CHAPTER 3

THE DOUBLE PRIOR SELECTION FOR THE PARAMETER OF RAYLEIGH LIFETIME MODEL UNDER TYPE-II CENSORING

3.1 Introduction

Rayleigh distribution was invented by Lord Rayleigh(1880) during a study of acoustical problems. It is widely useful in communication engineering (Dyer and Whisenand, (1973)), electro vacuum devices(Polovko(1968)), and some clinical studies dealing with Cancer patients (Bhattacharya and Tyagi, (1990)). This distribution was successfully employed as a radio wave power distribution (Siddiqui,(1962)). It is extremely useful in life testing, reliability performances and survival analysis.

For analyzing the lifetime data, the Bayesian approach has received large attention in the theory and practice for a couple of decades. In Bayesian approach, the parameter of the life time distribution is assumed to be random and its distribution, known as prior distribution, which is used with the data to obtained updated distribution of the parameter known as posterior distribution of the parameter of the life time distribution. In Bayes estimation usually single prior is used to obtain Bayes estimate of the parameter. Further detail can be obtain from Shah and Patel(2008\textsuperscript{a,b}), Sinha and Howlader(1983), Basu and Ebrahimi(1991), Javed and Saleem(2012), Howlader, and Hussain(1995), Aitchison, and Dunsmore(1975), Howlader(1985), etc.

In this chapter we have used Rayleigh life time distribution and Bayes estimation is done based on the joint priors for the parameter of the Rayleigh life time. The following three different types of joint prior and one single prior are used for the unknown parameter of the Rayleigh distribution.

(i) Hartigan (locally invariant,(1964)) and Inverted gamma priors.
(ii) Jeffrey’s and Inverted gamma priors.
(iii) General non informative and Inverted gamma priors
(iv) Inverted gamma priors.

We assume Rayleigh distribution as a life time model. The probability density function (pdf), the cumulative distribution function (cdf) and reliability function of this life time distribution are respectively given by

\[
f(x, \theta) = \frac{x}{\theta^2} \exp\left(\frac{-x^2}{2\theta^2}\right), \quad x > 0, \quad \theta > 0.
\] ..(1.1)

\[
F(x, \theta) = 1 - \exp\left(-\frac{x^2}{2\theta^2}\right)
\] ..(1.2)

and its reliability function at time \(t\) is given by

\[
R_t(\theta) = \exp\left(-\frac{t^2}{2\theta^2}\right), \quad t > 0.
\] ..(1.3)

Bayes estimate of parameter \(\theta\) and reliability at time \(t\) are obtained based on type-II censored sample (with fixed \(r\) failure) under squared error loss function based on the above prior distributions. Also Bayes predictive estimation and Bayes predictive equal tail interval are carried out. Prediction of the remaining \((n-r)\) failure times are done and their Bayes credible prediction interval are also derived. Section 2 covers the posterior distribution based on the above four priors. In section 3 Bayes estimator of \(\theta\) and reliability \(R(t)\) and Bayes credible interval for \(\theta\) and \(R(t)\) are derived. Section 4 deals with the Bayes predictive distributions and its Bayes credible predictive interval. In section 5 prediction of the remaining \((n-r)\) failure after observing the first \(r\) failure is obtained. Their Bayes credible intervals are constructed. A real life example is considered to exemplify the method in section 6. Section 7 include the simulation study and a comparison is made between the results obtained based on the above four different priors.

3.2 The Posterior Distribution of \(\theta\) under Different Prior Distributions:

Let \(n\) times are placed a test and the test is terminated at time of \(r\)-th failure, \(r\) is predetermined fixed number less than \(n\). consider the ordered failure time of \(r\) failure as
During the test failures are not replaced but the test is carried out with the remaining items on the test. Such censoring scheme is known as type-II censoring without replacement scheme.

