_1 = B_1 e^{\lambda_1 h} ; \quad \hat{C}_1 = C_1 e^{\lambda_2 h} ; \quad \hat{D}_1 = D_1 e^{\lambda_3 h} ; \quad \hat{B}_2 = B_2 e^{\lambda_1 h} \quad ; \quad \hat{C}_2 = C_2 e^{\lambda_2 h} \quad ; \quad \hat{D}_2 = D_2 e^{\lambda_3 h} \quad (4.8) \]
so that
\[ B_1 = \frac{\Delta_1}{\Delta} e^{-\lambda_1 h} ; \quad C_1 = \frac{\Delta_2}{\Delta} e^{-(\lambda_1 + \lambda_2) h} ; \quad D_1 = \frac{\Delta_3}{\Delta} e^{-(\lambda_1 + \lambda_3) h} \]
\[ B_2 = \frac{\Delta'_1}{\Delta} e^{-(\lambda_1 - \lambda_2) h} \quad C_2 = \frac{\Delta'_2}{\Delta} e^{-(\lambda_1 - \lambda_3) h} \quad D_2 = \frac{\Delta'_3}{\Delta} e^{-(\lambda_1 - \lambda_3) h} \quad (4.9) \]
where
\[
\Delta = \begin{vmatrix}
-1 & \bar{\lambda}_2 & \bar{\lambda}_3 & 1 & \bar{\lambda}'_2 & \bar{\lambda}'_3 \\
\bar{\lambda}_1 & -b_2 & -b_3 & \bar{\lambda}'_1 & b'_2 & b'_3 \\
-c_1 & c_2 & c_3 & c'_1 & c'_2 & c'_3 \\
2\bar{\lambda}_1 \mu_1 & -d_2 & -d_3 & 2\bar{\lambda}'_1 \mu_2 & d'_2 & d'_3 \\
0 & (\gamma + \varepsilon) a_2 \bar{\lambda}_2 & (\gamma + \varepsilon) a_3 \bar{\lambda}_3 & 0 & (\gamma' + \varepsilon') a'_2 \bar{\lambda}'_2 & (\gamma' + \varepsilon') a'_3 \bar{\lambda}'_3 \\
0 & a_2 & a_3 & 0 & -a'_2 & -a'_3
\end{vmatrix} \quad (4.10)
\]

\[ \Delta_1, \Delta_2, \Delta_3, \Delta'_1, \Delta'_2 \text{ and } \Delta'_3 \]
are obtained from \( \Delta \) replacing the first, second, third, .... column by the column
\[ [-k / \bar{\lambda}_1, -k, k(\rho_1 \omega^2 - 2\mu_1 k^2) / \bar{\lambda}_1, -2\mu_1 k, 0, 0] \]
respectively.

The system of equations (4.4) and (4.5), with the help of (4.9) represent the formal steady-state solution in the following
at $z > 0$

$$\phi_1 = \int_0^{\infty} \frac{k}{k_1} e^{-\lambda_1 |z-h|+\iota t} J_0 (kr) dk + \int_0^{\infty} \frac{\Delta_1}{k_1} e^{-\lambda_1 (z+h)+\iota t} J_0 (kr) dk$$

(4.11)

$$\psi_1 = \int_0^{\infty} \frac{\Delta_2}{\Delta} e^{-(\lambda_2 h+\lambda_1 h) + \iota t} + \frac{\Delta_3}{\Delta} e^{-(\lambda_3 h+\lambda_1 h) + \iota t} J_0 (kr) dk$$

(4.12)

$$\Gamma_1 = \int_0^{\infty} \frac{\alpha_2}{\Delta} e^{-(\lambda_2 h+\lambda_1 h) + \iota t} + \frac{\alpha_3}{\Delta} e^{-(\lambda_3 h+\lambda_1 h) + \iota t} J_0 (kr) dk$$

(4.13)

at $z < 0$

$$\phi_2 = \int_0^{\infty} \frac{\Delta_1'}{\Delta} e^{(\lambda_1 h+\lambda_2 h) + \iota t} J_0 (kr) dk$$

(4.14)

$$\psi_2 = \int_0^{\infty} \frac{\Delta_2'}{\Delta} e^{(\lambda_2 h+\lambda_1 h) + \iota t} + \frac{\Delta_3'}{\Delta} e^{(\lambda_3 h+\lambda_1 h) + \iota t} J_0 (kr) dk$$

(4.15)

$$\Gamma_2 = \int_0^{\infty} \frac{\alpha_2'}{\Delta} e^{(\lambda_2 h+\lambda_3 h) + \iota t} + \frac{\alpha_3'}{\Delta} e^{(\lambda_3 h+\lambda_1 h) + \iota t} J_0 (kr) dk$$

(4.16)

5. DISCUSSION

The first term in (4.11) represent the direct compressional waves. All other terms in (4.11) to (4.16) represent waves generated in both media by it. Different type of waves are determined by a set of branch line integrals corresponding to $k = \sigma_1, \sigma_2, \sigma_4, \sigma_1', \sigma_2', \sigma_4'$ and residues corresponding to the roots given by the equation

$$\Delta (k) = 0$$

(5.1)

Associated with the branch points of equations (4.11) to (4.16) the wave may be considered to travel along a path composed of three parts (i) source to interface (ii) along the interface and (iii) interface to reviewer. The coefficient of $h$ in the exponential indicates whether the first part of the path is traversed by compressional or shear waves, the coefficient of $z$ give the same information about the third part while the value of $k$ at the branch point indicates the mode of travel along the interface.

Now if the micropolar effect vanishes, we obtain the limiting value of the following expression.

Since the micropolar effects tend to zero, we have

$$\alpha, \beta, \gamma, \varepsilon \to 0 \quad \text{and so} \quad p \to 0, \quad v^2 \to 0, \quad \text{and} \quad c_2^2 \to c_2^2 = \mu / \rho.$$
Also \( \bar{\lambda}_3 \rightarrow (k^2 - \sigma_2\sigma_2) \) when \( \sigma_2^2 > \sigma_2^2 - \sigma_2^2 \)

Now \( a_3 \rightarrow 0 \), if the order of smallness of the numerator in the expression for \( a_3 \) is higher than that of \( \alpha \). Hence \( \Delta_2 \rightarrow 0 \) and so \( C_1(k) \rightarrow 0 \). Here

\[
\Delta \rightarrow \begin{vmatrix} -k/\bar{\lambda}_1 & -1 & -\lambda_3 & -\lambda'_3 \\ \bar{\lambda}'_1 & b_{03} & b'_{03} & 1 \\ (k/\bar{\lambda}_1)(2\mu_1k^2 - \rho_1\omega^2) & c'_{01} & c'_{03} & c'_{03} \\ 2k\mu_1 & 2\mu'_1\bar{\lambda}'_1 - d_{03} & d'_{03} & 2k\mu_1 & 2\mu'_1\bar{\lambda}'_1 - d_{03} & d'_{03} \end{vmatrix}
\]

where

\[
\bar{\lambda}_1 = \sqrt{k^2 - \sigma_1^2} \quad b_{03} = k^2 \quad c_{03} = 2\mu_1\lambda_3 \quad c_{01} = 2\mu_1k^2 - \rho_1\omega^2 = d_{03} \quad (5.2)
\]

\[
\lambda_3 = \sqrt{k^2 - \sigma_2^2} \quad \hat{\sigma}_2 = \frac{\omega}{\hat{c}_2}
\]

The above expressions in (5.2) are obtained from (4.7a) in the limiting case and similar are the expressions with dashes \( \Delta_2/\Delta \) and \( \Delta_3/\Delta \). can be also treated similarly. The above results are in complete agreement with the corresponding results as studied by Ewing et al [55]. Now in the above case the equation (5.1) reduces to

\[
4(\mu_2 - \mu_1)^2 \left[ k^2 - \frac{\omega^2(\rho_2 - \rho_1)}{2(\mu_2 - \mu_1)} \right]^2 - \frac{\rho_2\omega^2}{2(\mu_2 - \mu_1)} \left[ k^2 - \frac{\rho_2\omega^2}{2(\mu_2 - \mu_1)} \right]^2 - \lambda_3\lambda'_3
\]

\[
\left[ k^2 + \frac{\rho_1\omega^2}{2(\mu_2 - \mu_1)} \right]^2 - (\lambda_3'\lambda'_3 + \lambda_3\lambda'_3) \left[ \frac{\rho_1\rho_2\omega^4}{4(\mu_2 - \mu_1)^2} + \lambda_3'\lambda'_3\lambda_3'\lambda'_3k^2 \right] = 0 \quad (5.3)
\]

The above equation is equivalent to
\[ c^4 \left( (\rho_1 - \rho_2)^2 - (\rho_1 A_2 + \rho_2 A_1)(\rho_1 B_2 + \rho_2 B_1) \right) + 2Kc^2 [\rho_1 A_2 B_2 - \rho_2 A_1 B_1 - \rho_1 + \rho_2] \\
+ K^2 [A_2 B_2 - 1][A_1 B_1 - 1] = 0 \quad (5.4) \]

where

\[ c = \frac{\omega}{k} ; \quad A_1 = \left( 1 - \frac{c^2}{c_1^2} \right)^{\frac{1}{2}} ; \quad B_1 = \left( 1 - \frac{c^2}{c_2^2} \right)^{\frac{1}{2}} ; \quad A_2 = \left( 1 - \frac{c^2}{c_r^2} \right)^{\frac{1}{2}} \]

\[ B_2 = \left( 1 - \frac{c^2}{c_r'^2} \right)^{\frac{1}{2}} ; \quad K = 2 \left( \rho_1 c_2^2 - \rho_2 c_r'^2 \right) = 2(\mu_1 - \mu_2) \]

and

\[ \overline{\lambda}_1 = kA_1 ; \quad \overline{\lambda}_1' = kA_2 ; \quad \lambda_3 = kB_1 ; \quad \lambda_3' = kB_2 \]

The equation (5.4) is obtained by Stoneley [135] and known as frequency equation for stoneley waves.
1. INTRODUCTION

In micropolar elastic solid medium the action across an infinitesimal area element is equivalent to a force and a couple, both of them playing a definite role in the elastic behaviour of the solid. Two asymmetric tensors $\gamma_{ij}$, known as the deformation tensor, and $\chi_{ij}$, called the curvature twist tensor, describe the state of deformation of a solid body whether the state of stress is characterized by the asymmetric force stress tensor $\sigma_{ij}$, and the couple stress tensor, $\mu_{ij}$. Nowacki and others [54, 92, 93, 97] discussed several problems of waves and vibrations in a micropolar elastic solid medium. In classical theory of elasticity, Lamb [70], Cole and Huth [27] and Sneddon [131] studied the problem of steady-state response to moving loads in an elastic solid medium. The micropolar effect on the steady-state response to moving loads in an elastic solid medium have been discussed by Sengupta and Ghosh [117, 122]. Here is an endeavour by the authors to discuss the steady-state response to moving loads in a micropolar elastic solid medium under the influence of gravitational field.

2. GENERAL THEORY

In orthogonal Cartesian co-ordinate system $0-x_1 \ x_2 \ x_3$ in micropolar elastic semi-space $x_2 \geq 0$, the origin $0$ being any point on the free surface of the medium and $x_2$-axis pointing into the medium vertically downwards. Let us assume, a moving line load with a constant speed $U$ in the negative $x_1$-axis direction for an infinitely long time, so that a steady state prevails in the neighbourhood of the loading as seen by an observer moving with the load.

The dynamical equations of motion for a micropolar solid elastic medium with no body forces and body couples may be written as

$$
(\mu + \alpha)\nabla^2 \ddot{u} + (\lambda + \mu - \alpha)\nabla \text{div} \ddot{u} + 2\alpha \text{rot} \ddot{\omega} = \rho \ddot{u}
$$

$$
(\gamma + \varepsilon)\nabla^2 \ddot{\omega} + (\beta + \gamma - \varepsilon)\nabla \text{div} \ddot{\omega} - 4\omega + 2\text{rot} \ddot{u} = J \ddot{\omega}
$$

(2.1)

Where

$\rho$ : material density, $J$ : rotational inertia

$\lambda, \mu$ : Lame's elastic constants

$\alpha, \beta, \gamma, \varepsilon$ : material constants
\( \bar{u} \) : displacement vector  
\( \bar{\omega} \) : rotation vector
and dot denotes the time derivative.

In particular, Nowacki [87] generalized the problem of surface waves in a micropolar elastic medium in two special configurations:

(i) displacement vector is \( u = (u_1, u_2, 0) \) and rotation vector \( \omega = (0, 0, \omega_3) \)
(ii) the displacement is \( u = (0, 0, u_3) \) and the rotation \( \omega = (\omega_1, \omega_2, 0) \).

For the two systems, considering gravitational field as studied by Biot M.A [13] in the present plane strain problem equation (2.1) may be written as [1,85]

\[
(\mu + \alpha) \nabla^2 u_1 + (\lambda + \mu - \alpha) \Delta_{1} + 2\alpha \omega_{3,2} + g\rho u_{2,1} = \rho \ddot{u}_1 ,
\]

\[
(\mu + \alpha) \nabla^2 u_2 + (\lambda + \mu - \alpha) \Delta_{2} - 2\alpha \omega_{3,1} - g\rho u_{1,1} = \rho \ddot{u}_2 ,
\]

\[
(\gamma + \epsilon) \nabla^2 \omega_3 - 4\alpha \omega_3 + 2\alpha (u_{2,1} - u_{1,2}) = J\ddot{\omega}_3
\]

and

\[
(\gamma + \epsilon) \nabla^2 \omega_1 - 4\alpha \omega_1 + (\beta + \gamma - \epsilon) \Delta_{1,1} + 2\alpha u_{3,2} = J\ddot{\omega}_1
\]

\[
(\gamma + \epsilon) \nabla^2 \omega_2 - 4\alpha \omega_2 + (\beta + \gamma - \epsilon) \Delta_{2,2} - 2\alpha u_{3,1} = J\ddot{\omega}_2
\]

\[
(\mu + \alpha) \nabla^2 u_3 + 2\alpha (\omega_{2,1} - \omega_{1,2}) = \rho \ddot{u}_3
\]

(2.2)

where

\[
\nabla^2 = (\ )_{11} + (\ )_{22} ; \Delta = u_{1,1} + u_{2,2} ; \Delta' = \omega_{1,1} + \omega_{2,2}
\]

The relations between the state of stress and the state of strain are linear and are given by [90]

\[
\sigma_{ij} = (\mu + \alpha) \gamma_{ij} + (\mu - \alpha) \gamma_{ij} + \lambda \gamma_{kk} \delta_{ij}
\]

\[
\mu_{ij} = (\gamma + \epsilon) \chi_{ij} + (\gamma - \epsilon) \chi_{ij} + \beta \chi_{kk} \delta_{ij}
\]

where

\[
\gamma_{ij} = u_{i,j} - \epsilon_{kji} \omega_k \quad \text{and} \quad \chi_{ij} = \omega_{i,j}, \quad [i, j = 1,2,3]
\]

\( \epsilon_{kji} \) denotes the unit anti-symmetric tensor, \( \gamma_{ij} \), known as the deformation tensor and \( \chi_{ij} \) is called the curvature twist tensor.

We introduce Galilean transformation[58]

\[
x'_1 = x_1 + Ut \quad ; \quad x'_2 = x_2 \quad ; \quad t' = t
\]

(2.5)
3. FORMULATION OF THE PROBLEM

The displacement components \( u_1, u_2 \) \((u_3 = 0)\) can be expressed in terms of the potentials \( \phi \) and \( \psi \) as

\[
\begin{align*}
    u_1 &= \phi + \psi, \\
    u_2 &= \psi - \phi;
\end{align*}
\]

(3.1)

Where \( \phi, \psi \) are functions of the co-ordinates \( x_1, x_2 \) and time \( t \).

Substituting (3.1) in (2.2) we obtain the following differential equations:

\[
\begin{align*}
    \nabla^2 \left( \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right) \phi + \frac{g}{c_1^2} \frac{\partial \psi}{\partial x_1} &= 0, \\
    \nabla^2 \left( \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right) \psi - p \omega_3 - \frac{g}{c_2^2} \frac{\partial \phi}{\partial x_1} &= 0, \\
    \nabla^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \omega_3 + s \nabla^2 \psi &= 0
\end{align*}
\]

(3.2)

Where

\[
\begin{align*}
    c_1^2 &= (\lambda + 2\mu)/\rho; \\
    c_2^2 &= (\mu + \alpha)/\rho; \\
    c_4^2 &= (\gamma + \varepsilon)/J; \\
    \omega_3 &= \frac{4\alpha}{(\gamma + \varepsilon)}; \\
    v_0^2 &= \frac{2\alpha}{(\mu + \alpha)}; \\
    s &= \frac{2\alpha}{(\gamma + \varepsilon)}
\end{align*}
\]

(3.2a)

Eliminating \( \phi, \psi \) and \( \omega_3 \) from (3.2) we get

\[
\begin{align*}
\left\{ \left( \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right) \left( \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right) \left( \nabla^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) + v_1^2 \nabla^2 \right\} \left( \nabla^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \left( \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right) \frac{g}{c_4 c_2} \frac{\partial^2}{\partial x_1^2} \left( \nabla^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) (\phi, \psi, \omega_3) &= 0
\end{align*}
\]

(3.3)

where \( v_1^2 = p.s \)

In view of (2.5) equation (3.3) reduces to the following form.
\[
\left\{ \left( 1 - \frac{U^2}{c_i^2} \right) \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial x_j^2} \right\} \left\{ \left( 1 - \frac{U^2}{c_j^2} \right) \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial x_k^2} \right\} \left\{ \left( 1 - \frac{U^2}{c_k^2} \right) \frac{\partial^2}{\partial x_k^2} + \frac{\partial^2}{\partial x_l^2} - v_0^2 \right\} + \\
v_1^2 \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \right] + \left( \frac{g}{c_1 c_2} \right)^2 \frac{\partial^2}{\partial x_1^2} \left( \nabla^2 v_0^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \right\}(\phi, \psi, \omega_3) = 0 \tag{3.4}
\]

Introducing the Mach numbers [58]

\[ M_i = \frac{U}{c_i} \quad [i = 1, 2, 4] \tag{3.5} \]

and the parameters

\[ \bar{\beta}_i^2 = 1 - \frac{U^2}{c_i^2} = 1 - M_i^2 \quad \text{if} \quad M_i < 1 \]
\[ \check{\beta}_i^2 = \frac{U^2}{c_i^2} - 1 = M_i^2 - 1 \quad \text{if} \quad M_i > 1 \tag{3.6} \]

and considering the following three cases [122]

(i) supersonic \( M_1 > 1 \)
(ii) subsonic \( M_1 < M_2 < 1 \); \( M_4 > 1 \)
(iii) transonic \( M_1 < 1, M_2 > 1 \); \( M_4 > 1 \)

and assuming \( c_1 > c_2 > c_4 \) and \( c_4 \) very small we obtain the three sets of partial differential equations as [122]

\[
(i) \left( \bar{\beta}_1 \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) \left( \bar{\beta}_2 \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2} \right) \left( \bar{\beta}_3 \frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_4^2} \right) + v_1^2 \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \right] + \\
\left( \frac{g}{c_1 c_2} \right)^2 \frac{\partial^2}{\partial x_1^2} \left( \check{\beta}_4 \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} + v_0^2 \right) \right\}(\phi, \psi, \omega_3) = 0 \tag{3.7}
\]

\[
(ii) \left( \bar{\beta}_1 \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \left( \bar{\beta}_2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) \left( \check{\beta}_4 \frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_4^2} + v_0^2 \right) - v_1^2 \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \right] + \\
\left( \frac{g}{c_1 c_2} \right)^2 \frac{\partial^2}{\partial x_1^2} \left( \check{\beta}_4 \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} + v_0^2 \right) \right\}(\phi, \psi, \omega_3) = 0 \tag{3.8}
\]

\[
(iii) \left( \bar{\beta}_1 \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \left( \bar{\beta}_2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) \left( \check{\beta}_4 \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2} + v_0^2 \right) + v_1^2 \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \right] +
\]
The boundary conditions for a concentrated line load moving over the plane boundary of the semi-space are

\[ \sigma_{22} = -P \delta(x_1 + Ut) ; \quad \sigma_{21} = 0 ; \quad \mu_{23} = 0 \quad \text{at} \quad x_2 = 0 \]  \hspace{1cm} (4.1)

which may be written in terms of the transformations (2.5)

\[ \sigma_{22} = -P \delta(x_1') ; \quad \sigma_{21} = 0 ; \quad \mu_{23} = 0 \quad \text{at} \quad x_2' = 0 \]  \hspace{1cm} (4.2)

Using (3.2), (3.5) the boundary conditions may be written as [58,90]

at \( x_2' = 0 \)

\[ \frac{M_2^2}{M_1^2} \frac{\partial^2 \varphi}{\partial x_2'^2} + (2 - p) \frac{\partial^2 \psi}{\partial x_1' \partial x_2'} + \left( \frac{M_2^2}{M_1^2} + p - 2 \right) \frac{\partial^2 \varphi}{\partial x_1'^2} = -\frac{P}{\mu + \alpha} \delta(x_1') \]

\[ (2 - p) \frac{\partial^2 \Phi}{\partial x_1' \partial x_2'} + (1 - p) \frac{\partial^2 \psi}{\partial x_1'^2} - \frac{\partial^2 \psi}{\partial x_2'^2} + p \omega_3 = 0 \]  \hspace{1cm} (4.3)

\[ (\gamma + \epsilon) \frac{\partial \omega_3}{\partial x_2'} = 0 \]

5. SOLUTION OF THE PROBLEM

For the solution of the equations (3.7)-(3.9) we consider

\[ \varphi(x_1', x_2'), \psi(x_1', x_2'), \omega_3(x_1', x_2') = \{ \overline{\varphi}(x_2'), \overline{\psi}(x_2'), \overline{\omega}_3(x_2') \} e^{i \lambda x_1'} \]  \hspace{1cm} (5.1)

Case (i) supersonic \( M_i > 1 \) \( [i = 1, 2, 4] \)

