CHAPTER - 2

TRANSPORTATION PROBLEM

2.1 INTRODUCTION

The 21st century is built upon the word “transportation”. Mobility has made life fast and progressive. Life in tune with the changing trends is impossible in the traditional, slow fashion. International connectivity and information technology have changed the whole life of mankind. It is unthinkable today for families to produce food, clothing and other essential requirements. The markets have a wide range of novelties to offer to their customers which minimize the need of self production. Goods are produced in large scale in factories and farms and they reach the consumers within no time. The whole structure of modern society involves a trade-off between economies, group activities and transporting men and material from place to place. A problem that occurs in this kind of a society is that of transportation.

The transportation models or problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost.
Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales.

The transportation problem is a classic operations research problem where the objective is to determine the schedule for transporting goods from source to destination in a way that minimizes shipping cost, satisfying demand and supply constraints. Though this problem can be solved as a linear programming problem, other methods can also be used for bringing in an effective solution. The simplex algorithm that is used to solve a linear programming model is laborious in nature. To simplify the calculation many models have been developed. One such model is transportation model. The application of this model is not confined only to transportation and distribution alone, but can also be extended to problems such as machine assignment, plant location, product mix and so on.

At present transportation plays a acting vital role for financial development of the country. Mixture of transportation and mobility are directly involved with development of financial system of the country and for that mature transportation infrastructure required. Not only that, alteration in basic mathematical formation of transportation is necessary where simple objective function can be modified by multi-objective function.
2.2 THE ORIGIN

The transportation model made its first appearance in a study ‘The Distribution of a Product from Several Sources to Numerous Localities’ in 1941 by F. L. Hitchcock. After six years, another study titled ‘Optimum Utilization of the Transportation System’ by T. C. Koopmans was presented. These two studies contributed for the development of transportation model. A general transportation model involves ‘n’ number of supply sources and ‘n’ number of demand destinations. The objective of the problem is to minimize the cost of transportation from source to destination while meeting the requirement at the destinations. This can also be applied to maximize some total value or utility.

2.3. APPLICATION OF TRANSPORTATION PROBLEM

The objective of the transportation problem is to determine the shipping schedule that minimize that total shipping cost while satisfying supply and demand limits. The transportation model finds its application in industry, communication network, planning, scheduling transportation and allotment and so on. Transportation problem is a logistical problem for organizations especially for manufacturing and transport companies. This method is a useful tool in decision-making and the process of allocating problem in these organizations. The transportation model can also be used in making location decisions. The model helps in locating a new facility, a manufacturing plant or an office when two or more number of locations is under consideration. The total transportation cost, distribution cost or shipping cost and production costs are to be minimized by applying the model.
Transportation model is used for the following purpose:

- To decide the transportation of new materials from various centres to different manufacturing plants. In the case of multi-plant company this is highly useful.
- To decide the transportation of finished goods from different manufacturing plants to the different distribution centres. For a multi-plant-multi-market company this is useful.

### 2.4. A TRANSPORTATION PROBLEM

Transportation problem of a dairy production plant is considered. The dairy farm has three production plants and their daily production is given in Table 2.1.

**Table 2.1 Various plants and Production in litres**

<table>
<thead>
<tr>
<th>Plant</th>
<th>Production in Litres</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1</td>
<td>6 million</td>
</tr>
<tr>
<td>Plant 2</td>
<td>1 million</td>
</tr>
<tr>
<td>Plant 3</td>
<td>10 million</td>
</tr>
</tbody>
</table>

The minimum requirement at each centre is given Table 2.2:
Table 2.2 Distribution centre and its requirement in litres

<table>
<thead>
<tr>
<th>Distribution centre</th>
<th>Requirement in Litres</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7 million litres</td>
</tr>
<tr>
<td>2</td>
<td>5 million litres</td>
</tr>
<tr>
<td>3</td>
<td>3 million litres</td>
</tr>
<tr>
<td>4</td>
<td>2 million litres</td>
</tr>
</tbody>
</table>

The cost of transportation of the commodity from a plant to a distribution centre is different. The transportation tableau for the dairy form problem is given in table 2.3

Table 2.3 Illustration of distribution centre with different plants

<table>
<thead>
<tr>
<th>Plants</th>
<th>Distribution centre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