The likelihood function under such censoring scheme is given by,

\[ L(x, \theta) = c \prod_{i=1}^{r} f(x_{(i)}, \theta) \left[ 1 - F(x_{(i)}, \theta) \right]^{n-r}, \text{ where } c \text{ is constant depending on } n \text{ and } r. \]

Using (1.1) & (1.2), the likelihood function reduces to,

\[ L = L(x, \theta) \propto \theta^{-2r} \exp \left[ -\frac{(n-r)x_{(r)}^2 + s^2}{2\theta^2} \right]. \] (2.1)

Where \( s^2 = \sum_{i=1}^{r} x_{(i)}^2 \)

### 3.2.1 General Non Informative and Inverted Gamma Priors:

Let us consider the first general non informative prior distribution of \( \theta \) is,

\[ P_{11}(\theta) = \frac{1}{\theta^a}, \quad \theta > 0, \alpha > 0 \] (2.2)

and the second prior distribution for \( \theta \) is inverted gamma distribution having pdf,

\[ P_{12}(\theta) = \frac{2\lambda^\alpha}{\alpha} e^{-\frac{2\lambda}{\alpha} \theta^{2\alpha}}; \theta > 0, \alpha > 0, \lambda > 0 \] (2.3)

Combining (2.2) & (2.3), the double prior distribution for \( \theta \) can be defined as,

\[ P_{1}(\theta) \propto P_{11}(\theta) \cdot P_{12}(\theta) \]

\[ = K \frac{2\lambda^\alpha}{\alpha} e^{-\frac{1}{\alpha} \theta^{2\alpha} + a}; \theta > 0, \alpha > 0, \lambda > 0 \] (2.4)

where constant \( K \) is,

\[ K = \frac{\alpha^a}{\alpha + \frac{a}{2}} \] (2.5)
The posterior density of $\theta$ can be obtained by combining the likelihood function in (2.1) and double prior distribution in (2.4), as

$$
\pi_1(\theta \mid x) \propto L(x, \theta) \cdot P_1(\theta)
$$

$$
\propto \theta^{2r-2} \exp \left[ -\frac{(n-r)x^2_{(r)} + s^2 + 2\lambda}{2\theta^2} \right]
$$

$$
= 2 \left[ \frac{(n-r)x^2_{(r)} + s^2 + 2\lambda}{2} \right]^{\frac{\gamma_{r-\frac{1}{2}}}{2}} e^{-\frac{(n-r)x^2_{(r)} + s^2 + 2\lambda}{2\theta^2}} \cdot \theta^{-2d-2r+n-1}, \theta > 0
$$

$$
= \text{InGa}(\alpha + r + \frac{a}{2}, \frac{A}{2}), \text{where} A = (n-r)x^2_{(r)} + s^2 + 2\lambda, \theta > 0
$$

.. (2.6)

### 3.2.2 Inverted Gamma Prior Only:

Let us consider only inverted gamma prior $P_h(\theta)$ for the parameter of the Rayleigh distribution,

i.e. $P_h(\theta) = \frac{2\lambda^\gamma}{\alpha} \cdot e^{-\frac{\lambda}{\theta}} \cdot \theta^{2-\gamma-1}$. .. (2.7)

The posterior density of $\theta$ can be obtained by combining the likelihood function for in (2.1) and prior in (2.7), as

$$
\pi_h(\theta \mid x) \propto L(x, \theta) \cdot P_h(\theta)
$$

$$
\propto \theta^{-2r} \exp \left[ -\frac{(n-r)x^2_{(r)} + s^2}{2\theta^2} \right] e^{-\frac{\lambda}{\theta}} \cdot \theta^{-2d-2r+n-1}
$$

$$
= 2 \left[ \frac{(n-r)x^2_{(r)} + s^2 + 2\lambda}{2} \right]^{\gamma_{0}} e^{-\frac{(n-r)x^2_{(r)} + s^2 + 2\lambda}{2\theta^2}} \cdot \theta^{-2d-2r+n-1}, \theta > 0
$$

.. (2.8)
Thus in all the cases of different type of double prior distribution and in case of single prior distribution the posterior distribution of \( \theta \) given the data \( x \) becomes inverted gamma distribution. In case of \( i^{th} \) case the posterior distribution of \( \theta \) given the data \( x \) is given by

\[
\text{InGa}(\alpha_i, \beta_i), i=1,2,3,4; \text{ where } \alpha_i = \alpha + r + \frac{a}{2}, \quad \beta_i = \left[ \frac{(n-r)\chi^2_{\alpha_i} + s^2 + 2\hat{\lambda}}{2} \right]
\]

with pdf,

\[
\pi_i(\theta / x) = \frac{2\beta_i^{\alpha_i} \theta^{\alpha_i-1} e^{-\theta \beta_i}}{\Gamma(\alpha_i)} , \quad \theta > 0, \alpha_i > 0, \beta_i > 0
\]

Putting \( a = 3 \) and \( i = 1 \) we get posterior distribution of \( \theta \) under Hartigan and inverted gamma priors.