Substituting (5.1) in (3.7) we obtain

\[ \overline{\varphi} = \sum A_j e^{-\alpha_j x_2} ; \quad \overline{\psi} = \sum B_j e^{-\alpha_j x_2} ; \quad \overline{\omega_3} = \sum C_j e^{-\alpha_j x_2} \]  \hspace{1cm} (5.2)

The other possible solution of the form \( \exp[i \lambda_j x_2'] \) are rejected on the basis of the radiation condition at infinity.
Where
\[ (-\lambda^2_1) + (-\lambda^2_2) + (-\lambda^2_3) = \xi^2 (\beta^2_1 + \beta^2_2 + \beta^2_3) - v_0^2 + v_1^2 \]
\[ \sum (-1)^2 \lambda^2_1 \lambda^2_2 = \xi^4 (\beta^2_1 \beta^2_2 + \beta^2_3 \beta^2_2) - \xi^2 (\beta_1 v_0^2 + \beta_2 v_0^2 + v_1^2 + (g/c_1 c_2)^2) \]
\[ (-1)^3 \lambda^2_1 \lambda^2_2 \lambda^2_3 = (g/c_1 c_2)^2 \xi^2 (\beta^2_4 \xi^2 - v_0^2) - \beta^2_1 \xi^4 (\beta^2_1 \beta^2_4 \xi^2 - \beta^2_2 v_0^2 - v_1^2) \]  
(5.3)

Hence we may write the general solution in the form

\[ \phi(x_1', x_2') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{j=1}^{3} A_j e^{-i\alpha_j x^2} \right) e^{i\xi x^2} d\xi \]
\[ \psi(x_1', x_2') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{j=1}^{3} B_j e^{-i\alpha_j x^2} \right) e^{i\xi x^2} d\xi \]
\[ u_\delta(x_1', x_2') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{j=1}^{3} C_j e^{-i\alpha_j x^2} \right) e^{i\xi x^2} d\xi \]  
(5.4)

Using (5.4) and (3.2) we get on equating the co-efficient of
\[ e^{-i\alpha_j x^2} \]  
[j = 1, 2, 3]
\[ B_1 = a_1 A_1 \]
\[ C_1 = \chi_1 A_1 \]  
[i = 1, 2, 3]
\[ \alpha_j = \frac{c^2_1}{g_i \xi} (\lambda_j^2 - \beta^2_4 \xi^2) \]
\[ \chi_j = \frac{1}{p} \left[ (\beta^2_2 \xi^2 - \lambda_j^2) \alpha_j - \frac{gi \xi}{c^2_2} \right] \]  
(5.5)

Using (5.4), (4.3), (3.2a), (3.5) and (3.6) we obtain
\[ p_1 A_1 + p_2 A_2 + p_3 A_3 = i \xi P^*(\xi) \]
\[ q_1 A_1 + q_2 A_2 + q_3 A_3 = 0 \]
\[ r_1 A_1 + r_2 A_2 + r_3 A_3 = 0 \]  
(5.6)

where
\[ p_j = -\frac{M^2_2}{M^2_1} \lambda_j^2 \left( -\frac{M^2_2}{M^2_1} + p - 2 \right) \xi^2 + (2 - p) \xi \lambda_j a_j \]  
(j = 1, 2, 3)
\[ q_j = (2 - p) \xi \lambda_j - \alpha_j \left( (1 - p) \xi^2 + \lambda_j^2 \right) + p \chi_j \]
\[ r_j = \lambda_j \chi_j \]
and
iξ\hat{P}(ξ) is the Fourier transform of \(-\frac{P}{(\mu + \sigma)}\delta(x'_1)\), the representation of \(\delta(x'_1)\)

being \(Lt\lim_{\varepsilon \to 0}\frac{1}{2\pi \varepsilon} \int_{-\infty}^{\infty} \frac{\sin(\xi \varepsilon)}{\xi} e^{iz\xi} d\xi\) and \(\alpha\) is small

Case (ii) subsonic \(M_i < 1\) \(i = 1, 2\) \(M_i \geq 1\)

Substituting (5.1) in (3.8) we obtain \(i = 1, 2, 3\]

\[\bar{\phi} = \sum A_j e^{-\lambda_j|x'_2|} \quad ; \quad \bar{\psi} = \sum B_j e^{-\lambda_j|x'_2|} \quad ; \quad \omega_3 = \sum C_j e^{-\lambda_j|x'_2|} \quad (5.7)\]

Where

\[
(\lambda^2_1) + (\lambda^2_2) + (\lambda^2_3) = \xi^2 (\beta^2_1 + \beta^2_2 - \beta^2_4) + 4v_0^2 - v_1^2 \\
\sum \lambda^2_1 \lambda^2_2 = \xi^2 (\beta^2_1 \beta^2_2 - \beta^2_3 \beta^2_4 - \beta^2_4 \beta^2_1) + \xi^2 (\beta^2_1 v_0^2 + \beta^2_2 v_0^2 - v_1^2 + (g / c_1 c_2)^2) \\
\lambda^2_1 \lambda^2_2 \lambda^2_3 = (g / c_1 c_2)^2 \xi^2 (\beta^2_4 \xi^2 - v_0^2) + \beta^2_2 \xi^4 (\beta^2_2 v_0^2 - \beta^2_4 \xi^2 - v_1^2) \quad (5.8)\]

Hence we may write the general solution in the form

\[\varphi(x'_1, x'_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{j=1}^{3} A_j e^{-\lambda_j|x'_2|} \right) e^{ix\xi} d\xi \]

\[\psi(x'_1, x'_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{j=1}^{3} A_j e^{-\lambda_j|x'_2|} \right) e^{ix\xi} d\xi \]

\[\omega_3 (x'_1, x'_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{j=1}^{3} A_j e^{-\lambda_j|x'_2|} \right) e^{ix\xi} d\xi \quad (5.9)\]

Where

\[\alpha_j = \frac{c_1^2}{gi \xi} (\beta^2_1 \xi^2 - \lambda^2_j) \quad ; \quad X_j = \frac{1}{p} \left[ (\lambda^2_j - \beta^2_2 \xi^2) \alpha_j - \frac{g i \xi}{c_2^2} \right] \]
Using (5.9), (4.3), (3.2a), (3.5) and (3.6) we obtain

\[ p_1A_1 + p_2A_2 + p_3A_3 = i\xi P^\ast(\xi) \]
\[ q_1A_1 + q_2A_2 + q_3A_3 = 0 \]
\[ r_1A_1 + r_2A_2 + r_3A_3 = 0 \]  \hspace{1cm} (5.10)

where

\[ p_j = \frac{M_2^2}{M_1^2} \lambda_j^2 - \left( \frac{M_2^2}{M_1^2} + p - 2 \right) \xi^2 - (2 - p)2i\xi|\alpha_j| \quad (j = 1, 2, 3) \]
\[ q_j = -(2 - p)i\xi|\alpha_j| - \alpha_j ((1 - p)\xi^2 - \lambda^2_j) + px_j \]
\[ r_j = |\lambda_j| \xi_j \]

and

\[ i\xi P^\ast(\xi) \] is the Fourier transform of \[-\frac{P}{(\mu + \alpha)} \delta(x_1')\]

Case (iii) transonic \( M_1 < 1 \) \( [i = 2, 4] \quad M_i > 1 \)

Substituting (5.1) in (3.9) we obtain

\[ \phi(x_1', x_2') = \frac{1}{2\pi} \int_{-\infty}^{\infty} [A_1 e^{-|\lambda_1|\xi_2} + \left( \sum A_j e^{-|\lambda_j|\xi_2} \right)] e^{ix_1} d\xi \]  \hspace{1cm} [j = 2, 3] \]
\[ \psi(x_1', x_2') = \frac{1}{2\pi} \int_{-\infty}^{\infty} [B_1 e^{-|\lambda_1|\xi_2} + \left( \sum B_j e^{-|\lambda_j|\xi_2} \right)] e^{ix_1} d\xi \]
\[ \omega_3(x_1', x_2') = \frac{1}{2\pi} \int_{-\infty}^{\infty} [C_1 e^{-|\lambda_1|\xi_2} + \left( \sum C_j e^{-|\lambda_j|\xi_2} \right)] e^{ix_1} d\xi \]  \hspace{1cm} (5.11)

where

\[ \left( \lambda^2_1 \right) + (-\lambda^2_2) + (-\lambda^2_3) = \xi^2 \left( \beta^2_1 - \beta^2_2 - \beta^2_4 \right) - \nu_0^2 - \nu_1^2 \]
\[-\lambda^2_1\lambda^2_2 + \lambda^2_2\lambda^2_3 - \lambda^2_1\lambda^2_3 = \xi^4 \left( \beta^2_2\beta^2_4 - \beta^2_2\beta^2_1 - \beta^2_1\beta^2_4 \right) + \xi^2 \left( \beta^2_2\nu_0^2 - \beta^2_1\nu_0^2 - \nu_1^2 - (g/c_1c_2)^2 \right) \]
\[ \lambda^2_1\lambda^2_2\lambda^2_3 = \left( g/c_1c_2 \right)^2 \xi^2 \left( \beta^2_4\xi^2 - \nu_0^2 \right) + \left( \beta^2_1\xi^4 \right) \left( \beta^2_2\beta^2_4\xi^2 + \beta^2_2\nu_0^2 - \nu_1^2 \right) \]
where

\[
\alpha_i = \frac{c_i^2}{\text{gi} \xi} (\beta_i^2 \xi^2 - \lambda_i^2) ; \quad \chi_i = \frac{1}{\text{p}} \left[ (\lambda_i^2 - \beta_i^2 \xi^2) \alpha_i - \frac{\text{gi} \xi}{\text{c}_i^2} \right]
\]

\[
\alpha_j = \frac{c_j^2}{\text{gi} \xi} (\beta_j^2 \xi^2 + \lambda_j^2) ; \quad \chi_j = \frac{1}{\text{p}} \left[ (\beta_j^2 \xi^2 - \lambda_j^2) \alpha_j - \frac{\text{gi} \xi}{\text{c}_j^2} \right] ; \quad [j = 2, 3]
\]

(5.12)

Using (5.11), (4.3), (3.2a), (3.5) and (3.6) we obtain

\[
p_1A_1 + p_2A_2 + p_3A_3 = i\xi P^* (\xi)
\]

\[
q_1A_1 + q_2A_2 + q_3A_3 = 0
\]

\[
r_1A_1 + r_2A_2 + r_3A_3 = 0
\]

(5.13)

where

\[
p_1 = \frac{M_2^2}{M_1^2} \lambda_1 - \left( \frac{M_2^2}{M_1^2} + p - 2 \right) \xi^2 - (2 - p)2i\xi |\lambda_1| \alpha_1
\]

\[
p_j = -\frac{M_2^2}{M_1^2} \lambda_j - \left( \frac{M_2^2}{M_1^2} + p - 2 \right) \xi^2 + (2 - p)\xi \lambda_j \alpha_j \quad (j = 2, 3)
\]

\[
q_1 = (p - 2)\xi |\lambda_1| - \alpha_1 \left( (1 - p)\xi^2 - \lambda_1^2 \right) + p\chi_1
\]

\[
q_j = (2 - p)\xi \lambda_j - \alpha_j \left( (1 - p)\xi^2 + \lambda_j^2 \right) + p\chi_j
\]

\[
r_1 = |\lambda_1| \chi_1 \quad ; \quad r_j = \lambda_j \chi_j
\]

and

\[
i\xi P^* (\xi) \quad \text{is the Fourier transform of} \quad -\frac{P}{(\mu + \alpha)} \delta(x_i').
\]

From (5.6), (5.10) and (5.13), which are in the same form, we can evaluate

\[
A_1 = i\xi P^* (\xi) \Delta_1 / \Delta \quad ; \quad A_2 = i\xi P^* (\xi) \Delta_2 / \Delta \quad ; \quad A_3 = i\xi P^* (\xi) \Delta_3 / \Delta
\]

(5.14)

in which

\[
\Delta = p_1 \Delta_1 + p_2 \Delta_2 + p_3 \Delta_3
\]
and
\[ \Delta_1 = (q_2 r_3 - r_2 q_3) \quad ; \quad \Delta_2 = (q_3 r_1 - r_3 q_1) \quad ; \quad \Delta_3 = (q_1 r_2 - r_1 q_2) \]

Using (5.4), (5.9) and (5.11) we can calculate \( \phi \), \( \psi \) and \( \omega_3 \) with the help of (5.14) in the (i) supersonic (ii) subsonic (iii) transonic cases respectively, from which we can also calculate the displacement components.

If the micropolar parameters tend to zero and \( g \) tends to zero, the above results are in well agreement with the classical results. The above results are presented in the most general integral form to deduce it further and express the above expressions, if possible, in terms of real functions, considering particular cases.
1. INTRODUCTION

W. Nowacki and W. K. Nowacki [92] investigated the plane Lamb's problem in a micropolar elastic semi-space. Acharya and Sengupta [4] also studied the plane Lamb's problem under the influence of temperature. The plane problem of thermo-elasticity has been discussed by W Nowacki [87]. The thermal effect on Rayleigh waves has been discussed by Chadwick [23] who supposed that the heat is radiated from the free plane boundary surface of the solid and the maximum temperature difference across this surface being always small. Considering the concept of force stress and couple stress a series of research works have been published by Eringen [49,51,52]. It is noted that Eringen and Suhubi [54] studied the non-linear theory of micropolar elasticity. Here the authors deal with the solution of the coupled thermo-elastic problem due to the action of a harmonically varying heat source in micropolar elastic semi-space under the influence of gravity.

2. BASIC EQUATION AND FORMULATION OF THE PROBLEM

The dynamical equations of motion for a micropolar solid elastic medium under the influence of temperature with no body forces and body couples may be written as [91]

\[(\mu + \alpha)\nabla^2 \vec{u} + (\lambda + \mu - \alpha)\text{grad}div\vec{u} + 2\alpha\text{rot}\vec{\omega} - \gamma\text{grad}\theta = \rho\ddot{u}\]

\[(\gamma + \varepsilon)\nabla^2 \vec{\omega} + (\beta + \gamma - \varepsilon)\text{grad}div\vec{\omega} - 4\alpha\vec{\omega} + 2\alpha\text{rot}\vec{u} = J\ddot{\omega}
\]

Where

- \(\rho\) : material density
- \(\lambda, \mu\) : Lame's elastic constants
- \(\vec{u}\) : displacement vector
- \(\vec{\omega}\) : rotation vector
- \(\dot{v} = (3\lambda + 2\mu)\alpha_t\)
- \(\alpha_t\) : coefficient of linear expansion of solid
- \(\theta = T - T_0\) : absolute temperature - initial absolute temperature

dots denote the derivatives with respect to time \(t\).
Let us consider a set of orthogonal Cartesian co-ordinate system \(0-x_1 x_2 x_3\) in micropolar elastic semi-space. The origin \(0\) being any point on the free surface of the solid. The surface \(x_1 = 0\) is free from stresses and couple stresses and the heat source varies periodically with time.

Let us suppose that displacement vector \(\vec{u}\) and rotation vector \(\vec{\omega}\) do not depend on the variable \(x_3\) and

\[
\vec{u} = (u_1, u_2, 0) \quad ; \quad \vec{\omega} = (0, 0, \omega_3)
\]  

(2.2)

where \(u_1, u_2\) and \(\omega_3\) are functions of \(x_1, x_2\) and time \(t\) only.

The displacement potentials \(\varphi\) and \(\psi\) are related to the displacement as

\[
u_1 = \varphi_1 + \psi_2 \quad ; \quad u_2 = \varphi_2 - \psi_1
\]  

(2.3)

where comma denotes the partial differentiation with respect to space coordinate and from above we obtain

\[
\nabla^2 \varphi = \Delta; \quad \nabla^2 \psi = u_{1,2} - u_{2,1}; \quad \Delta = u_{1,1} + u_{2,2}
\]

(2.4)

Considering the gravitational field as viewed by Biot M. A [13], (2 1) takes the following form

\[
(\mu + \alpha)\nabla^2 u_1 + (\lambda + \mu - \alpha)\Delta_1 + 2\alpha \omega_{3,2} - \nu \theta_{1,1} + g\rho u_{2,1} = \rho \ddot{u}_1
\]

\[
(\mu + \alpha)\nabla^2 u_2 + (\lambda + \mu - \alpha)\Delta_2 - 2\alpha \omega_{3,1} - \nu \theta_{2,1} - g\rho u_{1,1} = \rho \ddot{u}_2
\]

\[
(\gamma + \varepsilon)\nabla^2 \omega_3 - 4\alpha \omega_3 + 2\alpha (u_{1,1} - u_{1,2}) = J\ddot{\omega}_3
\]

(2.5)

To determine \(\theta\), Fourier's law of heat conduction is used in the form

\[
k \nabla^2 \theta = \rho c_e \frac{\partial \theta}{\partial t} + T_0 \nu \frac{\partial}{\partial t} \left( \nabla^2 \varphi \right)
\]

(2.6)

where \(k\) is thermal conductivity, and \(c_e\) is specific heat at constant strain.

Using (2.3) and (2.4) into the equation (2.5), we obtain the following differential equations satisfied by \(\varphi, \psi, \theta\) and \(\omega_3\)
\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \varphi - q\theta + \frac{g}{c^2} \frac{\partial \psi}{\partial x_1} = 0
\]

\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \psi + p\omega_3 - \frac{g}{c^2} \frac{\partial \varphi}{\partial x_1} = 0
\]

\[
\left[ \nabla^2 - \nu_i^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \omega_3 - s\nabla^2 \psi = 0
\]

\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial}{\partial t} \right] \theta - r \frac{\partial}{\partial t} (\nabla^2 \varphi) = 0
\]

(2.7)

Eliminating \( \varphi, \psi, \theta \) and \( \omega_3 \) from (2.7) we get

\[
\left[ \left( \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right) \left( \nabla^2 - \frac{1}{c_2^2} \frac{\partial}{\partial t} \right) - \eta^2 \frac{\partial}{\partial t} \nabla^2 \right] \left( \nabla^2 - \frac{1}{c_3^2} \frac{\partial^2}{\partial t^2} \right) \left( \nabla^2 - \nu_i^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \xi^2 \nabla^2 \right] + \left( \frac{g}{c_1 c_2} \right)^2 \frac{\partial^2}{\partial x_1^2} \left( \nabla^2 - \frac{1}{c_3^2} \frac{\partial}{\partial t} \right) \left( \nabla^2 - \nu_i^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \left( \varphi \Theta, \psi, \omega_3 \right) = 0
\]

(2.8)

where

\[
\eta^2 = q r ; \quad \xi^2 = p s ; \quad \nu_i^2 = 4a / (\gamma + \varepsilon) ; \quad c_1^2 = (\lambda + 2\mu) / \rho ; \quad c_2^2 = (\mu + \alpha) / \rho ; \quad c_3^2 = k / \rho c_\varepsilon ; \quad c_4^2 = (\gamma + \varepsilon) / J ; \quad p = 2a / (\mu + \alpha) ; \quad q = v / (\lambda + 2\mu) ; \quad r = T_0 u / k ; \quad s = a / (\gamma + \varepsilon)
\]

The relations between the state of stress and the state of strain are linear and are given by Nowacki [87] as

\[
\sigma_{ij} = (\mu + \alpha)\gamma_{ij} + (\mu - \alpha)\gamma_{j1} + (\lambda\gamma_{kk} - \nu\theta)\delta_{ij}
\]

\[
\mu_{ij} = (\gamma + \varepsilon)\chi_{ij} + (\gamma - \varepsilon)\chi_{j1} + \beta\chi_{kk}\delta_{ij}
\]

where

\[
[\gamma_{ij} = u_{i,j} - \varepsilon_{kij}\omega_k] \quad \text{and} \quad [\chi_{ij} = \omega_{i,j}] \quad (2.9)
\]

\( \varepsilon_{kij} \) denotes the unit anti-symmetric tensor, \( \gamma_{ij} \), known as the deformation tensor and \( \chi_{ij} \) is called the curvature twist tensor.