The value in each cell is expressed as hundreds of rupees. Now the objective of the problem is to meet the requirement of each distribution centre by the three plants at minimum cost. The capacity of single plant may be enough for meeting the demand of a distribution and sometimes more than one plant may be utilized to satisfy the need of a single centre.
For example the demand of distribution centre 1 is 7 million litres. This demand is met by the capacity of plants 1 and 2. Plant 1 can contribute 6 million litres, while plant 2 can contribute the remaining 1 litre. Now it is seen that the demand of the distribution centre 1 is satisfied and the contribution of both the plants are within their capacity. But it cannot be concluded that this allocation can only yield the minimum cost of transportation. Other plants can also contribute for the allocation and may or may not yield a better solution. So this allocation of plants for each distribution centre is done in such a way that the overall cost of meeting the needs of the entire distribution centre is minimum.

2.5. REPRESENTATION OF TRANSPORTATION PROBLEM

2.5.1 TRANSPORTATION TABLEAU

The transportation problem is also represented by another common way called as tabular form or matrix form. A more general form of a transportation problem is given in figure 2.1

<table>
<thead>
<tr>
<th></th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>.....</th>
<th>$x_{1n}$</th>
<th>$a_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{11}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{m1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{m1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$x_{12}$</th>
<th>.....</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{12}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{m2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{m2}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$x_{1n}$</th>
<th>$a_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{1n}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_{mn}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_{mn}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_n$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.1 Tabular representation of the transportation problem
The tabular representation consists of squares called as “cells”. The cells are grouped together vertically and horizontally to form a matrix or table. Each cell has an address which is given by the combination of the headings of rows and columns. The cell on the top left corner can be expressed as (A,1), where A stands for the supply of plant A and 1 stands for the distribution centre 1. Each cell accommodates two values, one is the cost of transporting a unit of item (i.e. \( C_{ij} \)) between that plant and distribution centre. The second value is the number of units (i.e. \( X_{ij} \)) transported between the plant and warehouse. The sum of all the \( X_{ij} \) values in cells of each row gives the total supply of the plant (i.e. \( a_i \)) and the sum of all the \( X_{ij} \) values in cells of each column gives the total requirement or demand of the distribution centre (\( b_i \)).

![Figure 2.2 Representation of Transportation Problem](image)

**Figure 2.2 Representation of Transportation Problem**

The node represents a source or a destination and an arc that joins two nodes representing a shipping route through which commodity is shipped as shown in figure 2.2.
2.5.2. ASSUMPTIONS MADE IN TRANSPORTATION PROBLEM

1. The sum of the entire item available at different production plants is equal to the sum of requirements of all the distribution centres.

2. Any item or commodity can be transported from any plant to any distribution centre more conveniently.

3. The cost of transporting an item from source to destination is precisely known.

4. The transportation cost on any given route is directly proportionally to the number of units transported in that route.

5. The objective is to minimize the transportation cost of the company as a whole and not for individual plant and distribution centre.

2.6. MATHEMATICAL FORMULATION

Let us consider the following,

$S_1, S_2, \ldots, S_m$ are ‘m’ number of sources available.

$D_1, D_2, \ldots, D_n$ are ‘n’ number of destinations available.

$x_{ij}$ be the quantity to be transported from $S_i$ to $D_i$.

$c_{ij}$ be the cost of transport from $S_i$ to $D_i$.  

The problem is mathematically expressed as,

\[
\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}
\]  
(2.1)

Let \( a_i \) be the total availability of the commodity at the source \( S_i \)

\[ i.e. \quad x_{i1} + x_{i2} + \cdots + x_{im} = a_i, \text{ where } i = 1, 2, 3, \ldots, m. \]

and \( b_j \) be the total demand at the destination at \( D_j \).

\[ i.e. \quad x_{1j} + x_{2j} + \cdots + x_{mj} = b_j, \text{ where } j = 1, 2, 3, \ldots, n. \]

The constraint for the above objective function is the total demand which must be equal to the total availability.

\[ i.e. \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \]
(2.2)

Also the quantity of the commodity available at the source must be greater than 0.

\[ i.e. \quad x_{ij} \geq 0 \]  
(2.3)

### 2.7. SOLUTION TO TRANSPORTATION PROBLEM

The solution to transportation problem involves the following steps:

Step 1: Mathematical Model of Transportation Problem
The initial step in finding a solution to any transportation problem is creating a matrix. The supply from sources, demands at destinations and shipping cost between a source and destination are all expressed in this matrix.