For \( a = 1 \) and \( i = 2 \), we get posterior distribution of \( \theta \) under Jeffery’s and inverted gamma priors.

For \( a > 0 \) (other than 1 & 3) and \( i = 3 \), we get posterior distribution of \( \theta \) under general non informative and inverted gamma priors. For \( a = 0 \) and \( i = 4 \), we get posterior distribution of \( \theta \) under only single inverted gamma prior.

3.3 Bayes Estimator of \( \theta \) and Reliability \( R_d(\theta) \) at Time \( t \):

Here we assume squared error loss function defined as,

\[
L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2
\]

The Bayes estimator of \( \theta \) under squared error loss function is nothing but the posterior mean.

i.e. Mean of the posterior distribution.

\[
\hat{\theta} = E_p[\theta / x]
\]
3. A The Bayes estimator of \( \theta \) under \( i \)th posterior distribution \( \pi_i[\theta / x] \) can be obtained:

\[
\hat{\theta}(i) = E_{\pi}[\theta / x] = \int_0^\infty 2^{\alpha_i} \pi_i \frac{\beta_i^{\alpha_i}}{\alpha_i} e^{-\frac{\beta_i}{\alpha_i} \cdot \theta^{2\alpha_i-1}} d\theta
\]

\[
= \frac{\alpha_i^{\frac{1}{2}} \beta_i^{\frac{1}{2}}}{\alpha_i} \frac{\Gamma_m}{\Gamma_i} \quad \text{..(3.1)}
\]

and

\[
E_{\pi}[\theta^m / x] = \frac{\alpha_i^{\frac{1}{2}} \beta_i^{\frac{1}{2}}}{\alpha_i} \frac{\Gamma_m}{\Gamma_i} \quad \text{\( i=1,2,3,4. \) ..(3.2)}
\]

Bayes estimate of \( R(t) \) can be obtain as,

\[
\hat{R}_i(t) = E_{\pi}[R(t) / x] = \left[ 1 + \frac{t^2}{2\beta_i} \right]^{-\alpha_i} \quad \text{..(3.3)}
\]

3.3.1 Bayes Equal Tail Credible Interval for \( \theta \) and \( R(t) \):

Let \([ I_{1i}, I_{2i} \] be the Bayes equal tail credible interval for \( \theta \)

Here, \( \int_0^{I_{2i}} \pi_i(\theta / x) d\theta = \frac{\alpha_i}{2} = \int_{I_{1i}}^{\infty} \pi_i(\theta / x) d\theta \)

Consider, \( \int_0^{I_{2i}} \pi_i(\theta / x) d\theta = \frac{\alpha_i}{2} \)

\[
\Rightarrow \int_0^{I_{2i}} 2^{\alpha_i} \frac{\beta_i^{\alpha_i}}{\alpha_i} e^{-\frac{\beta_i}{\alpha_i} \cdot \theta^{2\alpha_i-1}} d\theta = \frac{\alpha_i}{2}
\]

Let \( w_i = \frac{\beta_i^{\alpha_i}}{\alpha_i} \Rightarrow d\theta = \frac{\beta_i^{\alpha_i}}{\alpha_i} dw_i \)
\[ \Rightarrow \int \frac{2\beta_i^{\gamma_i}}{\alpha_i} e^{-w_i} \left( \frac{\sqrt{w_i}}{\sqrt{\pi}} \right)^{-2} \left( -\frac{\beta_i^{1/2}}{2w_i^{1/2}} \right) dw_i = \frac{\gamma}{2} \]

\[ \Rightarrow \int \frac{e^{-w_i} w_i^{\gamma_i-1}}{\alpha_i} dw_i = \frac{\gamma}{2} \]

\[ \Rightarrow \int_0^\infty e^{-w_i} w_i^{\gamma_i-1} \frac{\beta_i}{\alpha_i} dw_i = 1 - \frac{\gamma}{2} \]

\[ \Rightarrow \frac{\beta_i^{\gamma_i}}{I_{2i}} = 1 - \frac{\gamma}{2} \quad \text{(3.4)} \]