**Boundary condition**: The mathematical formulation of the problem seeks the solution of (2.8) subject to the following boundary conditions...
\[ \sigma_{11} = \sigma_{12} = \sigma_{13} \quad \text{on} \quad x_1 = 0 \\
\theta_1 = e^{i\lambda t} \omega(x_2) \quad \text{on} \quad x_1 = 0 \] 

where

\[ \sigma_{11} = 2\mu [\varphi_{11} + \psi_{12}] + \lambda \nabla^2 \varphi - \nu \theta \\
\sigma_{12} = \mu [2\varphi_{12} + \psi_{22} - \psi_{11}] - \alpha (\nabla^2 \psi + 2\omega_3) \\
\mu_{13} = (\gamma + \varepsilon)\omega_{3,1} \] 

(2.10)

(2.11)

3. METHOD OF SOLUTION

We introduce double Fourier integral transform defined by

\[ \tilde{f}(x_1, \alpha_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x_1, x_2, t) e^{i(\alpha_2 x_2 + \omega t)} dx_2 dt \]

\[ f(x_1, x_2, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(x_1, \alpha_2, \omega) e^{-i(\alpha_2 x_2 + \omega t)} dx_2 d\omega \] 

(3.1)

Introducing (3.1) in (2.8) we obtain

\[ \varphi = \sum A_j e^{-\lambda_j x_3} ; \theta = \sum B_j e^{-\lambda_j x_3} ; \psi = \sum C_j e^{-\lambda_j x_3} ; \omega_3 = \sum D_j e^{-\lambda_j x_3} \quad (j = 1,2,3,4) \] 

(3.2)

where \( \lambda_j^2 \) are the roots of the equation \((j=1,2,3,4)\)

\[ \left[ (\lambda^2 - \alpha_2^2 + \sigma_2^2)(\lambda^2 - \alpha_2^2 + \sigma_4^2) + i\eta^2 \omega (\lambda^2 - \alpha_2^2 - \nu_1^2) \right] \left[ (\lambda^2 - \alpha_2^2 + \sigma_2^2)(\lambda^2 - \alpha_2^2 - \nu_1^2) + \zeta^2 \alpha_2^2 \right] - \frac{g_i^2 \alpha_2^2 \sigma_1^2 \sigma_2^2}{\omega^4} \left[ (\lambda^2 - \alpha_2^2 + \sigma_3^2)(\lambda^2 - \alpha_2^2 - \nu_1^2) + \sigma_4^2 \right] = 0 \] 

(3.3)

where \( \sigma_1^2 = \omega^2 / c_1^2 \quad (l = 1,2,4) ; \quad \sigma_3^2 = i\omega / c_3^2 \)

Also \( B_i, C_i \) and \( D_i \) are related to \( A_i \) as

\[ B_i = n_i A_i \quad ; \quad C_i = \gamma_i A_i \quad ; \quad D_i = \beta_i A_i \quad (i = 1,2,3,4) \] 

(3.4)

where

\[ n_j = i\omega(\alpha_2^2 - \lambda_j^2) \quad ; \quad \gamma_j = c_1^2 \frac{\left( \lambda_j^2 - \alpha_2^2 + \sigma_1^2 - qn_j \right)}{gi\alpha_2} \]

\[ \beta_j = s(\lambda_j^2 - \alpha_2^2) \gamma_j / (\lambda_j^2 - \alpha_2^2 + \sigma_4^2 - \nu_1^2) \quad (j = 1,2,3,4) \] 

(3.5)

Using (3.1) in the boundary conditions (2.10) we obtain
\[ p_1A_1 + p_2A_2 + p_3A_3 + p_4A_4 = \bar{\omega}(\alpha_2) \mathcal{S}(\lambda + \omega) \]
\[ q_1A_1 + q_2A_2 + q_3A_3 + q_4A_4 = 0 \]
\[ r_1A_1 + r_2A_2 + r_3A_3 + r_4A_4 = 0 \]
\[ s_1A_1 + s_2A_2 + s_3A_3 + s_4A_4 = 0 \]  
(3.6)

where
\[ p_j = \lambda_j n_j \quad ; \quad s_j = \lambda_j \beta_j \quad (j = 1, 2, 3, 4) \]
\[ q_j = 2\mu \left[ \lambda_j^2 + i\alpha_2 \lambda_j \gamma_j \right] + \lambda \left( \lambda_j^2 - \alpha_j^2 \right) - \nu \gamma_j \]
\[ r_j = 2\mu i\alpha_2 \lambda_j - \mu \gamma_j \left( \lambda_j^2 + \alpha_j^2 \right) - \alpha (\gamma_j \left[ \lambda_j^2 - \alpha_j^2 \right] + 2\beta_j) \]  
(3.7)

and since
\[ \delta(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{in\lambda} d\lambda \]

Solving the system of equations in (3.6) we obtain
\[ A_1 = \Delta_1 / \Delta \quad ; \quad A_2 = \Delta_2 / \Delta \quad ; \quad A_3 = \Delta_3 / \Delta \quad ; \quad A_4 = \Delta_4 / \Delta \]  
(3.8)

where
\[ \Delta = \sum p_i \Delta_i \quad (i = 1, 2, 3, 4) \]
\[ \Delta_1 = \bar{\omega}(\alpha_2) \delta(\lambda + \omega) \left[ q_1 (r_3 s_4 - s_3 r_4) - q_3 (r_2 s_4 - s_2 r_4) + q_4 (r_2 s_3 - s_2 r_3) \right] \]
\[ \Delta_2 = -\bar{\omega}(\alpha_2) \delta(\lambda + \omega) \left[ q_1 (r_3 s_4 - s_3 r_4) - q_3 (r_1 s_4 - s_1 r_4) + q_4 (r_1 s_3 - s_1 r_3) \right] \]
\[ \Delta_3 = \bar{\omega}(\alpha_2) \delta(\lambda + \omega) \left[ q_1 (r_2 s_4 - s_2 r_4) - q_2 (r_2 s_3 - s_2 r_3) + q_4 (r_2 s_1 - s_2 r_1) \right] \]
\[ \Delta_4 = -\bar{\omega}(\alpha_2) \delta(\lambda + \omega) \left[ q_1 (r_2 s_3 - s_2 r_3) - q_2 (r_1 s_3 - s_1 r_3) + q_3 (r_1 s_2 - s_1 r_2) \right] \]

Using (3.8) and (3.2) we easily obtain
\[ \phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int \left( \sum_{i=1}^{4} \frac{\Delta_i e^{-\lambda x_1}}{\Delta} \right) e^{-i(\alpha_2 x_2 + \omega t)} d\alpha_2 d\omega \]
\[ \theta = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int \left( \sum_{i=1}^{4} \frac{n_i \Delta_i e^{-\lambda x_1}}{\Delta} \right) e^{-i(\alpha_2 x_2 + \omega t)} d\alpha_2 d\omega \]
\[ \psi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int \left( \sum_{i=1}^{4} \frac{\gamma_i \Delta_i e^{-\lambda x_1}}{\Delta} \right) e^{-i(\alpha_2 x_2 + \omega t)} d\alpha_2 d\omega \]
\[ \omega_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \sum_{i=1}^{4} \frac{\beta_i \Delta_i e^{-\lambda_i x_i}}{\Delta} \right) e^{-i(\omega x_2 + \omega t)} d\alpha_2 d\omega \] (3.9)

Hence the displacement components \( u_1, u_2 \) and the component \( \mu_{13} \) may be written in the following form

\[ u_1 = -\frac{1}{2\pi} \int \left\{ \sum_{i=1}^{4} \left[ \frac{\lambda_i \Delta_i + i\sigma_2 \Delta_i}{\Delta} \right] e^{-\lambda_i x_i} \right\} e^{-i(\omega x_2 + \omega t)} d\alpha_2 d\omega \]
\[ u_2 = -\frac{1}{2\pi} \int \left\{ \sum_{i=1}^{4} \left[ \frac{\gamma_i \lambda_i \Delta_i + i\sigma_2 \Delta_i}{\Delta} \right] e^{-\lambda_i x_i} \right\} e^{-i(\omega x_2 + \omega t)} d\alpha_2 d\omega \]
\[ \mu_{13} = -(\gamma + \epsilon) \frac{1}{2\pi} \int \left\{ \sum_{i=1}^{4} \left[ \frac{\beta_i \lambda_i \Delta_i}{\Delta} \right] e^{-\lambda_i x_i} \right\} e^{-i(\omega x_2 + \omega t)} d\alpha_2 d\omega \] (3.10)

From the above results in (3.10) we can conclude that the displacement components \( u_1, u_2 \) and the couple stress component \( \mu_{13} \) obviously depend on the temperature field and gravity. With regard to the action of periodically varying heat source in presence of gravity in the micropolar theory of elasticity, it can be added as a concluding remark that the present analysis is sufficiently general and in addition, it incorporates other forms of heat source and interacting field of physical interest.
1. INTRODUCTION

Considerable attention has been given to the problems of wave propagation in a micropolar solid elastic medium in various configurations by several authors including Eringen [49,51, 52], Nowacki [87,88,90,91], De and Sengupta [39] and Das and Sengupta [32]. Most of these works deal with the displacement vector $u(x, t)$ and rotation vector $\omega(x, t)$ produced by the action across any infinitesimal surface element in a micropolar elastic medium. Further, two asymmetric tensors $\gamma_{ij}$, known as the deformation tensor, and $\chi_{ij}$, called the curvature twist tensor, describe the state of deformation of a solid body whether the state of stress is characterized by the asymmetric force stress tensor $\sigma_{ij}$, and/or the couple stress tensor, $\mu_{ij}$. In particular, Nowacki [87] generalized the problem of surface waves in a micropolar elastic medium in two special configurations:

(i) displacement vector is $u = (u_1, 0, u_3)$ and rotation vector $\omega = (0, \omega_2, 0)$
(ii) the displacement is $u = (0, u_2, 0)$ and the rotation $\omega = (\omega_1, 0, \omega_3)$.

In spite of this works, more work is needed for an understanding of the problems of waves and vibrations in magneto- thermo-elastic medium. So the main purpose of this paper is to consider such problems in the presence of a constant magnetic field. Special attention will be given to the velocities of various waves generated in the medium.

2. FUNDAMENTAL RELATIONS AND EQUATIONS OF MOTION

The relations between the state of stress and the state of strain are linear and are given by Nowacki [87] as

$$\sigma_{ij} = (\mu + \alpha)\gamma_{ij} + (\mu - \alpha)\gamma_{ij} + (\lambda\gamma_{kk} - \nu\theta)\delta_{ij}$$
$$\mu_{ij} = (\gamma + \varepsilon)\chi_{ij} + (\gamma - \varepsilon)\chi_{ij} + \beta\chi_{kk}\delta_{ij}$$

where

$$\gamma_{ij} = u_{i,j} - \varepsilon_{kij}\omega_k$$
$$\chi_{ij} = \omega_{i,j}$$

$[i, j = 1,2,3]$}

$\lambda, \mu, \alpha, \beta, \gamma$ and $\varepsilon$ are material constants, $\varepsilon_{kij}$ denotes the unit anti-symmetric tensor, $\nu = (3\lambda + 2\mu)/\alpha_t$ and $\alpha_t$ is the co-efficient of linear expansion of solid and the tensorial index notation in the
rectangular coordinate system O-X₁ X₂ X₃ is used.

Following Nowacki [87], the basic equations of motion for a micropolar elastic medium under the constant primary magnetic field with no body forces and body couples may be written as

\[
\begin{align*}
(\mu + \alpha)\nabla^2 \ddot{u} + (\lambda + \mu - \alpha) \text{grad} \text{div} \ddot{u} + 2\alpha \text{rot} \ddot{\omega} + (\mu_0/c) (J \times \bar{H}) &= \rho \ddot{u} \\
(\gamma + \varepsilon)\nabla^2 \ddot{\omega} + (\beta + \gamma - \varepsilon) \text{grad} \text{div} \ddot{\omega} - 4\alpha \ddot{\omega} + 2\alpha \text{rot} \ddot{u} &= J \ddot{\omega}
\end{align*}
\]  

(2.4)

Where \( \rho \) is material density, \( J \) is rotational inertia, \( \mu_0 \) is magnetic permeability factor, \( j \) current density vector, \( \omega \) is the velocity of light, dots denote the derivatives with respect to time \( t \) and comma denotes the partial differentiation with respect to space coordinate.

3. SOLUTION OF THE PROBLEM

We consider two micropolar elastic solid media \( M \) and \( M' \) which are either welded in contact or sufficiently rough enough to prevent any sliding on the common surface of separation, which is a plane horizontal boundary extended to infinitely great distance from the origin \( o \). Both the media are subjected to a constant strong primary magnetic field \( H \) and thermal field \( \theta \). It is assumed that prior to existence of any disturbance both the media are everywhere at the constant absolute temperature \( T_0 \). The medium \( M' \) is above the medium \( M \), the origin \( o \) being any point on the plane horizontal boundary and \( ox_3 \) pointing normally into the medium \( M \).

If we assume that (i) the disturbance is largely confined to the neighborhood of the boundary and (ii) at any instant all particles in any line parallel to the \( x_2 \)-axis have the same displacements, then assumption (i) asserts that the wave is a surface wave and (ii) admits that all partial derivatives with respect to \( x_2 \)-axis are zero.

The displacement components \( u_1, u_3 \) and the rotation components \( \omega_1 \) and \( \omega_3 \) can be expressed in terms of displacement potentials \( \varphi \) and \( \psi \) as

\[
\begin{align*}
\varphi &= \varphi_1 + \psi_3 ; u_3 = \varphi_3 - \psi_1 ; \quad \omega_1 = \varphi_1 + \omega_3 ; \quad \omega_3 = \varphi_3 - \omega_1
\end{align*}
\]  

(3.1a - d)

In a micropolar elastic medium under the influence of the specified magnetic field and thermal field as stated in section 2 with no body forces and body couples, the equations of motion for the first system (i) \( u = (u_1, 0, u_3) \) and \( \omega = (0, \omega_2, 0) \) are given by

\[
\begin{align*}
(\mu + \alpha)\nabla^2 u_1 + (\lambda + \mu - \alpha + \alpha_0^2 \rho) \varepsilon_1 - 2\alpha \omega_{2,3} + \alpha_0^2 \rho (u_{1,33} - u_{3,13}) - \nu \theta_1 &= \rho \ddot{u}_1 \\
(\mu + \alpha)\nabla^2 u_3 + (\lambda + \mu - \alpha) \varepsilon_3 + 2\alpha \omega_{2,1} - \nu \theta_3 &= \rho \ddot{u}_3 \\
(\gamma + \varepsilon)\nabla^2 \omega_2 - 4\alpha \omega_{2,1} + 2\alpha (u_{1,3} - u_{3,1}) &= J \ddot{\omega}_2
\end{align*}
\]  

(3.2) and (3.3) and (3.4)
To determine $\theta$, Fourier’s law of heat conduction is used in the form

$$k \nabla^2 \theta = \rho c_e \frac{\partial \theta}{\partial t} + T_0 v \frac{\partial}{\partial t} (\nabla^2 \varphi)$$  \hspace{1cm} (3.5)

For the second system $u = (0, u_2, 0)$ and $\omega = (\omega_1, 0, \omega_3)$, equations of motion are given by

$$(\mu + \alpha) \nabla^2 u_2 + 2\alpha (\omega_{1,3} - \omega_{3,1}) + \alpha_0 \rho u_{3,3} = \rho \ddot{u}_2$$ \hspace{1cm} (3.6)

$$(\gamma + \varepsilon) \nabla^2 \omega_1 - 4\alpha \omega_1 + (\beta + \gamma - \varepsilon) e'_{1,1} - 2\alpha u_{2,3} = J \dot{\omega}_1$$ \hspace{1cm} (3.7)

$$(\gamma + \varepsilon) \nabla^2 \omega_3 - 4\alpha \omega_3 + (\beta + \gamma - \varepsilon) e'_{3,3} + 2\alpha u_{2,1} = J \dot{\omega}_3$$ \hspace{1cm} (3.8)

Where

$$\nabla^2 = \begin{pmatrix} \omega_{11} + \omega_{33} \\ \omega_{13} + \omega_{31} \end{pmatrix}, \ e = u_{1,1} + u_{3,3}, \ e' = \omega_{1,1} + \omega_{3,3}$$ \hspace{1cm} (3.9)

$\theta$ denotes the temperature increment measured from the reference state $T_0$, $k$ is thermal conductivity, and $c_e$ is specific heat at constant strain, $\alpha_0 = \mu_0 H_0^2 / 4 \pi \rho$, $\alpha_0$ denotes the Alfven wave velocity. Substituting (3 1a-d) in system of equations (3.2)-(3.5) and (3.6)-(3.9), respectively we get

$$\left[ \left( 1 + \frac{\alpha_0^2}{c_1^2} \right) \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right] \varphi - q \theta = 0$$ \hspace{1cm} (3.10)

$$\left[ \left( 1 + \frac{\alpha_0^2}{c_2^2} \right) \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right] \psi - p \omega_2 = 0$$ \hspace{1cm} (3.11)

$$\nabla^2 - u_2^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right] \omega_2 + s \nabla^2 \psi = 0$$ \hspace{1cm} (3.12)

$$\nabla^2 - \frac{1}{c_2^2} \frac{\partial}{\partial t} \left( \theta - r \frac{\partial}{\partial t} (\nabla^2 \varphi) = 0 \right.$$

and

$$\left( \nabla^2 - u_1^2 - \frac{1}{c_3^2} \frac{\partial^2}{\partial t^2} \right) \Gamma = 0 \hspace{1cm} (3.14)$$

$$\left( \nabla^2 - u_2^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \chi - s \dot{u}_2 = 0 \hspace{1cm} (3.15)$$
\[
\left( \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} + \frac{\alpha_0^2}{c_2^2} \frac{\partial^2}{\partial x_3^2} \right) u_2 + p \nabla^2 \chi = 0 \]  
(3.16)

where

\[
\begin{align*}
    c_1^2 &= (\lambda + 2\mu) / \rho; \\
    c_2^2 &= (\mu + \alpha) / \rho; \\
    c_3^2 &= (\beta + 2\gamma) / J; \\
    c_4^2 &= (\gamma + \epsilon) / J; \\
    c_5^2 &= k / \rho c_e; \\
    \nu_1^2 &= 4\alpha / (\beta + 2\gamma); \\
    \nu_2^2 &= 4\alpha / (\gamma + \epsilon); \\
    p &= 2\alpha / (\mu + \alpha); \\
    q &= \nu / (\lambda + 2\mu); \\
    r &= T_0 \nu / k; \\
    s &= 2\alpha / (\gamma + \epsilon).
\end{align*}
\]

\[
\nabla^2 \phi = e; \quad \nabla^2 \psi = u_{1,3} - u_{3,1}; \quad \nabla^2 \Gamma = e'; \quad \nabla^2 \chi = \omega_{1,3} - \omega_{3,1} \]  
(3.17)

Eliminating \( \phi \) and \( \theta \) from (3.10) and (3.13) and \( \psi \) and \( \omega_2 \) from (3.11) and (3.12), we get, respectively

\[
\left[ \left( 1 + \frac{\alpha_0^2}{c_1^2} \right) \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right] \left[ \nabla^2 - \frac{1}{c_2^2} \frac{\partial}{\partial t} \right] \left[ \nabla^2 - \frac{1}{c_2^2} \frac{\partial}{\partial t} \right] - \tau^2 \frac{\partial}{\partial t} \nabla^2 \right)(\phi, \theta) = 0 \]  
(3.18)

\[
\left[ \left( 1 + \frac{\alpha_0^2}{c_2^2} \right) \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right] \left[ \nabla^2 - \nu_1^2 - \frac{1}{c_2^2} \frac{\partial}{\partial t} \right] + \zeta^2 \nabla^2 \right)(\psi, \omega_2) = 0 \]  
(3.19)

Eliminating \( \chi \) and \( u_2 \) from (3.15) and (3.16), we get

\[
\left[ \left( \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} + \frac{\alpha_0^2}{c_2^2} \frac{\partial^2}{\partial x_3^2} \right) \left( \nabla^2 - \nu_2^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) + \zeta^2 \nabla^2 \right](\chi, u_2) = 0 \]  
(3.20)

where

\[
\tau^2 = q.r; \quad \zeta^2 = p.s \]  
(3.21)

We seek solutions of differential equations (3.14), (3.18), (3.19) and (3.20) in the following form

\[
(\phi, \theta, \psi, \omega_2, \Gamma, \chi, u_2) = \\
\left[ \phi (x_3), \theta (x_3), \psi (x_3), \omega_2 (x_3), \Gamma (x_3), \chi (x_3), u_2 (x_3) \right] \exp [i\xi (x_1 - ct)] \]  
(3.22)

Substituting the values \( \phi, \theta, \psi, \omega_2, \Gamma, \chi, u_2 \) from (3.22) in equations (3.14), (3.19) and (3.20) respectively, and assuming that \( \phi, \theta, \psi, \omega_2, \Gamma, \chi, u_2 \) tend to zero as \( x_3 \to \infty \), we get the
solution in the medium \( M \) as

\[
\varphi = [A e^{-i\varphi_1 x_3} + B e^{-i\varphi_2 x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\theta = [\alpha_1 A e^{-i\varphi_1 x_3} + \alpha_2 B e^{-i\varphi_2 x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\psi = [C e^{-i\varphi_3 x_3} + D e^{-i\varphi_4 x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\omega_2 = [\alpha_3 C e^{-i\varphi_3 x_3} + \alpha_4 D e^{-i\varphi_4 x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\chi = [E e^{-i\varphi_5 x_3} + F e^{-i\varphi_6 x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
u_2 = [\alpha_5 E e^{-i\varphi_5 x_3} + \alpha_6 F e^{-i\varphi_6 x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\Gamma = G \exp\{i\xi(x_1 - ct) - \lambda_7 x_3\}
\]

and in the medium \( M' \) as

\[
\varphi' = [A' e^{i\varphi_1' x_3} + B' e^{i\varphi_2' x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\theta' = [\alpha_1' A' e^{i\varphi_1' x_3} + \alpha_2' B' e^{i\varphi_2' x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\psi' = [C' e^{i\varphi_3' x_3} + D' e^{i\varphi_4' x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\omega'_2 = [\alpha_3' C' e^{i\varphi_3' x_3} + \alpha_4' D' e^{i\varphi_4' x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\chi' = [E' e^{i\varphi_5' x_3} + F' e^{i\varphi_6' x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
u'_2 = [\alpha_5' E' e^{i\varphi_5' x_3} + \alpha_6' F' e^{i\varphi_6' x_3}] \exp\{i\xi(x_1 - ct)\}
\]

\[
\Gamma' = G' \exp\{i\xi(x_1 - ct) - \lambda_7' x_3\}
\]

Where primes denote the properties for the medium \( M' \) i.e \( \alpha, \beta, \gamma, \epsilon, \rho \ldots \) etc in the medium \( M \) are replaced by \( \alpha', \beta', \gamma', \epsilon', \rho' \ldots \) etc in the medium \( M' \), and

\[
\lambda_1^2, \lambda_2^2 = \frac{c^2}{2(c^2 + \alpha^2)} \left[ \left( \zeta_1^2 + \zeta_2^2 + \frac{\alpha^2}{c^2} (-1 + \zeta_2^2) + \frac{\tau^2 i c}{\xi} \right) \right] \pm
\]

\[
\sqrt{\left[ \left( \zeta_1^2 + \zeta_2^2 + \frac{\alpha^2}{c^2} (-1 + \zeta_2^2) + \frac{\tau^2 i c}{\xi} \right)^2 - 4 \left[ 1 + \frac{\alpha^2}{c^2} \right] \left( \zeta_1^2 - \frac{\alpha^2}{c^2} \zeta_2^2 - \frac{\tau^2 i c}{\xi} \right) } \] (3.28)

\[
\lambda_3^2, \lambda_4^2 = \frac{c^2}{2(c^2 + \alpha^2)} \left[ \left( \zeta_3^2 + \zeta_4^2 + \frac{\alpha^2}{c^2} (-1 + \zeta_4^2) + \frac{\zeta_2^2}{\xi^2} \right) \right] \pm
\]
\[
\sqrt{\left[\zeta_3^2 + \zeta_4^2 + \frac{\alpha_0^2}{c_2^2} (-1 + \zeta_4^2) + \frac{\zeta_2^2}{\xi^2}\right]^2 - 4 \left(1 + \frac{\alpha_0^2}{c_2^2}\right) \left[\zeta_3^2 - \frac{\alpha_0^2}{c_2^2}\zeta_4^2 - \frac{\zeta_2^2}{\xi^2}\right]} \tag{3.29}
\]

\[
\lambda_5^2, \lambda_6^2 = \frac{c_2^2}{2(c_2^2 + \alpha_0^2)} \left[\left(\zeta_3^2 + \zeta_4^2 + \frac{\alpha_0^2}{c_2^2} \zeta_4^2 + \frac{\zeta_2^2}{\xi^2}\right) + \right]
\]

\[
\sqrt{\left[\zeta_3^2 + \zeta_4^2 + \frac{\alpha_0^2}{c_2^2} \zeta_4^2 + \frac{\zeta_2^2}{\xi^2}\right]^2 - 4 \left(1 + \frac{\alpha_0^2}{c_2^2}\right) \left[\zeta_3^2 \zeta_4^2 - \frac{\zeta_2^2}{\xi^2}\right]} \tag{3.30}
\]

\[
\lambda_7^2 = \frac{c_2^2 - \nu_1^2}{c_3^2} - 1
\]

\[
\alpha_j = -\frac{\xi_1}{q} \left[\frac{\alpha_0^2}{c_1^2} (\lambda_j^2 + 1) + \lambda_j^2 - \zeta_1^2\right] ; \quad \zeta_1^2 = -1 + \frac{c_2^2}{c_1^2} \quad (j = 1, 2)
\]

\[
\alpha_1 = -\frac{\xi_1}{p} \left[\frac{\alpha_0^2}{c_2^2} (\lambda_1^2 + 1) + \lambda_1^2 - \zeta_3^2\right] ; \quad \zeta_2^2 = -1 + \frac{ic}{\xi_2^2} \quad (l = 3, 4)
\]

\[
\alpha_m = \frac{p(\lambda_m^2 + 1)}{-\zeta_3^2 + \lambda_m^2 + \frac{\alpha_0^2}{c_2^2} \lambda_m^2} ; \quad \zeta_3^2 = -1 + \frac{c_2^2}{c_3^2} \quad (m = 5, 6)
\]

\[
\zeta_4^2 = -1 - \frac{\nu_2^2}{\xi^2} + \frac{c_2^2}{c_4^2} \tag{3.31}
\]

4. BOUNDARY CONDITIONS

To obtain the frequency equation, we consider the following boundary conditions:
A(i) the displacement components \(u_1, u_3\), the rotational component \(\omega_2\), temperature and temperature flux are continuous on the boundary surface of separation \(x_3 = 0\),
A(ii) the stress components \(\sigma_{31}, \sigma_{33}\) and the couple-stress component \(\mu_{32}\) are continuous on the boundary surface of separation \(x_3 = 0\),
B(i) the displacement component \(u_2\), and the rotation components \(\omega_1, \omega_3\) are continuous on the boundary surface of separation \(x_3 = 0\),
B(ii) the stress component $\sigma_{32}$ and the couple-stress components $\mu_{31}, \mu_{33}$ are continuous on the boundary surface of separation $x_3 = 0$.