Step 2: Finding initial basic feasible solution

Problem formulation is followed by obtaining an initial feasible solution. The initial solution must satisfy certain important conditions. First, the initial solution should satisfy the rim condition that is it must satisfy all the supply and demand constraints. Some of the most common methods that are used to find initial feasible solutions are listed below.

- North-West Corner method
- Row Minima method
- Column Minima method
- Least Cost method
- Vogel’s Approximation method (VAM)

Step 3: Perform Optimality Test

The obtained initial feasible solution is subject to optimality test methods such as stepping stone method and modified distribution (MODI) method are usually applied to check the optimality to the solution.

2.8. METHODS TO OBTAIN INITIAL BASIC FEASIBLE SOLUTION

Initial basic feasible solution can be obtained by using anyone of the following methods:
• **North-West Corner Method**

   It is a simple method to obtain an initial solution. The method is carried out by selecting the basic variable from the North-West corner. This method does not take into account the cost of transportation on any route and this result in poor initial solutions.

• **Least Cost method / Matrix minima method**

   The main objective of this method is to minimize the total transportation cost. The main advantage of this method is it concentrates on the cheapest routes.

• **Row Minima method**

   The allocation in this method is done by selecting the cheapest cost cell of the first row of the matrix.

• **Column Minima method**

   This method is similar to the previous one, where the only difference is the cheapest cost cell of the first column is selected instead of selecting from row.

• **Vogel’s Approximation / Penalty method**

   The objective of this method is to minimize the penalty cost during the allocation. Here a penalty is given when an allocation doesn’t result in minimized transportation cost. This method returns a solution which is very closer to optimal solution or sometimes the optimal solution itself. Consequently, if the initial solution is used that is obtained by Vogel’s
Approximation method and proceeds to solve for the optimum solution, the amount of time required to arrive at the optimum solution is greatly reduced.

2.9. FINDING AN OPTIMAL SOLUTION

An optimal solution is obtained by making successive improvements to initial basic feasible solution until no further decrease in the transportation cost is possible. An optimal solution is one where there is no other set of transportation routes that will further reduce the total transportation cost.

The widely used methods for finding an optimal solution are as follows:

- Stepping stone method.
- Modified Distribution (MODI) method.

![Figure 2.3 Flowchart of the Optimal Solution Method](image-url)
2.10. VARIATION IN TRANSPORTATION PROBLEM

A real time transportation problem is not free from abnormalities. Some of the possible abnormalities or variations in transportation problem are listed below:

2.10.1. UNBALANCED TRANSPORTATION PROBLEM

It is know that for a feasible solution to exist, the total supply must be equal to the total demand. But in real time, there are possibilities where the total supply may not be sufficient enough to satisfy the demand or the production at a plant may be more than the requirement at the destination. Both the cases can occur in a transportation problem which is called as unbalanced transportation problem.

The unbalanced condition is solved by the introduction of a new column or new row to the existing matrix. If the supply exceeds the demand, an additional column is added to the table in order to absorb the excess supply. This new demand centre is called as dummy demand centre. The cost of units in the cells of this column is made zero, so these represents items that are not produced and not transported.

If the total demand exceeds the supply, then the problem is solved by adding a new row to the existing table. The additional row accounts for the excess demand. Similar to the previous condition the transportation cost for the cells in this dummy row is also set to zero.
2.11. CONCLUSION

The method of finding the initial feasible solution is the most important step in a transportation problem, because this solution decides the number of iterations to be carried out to obtain a final optimal solution. If the initial solution is closer to the optimal solution then the convergence is quicker. Thus various methods are applied and still new ones are being found out to obtain a more optimal solution. It can also be stated that every method cannot be applied for every transportation problem. With the additional objective solve under additional constraints and increase in decision variables may make the problem to look like a realistic one, but on the other hand the problem also becomes more complex in nature. The conventional methods are very good at solving an approximate transportation problem rather than a practical one. Thus new methods are being developed and new transportation problems are solved to make it closer to the real one. North east method is one such method proposed in this mind with the idea of approaching a transportation problem more closely to the real-world problem. The method solves the problem with some of its variance included. The method is also tested by taking real life problem and compared with the other conventional methods to show its superiority over them. The following chapters analyze the literature to show the aspects covered by the conventional methods and the space in this field which they left untouched. This work is made as an attempt to try those areas by the proposed method and compare its way of finding the optimal solution with other methods in terms of time and space.