Similarly, for \[ \Rightarrow \int \frac{\beta_i^{\gamma_i}}{I_{2i}} = \frac{\gamma}{2} \quad \text{..(3.5)} \]

Bayes equal tail credible interval for \( R(t) = e^{-t^2/2\theta^2} \):

First, we obtain posterior distribution of \( R(t) \) using the posterior distribution of \( \theta \).

Using \( R_t = e^{-t^2/2\theta^2} \Rightarrow \ln R(t) = -t^2/2\theta^2 \Rightarrow \frac{d\theta}{dR_t} = \frac{\theta^3}{t^2 R_t} \Rightarrow \theta^2 = -\frac{t^2}{2\ln R_t} \)

Here, \( \pi_t (R_t / \theta) = \frac{2\beta_i^{\gamma_i}}{\alpha_i} e^{-t^2/2\theta^2} \left( \frac{t}{2\ln R_t} \right)^{-2\gamma_i-1} \left( -\frac{t^2}{2\ln R_t} \right)^{2\gamma_i - 1} \)

\[ = \frac{2\beta_i^{\gamma_i}}{\alpha_i} (R_t)^{2\gamma_i - 1} \left( \frac{2\gamma_i + \frac{1}{2} \frac{3}{2}}{t^{2\gamma_i - 1 - \frac{3}{2}}} \right) \frac{1}{(-\ln R_t)^{\frac{1}{2} + \frac{1}{2}}} \]
\[
= \frac{\beta_i^{\alpha_i} 2^{\gamma_i}}{\alpha_i (t^2)^{\gamma_i}} \left( R_i \right)^{\frac{2\beta_i}{t^2} - 1} (-\ln R_i)^{a_i-1}, \quad 0 < R_i < 1
\]

Hence Bayes equal tail credible interval for \( R_i \), given by \([ h_{l_i} , h_{r_i} ]\) can be obtained by solving the equations.

\[
\int_0^{h_{l_i}} \pi_i (R_i / x) dR_i = \frac{\gamma_i}{2} = \int_{h_{r_i}}^1 \pi_i (R_i / x) dR_i
\]

Consider, \( \int_0^{h_{l_i}} \pi_i (R_i / x) dR_i = \frac{\gamma_i}{2} \)

\[
\Rightarrow \int_0^{h_{l_i}} \frac{\beta_i^\alpha_i}{\alpha_i \left( t^2 \right)^\gamma_i} \left( R_i \right)^{\frac{2\beta_i}{t^2} - 1} (-\ln R_i)^{a_i-1} dR_i = \frac{\gamma_i}{2}
\]

Let, \( y = -\ln R_i \Rightarrow R_i = e^y \& \Rightarrow dR_i = -e^{-y} dy \)

\[
\Rightarrow \int_{-\ln h_{l_i}}^{0} \frac{\beta_i^\alpha_i}{\alpha_i \left( t^2 \right)^\gamma_i} \left( e^y \right)^{\frac{2\beta_i}{t^2} - 1} e^{-y} e^{-y} dy = \frac{\gamma_i}{2}
\]

\[
\Rightarrow \int_{-\ln h_{l_i}}^{\infty} \frac{\beta_i^\alpha_i}{\alpha_i \left( t^2 \right)^\gamma_i} \left( e^y \right)^{\frac{2\beta_i}{t^2} - 1} e^{-y} dy = \frac{\gamma_i}{2}
\]

Let \( u = \frac{y 2\beta_i}{t^2} \Rightarrow du = \frac{2\beta_i}{t^2} dy \)

\[
\Rightarrow \int_{-\ln h_{l_i}}^{\infty} \frac{\beta_i^\alpha_i}{\alpha_i \left( t^2 \right)^\gamma_i} \left( e^u \right)^{\frac{2\beta_i}{t^2} - 1} e^{-u} e^{-u} \frac{2\beta_i}{t^2} du = \frac{\gamma_i}{2}
\]

\[
\Rightarrow \int_{0}^{\frac{(-\ln h_{l_i}) 2\beta_i}{t^2}} \frac{\alpha_i}{\alpha_i \left( t^2 \right)^\gamma_i} \left( e^u \right)^{\frac{2\beta_i}{t^2} - 1} du = 1 - \frac{\gamma_i}{2}
\]

\[
\Rightarrow \frac{(-\ln h_{l_i}) 2\beta_i}{t^2} \frac{\Gamma_i}{\alpha_i} = 1 - \frac{\gamma_i}{2}
\]