Where

$$
\begin{align*}
\sigma_{31} &= \mu \left(2\phi_{13} + \psi_{33} - \psi_{11}\right) + a \left(\nabla^2 \psi - 2\omega_2 \right) \\
\sigma_{33} &= 2\mu \left(\phi_{33} - \psi_{13}\right) + \lambda \nabla^2 \phi - \nu \theta \\
\mu_{32} &= (\gamma + \varepsilon) \omega_{2,3} \\
\sigma_{32} &= (\mu + a) u_{2,3} + 2a \left(\Gamma_{23} + \chi_{33}\right) \\
\mu_{31} &= \gamma \left(2\Gamma_{13} + \chi_{33} - \chi_{11}\right) + \varepsilon \nabla^2 \chi \\
\mu_{33} &= (2\gamma + \beta) \left[\Gamma_{33} - \chi_{13}\right] + \beta \left(\Gamma_{11} + \chi_{13}\right)
\end{align*}
$$

(4.1)

Commas denote the partial differentiation with respect to space co-ordinates. Substituting the values of $\phi$, $\theta$, $\psi$ and $\omega_2$ from (3.23) and (3.26) in the boundary condition $A$, we get eight simultaneous equations involving eight unknown constants $A, B, C, D, A', B', C', D'$. Eliminating the unknown constants we get the frequency equation as

$$
\begin{pmatrix}
1 & 1 & -\lambda_3 & -\lambda_4 & 1 & 1 & -\lambda_3' & -\lambda_4' \\
\lambda_1 & \lambda_2 & 1 & 1 & \lambda_1' & \lambda_2' & 1 & 1 \\
0 & 0 & \alpha_3 & \alpha_4 & 0 & 0 & -\alpha_3' & -\alpha_4' \\
\alpha_1 & \alpha_2 & 0 & 0 & -\alpha_1' & -\alpha_2' & 0 & 0 \\
k\lambda_1\alpha_1 & k\lambda_2\alpha_2 & 0 & 0 & k\lambda_1'\alpha_1' & k\lambda_2'\alpha_2' & 0 & 0 \\
p_1 & p_2 & p_3 & p_4 & p_1' & p_2' & -p_3' & -p_4' \\
q_1 & q_2 & q_3 & q_4 & -q_1' & -q_2' & q_3' & q_4' \\
0 & 0 & r_3\lambda_3 & r_4\lambda_4 & 0 & 0 & r_3'\lambda_3' & r_4'\lambda_4'
\end{pmatrix}
= 0
$$

(4.2)

where

$$
\begin{align*}
p_1 &= 2\mu \lambda_i \\
p_1' &= 2\mu \lambda_i' \\
q_1 &= 2\mu \lambda_i^2 + \lambda \left(\lambda_i^2 + 1\right) + \nu a_i \\
q_1' &= 2\mu \lambda_i'^2 + \lambda \left(\lambda_i'^2 + 1\right) + \nu' a_i' \\
p_j &= -\left(\mu + a\right) \lambda_j + \left(\mu - a\right) - 2\alpha a_j \lambda_j / \xi^2 \\
p_j' &= \left(\mu' + a'\right) \lambda_j'^2 + \left(\mu' - a'\right) - 2\alpha' a_j' / \xi^2 \\
q_j &= 2\mu \lambda_j \\
q_j' &= 2\mu \lambda_j' \\
r_j &= \left(\gamma + \varepsilon\right) a_j \\
r_j' &= \left(\gamma' + \varepsilon'\right) a_j' \\
(i = 1, 2) \\
(j = 3, 4)
\end{align*}
$$

(4.3)

Substituting the values of $\chi$, $u_2$, $\Gamma$ from (3.24), (3.25) and (3.27) in the boundary condition $B$, we get six simultaneous equations involving six unknown constants $E, F, G, E', F', G'$. Eliminating the unknown constants we get the frequency equation as
\[ \Delta_3 = \begin{vmatrix} a_5 & a_6 & 0 & -\alpha'_5 & -\alpha'_6 & 0 \\ \lambda_5 & \lambda_6 & -1 & \lambda'_5 & \lambda'_6 & 1 \\ -1 & -1 & -\lambda_7 & 1 & 1 & -\lambda'_7 \\ b_2 & b_3 & 2\alpha & b'_2 & b'_3 & -2\alpha' \\ d_2 & d_3 & 2\gamma \lambda_7 & -d'_2 & -d'_3 & 2\gamma \lambda'_7 \\ 2\gamma \lambda_5 & 2\gamma \lambda_6 & e_1 & 2\gamma \lambda'_5 & 2\gamma \lambda'_6 & -e'_1 \end{vmatrix} = 0 \quad (4.4) \]

where
\[ b_2 = -\lambda_5 \left[ (\mu + \alpha) a_5 + 2\alpha \right] ; \quad b_3 = -\lambda_6 \left[ (\mu + \alpha) a_6 + 2\alpha \right] ; \quad e_1 = (2\gamma + \beta) \lambda_7^2 + \beta' \]
\[ d_2 = -(\gamma + \varepsilon) \lambda_5^2 + (\gamma - \varepsilon) ; \quad d_3 = -(\gamma + \varepsilon) \lambda_6^2 + (\gamma - \varepsilon) ; \quad e'_1 = (2\gamma' + \beta') \lambda'_7^2 + \beta'' \]
\[ b'_2 = -\lambda'_5 \left[ (\mu' + \alpha') a'_5 + 2\alpha' \right] ; \quad b'_3 = -\lambda'_6 \left[ (\mu' + \alpha') a'_6 + 2\alpha' \right] \]
\[ d'_2 = -(\gamma' + \varepsilon') \lambda'^2_5 + (\gamma' - \varepsilon') ; \quad d'_3 = -(\gamma' + \varepsilon') \lambda'^2_6 + (\gamma' - \varepsilon') \quad (4.5) \]

5. PARTICULAR CASES

**Rayleigh waves:** For the existence of Rayleigh-type waves, the plane boundary is to be a free surface so that \( M' \) is replaced by vacuum. The prescribed boundary values of the components of force stresses and couple stress associated with \( u_1, u_2 \) and \( \omega_2 \) are \( \sigma_{31}, \sigma_{33}, \mu_{32} \). Hence, in this case, we take the conditions for the first system as
\[ \sigma_{31} = \sigma_{33} = \mu_{32} = 0 \quad \text{on} \quad x_3 = 0 \quad (5.1) \]
associated with a thermal boundary condition as
\[ \theta_3 + h \theta = 0 \quad \text{on} \quad x_3 = 0 ; \quad h = \text{Plank's constant} \quad (5.2) \]
Using (3.23) in (5.1) and (5.2) and eliminating \( A, B, C, D \), we get the wave velocity equation for magneto-thermo-elastic Rayleigh waves in micropolar elastic medium as
\[ \Delta_4 = \begin{vmatrix} p_1 & p_2 & p_3 & p_4 \\ q_1 & q_2 & q_3 & q_4 \\ 0 & 0 & r_3 \lambda_3 & r_4 \lambda_4 \\ (h - \lambda_1) \alpha_1 & (h - \lambda_2) \alpha_2 & 0 & 0 \end{vmatrix} = 0 \quad (5.3) \]

Similarly, for the second system, the boundary conditions are
\[ \sigma_{32} = \mu_{31} = \mu_{33} = 0 \quad \text{on} \quad x_3 = 0 \quad (5.4) \]
Using (3.24) and (3.25) in (5.4) we get the wave velocity equation for magneto-thermo-elastic Rayleigh waves in micropolar elastic medium for the second system as

$$\Delta_5 = \begin{vmatrix} b_2 & b_3 & 2\alpha \\ d_2 & d_3 & -2\gamma \lambda_7 \\ 2\gamma \lambda_5 & 2\gamma \lambda_6 & e_1 \end{vmatrix} = 0$$  \hspace{1cm} (5.5)$$

We use equation (5.3) to consider the following cases

(a) For a weak magnetic field, that is, \( c_0 \to 0 \), we obtain the following results from equations (3.28) to (3.31) in the form

$$\lambda_1^2, \lambda_2^2 = \frac{1}{2} \left[ \zeta_1^2 + \zeta_2^2 + \frac{\tau^2}{\xi} \right] \mp \sqrt{\left\{ \zeta_1^2 + \zeta_2^2 + \frac{\tau^2}{\xi} \right\}^2 - 4 \left\{ \zeta_1^2 \zeta_2^2 - \frac{\tau^2}{\xi} \right\}}$$

$$\lambda_3^2, \lambda_4^2 = \frac{1}{2} \left[ \zeta_3^2 + \zeta_4^2 + \frac{\tau^2}{\xi} \right] \mp \sqrt{\left\{ \zeta_3^2 + \zeta_4^2 + \frac{\tau^2}{\xi} \right\}^2 - 4 \left\{ \zeta_3^2 \zeta_4^2 - \frac{\tau^2}{\xi} \right\}}$$

$$\alpha_j = -\frac{\xi^2}{q} (\lambda_j^2 - \zeta_j^2), \quad (j = 1, 2) \quad \alpha_1 = -\frac{\xi^2}{p} (\lambda_1^2 - \zeta_1^2), \quad (l = 3, 4)$$  \hspace{1cm} (5.6)$$

(b) If there are no magnetic field and temperature field, that is, \( \nu = 0 \) and from (3.21) \( \tau^2 = 0 \), we obtain the following results from equation (5.6) in the form, as \( \theta \to 0 \)

$$\lambda_2^2 = \zeta_1^2 \quad ; \quad \alpha_2 = 0$$

$$\lambda_3^2, \lambda_4^2 = \frac{1}{2} \left[ \zeta_3^2 + \zeta_4^2 + \frac{\tau^2}{\xi} \right] \mp \sqrt{\left\{ \zeta_3^2 + \zeta_4^2 + \frac{\tau^2}{\xi} \right\}^2 - 4 \left\{ \zeta_3^2 \zeta_4^2 - \frac{\tau^2}{\xi} \right\}}$$

$$\alpha_1 = -\frac{\xi^2}{p} (\lambda_1^2 - \zeta_3^2), \quad (l = 3, 4)$$  \hspace{1cm} (5.7)$$

(c) If in addition, the micropolar effects are absent, then \( \alpha = \beta = \gamma = \varepsilon = 0 \Rightarrow \zeta^2 \) and hence (5.6) and (5.7) yield the following results

$$\lambda_2^2 = \zeta_1^2 \quad ; \quad \alpha_2 = 0 \quad ; \quad \lambda_3^2 = \zeta_3^2 \quad ; \quad \alpha_4 = 0$$  \hspace{1cm} (5.8)$$

From (3.4), we find \( r_4 = 0 \), and then using (5.8) and (4.3), we get from (5.3)
\[ p_2 q_4 - q_2 p_4 = 0 \] (5.9)

where
\[ p_2 = 2\mu \zeta_1 ; \quad p_4 = \mu(1 - \zeta_1^2) ; \quad q_4 = 2\mu \zeta_3 ; \quad q_2 = 2\mu \zeta_1 + \lambda(1 + \zeta_1^2) \] (5.10)

More explicitly, the use of (5.10) in (5.9) leads to the classical equation for the Rayleigh wave velocity in the form
\[ \left( 2 - \frac{c^2}{\hat{c}_2^2} \right)^2 = 4 \left( 1 - \frac{c^2}{c_1^2} \right)^2 \left( 1 - \frac{c^2}{\hat{c}_2^2} \right)^2 \quad ; \quad [\hat{c}_2^2 = \mu / \rho] \] (5.11)

**Love waves:**

For the existence of love waves, we assume that the medium \( M' \) is bounded by two horizontal plane surfaces at a finite distance \( H \) apart, the upper plane surface being free while the lower plane surface forms the medium \( M \) which is extended beyond finite region. Here it is sufficient to consider the component \( u_2 \) of the displacement vector \( u \) and the components \( \omega_1, \omega_3 \) of rotation vector \( \omega \). Here the notable fact is that \( u \) and \( \omega \) in \( M' \) may no longer diminish with the distance from \( x_3 = 0 \). Thus for the medium \( M' \), we assume the full solutions as
\[
\begin{align*}
\chi &= [E'e^{-i\xi_3 x_3} + E'e^{i\xi_3 x_3} + F''e^{-i\xi_3 x_3} + F'e^{i\xi_3 x_3}] \exp\{i\xi(x_1 - ct)\} \\
u_2 &= [\alpha_5 E'e^{-i\xi_3 x_3} + \alpha_5' E'e^{i\xi_3 x_3} + \alpha_6 F''e^{-i\xi_3 x_3} + \alpha_6' F'e^{i\xi_3 x_3}] \exp\{i\xi(x_1 - ct)\} \\
\Gamma &= [G'e^{-i\xi_3 x_3} + G'e^{i\xi_3 x_3}] \exp\{i\xi(x_1 - ct)\}
\end{align*}
\] (5.12)
In addition to the boundary conditions (B), for general surface waves we have the condition that there shall be no stress and couple stress across the free surface \( x_3 = -H \). Applying the boundary conditions and using (5.12), we obtain the wave velocity equation for Love waves in a micropolar elastic medium with the influence of temperature and magnetic field by eliminating the constants \( E, F, G, E', F', G', E'', F'', G'' \) as

\[
\text{det}[M_{jk}] = 0 \quad ; \quad j, k = 1, 2, 3, \ldots, (5.13)
\]

where

\[
\begin{align*}
M_{11} &= \alpha_5 \quad ; \quad M_{12} = \alpha_6 \quad ; \quad M_{14} = -\alpha'_5 = M_{15} \quad ; \quad M_{16} = -\alpha'_6 = M_{17} \quad ; \quad M_{21} = \lambda_5 \\
M_{22} &= \lambda_6 \quad ; \quad M_{24} = \lambda'_5 \quad ; \quad M_{25} = -\lambda'_5 \quad ; \quad M_{26} = \lambda'_6 \quad ; \quad M_{27} = -\lambda'_6 \quad ; \quad M_{23} = \\
M_{31} &= M_{32} = -1 \quad ; \quad M_{33} = -\lambda_7 \quad ; \quad M_{28} = M_{29} = M_{34} = M_{35} = M_{36} = M_{37} = 1 \\
M_{38} &= -\lambda'_7 \quad ; \quad M_{39} = \lambda'_7 \quad ; \quad M_{41} = b_2 \quad ; \quad M_{42} = b_3 \quad ; \quad M_{43} = 2a \quad ; \quad M_{44} = b'_2 \\
M_{45} &= -b'_2 \quad ; \quad M_{46} = b'_3 \quad ; \quad M_{47} = -b'_4 \quad ; \quad M_{48} = M_{49} = -2a' \quad ; \quad M_{51} = d_2 \\
M_{52} &= d_3 \quad ; \quad M_{53} = 2\gamma \lambda_7 \quad ; \quad M_{54} = -d'_2 = M_{56} \quad ; \quad M_{55} = -d'_3 = M_{57} \quad ; \quad M_{58} = 2\gamma \lambda'_7 \\
M_{61} &= 2\gamma \lambda_5 \quad ; \quad M_{62} = 2\gamma \lambda_6 \quad ; \quad M_{63} = e_1 \quad ; \quad M_{64} = 2\gamma \lambda'_5 \quad ; \quad M_{65} = -2\gamma \lambda'_5 \\
M_{66} &= 2\gamma \lambda'_6 \quad M_{67} = -2\gamma \lambda'_6 \quad ; \quad M_{68} = -e'_1 = M_{69} \quad ; \quad M_{74} = -S^{-1}b'_2 \quad ; \quad M_{75} = Sb'_2 \\
M_{76} &= -T^{-1}b'_3 \quad ; \quad M_{77} = Tb'_3 \quad ; \quad M_{78} = 2a'v^{-1} \quad ; \quad M_{79} = 2a'v \quad ; \quad M_{84} = S^{-1}d'_2 \\
M_{85} &= Sd'_2 \quad ; \quad M_{86} = T^{-1}d'_3 \quad ; \quad M_{87} =Td'_3 \quad ; \quad M_{88} = -2\gamma \lambda'_7 v^{-1} \quad ; \quad M_{89} = 2\gamma \lambda'_7 v \\
M_{94} &= 2\gamma \lambda'_5 S^{-1} \quad ; \quad M_{95} = -2\gamma \lambda'_5 S \quad ; \quad M_{96} = 2\gamma \lambda'_5 T^{-1} \quad ; \quad M_{97} = -2\gamma \lambda'_6 T \\
M_{98} &= -e'_1 v^{-1} \quad ; \quad M_{99} = -e'_1 v \quad ; \quad M_{13} = M_{18} = M_{19} = M_{71} = \\
M_{72} &= \quad M_{73} = \quad M_{81} = \quad M_{82} = \quad M_{83} = \quad M_{91} = \quad M_{92} = \quad M_{93} = 0
\end{align*}
\]

and

\[
S = \exp(i\xi \lambda'_7 H) \quad ; \quad T = \exp(i\xi \lambda'_6 H) \quad ; \quad v = \exp(i\xi \lambda'_5 H)
\]

Moreover, when the upper medium extends to infinity i.e \( H \rightarrow \infty \), we get from (5.13) the following equation
This is identically same as the equation given in (4.4).

\[
\begin{array}{cccccc}
\alpha_5 & \alpha_6 & 0 & -\alpha'_5 & -\alpha'_6 & 0 \\
\lambda_5 & \lambda_6 & -1 & \lambda'_5 & \lambda'_6 & 1 \\
-1 & -1 & -\lambda_7 & 1 & 1 & -\lambda'_7 \\
b_2 & b_3 & 2\alpha & b'_2 & b'_3 & -2\alpha' \\
d_2 & d_3 & 2\gamma\lambda_7 & -d'_2 & -d'_3 & 2\gamma\lambda'_7 \\
2\gamma\lambda_5 & 2\gamma\lambda_6 & e_1 & 2\gamma\lambda'_5 & 2\gamma\lambda'_6 & -e'_1 \\
\end{array}
= 0 \quad (5.14)
\]

**Stoneley waves:** In classical theory of elasticity, the generalised form of Rayleigh waves are the Stoneley waves which propagate in the vicinity of the interface of two semi-infinite medium M and M'. Thus, in a micropolar elastic medium under the influence of temperature and magnetic field, the wave velocity equation for Stoneley waves along the common boundary of M and M' are given by the equations (4.2) and (4.4).

6. DISCUSSIONS

The above analysis reveals that the equations (4.2) and (4.4) represent the wave velocity equations for the general surface waves in a micropolar – magneto-thermoelastic medium. These velocities depend on the wave number $\xi$ confirming that waves are dispersive. Moreover, equations (5.3), (5.5) (5.13) also reveal the fact that the Rayleigh waves are affected by both magnetic and thermal fields whereas the Love waves are affected by the magnetic field. The explicit solutions of these wave velocity equations can not be determined by simple analytical methods. However, these equations can be solved numerically by a suitable choice of physical parameters involved in both the mediums M and M'.
1. INTRODUCTION
A series of research works have been published by Nowacki and Nowacki [92-95] and Eringen [51,52] dealing with the theory of micropolar elasticity. Eringen and Suhubi [54] investigated some problems of non-linear theory of micropolar elastic solids. Problems of waves and vibrations in micropolar elastic solid media have also been discussed by Sengupta and his research collaborators [30,32,39].

Problems of waves and vibrations in a micropolar elastic solid medium, situated in a constant primary magnetic field, are receiving greater attention by many investigators [32,88]. The generalized problem of surface waves in micropolar elastic solid medium has been separated by Nowacki [88] into two parts, the first one generating the displacement $u = (u_1, 0, u_3)$ and rotation vector $\omega = (0, \omega_2, 0)$, while the second includes the displacement $u = (0, u_2, 0)$ and the rotation $\omega = (\omega_1, 0, \omega_3)$.

2. GENERAL THEORY
We consider two homogeneous, isotropic, centro-symmetric micropolar semi-infinite elastic medium $M$ and $M'$ which are either welded in contact or sufficiently rough enough to prevent any sliding on the common surface of separation, which is a plane horizontal boundary extending to infinity. Both the initially stressed conducting medium are placed within constant strong primary magnetic field $H$. We consider the orthogonal Cartesian co-ordinate system $0-x_1 x_2 x_3$ in micropolar elastic solid media. The medium $M'$ is above the medium $M$, the origin $o$ being any point on the plane horizontal boundary and $ox_3$ pointing normally into the medium $M$.

If we assume that (i) the disturbance is largely confined to the neighborhood of the boundary and (ii) at any instant all particles in any line parallel to the $x_2$-axis have the same displacements, then assumption (i) asserts that the wave is a surface wave and (ii) admits that all partial derivatives with respect to $x_2$-axis are zero.

Following Nowacki [87], the basic equations of motion for a micropolar elastic medium under the constant primary magnetic field with no body forces and body couples may be written as
\[(\mu + \alpha) \nabla^2 \ddot{u} + (\lambda + \mu - \alpha) \nabla \nabla \ddot{u} + 2\alpha \dot{\omega} + \frac{\mu_0}{c}(j \times \overrightarrow{H}) = \rho \ddot{u} \tag{2.1}\]
\[(\gamma + \epsilon) \nabla^2 \ddot{\omega} + (\beta + \gamma - \epsilon) \nabla \nabla \ddot{\omega} - 4\alpha \ddot{\omega} + 2\alpha \dot{\omega} = J \ddot{\omega} \tag{2.2}\]

Where \(\rho\) is material density, \(\lambda, \mu, \alpha, \beta, \gamma, \epsilon\) are material constants, \(J\) is rotational inertia, \(\mu_0\) is magnetic permeability factor, \(j\) is current density vector, \(c\) is the velocity of light, dots denote the derivatives with respect to time \(t\) and comma denotes the partial differentiation with respect to space coordinate.

The equations of motion for the first system \(\mathbf{u} = (u_1, 0, u_3)\) and \(\mathbf{\omega} = (0, \omega_2, 0)\) in perfectly micropolar elastic solid under uniaxial tension or compression in a uniform magnetic field are given by

\[(\mu + \alpha) \nabla^2 u_1 + (\lambda + \mu - \alpha + \alpha_0^2 \rho) \Delta_{1,1} - 2\alpha \omega_{2,3} + \alpha_0^2 \rho (u_{1,33} - u_{3,13}) + T_0 u_{1,11} = \rho \ddot{u}_1\]
\[(\mu + \alpha) \nabla^2 u_3 + (\lambda + \mu - \alpha) \Delta_{3,3} + 2\alpha \omega_{2,1} + T_0 u_{3,11} = \rho \ddot{u}_3\]
\[(\gamma + \epsilon) \nabla^2 \omega_2 - 4\alpha \omega_2 + 2\alpha (u_{1,3} - u_{3,1}) = J \ddot{\omega}_2 \tag{2.3a}\]

For the second system \(\mathbf{u} = (0, u_2, 0)\) and \(\mathbf{\omega} = (\omega_1, 0, \omega_3)\), equations of motion are given by

\[(\mu + \alpha) \nabla^2 u_2 + 2\alpha (\omega_{1,3} - \omega_{3,1}) + \alpha_0^2 \rho u_{2,13} + T_0 u_{2,11} = \rho \ddot{u}_2\]
\[(\gamma + \epsilon) \nabla^2 \omega_1 - 4\alpha \omega_1 + (\beta + \gamma - \epsilon) \Delta_{1,1} - 2\alpha u_{2,3} = J \ddot{\omega}_1\]
\[(\gamma + \epsilon) \nabla^2 \omega_3 - 4\alpha \omega_3 + (\beta + \gamma - \epsilon) \Delta_{3,3} + 2\alpha u_{2,1} = J \ddot{\omega}_3 \tag{2.3b}\]

We get similar differential equations for the medium \(M'\) in which physical quantities with dashes denotes the properties for \(M'\). \(T_0\) denotes the tension or compression in the direction of \(x_1\) axis according as \(T_0 > 0\) or \(T_0 < 0\) and \(\alpha^2 = \frac{\mu_0 H_0^2}{4\pi \rho}\), \(\alpha_0\) denotes the Alfvén wave velocity.

The displacement components \(u_1, u_3\) and the rotation components \(\omega_1\) and \(\omega_3\) can be expressed in terms of displacement potentials \(\varphi\) and \(\psi\) as

\[u_1 = \varphi_{x_1} + \psi_{x_3}; u_3 = \varphi_{x_3} - \psi_{x_1}; \quad \omega_1 = \theta_{x_1} + \chi_{x_3}; \quad \omega_3 = \theta_{x_3} - \chi_{x_1}\tag{2.4}\]

and

\[\nabla^2 \varphi = \Delta; \quad \nabla^2 = \left( \begin{array}{c} \cdot_{11} + \cdot_{33} \\ \cdot_{13} + \cdot_{31} \end{array} \right); \quad \nabla^2 \psi = u_{1,3} - u_{3,1}; \quad \Delta = u_{1,1} + u_{3,3}\tag{2.5}\]
\[\nabla^2 \theta = \Delta'; \quad \Delta' = \omega_{1,1} + \omega_{3,3}; \quad \nabla^2 \chi = \omega_{1,3} - \omega_{3,1}\]
The relations between the state of stress and the state of strain are linear and are given by [88]

\[ \sigma_{ji} = (\mu + \alpha)\gamma_{ji} + (\mu - \alpha)\mu_{ji} + \lambda\gamma_{kk}\delta_{ij} \]

\[ \mu_{ji} = (\gamma + \varepsilon)\lambda_{ji} + (\gamma - \varepsilon)\mu_{ji} + \beta\lambda_{kk}\delta_{ij} \]

where

\[ \gamma_{ji} = u_{i,j} - \varepsilon_{kj} \omega_{k} \quad \text{and} \quad \lambda_{ji} = \omega_{i,j}, \quad (2.6) \]

\( \varepsilon_{kj} \) denotes the unit anti-symmetric tensor, \( \gamma_{ij} \), known as the deformation tensor and \( \lambda_{ij} \) is called the curvature twist tensor.

Using (2.4) and (2.5) the system of equations in (2.3a) and (2.3b) reduces to the following form

\[ \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} + \frac{T_0}{\rho c_1^2} \frac{\partial^2}{\partial x_1^2} \] \[ + \frac{\alpha_0^2}{c_1^2} \frac{\partial}{\partial x_1} \left( \frac{\partial \phi}{\partial x_1} + \frac{\partial \psi}{\partial x_3} \right) = 0 \]

\[ \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} + \frac{T_0}{\rho c_2^2} \frac{\partial^2}{\partial x_1^2} \] \[ - \rho \omega_2 + \frac{\alpha_0^2}{c_2^2} \frac{\partial}{\partial x_3} \left( \frac{\partial \phi}{\partial x_1} + \frac{\partial \psi}{\partial x_3} \right) = 0 \]

\[ \nabla^2 - \psi^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \] \[ \omega_2 + s \nabla^2 \psi = 0 \quad (2.7a) \]

\[ \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} + \frac{T_0}{\rho c_2^2} \frac{\partial^2}{\partial x_1^2} \] \[ + \alpha_0^2 \frac{\partial^2}{\partial x_1^2} \] \[ u_2 + p \nabla^2 \lambda = 0 \]

\[ \left( \nabla^2 - \psi^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \lambda - s u_2 = 0 \]

\[ \left( \nabla^2 - \psi^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) \theta = 0 \quad (2.7b) \]

where

\[ \psi_1 = 4\alpha/(\beta + 2\varepsilon) \quad ; \quad c_1^2 = (\lambda + 2\mu)/\rho \quad ; \quad c_2^2 = (\mu + \alpha)/\rho \quad ; \quad \psi_2 = 4\alpha/(\gamma + \varepsilon) \]

\[ c_3^2 = (\beta + 2\gamma)/J \quad ; \quad c_4^2 = (\gamma + \varepsilon)/J \quad ; \quad p = 2\alpha/(\mu + \alpha) \quad ; \quad s = 2\alpha/(\gamma + \varepsilon) \]
Eliminating $\phi, \psi, \omega_2$ from (2.7a) we obtain

$$
\left\{ \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} + \frac{1}{c_1^2} \left( \frac{T_0}{\rho} + \alpha_0 \right) \frac{\partial^2}{\partial x_1^2} \right\} \left\{ \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} + \frac{1}{c_2^2} \left( \frac{T_0}{\rho} \frac{\partial^2}{\partial x_1^2} + \alpha_0^2 \frac{\partial^2}{\partial x_2^2} \right) \right\} (\phi, \psi, \omega_2) = 0 \quad (2.8)
$$

where

$$
\eta^2 = \mu s
$$

Eliminating $u_2$ and $\chi$ from (2.7b) we obtain

$$
\left\{ \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} + \frac{1}{c_2^2} \left( \frac{T_0}{\rho c_2^2} \right) \frac{\partial^2}{\partial x_1^2} + \frac{\alpha_0^2}{c_2^2} \frac{\partial^2}{\partial x_2^2} \right\} \left\{ \nabla^2 - \frac{u_2^2}{c_4^2} \frac{\partial^2}{\partial t^2} + \eta^2 \nabla^2 \right\} (u_2, \chi) = 0 \quad (2.9)
$$

If we neglect the term containing $\alpha_0^4$ ($\alpha_0$ being small) we obtain from equation (2.8) and (2.9)

$$
\left\{ \nabla^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} + \frac{1}{c_1^2} \left( \frac{T_0}{\rho c_1^2} \right) \frac{\partial^2}{\partial x_1^2} \right\} \phi = 0 \quad (2.10)
$$

$$
\left\{ \nabla^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} + \frac{T_0}{\rho c_2^2} \frac{\partial^2}{\partial x_1^2} + \frac{\alpha_0^2}{c_2^2} \frac{\partial^2}{\partial x_2^2} \right\} \left\{ \nabla^2 - \frac{u_2^2}{c_4^2} \frac{\partial^2}{\partial t^2} + \eta^2 \nabla^2 \right\} (\psi, \omega_2, u_2, \chi) = 0 \quad (2.11)
$$

3. SOLUTION OF THE PROBLEM

We seek solution of differential equations (2.10), (2.11) and last equation of (2.7b) as

$$(\phi, \psi, \omega_2, \chi, u_2, \theta) =$$

$$[\phi (x_1, x_2, \omega_2 (x_3), \chi (x_3), u_2 (x_3), \theta (x_3)) \exp[i\xi(x_1 - ct)]$$

$$\quad (3.1)$$

Substituting the values $\phi, \psi, \omega_2, \chi, u_2, \theta$ from (3.1) in the above equations respectively and assuming that $\phi, \psi, \omega_2, \chi, u_2, \theta$ tend to zero as $x_3 \to \infty$, we get for the medium $M$
\[ \varphi = [A e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \psi = [B e^{-i \xi x_3} + C e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \omega_2 = [v_3 B e^{-i \xi x_3} + v_4 C e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \theta = [F e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \chi = [D e^{-i \xi x_3} + E e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ u_2 = [v_5 D e^{-i \xi x_3} + v_6 E e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]

and in the medium M' as
\[ \varphi = [A' e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \psi = [B' e^{-i \xi x_3} + C' e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \omega_2 = [v_3' B' e^{-i \xi x_3} + v_4' C' e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \theta = [F' e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ \chi = [D' e^{-i \xi x_3} + E' e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]
\[ u_2 = [v_5' D' e^{-i \xi x_3} + v_6' E' e^{-i \xi x_3} \exp \{ i \xi (x_1 - ct) \}] \]

where dashes denote the properties for the medium M and

\[ \lambda_1^2 = \frac{c^2}{c_i^2} - \frac{1}{c_i^2} \left( \frac{T_0}{\rho} + \alpha_0^2 \right) - 1 ; \quad \lambda_4^2 = \left( \frac{c^2}{c_i^2} - \frac{v_1^2}{\xi^2} - 1 \right) \]

\[ \xi^2 (\lambda_3^2 + \lambda_3^2) = \left[ \xi^2 c^2 c_2^2 \left( \frac{1}{c_2^2} + \frac{1}{c_4^2} \right) - c_2^2 (2 \xi^2 + v_2^2 - \eta^2) + \alpha_0^2 \left( \xi^2 \left( \frac{c^2}{c_i^2} - 1 \right) - v_2^2 \right) - \frac{T_0 \xi^2}{\rho} \right] + (\alpha_0^2 + c_2^2) \]

\[ \xi^2 (\lambda_3^2 \lambda_3^2) = \left[ c^2 \left( \frac{\xi^2 c^2}{c_4^2} - v_2^2 \right) - \xi^2 c^2 c_2^2 \left( \frac{1}{c_2^2} + \frac{1}{c_4^2} \right) + c_2^2 (\xi^2 + v_2^2 - \eta^2) - \frac{T_0}{\rho} \left( \xi^2 \left( \frac{c^2}{c_i^2} - 1 \right) - v_2^2 \right) \right] + (\alpha_0^2 + c_2^2) \]

\[ v_3 = \frac{\xi^2}{\rho} \left[ \frac{c^2}{c_2^2} - (1 + \lambda_2^2) - \frac{1}{c_2^2} \left( \frac{T_0}{\rho} + \alpha_0^2 \lambda_2^2 \right) \right] ; \quad v_4 = \frac{\xi^2}{\rho} \left[ \frac{c^2}{c_2^2} - (1 + \lambda_3^2) - \frac{1}{c_2^2} \left( \frac{T_0}{\rho} + \alpha_0^2 \lambda_3^2 \right) \right] \]
\[
\nu_5 = \frac{p(1+\lambda_2^2)}{[c^2/c_2^2 - (1 + \lambda_2^2) - (1/c_2^2)(T_0/p + \alpha_0^2 \lambda_2^2)]} \quad \nu_6 = \frac{p(1+\lambda_3^2)}{[c^2/c_2^2 - (1 + \lambda_3^2) - (1/c_2^2)(T_0/p + \alpha_0^2 \lambda_3^2)]}
\]

Similar results hold for \( M' \).

**BOUNDARY CONDITIONS:**

To obtain the frequency equations we consider the following boundary conditions for the systems 3.2(a) and 3.3(a)

(A) (i) The displacement components \( u_1 \), \( u_3 \) and rotational component \( \omega_2 \) are continuous on the boundary surface of separation \( x_3=0 \)

(ii) The stress components \( \sigma_{31} \), \( \sigma_{33} \) and the couple stress component \( \mu_{32} \) are continuous on \( x_3=0 \).

and for the systems 3.2(b) and 3.3(b)

(B) (i) The displacement component \( u_2 \) and rotational components \( \omega_1 \), \( \omega_3 \) are continuous on \( x_3=0 \)

(ii) The stress component \( \sigma_{32} \) and the couple stress components \( \mu_{31} \), \( \mu_{33} \) are continuous on \( x_3=0 \).

where

\[
\begin{align*}
\sigma_{31} &= \mu \left[ 2\varphi_{13} + \psi_{33} - \psi_{11} \right] + \alpha \left[ \nabla^2 \psi - 2\omega_2 \right] \\
\sigma_{33} &= 2\mu \left[ \varphi_{33} - \psi_{11} \right] + \lambda \nabla^2 \varphi \\
\mu_{32} &= (\gamma + \varepsilon) \omega_{23} \\
\sigma_{32} &= (\mu + \alpha) u_{2,3} + 2\alpha \left( \theta_{11} + \chi_3 \right) \\
\mu_{31} &= \gamma \left[ 2\theta_{13} + \chi_{33} - \chi_{11} \right] + \varepsilon \nabla^2 \chi \\
\mu_{33} &= (2\gamma + \beta) \left[ \theta_{33} - \chi_{33} \right] + \beta \left[ \theta_{11} - \chi_{11} \right] 
\end{align*}
\]

3.4(a)

Now using the boundary condition (A) we have

\[
A - \lambda_2 B - \lambda_3 C - A' - \lambda_2' B' - \lambda_3' C' = 0 \\
\lambda_1 A + B + C + \lambda_1' A' - B' - C' = 0 \\
\nu_3 B + \nu_4 C - \nu_3' B' - \nu_4' C' = 0 \\
2\mu \lambda_1 A + A_2 B + A_3 C + 2\mu \lambda_1' A' - A_2' B' - A_3' C' = 0 \\
C_1 A + 2\mu \lambda_3 B + 2\mu \lambda_3' C - C_1' A' + 2\mu \lambda_3' B' + 2\mu \lambda_3' C' = 0 \\
D_2 B + D_3 C + D_2' B' + D_3' C' = 0
\]

3.5(a) 3.5(b) 3.5(c) 3.5(d) 3.5(e) 3.5(f)

and using the boundary condition (B) we have

\[
\nu_5 D - \nu_6 E - \nu_5' D' - \nu_6' E' = 0
\]

3.6(a)
\[
\begin{align*}
\lambda_2 D + \lambda_3 E - F + \lambda_2' D' + \lambda_3' E' + F' &= 0 \\
D - E - \lambda_4 F + D' + E' - \lambda_4' F' &= 0 \\
B_2 D + B_3 E + 2\alpha F + B_2' D' + B_3' E' - 2\alpha' F' &= 0 \\
E_2 D + E_3 E + 2\gamma \lambda_4 F - E_2' D' - E_3' E' + 2\gamma' \lambda_4' F' &= 0 \\
2\gamma \lambda_2 D + 2\gamma \lambda_3 E + F_3 F + 2\gamma' \lambda_2' D' + 2\gamma' \lambda_3' E' - F_3' F &= 0
\end{align*}
\]

Eliminating \(A, B, C, A', B', C'\) from system of equations (3.5) we have

\[
\Delta_2 = \begin{vmatrix}
1 & -\lambda_2 & -\lambda_3 & -1 & -\lambda_2' & -\lambda_3' \\
\lambda_1 & 1 & \lambda_1' & -1 & -1 \\
0 & v_3 & v_4 & 0 & -v_3' & -v_4' \\
2\mu \lambda_1 & A_2 & A_3 & 2\mu \lambda_2 & -A_2' & -A_3' \\
C_1 & 2\mu \lambda_2 & 2\mu \lambda_3 & -C_1' & 2\mu \lambda_2' & 2\mu \lambda_3' \\
0 & D_2 & D_3 & 0 & D_2' & D_3'
\end{vmatrix} = 0
\] (3.7)

Eliminating \(D, E, F, D', E', F'\) from system of equations (3.6) we have

\[
\Delta_3 = \begin{vmatrix}
v_5 & v_6 & 0 & -v_5' & -v_6' & 0 \\
v_2 & \lambda_2 & -1 & \lambda_2' & 1 \\
-1 & -1 & -\lambda_4 & 1 & 1 & -\lambda_4' \\
B_2 & B_3 & 2\alpha & B_2' & -B_3' & -2\alpha' \\
E_2 & E_3 & 2\gamma \lambda_4 & -E_2' & -E_3' & 2\gamma' \lambda_4' \\
2\gamma \lambda_2 & 2\gamma \lambda_3 & F_3 & 2\gamma' \lambda_2' & 2\gamma' \lambda_3' & -F_3'
\end{vmatrix} = 0
\] (3.8)

where

\[
\begin{align*}
A_2 &= (\mu - \alpha) - \lambda_2^2 (\mu + \alpha) - (2\alpha v_3 / \xi^2) \\
A_3 &= (\mu - \alpha) - \lambda_3^2 (\mu + \alpha) - (2\alpha v_4 / \xi^2) \\
A_2' &= (\mu' - \alpha') - \lambda_2'^2 (\mu' + \alpha') - (2\alpha' v_3' / \xi^2) \\
A_3' &= (\mu' - \alpha') - \lambda_3'^2 (\mu' + \alpha') - (2\alpha' v_4' / \xi^2) \\
C_1 &= 2\mu \lambda_1^2 + \lambda(1 + \lambda_1^2) \\
C_1' &= 2\mu \lambda_1'^2 + \lambda'(1 + \lambda_1'^2) \\
D_2 &= (\gamma + \varepsilon) \lambda_2 v_3 \\
D_2' &= (\gamma' + \varepsilon') \lambda_2' v_3' \\
D_3 &= (\gamma + \varepsilon') \lambda_3 v_4 \\
D_3' &= (\gamma' + \varepsilon') \lambda_3' v_4' \\
B_2 &= -\lambda_2 \{2\alpha + v_3 (\mu + \alpha)\} \\
B_2' &= -\lambda_2' \{2\alpha' + v_3' (\mu' + \alpha')\} \\
B_3 &= -\lambda_3 \{2\alpha + v_6 (\mu + \alpha)\} \\
B_3' &= -\lambda_3' \{2\alpha' + v_6' (\mu' + \alpha')\} \\
E_2 &= \gamma(1 - \lambda_2^2) - \varepsilon(1 + \lambda_2^2) \\
E_2' &= \gamma'(1 - \lambda_2'^2) - \varepsilon'(1 + \lambda_2'^2) \\
E_3 &= \gamma(1 - \lambda_3^2) - \varepsilon(1 + \lambda_3^2) \\
E_3' &= \gamma'(1 - \lambda_3'^2) - \varepsilon'(1 + \lambda_3'^2) \\
F_3 &= (2\gamma + \beta) \lambda_4^2 + \beta \\
F_3' &= (2\gamma' + \beta') \lambda_4'^2 + \beta'
\end{align*}
\] (3.9)
Both the frequency equation (3.7) and (3.8) contain magnetic constraints and are thus affected by magnetic field. The wave velocity equations (4.7) and (4.8) for general surface waves in initially stressed micropolar magneto-elastic medium depend on the particular value of $\xi$, creating the dispersions of the general wave form.