..(3.6)
Similarly, for 
\[ \int_{h_{21}}^{1} \pi_{1}(R_{t} / x) dR_{t} = \frac{\gamma}{2} \]

\[\Rightarrow \frac{(-\ln h_{22})2^{/\beta_{i}}}{t^{2}} \frac{e^{-u} e^{-\alpha_{i}-1}}{\alpha_{i}} du = \frac{\gamma}{2}\]

\[\Rightarrow \frac{(-\ln h_{22})2^{\beta_{i}} \Gamma_{i}}{t^{2}} = \frac{\gamma}{2} \]

..(3.7)

### 3.4 Bayes Predictive Estimator and (1-\(\alpha\))100% Equal Tail Credible Interval for a Future Observation:

Let \(Z\) be a future observation which has already survived \(X_{(i)}\) and let \(y=Z-X_{(i)}\). Given the data \(X\), the conditional joint pdf of \(y\) and \(\theta\) is,

\[h_{i}(y,\theta / x) = f(y / \theta \times) \cdot \pi_{i}(\theta / x)\]

\[= \frac{y}{\theta} e^{-2/\theta^{2}} \frac{2^{\beta_{i}}}{\alpha_{i}} \frac{e^{-\beta_{i}}}{e^{-\beta_{i}} \cdot \theta^{-2/\alpha_{i}-1}}\]

\[= \frac{2^{\beta_{i}}}{\alpha_{i}} y \theta^{-2/\alpha_{i}-3} \frac{e^{-2/\theta^{2}}}{2\theta^{2}} \cdot \theta^{-2/\alpha_{i}-1}\]

Integrating out \(\theta\) and restoring the normalizing constant, the predictive density of \(y\) is,

\[P(y / x) \propto \int_{0}^{\infty} \theta^{-2/\alpha_{i}-3} e^{-2/\theta^{2}} d\theta \frac{2^{\beta_{i}}}{\alpha_{i}} \]

50
\[
\frac{y^\alpha_i+1}{2} \cdot \frac{2 \beta_i^\alpha}{(2 \beta_i + y^2)^{\gamma+1}} \cdot \frac{y}{(2 \beta_i + y^2)^{\gamma+1}}, \quad y > 0
\]

Here \( y^2 \) has inverted beta distribution \( \text{Inbe}(1, \alpha_i, 2 \beta_i) \).

Hence the Bayes estimator of \( y \) under squared error loss function is,

\[
y^* = E\left[ \frac{y}{x} \right]
\]

\[
= \text{const.} \int_0^\infty \frac{y^2}{(2 \beta_i + y^2)^{\gamma+1}} dy
\]

\[
= \text{const.} \int_0^\infty \frac{y^2}{(2 \beta_i + y^2)^{\gamma+1}} \left(1 + \frac{y^2}{2 \beta_i}\right)^{-\gamma/2} dy
\]

let \( \frac{y^2}{2 \beta_i} = u \Rightarrow dy = \frac{\beta_i}{2 \beta_i u} du
\]

\[
y^* = \text{const.} \int_0^\infty \frac{2 \beta_i u}{(2 \beta_i)^{\gamma+1}} \left(1 + u\right)^{-\gamma/2} \sqrt{2 \beta_i u} du
\]

\[
= \text{const.} \sqrt{2 \beta_i} \int_0^{\sqrt{u}} \frac{\sqrt{u}}{(2 \beta_i)^{\gamma+1}} dy
\]

\[
= \frac{2 (2 \beta_i)^{\gamma}}{(\beta(\alpha_i, 1))^{\gamma+1}} \left(2 \beta_i\right)^{\gamma+1} \left(\frac{\beta_i}{2 \beta_i^2 + 1}\right)\frac{1}{2}
\]
\[
- \sqrt{\frac{2 \beta_i}{\gamma_i}} \frac{3}{2} \alpha_i - \frac{1}{2}
\]  

..(4.1)

Hence the Bayes predictive estimator of \( Z \) becomes \( Z^* = y^* + X(a) \).  