4. RAYLEIGH WAVES

Particular case: In the present case for the Rayleigh type of waves the plane boundary must be a free surface so that $M'$ is replaced by vacuum.

So from 3.5(d), 3.5(e), 3.5(f) and 3.6(d), 3.6(e), 3.6(f) we get the following set of equations

\[
\begin{align*}
2\mu \lambda_1 A + A_2 B + A_3 C &= 0 \\
C_1 A + 2\mu \lambda_2 B + 2\mu \lambda_3 C &= 0 \\
D_2 B + D_3 C &= 0 \\
B_2 D + B_3 E + 2\alpha F &= 0 \\
E_2 D + E_3 E + 2\gamma \lambda_4 F &= 0 \\
2\gamma \lambda_2 D + 2\gamma \lambda_3 E + F_3 F &= 0
\end{align*}
\]

4.1(a)

\[
\begin{align*}
C_1 A + 2\mu \lambda_2 B + 2\mu \lambda_3 C &= 0 \\
D_2 B + D_3 C &= 0 \\
B_2 D + B_3 E + 2\alpha F &= 0 \\
E_2 D + E_3 E + 2\gamma \lambda_4 F &= 0 \\
2\gamma \lambda_2 D + 2\gamma \lambda_3 E + F_3 F &= 0
\end{align*}
\]

4.1(b)

Eliminating $A$, $B$, $C$ from 4.1(a) and $D$, $E$, $F$ from 4.1(b) we get the wave velocity equations for Rayleigh waves in the present case as

\[
\Delta_4 = \begin{vmatrix}
2\mu \lambda_1 & A_2 & A_3 \\
C_1 & 2\mu \lambda_2 & 2\mu \lambda_3 \\
0 & \lambda_2 v_3 & \lambda_3 v_4
\end{vmatrix} = 0
\]

\[
\Delta_5 = \begin{vmatrix}
B_2 & B_3 & 2\alpha \\
E_2 & E_3 & 2\gamma \lambda_4 \\
2\gamma \lambda_2 & 2\gamma \lambda_3 & F_3
\end{vmatrix} = 0
\]

(4.2)

In this case also the equations $\Delta_4 = 0$ and $\Delta_5 = 0$ contain magnetic constraints.

Now if we put $T_0 = 0$ in (3.7) we easily obtain the results of Das, Acharya and Sengupta [32] who studied surface waves in the micropolar magneto elasticity.

If we neglect the magnetic field by employing the condition $\omega_0 \to 0$ and $T_0 = 0$ we obtain the wave velocity equations for Rayleigh waves in a micropolar elastic medium as

\[
\bar{\Delta}_4 = \begin{vmatrix}
2\mu \bar{\lambda}_1 & \bar{A}_2 & \bar{A}_3 \\
C_1 & 2\mu \bar{\lambda}_2 & 2\mu \bar{\lambda}_3 \\
0 & \bar{\lambda}_2 \bar{v}_3 & \bar{\lambda}_3 \bar{v}_4
\end{vmatrix} = 0
\]

(4.4)
\[ \Delta_3 = \begin{vmatrix} B_2 & B_3 & 2\alpha \\ E_2 & E_3 & 2\gamma \bar{\lambda}_4 \\ 2\gamma \bar{\lambda}_2 & 2\gamma \bar{\lambda}_3 & \bar{F}_3 \end{vmatrix} = 0 \] (4.5)

where

\[ \bar{\lambda}_1 = 2/C_1^2 - 1; \quad \bar{\omega}_3 = (\xi/z/p)((C_2^2/C_1^2) - (1 + \lambda_2^2)); \quad \bar{\omega}_4 = (\xi/z/p)((C_2^2/C_2^2) - (1 + \lambda_2^2)) \]

\[ \xi^2(\bar{\lambda}_2^2 + \bar{\lambda}_3^2 + 2) = \xi^2 C_2^2 (1/C_2^2 + 1/C_4^2) + \eta^2 - \nu_2^2 \]

\[ \xi^2(\bar{\lambda}_2^2 + 1)(\bar{\lambda}_3^2 + 1) = (C_2^2/C_2^2) (\xi^2 C_2^2/C_4^2 - \nu_2^2) \] (4.6)

\[ A_2, A_3, C_1, B_2, B_3, E_2, E_3, \bar{f}_3 \] can be easily calculated from (3.9) using (4.6)

The above result is in well agreement with that of De and Sengupta [39], who studied the surface waves in micropolar elastic medium.

Again employing the conditions \( \nu_4, \lambda_3 \to \infty \) and \( \nu_3 \to 0 \) in (4.4) we have the classical Rayleigh wave velocity equation for elastic medium as in the following

\[ 4\sqrt{1 - C_2^2/C_1^2} \sqrt{1 - C_2^2/C_2^2} = \left[ 2 - C_2^2/C_2^2 \right]^2 \] (4.7)

where \( C_2^2 = \mu/\rho \)

Moreover making \( \nu_4, \lambda_3 \to \infty \) and \( \nu_3 \to 0 \) in (4.2) we get the Rayleigh wave velocity equation in an elastic medium which is placed in a constant primary magnetic field \( H = (0, 0, H_0) \) as

\[ 4\sqrt{1 - C_2^2/C_1^2 + C_2^2/C_2^2} \sqrt{1 - C_2^2/C_2^2} = \left[ 1 + \frac{\alpha_0^2}{C_2^2} \right] \left[ 2 - C_2^2/C_2^2 + \frac{\alpha_0^2}{C_2^2} \right]^2 \] (4.8)

If we put \( \alpha_0 = 0 \) in (4.8) then (4.8) becomes completely identical with the equation (4.7)

5. LOVE WAVES

For Love type of waves the medium \( M' \) should be replaced by a layer. Let us assume that the medium \( M' \) is bounded by two horizontal plane surfaces at a finite distance \( H \) apart the upper plane surface being free while the lower plane surface forms the medium \( M \). For the medium \( M' \) we preserve the full solution as

\[ \theta = \left[ F'e^{-\bar{\xi}_3 x_3} + F'e^{i\bar{\xi}_3 x_3} \right] e^{ix_1(C_1 - Ct)} \]

\[ \chi = \left[ D'e^{-i\bar{\xi}_3 x_3} + D'e^{i\bar{\xi}_3 x_3} + E'e^{i\bar{\xi}_3 x_3} + E'e^{i\bar{\xi}_3 x_3} \right] e^{ix_1(C_1 - Ct)} \]

\[ u_2 = \left[ \nu_2 D'e^{-i\bar{\xi}_3 x_3} + \nu_2 D'e^{i\bar{\xi}_3 x_3} + \nu_6 E'e^{i\bar{\xi}_3 x_3} + \nu_6 E'e^{i\bar{\xi}_3 x_3} \right] e^{ix_1(C_1 - Ct)} \] (5.1)

For the medium \( M \) the solutions are as 3.2(b) and \( \lambda_4, \lambda_2, \lambda_3, u_2, u_6 \) defined earlier.
In addition to the boundary condition (B) for general surface waves we have the condition that there shall be no stress and couple stress across the free surface $x_3=-H$. Applying the boundary conditions stated above and using (5.1) and 3.2(b) we get the wave velocity equation for Love waves in initially stressed micropolar elastic medium under the influence of magnetic field by eliminating the constants $D', E', F', D'', E'', F''$ or

$$|b_{jk}| = 0 \quad j, k = 1,2, \ldots, 9$$

(5.2)

where expanding $b_{jk}$ we have

$$
\begin{array}{cccccccc}
\nu_5 & \nu_6 & 0 & -\nu'_5 & -\nu'_5 & -\nu'_5 & -\nu'_6 & 0 & 0 \\
\lambda_2 & \lambda_3 & -1 & \lambda'_2 & -\lambda'_2 & \lambda'_3 & -\lambda'_3 & 1 & 1 \\
-1 & -1 & -\lambda_4 & 1 & 1 & 1 & 1 & -\lambda'_4 & \lambda'_4 \\
B_2 & B_3 & 2\alpha & B'_2 & -B'_2 & B'_3 & -B'_3 & -2\alpha' & -2\alpha' \\
E_2 & E_3 & 2\gamma\lambda_4 & -E'_2 & -E'_3 & -E'_2 & -E'_3 & 2\gamma\lambda'_4 & -2\gamma\lambda'_4 \\
2\gamma\lambda_2 & 2\gamma\lambda_3 & F_3 & 2\gamma'\lambda'_2 & -2\gamma'\lambda'_2 & 2\gamma'\lambda'_3 & -2\gamma'\lambda'_3 & -F'_3 & -F'_3 \\
0 & 0 & 0 & -\chi^{-1}B'_2 & \chi B'_2 & -Y^{-1}B'_3 & YB'_3 & 2Z^{-1}\alpha' & 2\alpha'Z \\
0 & 0 & 0 & -\chi^{-1}E'_2 & \chi E'_2 & Y^{-1}E'_3 & YE'_3 & -2\gamma'\lambda'_4Z^{-1} & 2\gamma'\lambda'_4Z \\
0 & 0 & 0 & 2\gamma'\lambda'_2\chi^{-1} & -2\gamma'\lambda'_2\chi & 2\gamma'\lambda'_3Y^{-1} & -2\gamma'\lambda'_3Y & -F'_3Z^{-1} & -F'_3Z \\
\end{array}
$$

where $\chi = e^{i\phi_3H}$; $Y = e^{i\phi_3H}$; $Z = e^{i\phi_3H}$
From the equation (5.2) it is clear that the primary magnetic field influences the wave velocity equation for Love waves in an initially stressed micropolar magneto-elastic medium. Moreover, when the upper medium extends to infinity then making $H \to \infty$ we get from (5.2) the following equation:

$$
\begin{vmatrix}
\nu_5 & \nu_6 & 0 & -\nu'_5 & -\nu'_6 & 0 \\
\lambda_2 & \lambda_3 & -1 & \lambda'_2 & \lambda'_3 & 1 \\
-1 & -1 & -\lambda_4 & 1 & 1 & -\lambda'_4 \\
B_2 & B_3 & 2\alpha & B'_2 & B'_3 & -2\alpha' \\
E_2 & E_3 & 2\gamma\lambda_4 & -E'_2 & -E'_3 & 2\gamma'\lambda'_4 \\
2\gamma\lambda_2 & 2\gamma\lambda_3 & F_3 & 2\gamma'\lambda'_2 & 2\gamma'\lambda'_3 & -F'_3 \\
\end{vmatrix} = 0
$$

(5.3)

The above equation (5.3) is in complete agreement with the result (3.8), obtained for the general surface waves. If $T = \omega_0 = 0$ in this case then we also get the result of De and Sengupta [39].

6. STONELEY WAVES

The Stoneley waves are the generalized form of Rayleigh waves which are assumed to propagate in the vicinity of the interface of two semi-infinite media $M$ and $M'$. Thus in an initially stressed micropolar magneto-elastic medium the wave velocity equation for Stoneley waves are given by the equations (3.7) and (3.8). Making suitable choice of Physical constants of both media $M$ and $M'$ and applying numerical techniques, the determinantal equations (3.7) and (3.8) may be solved.
1. INTRODUCTION
In micropolar theory of elasticity the action across any infinitesimal surface element within a solid produces displacement $\mathbf{u}(x, t)$ and rotation $\omega(x, t)$ in general. The state of deformation is described by two asymmetric tensors, $\gamma_{ij}$ (strain tensor) and $\chi_{ij}$ (curvature twist tensor) respectively. Also the state of stress is characterized by two asymmetric tensors, $\sigma_{ij}$ (force stress tensor) and $\mu_{ij}$ (couple stress tensor) respectively. Propagation of mono-cromatic waves in micropolar theory of elasticity has been studied in details by Nowacki [90]. Problems of waves and vibrations in micropolar elastic medium have been investigated by Nowacki [88], Das et al.[32]. Here is an endeavour to study the propagation of mono-cromatic waves in an infinite micropolar plate under the influence of magnetic field (Lamb's problem).

2. BASIC EQUATION
The basic equations of motion for a micropolar elastic medium under the influence of constant primary magnetic field with no body forces and body couples may be written as [87]

$$
(\mu + \alpha)\nabla^2 \mathbf{u} + (\lambda + \mu - \alpha)\text{grad div}\mathbf{u} + 2\alpha \text{rot}\mathbf{\omega} + (\mu_0 / c) (\mathbf{j} \times \mathbf{H}) = \rho \ddot{\mathbf{u}} \tag{2.1}
$$

$$
(\gamma + \epsilon)\nabla^2 \mathbf{\omega} + (\beta + \gamma - \epsilon)\text{grad div}\mathbf{\omega} - 4\alpha \mathbf{\omega} + 2\alpha \text{rot}\mathbf{u} = J \ddot{\mathbf{\omega}} \tag{2.2}
$$

Where $\rho$ is material density, $\lambda$, $\mu$, $\alpha$, $\beta$, $\gamma$, $\epsilon$ are material constants, $J$ is rotational inertia, $\mu_0$ is magnetic permeability factor, $\mathbf{j}$ is current density vector, $c$ is the velocity of light and dots denote the derivatives with respect to time $t$.

3. GENERAL THEORY AND SOLUTION OF THE PROBLEM
We consider a homogeneous, conducting, micropolar elastic plate of thickness $2H$ situated under the influence of constant strong magnetic field $\mathbf{H}$. Let us consider mono-cromatic waves propagate along the $x_3$-axis.

Boundary Conditions: We consider the boundaries of the plate $x_3 = \pm H$ are free of stresses. So the following conditions are satisfied on these edges
we assume that displacement \( \overrightarrow{u} \) and rotation \( \overrightarrow{\omega} \) do not depend on \( x_2 \) and
\[ \overrightarrow{u} = (u_1, 0, u_3) ; \quad \overrightarrow{\omega} = (0, \omega_2, 0), \] (3.2)
The displacement components \( u_1 \) and \( u_3 \) can be expressed in terms of displacement potentials \( \varphi \) and \( \psi \) as
\[ u_1 = \varphi_1 + \psi_3 ; \quad u_3 = \varphi_3 - \psi_1 \] (3.3)
comma denotes the partial differentiation with respect to the space co-ordinate and
\[ \nabla^2 \varphi = e_3 ; \quad \nabla^2 \psi = u_{1,3} - u_{3,1} ; \quad e = u_{1,1} + u_{3,3} \] (3.4)

Since we are considering a plane strain problem under the above assumptions, the equations of motion in a micropolar magneto-elastic solid medium may be written from (2.1) and (2.2) as
\[
(\mu + \alpha) \nabla^2 u_1 + (\lambda + \mu - \alpha + \alpha_0 \rho)e_{31} - 2\alpha \omega_{1,3} + \alpha_0 \rho (u_{1,33} - u_{3,13}) = \rho \ddot{u}_1 
\] (3.5)
\[
(\mu + \alpha) \nabla^2 u_3 + (\lambda + \mu - \alpha)e_{33} + 2\alpha \omega_{2,1} = \rho \ddot{u}_3 
\] (3.6)
\[
(\gamma + \epsilon) \nabla^2 \omega_2 - 4\alpha \omega_2 + 2\alpha (u_{1,3} - u_{3,1}) = J \ddot{\omega}_2 
\] (3.7)

Introducing (3.3) into the equations (3.5), (3.6) and (3.7) we get
\[
(\lambda + 2\mu + \alpha_0 \rho) \nabla^2 \varphi - \rho \ddot{\varphi} = 0 
\] (3.8)
\[
(\mu + \alpha + \alpha_0 \rho) \nabla^2 \psi - \rho \ddot{\psi} - 2\alpha \omega_2 = 0 
\] (3.9)
\[
(\gamma + \epsilon) \nabla^2 \omega_2 - 4\alpha \omega_2 - J \ddot{\omega}_2 + 2\alpha \nabla^2 \psi = 0 
\] (3.10)

Eliminating \( \psi \) and \( \omega_2 \) (3.9) and (3.10) we obtain
\[
\left\{ \left[ (\mu + \alpha + \alpha_0 \rho) \nabla^2 - \rho \frac{\partial^2}{\partial t^2} \right]\left[ (\gamma + \epsilon) \nabla^2 - 4\alpha - J \frac{\partial^2}{\partial t^2} \right] + 4\alpha \nabla^2 \right\} (\varphi, \omega_2) = 0 
\] (3.11)

We seek solution of the equations (3.8) and (3.11) in the form
\[ \varphi, \psi, \omega_2 = \tilde{Q}(x_3), \tilde{\psi}(x_3), \tilde{\omega}_2(x_3) e^{i(kx_2 - \omega t)} \] (3.12)
Using (3.12) in the equations (3.8) and (3.11) we obtain the solution as

\[ \bar{\varphi} = A \sinh \delta x_3 + B \cosh \delta x_3 \]  

(3.13)

\[ \bar{\psi} = C \sinh \lambda_1 x_3 + D \cosh \lambda_1 x_3 + E \sinh \lambda_2 x_3 + F \cosh \lambda_2 x_3 \]  

(3.14)

\[ \bar{\omega}_2 = C' \sinh \lambda_1 x_3 + D' \cosh \lambda_1 x_3 + E' \sinh \lambda_2 x_3 + F' \cosh \lambda_2 x_3 \]  

(3.15)

where

\[ \delta^2 = k^2 - \left\{ \frac{C_i^2 \alpha_i^2}{(C_i^2 + \alpha_i^2)} \right\} \]  

(3.16)

\[ \lambda_{i,2} = k^2 + \frac{C_2}{2(C_2 + \alpha_2)} \left\{ \left[ \left( 1 + \frac{\alpha_0^2}{C_2^2} \right) \left( \eta_0^2 - \sigma_2^2 \right) - \sigma_2^2 - \eta_0^2 \right]^{1/2} + \left[ \left( 1 + \frac{\alpha_0^2}{C_2^2} \right) \left( \eta_0^2 + \sigma_2^2 \right) + \sigma_2^2 + \eta_0^2 \right]^2 - 4 \left( 1 + \frac{\alpha_0^2}{C_2^2} \right) \sigma_2^4 \right\}^{1/2} \]

and

\[ C_i^2 = \left( \frac{\omega_0^2 + \mu}{\rho} \right), \quad C_2 = \left( \frac{\sigma_0 + \alpha}{\rho} \right), \quad C_4 = \left( \frac{\gamma + \epsilon}{\mu} \right), \quad \eta_0^2 = 4 \alpha^2 / (\gamma + \epsilon)(\mu + \alpha) \]

The solutions (3.14) and (3.15) are connected through equations (3.9) and (3.10) respectively.

Similarly as in the classical elasto-kinetics the general problems of propagation of waves may be reduced to the solution of two simple problems i.e. to the consideration of symmetric and anti-symmetric vibrations.

(i) Symmetric vibration : Symmetric vibrations are characterized by the symmetry of displacement \( u_1 \) and stresses \( \sigma_{31} \), \( \sigma_{33} \), \( \mu_{32} \) with respect to the plane \( x_3 = 0 \). In this case we have to put in the expressions (3.13) to (3.15) \( A = D = F = F' = 0 \). In view of equation (3.9) we get

\[ C' = \chi_1 C \quad \text{and} \quad E' = \chi_2 E \]  

(3.17)

where

\[ \chi_j = \frac{1}{S} \left[ \frac{\sigma_j^2}{\left( 1 + \frac{\alpha_0^2}{C_j^2} \right) \left( k^2 - \lambda_j^2 \right)} \right] \]  

(3.18)

and

\[ S = 2\alpha / (\mu + \alpha) \]

By expressing the boundary condition (3.1) by the function \( \tilde{\phi} \), \( \tilde{\psi} \) and \( \tilde{\omega}_2 \) we obtain a system of three homogeneous equations. Making equal to zero the determinant of this system, we obtain the following characteristic equation in micropolar elastic plate which is situated under the said magnetic field as

\[ \frac{\tanh \delta H}{\tanh (\lambda_1 H)} = \frac{a_1 \chi_2 - a_2 \chi_1}{\lambda_2 \tanh (\lambda_1 H)} \left[ \frac{(2\mu + \lambda_0) \delta^2 - k^2 \lambda_0}{\chi_2 - \chi_1} \right] \left[ \frac{4\mu^2 k^2 \lambda_0^2 \delta}{\chi_2^2 - \chi_1^2} \right] \]  

(3.19)

where \( \delta, \lambda_1, \lambda_2, \chi_1, \chi_2 \) have already defined in (3.16) and (3.18) and

\[ a_j = \mu [k^2 + \lambda_j^2] + \alpha (\lambda_j^2 - k^2) - 2\alpha \chi_j \quad \quad [j = 1, 2] \]
The quantity $c = k/\omega$ is the phase velocity sought for. From the transcendental equation (3.19) we obtain an infinite number of roots. To each of these roots there corresponds a definite form of vibration.

Let us now consider a particular case. We assume that the wavelength is small as compared with the thickness of the plate $2H$. Then the quantities $\delta H$, $\lambda_1H$ and $\lambda_2H$ are large such that we can assume hyperbolic tangents as equal to unity and then we obtain from (3.19)

$$\frac{\chi_2 a_1}{\chi_2 - \chi_1} - \frac{a_2 \lambda_1}{\lambda_2} \frac{\chi_1}{\chi_2 - \chi_1} = \frac{4\mu^2 k^2 \lambda_1 \delta}{(2\mu + \lambda)^2 \delta^2 - k^2 \lambda}$$

(3.20)

Which is the Rayleigh type of surface waves in micropolar elastic medium under the influence of magnetic field where the values of $\delta$, $\lambda_1$, $\lambda_2$, $\chi_1$, $\chi_2$ are defined in (3.16) and (3.18).

Anti-symmetric vibration: Let us now consider another special type of vibration in which the waves are anti-symmetric in nature with respect to the plane $x_3 = 0$ and in this case we have only to retain those terms which are opposite to the sign if we replace $x_3$ by $-x_3$. Therefore from equations (3.13) to (3.15), we consider $B = C = B' = C' = E' = 0$ and

$$D' = \chi_0 D; F' = \chi_2 F$$

(3.21)

Making use of the boundary conditions (3.1) we arrive at the transcendental equation in micropolar elastic plate under the influence of magnetic field as

$$\left[ \frac{a_1 \chi_2 \lambda_2}{\tanh (\lambda_1 H)} - \frac{a_2 \chi_1 \lambda_1}{\tanh (\lambda_2 H)} \right] \tanh (\delta H) = \frac{4\mu^2 k^2 \delta^2 \lambda_1 \lambda_2 (\chi_2 - \chi_1)}{(2\mu + \lambda)^2 \delta^2 - k^2 \lambda}$$

(3.22)

If the length of the waves is very small compared with the thickness of the plate $2H$ the equation (3.22) reduces to (3.20). If on the contrary the length of the wave is large as compared with the thickness of the plate then expanding the hyperbolic tangent into a series and retaining two terms of the expansion, we obtain the equation

$$\left( 1 - \frac{\delta^2 H^2}{3} \right) \left[ \frac{a_1 \chi_2}{\lambda_1^2 (1 - \chi_2 H^2/3)} - \frac{a_2 \chi_1}{\lambda_2^2 (1 - \chi_2 H^2/3)} \right] = \frac{4\mu^2 k^2 (\chi_2 - \chi_1)}{(2\mu + \lambda)^2 \delta^2 - k^2 \lambda}$$

(3.23)

4. DISCUSSION

If the magnetic field is weak or absent, then $\omega_0 \to 0$ and the values of $\delta$, $\lambda_1$, $\lambda_2$, $\chi_1$, $\chi_2$ which are present in the equations (3.19), (3.20), (3.22) and (3.23) and defined in (3.16) and (3.18) reduce to the following forms
\[
\delta^2 = k^2 - \sigma_1^2; \quad \lambda_{i,2}^2 = k^2 + 1/2 \left( \nu_0^2 - \eta_0^2 - \sigma_2^2 - \sigma_4^2 \right) \pm \sqrt{\left( \sigma_2^2 + \sigma_4^2 + \eta_0^2 - \nu_0^2 \right)^2 - 4\sigma_2^2 \left( \sigma_4^2 - \nu_0^2 \right)^{1/2}} \\
\]

\[
\chi_j = 1/S \left[ \sigma_j^2 - \left( k^2 - \lambda_j^2 \right) \right] \quad [j = 1,2] \tag{4.1}
\]

The values are in well agreement with the corresponding values in the problem of 'propagation of monochromatic waves in an infinite micropolar elastic plate' as presented by Nowacki [90]. If in addition with the above assumption we take \( \lambda \to 0 \) then the equations (3.19), (3.20), (3.22) and (3.23) reduce to the following form of classical elasticity.

The equation (3.19) reduces to the transcendental equation for Lamb's waves

\[
\frac{\tanh \left( kH \sqrt{1 - \left( c^2 / C_1^2 \right)} \right)}{\tanh \left( kH \sqrt{1 - \left( c^2 / C_2^2 \right)} \right)} = \left( 2 - \frac{\nu_1}{C_2^2} \right)^2 / 4 \sqrt{1 - \left( c^2 / C_1^2 \right)} \sqrt{1 - \left( c^2 / C_2^2 \right)} ; \quad \hat{C}_2^2 = \frac{\mu}{\rho} \tag{4.2}
\]

The equation (3.20) reduces to characteristic equation for Rayleigh wave

\[
\left( 2 - \frac{\nu_1}{C_2^2} \right)^2 = 4 \sqrt{1 - \left( c^2 / C_1^2 \right)} \sqrt{1 - \left( c^2 / \hat{C}_2^2 \right)} \quad ; \quad \hat{C}_2^2 = \frac{\mu}{\rho} \tag{4.3}
\]

The equations (3.22) and (3.23) reduce to the transcendental equation of classical elastokinetics respectively as

\[
\frac{\tanh \left( kH \sqrt{1 - \left( c^2 / C_1^2 \right)} \right)}{\tanh \left( kH \sqrt{1 - \left( c^2 / C_2^2 \right)} \right)} = 4 \sqrt{1 - \left( c^2 / C_1^2 \right)} \sqrt{1 - \left( c^2 / \hat{C}_2^2 \right)} / \left( 2 - \frac{\nu_1}{C_2^2} \right)^2 \tag{4.4}
\]

\[
c^2 = \frac{4}{3} \left( kH \hat{C}_2^2 \left( 1 - \frac{\hat{C}_2^2}{C_1^2} \right) \right) \tag{4.5}
\]
1 INTRODUCTION
In micropolar theory of elasticity the state of deformation depends on two vectors, namely, the displacement vector and rotation vector. Depending on these two vectors, the deformation is characterized by means of two asymmetric tensors, namely, the strain tensor and curvature twist tensor, both of which are tensors of rank two. Similarly, the state of stress is characterized by means of two asymmetric tensors - force stress tensor and couple stress tensor respectively. The generalized Hooke’s law is a set of equations connecting the force-stress and couple-stress with strain tensor and curvature twist tensor. The gradual development of micropolar theory of elasticity as evidenced by the work of Nowacki [90], Eringen [51] and Eringen and Suhubi [54], in recent years De and Sengupta [39], Acharya and Sengupta [2], Sengupta and Ghosh [117] and Sengupta and Chakraborty [126-128] investigated a good number of research problems in micropolar theory of elasticity.

2 FORMULATION OF THE PROBLEM
We consider a homogeneous, micropolar, visco-elastic plate of thickness 2h through which a monocromatic wave propagates along x3-axis under the influence of gravity. Again we consider the boundary of the layer x3 = ±h are free of stresses.

Boundary Conditions: The following conditions should be satisfied on these edges

\[ \sigma_{31} = \sigma_{33} = \mu_{23} = 0 \quad \text{at} \quad x_3 = \pm h \]  

(2.1)

we assume that displacement \( \mathbf{u} \) and rotation \( \mathbf{\omega} \) do not depend on \( x_2 \) and

\[ \mathbf{u} = (u_1, 0, u_3) \quad ; \quad \mathbf{\omega} = (0, \omega_2, 0) \]  

(2.2)

The displacement components \( u_1 \) and \( u_3 \) can be expressed in terms of displacement potentials \( \phi \) and \( \psi \) as
\( u_1 = \phi_{21} + \psi_{33} ; u_3 = \phi_{33} - \psi_{11} \) 

(2.3)

so that

\[
\nabla^2 \varphi = \Delta; \quad \nabla^2 \psi = \psi_{1,3} - \psi_{3,1} , \quad \Delta = u_{1,1} + u_{3,3}
\]

(2.4)

Since we are considering a plane strain problem under the above assumptions, the equations of motion in a micropolar, visco-elastic solid medium under the influence of gravity may be written in the following form [13,143]

\[
(D_{\mu} + D_{\alpha}) \nabla^2 u_1 + (D_{\lambda} + D_{\mu} - D_{\alpha}) \Delta_{1,1} - 2D_{\alpha} \omega_{2,3} + \rho g u_{3,1} = \rho \ddot{u}_1
\]

\[
(D_{\mu} + D_{\alpha}) \nabla^2 u_3 + (D_{\lambda} + D_{\mu} - D_{\alpha}) \Delta_{3,3} + 2D_{\alpha} \omega_{2,1} - \rho g u_{1,1} = \rho \ddot{u}_3
\]

\[
(D_{\gamma} + D_{\epsilon}) \nabla^2 \omega_2 - 4D_{\alpha} \omega_2 + 2D_{\alpha} (u_{1,3} - u_{3,1}) = J \ddot{\omega}_2
\]

(2.5)

where

\[
D_{\mu} = \sum_{i=0}^{n} \mu_i \frac{\partial^l}{\partial t^l} ; \quad D_{\alpha} = \sum_{i=0}^{n} \alpha_i \frac{\partial^l}{\partial t^l} ; \quad D_{\lambda} = \sum_{i=0}^{n} \lambda_i \frac{\partial^l}{\partial t^l}
\]

\[
D_{\gamma} = \sum_{i=0}^{n} \gamma_i \frac{\partial^l}{\partial t^l} ; \quad D_{\epsilon} = \sum_{i=0}^{n} \epsilon_i \frac{\partial^l}{\partial t^l}
\]

Introducing (2.3) and (2.4) into (2.5) we obtain the following set of equations

\[
\begin{align*}
\left[ D_{\gamma} \nabla^2 - \frac{\partial^2}{\partial t^2} \right] \varphi + g \frac{\partial \psi}{\partial x_1} &= 0 \\
\left[ D_{\alpha} \nabla^2 - \frac{\partial^2}{\partial t^2} \right] \psi + D_{\rho} \omega_2 - g \frac{\partial \varphi}{\partial x_1} &= 0 \\
\left[ D_{\gamma} \nabla^2 - D_{w} - \frac{\partial^2}{\partial t^2} \right] \omega_2 + \frac{1}{2} D_{w} \nabla^2 \psi &= 0
\end{align*}
\]

(2.6)

Eliminating \( \varphi, \psi \) and \( \omega_2 \) from (2.6) we get
\[
\left\{ \begin{array}{l}
D_T \nabla^2 - \frac{\partial^2}{\partial t^2} [D_s \nabla^2 - \frac{\partial^2}{\partial t^2} [D_Q \nabla^2 - D_w - \frac{\partial^2}{\partial t^2}]] - \frac{1}{2} D_p D_w \nabla^2 \left[ D_T \nabla^2 - \frac{\partial^2}{\partial t^2} \right] \\
+ g^2 \frac{\partial^2}{\partial x_1^2} [D_Q \nabla^2 - D_w - \frac{\partial^2}{\partial t^2}]] \end{array} \right\} (\phi, \psi, \omega_2) = 0
\]  

(2.7)

Where

\begin{align*}
D_T &= \sum_{l=0}^{n} V_{II} \frac{\partial^{l}}{\partial t^{l}} ; & D_S &= \sum_{l=0}^{n} V_{ls} \frac{\partial^{l}}{\partial t^{l}} ; & D_P &= \sum_{l=0}^{n} V_{ip} \frac{\partial^{l}}{\partial t^{l}} ; & D_W &= \sum_{l=0}^{n} V_{lw} \frac{\partial^{l}}{\partial t^{l}} \\
D_Q &= \sum_{l=0}^{n} V_{IQ} \frac{\partial^{l}}{\partial t^{l}} ; & V_{II} &= \frac{\lambda_1 + 2 \mu_1}{\rho} ; & V_{ls} &= \frac{\mu_1 + \alpha_1}{\rho} ; & V_{ip} &= \frac{2 \alpha_1}{\rho} \\
V_{IQ} &= \frac{\gamma_1 + \epsilon_1}{\rho} ; & V_{lw} &= \frac{4 \alpha_1}{\rho} 
\end{align*}

Now from the boundary conditions (2.1), we get on \( x_3 = \pm h \)

\begin{align*}
\sigma_{33} &= 2D_\mu (\phi_{33} - \psi_{33}) + D_\lambda \nabla^2 \phi = 0 \\
\sigma_{31} &= D_\mu (2\phi_{31} - \psi_{33} + \psi_{11}) - D_\alpha (\nabla^2 \psi + 2\omega_2) = 0 \\
\mu_{32} &= (D_\mu + D_\epsilon)\omega_{2,3} = 0
\end{align*}

(2.8)

3. SOLUTION OF THE PROBLEM

We seek solutions in the form

\[
\phi, \psi, \omega_2 = \bar{\phi} \cdot (x_3), \bar{\psi} \cdot (x_3), \bar{\omega}_2 \cdot (x_3) e^{i(\xi x_1 - \eta_1)}
\]  

(3.1)

Substituting (3.1) in (2.7) we obtain

\[
\bar{\phi} = A \sinh \lambda_1 x_3 + B \cosh \lambda_1 x_3 + C \sinh \lambda_2 x_3 + D \cosh \lambda_2 x_3 + E \sinh \lambda_3 x_3 + F \cosh \lambda_3 x_3 \\
\bar{\psi} = A_1 \sinh \lambda_1 x_3 + B_1 \cosh \lambda_1 x_3 + C_1 \sinh \lambda_2 x_3 + D_1 \cosh \lambda_2 x_3 + E_1 \sinh \lambda_3 x_3 + \\
+ F_1 \cosh \lambda_3 x_3
\]
\[ \Phi_2 = A_2 \sinh \lambda_1 x_3 + B \cosh \lambda_1 x_3 + C_2 \sinh \lambda_2 x_3 + D_2 \cosh \lambda_2 x_3 + E_2 \sinh \lambda_3 x_3 + F_2 \cosh \lambda_3 x_3 \]  
(3.2)

where,
\[ \sum \lambda_i^2 = \sum K_i^2 + ps, \]
\[ \sum \lambda_i^2 \lambda_j^2 = \sum K_i^2 K_j^2 - \left( g^2 k^2 \right) \left( C_1^2 C_2^2 \right) + ps \left( K_1^2 + k^2 \right) \]
\[ \lambda_i^2 \lambda_j^2 \lambda_3^2 = K_j^2 \left[ K_i^2 K_j^2 - \left( g^2 k^2 \right) \left( C_1^2 C_2^2 \right) \right] + ps k^2 K_i^2 \]
(3.3)
in which,
\[ K_1^2 = k^2 - \eta^2 / C_1^2, \quad K_2^2 = k^2 - \eta^2 / C_2^2, \quad K_3^2 = k^2 + \gamma_1^2 - \eta^2 / C_4^2, \]
\[ C_1^2 = \sum_{i=0}^{n} v_i^2 \left( - in \right), \quad C_2^2 = \sum_{i=0}^{n} v_i^2 \left( - in \right), \quad C_4^2 = \sum_{i=0}^{n} v_i^2 \left( - in \right), \]
\[ ps = \frac{1}{2} \sum_{i=0}^{n} v_i^2 \left( - in \right), \quad \gamma_1^2 = \sum_{i=0}^{n} v_i^2 \left( - in \right), \quad p = \sum_{i=0}^{n} v_i^2 \left( - in \right) \]
(3.4)

Also, \( A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, E_1, E_2, F_1, F_2, F_3 \) are connected by the following relations
\[ A_1 = p_1 A, \quad B_1 = p_1 B, \quad C_1 = p_2 C, \quad D_1 = p_2 D, \quad E_1 = p_3 E, \quad F_1 = p_3 F, \quad A_2 = q_1 A, \]
\[ B_2 = q_1 B, \quad C_2 = q_2 C, \quad D_2 = q_2 D, \quad E_2 = q_3 E, \quad F_2 = q_3 F \]
(3.5)

where
\[ p_j = \frac{i C_1^2 \left( \lambda_j^2 - K_j^2 \right)}{k g}, \]
\[ q_j = \left( i / p \right) \left\{ \left( \lambda_j^2 - K_j^2 \right) p_j - i k g / C_2^2 \right\} \]
\[
\left[ j = 1, 2, 3 \right] \]
(3.6)

We now consider the following two cases

**Case I**

*Symmetric vibration*: Symmetric vibrations are characterized by the symmetry of displacement \( u_1 \) and stresses \( \sigma_{31}, \sigma_{33} \) and \( \mu_{32} \) with respect to the plane \( x_3 = 0 \)

From expressions given in (3.2) we have,
\[ A = C = E = B_1 = D_1 = F_1 = B_2 = D_2 = F_2 = 0 \]
(3.7)

In view of the equations (2.6) and (2.8) we have
\[ B = A_1 \xi_1, \quad D = C_1 \xi_2, \quad F = E \xi_3, \quad A_2 = A_1 \eta_1, \quad C_2 = C_1 \eta_2, \quad E_2 = E_1 \eta_3 \]

where
\[ \xi_j = \tanh \lambda_j x_3 \cdot g k / C_1^2 \left( k^2 - \lambda_j^2 \right) \]
\[ \eta_j = s \left( k^2 - \lambda_j^2 \right) / \left( \lambda_j^2 - K_j^2 \right) \]
\[
\left[ j = 1, 2, 3 \right] \]
(3.8)

In this case if we put the solutions from (3.2) considering (3.7) in the boundary conditions, given in (2.8), we obtain a system of homogeneous equations
\begin{align*}
A_1 l_1 c_1 + C_1 l_2 c_2 + E_1 l_3 c_3 = 0 \\
A_1 m_1 s_1 + C_1 m_2 s_2 + E_1 m_3 s_3 = 0 \\
A_1 n_1 c_1 + C_1 n_2 c_2 + E_1 n_3 c_3 = 0
\end{align*}

where

\begin{align*}
l_j &= \frac{1}{n} \left( \sum_{i=0}^{n} (\lambda_i + 2\mu_i)(-i\eta)^2 \lambda_j^2 - \sum_{i=0}^{n} \lambda_i (-i\eta)^2 k^2 \right) \xi_j - 2 \sum_{i=0}^{n} \mu_i (-i\eta)^2 i k \lambda_j \\
m_j &= 2 \sum_{i=0}^{n} \mu_i (-i\eta)^2 i k \lambda_j \lambda_j + \sum_{i=0}^{n} (\lambda_i + \mu_i)(-i\eta)^2 \lambda_j^2 + \sum_{i=0}^{n} (\mu_i - \lambda_i)(-i\eta)^2 k^2 \\
& \quad - 2 \sum_{i=0}^{n} a_i (-i\eta)^2 \eta_j \\
n_j &= \eta_j \lambda_j, \quad c_j = \cosh \lambda_j h, \quad s_j = \sinh \lambda_j h; \quad (j = 1, 2, 3)
\end{align*}

(3.10)

Eliminating the independent constants $A_1$, $C_1$, $E_1$ from the set of equations (3.9) we have

\begin{align*}
\begin{vmatrix}
1_l c_1 & 1_2 c_2 & \cdots & 1_3 c_3 \\
1_m s_1 & 1_m s_2 & \cdots & 1_m s_3 \\
1_n c_1 & 1_n c_2 & \cdots & 1_n c_3
\end{vmatrix} = 0
\end{align*}

(3.11)

Case II

Anti-symmetric vibrations:

Let us now consider another special type of vibration in which the waves are anti-symmetric in nature with respect to the plane $x_3 = 0$ and in this case, we have only to retain those terms which are opposite to the sign if we replace $x_3$ by $-x_3$. Therefore, from equations (3.2) we have,

\begin{align*}
B = D = F = A_1 = C_1 = E_1 = A_2 = C_2 = E_2 = 0
\end{align*}

(3.12)

where

\begin{align*}
A &= B_1 \xi_1' \quad C = D_1 \xi_2' \quad E = F_1 \xi_3' \quad B_2 = B_1 \eta_1' \quad D_2 = D_1 \eta_2' \quad F_2 = F_1 \eta_3' \\
\text{and} \quad \xi_j' &= \coth \lambda_j \eta_{.j} \quad \frac{g_{ik}}{C_1^2 (k^2 - K_{i,2}^2)} \\
\eta_j' &= s \left( k^2 - \lambda_j^2 \right) \left( \lambda_j^2 - K_{j,3}^2 \right) \quad ; \quad j = 1, 2, 3
\end{align*}

(3.13)

Expressing the boundary conditions with the functions $\varphi$, $\psi$ and $\omega_2$, as in case (1) we get the following set of equations in the unknowns $B_1$, $D_1$, $F_1$ as
\[ B_1 l'_1 s_1 + D_1 l'_2 s_2 + F_1 l'_3 s_3 = 0 \]
\[ B_1 m'_1 c_1 + D_1 m'_2 c_2 + F_1 m'_3 c_3 = 0 \]
\[ B_1 n'_1 s_1 + D_1 n'_2 s_2 + F_1 n'_3 s_3 = 0 \]  \hspace{1cm} (3.14)

where

\[ l'_j = \sum_{l=0}^{n} (\lambda_j + 2\mu_j) (-\eta)^l \lambda_j^2 - \sum_{l=0}^{n} \lambda_j (-\eta)^l k^2 \xi'_j - 2ik \sum_{l=0}^{n} \mu_j (-\eta)^l \lambda_j \]

\[ m'_j = 2ik \sum_{l=0}^{n} \mu_j (-\eta)^l \xi'_j \lambda_j + \sum_{l=0}^{n} (\mu_j + \varepsilon_j) (-\eta)^l \lambda_j^2 + \sum_{l=0}^{n} (\mu_j - \alpha_j) (-\eta)^l k^2 \]

\[ -2 \sum_{l=0}^{n} \alpha_i (-\eta)^l \eta'_i \]  \hspace{1cm} (3.15)

Eliminating the constants \(B_1, D_1, F_1\) from the set of equations (3.14) we get

\[ \begin{vmatrix} l'_1 s_1 & l'_2 s_2 & l'_3 s_3 \\ m'_1 c_1 & m'_2 c_2 & m'_3 c_3 \\ n'_1 s_1 & n'_2 s_2 & n'_3 s_3 \end{vmatrix} = 0 \]  \hspace{1cm} (3.16)

If the gravity field and viscous field diminished from both symmetric and anti-symmetric cases mentioned above, the results are in fair agreement with the micropolar theory of elasticity which are presented by Nowacki [90]. Whenever, the micropolar field is removed, the results are in fair agreement with the corresponding classical isotropic theory of elasticity.
1 INTRODUCTION

In the micropolar theory of elasticity we generally deal with the displacement vector \( u(x, t) \) and rotation vector \( \omega(x, t) \) produced by the action across any infinitesimal surface element in a solid elastic medium. Further, two asymmetric tensors \( \gamma_{ij} \), known as the deformation tensor, and \( \chi_{ij} \), called the curvature twist tensor, describe the state of deformation of a solid body whether the state of stress is characterized by the asymmetric force stress tensor \( \sigma_{ij} \), and the couple stress tensor, \( \mu_{ij} \). In particular, Nowacki [87] generalized the problem of surface waves in a micropolar elastic medium in two special configurations

(i) displacement vector is \( u = (u_1, 0, u_3) \) and rotation vector \( \omega = (0, \omega_2, 0) \)

(ii) the displacement is \( u = (0, u_2, 0) \) and the rotation \( \omega = (\omega_1, 0, \omega_3) \).

In spite of the previous works more work is needed for an understanding of the problems of waves and vibrations in coupled micropolar theory of elasticity. As for example we may mention the research paper entitled 'surface waves in micropolar thermo-elasticity under the influence of gravity' investigated by Das and Sengupta [30].

2. GENERAL THEORY

Let \( M_1 \) and \( M_2 \) be two homogeneous, micropolar, visco-elastic solid mediums under the influence of temperature and gravity are welded in contact and separated by a plane horizontal boundary extending to infinity. We consider the orthogonal Cartesian co-ordinate system \( 0-x_1 \ x_2 \ x_3 \) in micropolar elastic solid media. The medium \( M_2 \) is above the medium \( M_1 \), the origin \( O \) being any point on the plane horizontal boundary and \( ox_3 \) pointing normally into the medium \( M_2 \).

If we assume that (i) the disturbance is largely confined to the neighborhood of the boundary and (ii) at any instant all particles in any line parallel to the \( x_2 \)-axis have the same displacements, then assumption (i) asserts that the wave is a surface wave and (ii) admits that all partial derivatives with respect to \( x_2 \)-axis are zero.

The displacement components $u_i$ and the rotation components $\omega_i$ can be expressed in the form

$$u_i = \varphi_i + \psi_i, \quad u_3 = \varphi_3 - \psi_1, \quad \omega_i = \Gamma_i + \chi_i, \quad \omega_3 = \Gamma_3 - \chi_1$$  \hspace{1cm} \text{(2.1)}$$

Where $\varphi, \psi, \Gamma$ and $\chi$ are functions of the co-ordinates $x_i, x_5$ and time $t$ and

$$\nabla^2 \varphi = \Delta, \quad \nabla^2 \psi = \nabla \omega \cdot \nabla \omega; \quad \Delta = u_{1,1} + u_{3,3}$$

$$\Delta' = \omega_{1,1} + \omega_{3,3}; \quad \nabla^2 \chi = \omega_{1,3} - \omega_{3,1}$$

The equations of motion in micropolar visco-elastic solid under the influence of temperature may be written as [91,143]

$$\left( D_{\mu} + D_{\alpha} \right) \nabla^2 \vec{u} + \left( D_{\lambda} + D_{\mu} - D_{\alpha} \right) \text{grad} \text{div} \vec{u} + 2D_{\alpha} \text{rot} \vec{\omega} - \left( 3D_{\lambda} + 2D_{\mu} \right) \vec{a} \text{grad} \theta = \rho \ddot{u}$$

$$\left( D_{\gamma} + D_{\varepsilon} \right) \nabla^2 \vec{\omega} + \left( D_{\gamma} + D_{\beta} - D_{\varepsilon} \right) \text{grad} \text{div} \vec{\omega} - 4D_{\varepsilon} \vec{\omega} + 2D_{\alpha} \text{rot} \vec{u} = J \ddot{\omega}$$

The equations of motion in micropolar visco-elastic solid under the influence of temperature and gravity for the systems (i) $u = (u_1, 0, u_3)$ and $\omega = (0, \omega_2, 0)$ and (ii) $u = (0, u_2, 0)$ and $\omega = (\omega_1, 0, \omega_3)$ may be written as

$$\left( D_{\mu} + D_{\alpha} \right) \nabla^2 u_1 + \left( D_{\lambda} + D_{\mu} - D_{\alpha} \right) \Delta_1 - 2D_{\alpha} \omega_{2,3} - \left( 3D_{\lambda} + 2D_{\mu} \right) \vec{a}_1 \theta_1 + \rho g u_{3,1} = \rho \ddot{u}_1$$

$$\left( D_{\mu} + D_{\alpha} \right) \nabla^2 u_3 + \left( D_{\lambda} + D_{\mu} - D_{\alpha} \right) \Delta_3 + 2D_{\alpha} \omega_{2,1} - \left( 3D_{\lambda} + 2D_{\mu} \right) \vec{a}_1 \theta_3 - \rho g u_{3,1} = \rho \ddot{u}_3$$

$$\left( D_{\gamma} + D_{\varepsilon} \right) \nabla^2 \omega_2 - 4D_{\varepsilon} \omega_2 + 2D_{\alpha} \left( u_{1,3} - u_{3,1} \right) = J \ddot{\omega}_2$$

$$k \nabla^2 \theta = \rho e c_e \frac{\partial \theta}{\partial t} + T_0 \left( 3D_{\lambda} + 2D_{\mu} \right) \frac{\partial}{\partial t} \left( \nabla^2 \varphi \right)$$  \hspace{1cm} \text{(2.4a)}$$

and

$$\left( D_{\mu} + D_{\alpha} \right) \nabla^2 u_2 + 2D_{\alpha} \left( \omega_{1,3} - \omega_{3,1} \right) = \rho \ddot{u}_2$$

$$\left( D_{\gamma} + D_{\varepsilon} \right) \nabla^2 \omega_1 + \left( D_{\gamma} + D_{\beta} - D_{\varepsilon} \right) \Delta_{1,1} - 4D_{\varepsilon} \omega_1 - 2D_{\alpha} u_{2,3} = \rho \ddot{\omega}_1$$

$$\left( D_{\gamma} + D_{\varepsilon} \right) \nabla^2 \omega_3 + \left( D_{\gamma} + D_{\beta} - D_{\varepsilon} \right) \Delta_{3,3} - 4D_{\varepsilon} \omega_3 + 2D_{\alpha} u_{2,1} = J \ddot{\omega}_3$$  \hspace{1cm} \text{(2.4b)}$$
Where

\[ D_{\mu} = \sum_{l=0}^{n} \mu_l \frac{\partial^{l}}{\partial t^{l}} \]  
\[ D_{\alpha} = \sum_{l=0}^{n} \alpha_l \frac{\partial^{l}}{\partial t^{l}} \]  
\[ D_{\lambda} = \sum_{l=0}^{n} \lambda_l \frac{\partial^{l}}{\partial t^{l}} \]  
\[ D_{\gamma} = \sum_{l=0}^{n} \gamma_l \frac{\partial^{l}}{\partial t^{l}} \]  
\[ D_{\beta} = \sum_{l=0}^{n} \beta_l \frac{\partial^{l}}{\partial t^{l}} \]  
\[ D_{\epsilon} = \sum_{l=0}^{n} \epsilon_l \frac{\partial^{l}}{\partial t^{l}} \]

and

\( \lambda_0, \mu_0 \) are elastic constants, \( \alpha_0, \beta_0, \gamma_0, \epsilon_0 \) are other material constants, \( \rho \) is the density of the solid, \( \lambda_1, \mu_1, \alpha_1, \beta_1, \gamma_1, \epsilon_1 \) (1 = 1, 2, ..., n) are the parameters representing the effect of viscosity, \( \alpha_t \) is the coefficient of linear expansion of solid, \( J \) is the rotational inertia, \( \theta \) denotes the temperature increment measured from the reference state \( T_0 \), \( k \) is thermal conductivity, and \( c_e \) is specific heat at constant strain.

Using (2.1) and (2.2) in (2.4a) and (2.4b) we obtain

\[
\begin{align*}
\left[ D_T \nabla^2 - \frac{\partial^2}{\partial t^2} \right] \varphi - D_u \Theta - g \frac{\partial \psi}{\partial x_1} &= 0 \\
\left[ D_s \nabla^2 - \frac{\partial^2}{\partial t^2} \right] \psi - D_p \omega_2 + g \frac{\partial \phi}{\partial x_1} &= 0 \\
\left[ D_Q \nabla^2 - D_w - \frac{\partial^2}{\partial t^2} \right] \omega_2 + \frac{1}{2} D_w \nabla^2 \psi &= 0 \\
\left( c_3^2 \nabla^2 - \frac{\partial}{\partial t} \right) \Theta - D_u r \frac{\partial}{\partial t} \nabla^2 \varphi &= 0 \quad \text{(2.5a)}
\end{align*}
\]

and

\[
\begin{align*}
\left[ D_s \nabla^2 - \frac{\partial^2}{\partial t^2} \right] u_2 + D_p \nabla^2 \chi &= 0 \\
\left[ D_Q \nabla^2 - D_w - \frac{\partial^2}{\partial t^2} \right] \chi - \frac{1}{2} D_w u_2 &= 0 \\
\left[ D_L \nabla^2 - D_w - \frac{\partial^2}{\partial t^2} \right] \Gamma &= 0 \quad \text{(2.5b)}
\end{align*}
\]

where
\[ D_T = \sum_{i=0}^{n} V_{IT}^2 \frac{\partial^1}{\partial t^1} ; \quad D_S = \sum_{i=0}^{n} V_{IS}^2 \frac{\partial^1}{\partial t^1} ; \quad D_P = \sum_{i=0}^{n} V_{IP}^2 \frac{\partial^1}{\partial t^1} ; \quad D_W = \sum_{i=0}^{n} V_{IW}^2 \frac{\partial^1}{\partial t^1} \]

\[ D_Q = \sum_{i=0}^{n} V_{IQ}^2 \frac{\partial^1}{\partial t^1} ; \quad D_u = \sum_{i=0}^{n} V_{IU}^2 \frac{\partial^1}{\partial t^1} ; \quad D_L = \sum_{i=0}^{n} V_{IL}^2 \frac{\partial^1}{\partial t^1} \]

\[ V_{IT}^2 = \frac{\lambda_1 + 2\mu_1}{\rho} ; \quad V_{IS}^2 = \frac{\mu_1 + \alpha_1}{\rho} ; \quad V_{IP}^2 = \frac{2\alpha_1}{\rho} ; \quad V_{IQ}^2 = \frac{(3\lambda_1 + 2\mu_1)\alpha_1}{\rho} \]

\[ V_{IW}^2 = \frac{\gamma_1 + \epsilon_1}{J} ; \quad V_{IL}^2 = \frac{4\alpha_1}{J} ; \quad V_{IQ}^2 = \frac{2\gamma_1 + \beta_1}{J} ; \quad c_3^2 = \frac{k}{\rho c_e} ; \quad r = \frac{T_0}{c_e} \]

Eliminating \( \phi, \psi, \theta \) and \( \omega_2 \) from (2.5a) we get

\[ \left\{ \left[ \left( D_S V^2 - \frac{\partial^2}{\partial t^2} \right) \left[ D_Q V^2 - D_W - \frac{\partial^2}{\partial t^2} \right] + \frac{1}{2} D_P D_W V^2 \right] \right\} \left( D_T V^2 - \frac{\partial^2}{\partial t^2} \right) \left[ \frac{c_3^2 V^2}{\partial t^2} \right] - rD_u \frac{\partial^2}{\partial t^2} V^2 + g^2 \frac{\partial^2}{\partial t^2} \left[ c_3^2 V^2 - \frac{\partial^2}{\partial t^2} \left[ D_Q V^2 - D_W - \frac{\partial^2}{\partial t^2} \right] \right] \left( \phi, \psi, \theta, \omega_2 \right) = 0 \tag{2.6a} \]

Eliminating \( u_2 \) and \( \chi \) from (2.5b) we get

\[ \left\{ \left[ \left( D_s V^2 - \frac{\partial^2}{\partial t^2} \right) \right] \left[ D_Q V^2 - D_W - \frac{\partial^2}{\partial t^2} \right] + \frac{1}{2} D_P D_W V^2 \right\} \left( \chi, u_2 \right) = 0 \tag{2.6b} \]

We seek solution of (2.6a), (2.6b) and last equation of (2.5b) in the form

\[ \varphi, \theta, \psi, \omega_2, u_2, \chi, \Gamma = \left[ \varphi(x_3), \theta(x_3), \psi(x_3), \omega(x_3), \bar{\omega}_2(x_3), \bar{\chi}(x_3), \bar{\Gamma}(x_3) \right] \]

\[ e^{\xi(x_3 - \alpha)} \tag{2.7} \]

In the medium \( M_1 \) and \( M_2 \) solutions are respectively as

\[ \varphi = \sum A_j e^{-\lambda_j x_3} ; \quad \bar{\theta} = \sum m_j A_j e^{-\lambda_j x_3} ; \quad \psi = \sum n_j A_j e^{-\lambda_j x_3} ; \quad \omega_2 = \sum o_j A_j e^{-\lambda_j x_3} \]

\[ \bar{\chi} = E_1 e^{-\lambda_1 x_3} + E_2 e^{-\lambda_2 x_3} ; \quad \bar{u}_2 = m_3 E_1 e^{-\lambda_3 x_3} + m_6 E_2 e^{-\lambda_6 x_3} ; \quad \bar{\Gamma} = F e^{-\lambda_7 x_3} \tag{2.8a} \]

and
\[ \bar{\phi} = \sum A_i e^{-\lambda_j x_3}, \quad \bar{\theta} = \sum m_i A_i e^{-\lambda_j x_3}, \quad \bar{\psi} = \sum n_i A_i e^{-\lambda_j x_3}, \quad \bar{\omega} = \sum o_i A_i e^{-\lambda_j x_3} \]
\[ \bar{\chi} = E_1 e^{-\lambda_j x_3} + E_2 e^{-\lambda_j x_3}, \quad \bar{\Gamma} = \Gamma e^{-\lambda_j x_3}, \quad u_2 = m_3 E_1 e^{-\lambda_j x_3} + m_6 E_2 e^{-\lambda_j x_3} \]

\[ [j = 1, 2, 3, 4] \]

Where \( \lambda_j^2 \) are the roots of the equation

\[ \left[ \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda^2 \right) + \eta^2 \left( \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda^2 \right) - \sum_{i=0}^{n} V^2_i (-i\eta)^j + \eta^2 \right) \right. \]
\[ + \frac{1}{2} \sum_{i=0}^{n} V^2_i (-i\eta)^j \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda^2 \right) \]

And \( \lambda_5^2, \lambda_6^2 \) are the roots of the equation

\[ \left[ \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda^2 \right) - \sum_{i=0}^{n} V^2_i (-i\eta)^j + \eta^2 \left( \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda^2 \right) \right) \right. \]
\[ + \frac{1}{2} \sum_{i=0}^{n} V^2_i (-i\eta)^j \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda^2 \right) \]

\[ = 0 \]

Also \( n_j = \frac{1}{gi\xi} \left[ \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda_j^2 \right) + \eta^2 \left( \sum_{i=0}^{n} V^2_i (-i\eta)^j \right) \right] \]
\[ \left[ \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda_j^2 \right) \right] \]

\[ = \frac{1}{2} \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( \lambda_j^2 - \xi^2 \right) \]

\[ m_j = -\frac{1}{C_j \left( -\xi^2 + \lambda_j^2 \right)} \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda_j^2 \right) + \eta^2 \]

\[ \sigma_j = \frac{1}{2} \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( \lambda_j^2 - \xi^2 \right) - \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda_j^2 \right) + \eta^2 \]

\[ m_l = \frac{1}{2} \sum_{i=0}^{n} V^2_i (-i\eta)^j \left( -\xi^2 + \lambda_l^2 \right) \]

\[ [l = 5, 6] \]

 similary solutions can be derived for the Medium M2

3. BOUNDARY CONDITIONS AND SOLUTION OF THE PROBLEM

(A) The boundary conditions for the system (l) are (i) The displacement vector \( \bar{\xi} (u_1, o_1, u_3) \) and rotation vector \( \bar{\omega}(\sigma, \omega_2, \sigma) \), temperature and its flux at the boundary surface must be continuous at all times and places (ii) The components \( \sigma_{31}, \sigma_{33} \) and \( \mu_{32} \) across the boundary surface are continuous at all times and places.
(B) The boundary conditions for the system (2) are (iii) the displacement u_2 and the rotations \( \omega_1, \omega_3 \) at the boundary surface must be continuous at all times and places (iv) the components \( \sigma_{32}, \mu_{31} \) and \( \mu_{33} \) across the boundary surface are continuous at all times and places.

where

\[
\begin{align*}
\sigma_{33} &= 2D_\mu [p_{33} - \psi_{13}] + D_\lambda \nabla^2 \varphi - (3D_\lambda + 2D_\mu) \alpha \theta \\
\sigma_{31} &= D_\mu [2\psi_{13} + \psi_{33} - \psi_{11}] + D_\alpha \left( \nabla^2 \psi - 2\omega_2 \right) \\
\mu_{32} &= (D_\mu + D_e) \omega_{2,3} \\
\sigma_{32} &= (D_\mu - D_a) u_{2,3} - 2D_a (\gamma_{s1} + \chi_{s3}) \\
\mu_{31} &= D_\gamma [2\gamma_{s1} + \chi_{33} - \chi_{11}] + D_e \nabla^2 \psi \\
\mu_{33} &= (2D_\gamma + D_\beta) [\gamma_{s3} - \chi_{s1}] + D_\beta [\gamma_{11} - \chi_{s3}]
\end{align*}
\]

Using (2.1), (2.7), 2.8(a), (3.1) and boundary conditions (A) we obtain eight simultaneous equations from which we get on eliminating the unknown constants \( A_j, A'_j \) \( j=1,2,3,4 \) the frequency equations for the first system as

\[
\Delta_2 \equiv |a_j| = 0
\]

Using (2.1), (2.7), 2.8(a), (3.1) and boundary conditions (B) we get six simultaneous equations from which we obtain on eliminating the constants \( E_1, E_2, F, E'_1, E'_2, F' \) the frequency equations for the second system as

\[
\Delta_3 \equiv |b_j| = 0
\]

where,

\[
\begin{align*}
a_{ij} &= n_j \lambda_j - i\xi_j, \\
a_{im} &= i\xi_j + \lambda'_m n_m, \\
a_{2j} &= \lambda_j + i\xi_j, \\
a_{2m} &= \lambda'_m + i\xi_n \\
[m=5,6,7,8] \text{ and } a_{6j}, a_{6m}, \text{ etc. can be evaluated similarly}
\end{align*}
\]

\[
\begin{align*}
b_{11} &= m_5, \\
b_{12} &= m_6, \\
b_{13} &= 0, \\
b_{14} &= -m'_5, \\
b_{15} &= -m'_6, \\
b_{16} &= 0, \\
b_{21} &= \lambda_5, \\
b_{22} &= \lambda_6, \\
b_{23} &= -i\xi_5, \\
b_{24} &= \lambda'_5, \\
b_{25} &= \lambda'_6, \\
b_{26} &= i\xi_6, \\
b_{27} &= i\xi = b_{32}, \\
b_{28} &= \lambda_7, \\
b_{34} &= i\xi = b_{35}, \\
b_{36} &= \lambda'_7, \\
b_{37} &= \lambda_7, \\
b_{41} &= b_{42} = b_{43} = b_{44} = b_{45} = b_{46} = b_{47} = b_{48} = 0
\end{align*}
\]

and \( b_{4j}, b_{5j} \) \( j=1 \text{ to } 6 \), etc. can be evaluated similarly.

4. PARTICULAR CASES

RAYLEIGH WAVES For the existence of Rayleigh waves, the plane boundary is to be a free surface so that \( M_2 \) is replaced by Vacuum. Here the boundary conditions are \( \sigma_{33}=\sigma_{31}=\mu_{32}=0 \) on the plane boundary of \( M_1 \) with an additional thermal boundary condition
Using the above boundary conditions we get the wave velocity equation of Rayleigh waves in the present medium for the first system as,

\[
\Delta_4 = \begin{vmatrix}
    a_{61} & a_{62} & a_{63} & a_{64} \\
    a_{71} & a_{72} & a_{73} & a_{74} \\
    \lambda_1 a_{31} & \lambda_2 a_{32} & \lambda_3 a_{33} & \lambda_4 a_{34} \\
    (h - \lambda_1) a_{41} & (h - \lambda_2) a_{42} & (h - \lambda_3) a_{43} & (h - \lambda_4) a_{44}
\end{vmatrix} = 0
\]  

(4.2)

where

\[a_{31} = 0_1, \quad a_{32} = 0_2, \quad a_{33} = 0_3, \quad a_{34} = 0_4, \quad a_{41} = m_1, \quad a_{42} = m_2, \quad a_{43} = m_3, \quad a_{44} = m_4\]

For the second system the boundary conditions are \(\sigma_{32} = \mu_{31} = \mu_{33} = 0\) over the plane boundary of \(M_1\), proceeding above we obtain the wave velocity equations of Rayleigh waves in the present medium for the second system as

\[
\Delta_5 = \begin{vmatrix}
    b_{41} & b_{42} & b_{43} \\
    b_{51} & b_{52} & b_{53} \\
    b_{61} & b_{62} & b_{63}
\end{vmatrix} = 0
\]

(4.3)

LOVE WAVES: For the Love type of surface waves, we assume that \(M_2\) is bounded by two horizontal plane surface at a finite distance \(H\) apart. The upper plane surface being free while the lower plane surface forms the medium \(M_1\) and extends to infinity.
Here it is sufficient to consider the component $u_2$ of displacement vector $\mathbf{u}$ and $\omega_1$, $\omega_3$ of the rotation vector $\omega$. Here $\mathbf{u}$ and $\omega$ in $M_2$ may no longer diminish with the distance from the boundary between $M_1$ and $M_2$ i.e for the medium $M_2$ we preserve the full solution. The frequency equation can be constructed as in the previous papers in this thesis.

STONELEY WAVES In the classical theory, Stoneley waves are the generalized form of Rayleigh waves propagating along the common boundary of $M_1$ and $M_2$. The Stoneley waves for the present problem are determined by the roots of the frequency equations (3.2) and (3.3). If we neglect all the fields, the above results are in well agreement with that of De and Sengupta [39] who studied the surface waves in micropolar elastic medium.

5. DISCUSSION

It is important to note that in the present problem, Rayleigh waves are affected by all the fields but the Love waves are affected only by viscous field. Making suitable choice of physical constants of both media $M_1$ and $M_2$ and applying numerical techniques the determinantal equations (3.2) and (3.3) may be solved. It is mentioned that we are mainly interested in general coupled theory of micropolar elasticity rather than some mathematical calculations and on this basis an attempt has been made throughout this thesis.