..(4.2)

Bayes credible equal tail interval for \( y \):

Let \( (h_1^*, h_2^*) \) be the Bayes equal tail \( \left( \frac{y}{2} \right) \) credible interval for \( y \) then,

\[
P(y < h_1^*) = \frac{y}{2} = P(y > h_2^*)
\]

Consider \( P(y < h_1^*) = \frac{y}{2} \)

\[
\Rightarrow \int_0^n P(y/x) dy = \frac{y}{2}
\]

\[
\Rightarrow h_1^* \left[ \int_0^n 2(2 \beta_i)^{\alpha_i} \frac{y}{(2 \beta_i + y^2)^{\alpha_i + 1}} dy = \frac{y}{2} \right]
\]

Let \( y^2 + 2 \beta_i = u \Rightarrow 2y dy = du 

\[
\Rightarrow \int_{h_1^*}^{h_2^*} \left( \frac{2 \beta_i}{y^2} \right)^{\alpha_i} \frac{y}{u^{\alpha_i + 1}} du = \frac{y}{2}
\]

\[
\Rightarrow 1 - \left( \frac{2 \beta_i}{(h_1^*)^2 + 2 \beta_i} \right)^{\alpha_i} = \frac{y}{2}
\]
\[
\Rightarrow h_1^* = \sqrt{\frac{2\beta_i}{\frac{1}{\alpha_i} - 2\beta_i}} \sqrt{\left(1 - \frac{\nu}{2}\right)}
\]

..(4.3)

Similarly for, \( P(y > h_2^*) = \frac{\nu}{2} \)

\[
\Rightarrow P(y < h_2^*) = 1 - \frac{\nu}{2}
\]

\[
\Rightarrow h_2^* = \sqrt{\frac{2\beta_i}{\frac{1}{\alpha_i} - 2\beta_i}} \sqrt{\left(1 - \frac{\nu}{2}\right)}
\]

..(4.4)

### 3.5 Bayes Predictive Estimator for the Remaining \((n-r)\) Order Statistics

**Truncated at** \(X(r)\) **and Their \((1-\alpha)100\%\) Equal Tail Credible Interval:**

Let \(X(s), r+1 \leq S \leq n\) denote the failure time of \(S^{th}\) unit to fail. The conditional pdf of \(w = X(s) - X(r)\) from pdf truncated at \(X(r)\) is given by,

\[
f(w|\theta) = \frac{(F(w))^{\nu-1}}{\beta_{(3-r,n-r-1)}} \left[1 - F(w)\right]^{\nu-2} \cdot f(w) ; \quad w \geq 0, \quad r+1 \leq S \leq n
\]

Using the pdf &cdf of \(X\), from (1.1) and (1.2) we get

\[
f(w|\theta) = \frac{w^{(n-r-1)w^2}}{\beta_{(3-r,n-r-1)}} \left[1 - e^{-\frac{w}{2\sigma^2}}\right]^{\nu-2} ; \quad w \geq 0, \quad r+1 \leq S \leq n.
\]

Given \(X\), the conditional joint pdf \(w\) and \(\theta\) is.
\[ f_i(w, \theta / x) = f(w / \theta) \cdot \pi_i(\theta / x) \]

\[ w \cdot e^{-(\frac{n-x+1)}{2\beta^2}}} \sum_{j=0}^{r-1} (-1)^j (s-r-j)! \frac{2 \beta_i^{2q+1}}{\alpha_i} \cdot \frac{e^{-(A_i + jw^2) / 2\theta^2}}{\theta^{2\alpha_i-1}} \]

Integrating out \( \theta \), the predictive density of \( w \) is,

\[ f_i(w / x) = \frac{2 \beta_i^{2q} \cdot w \cdot \sum_{j=0}^{r-1} (-1)^j (s-r-j)! \frac{\alpha_i}{\beta_i}}{\beta_i(s-r-n+1) \cdot \alpha_i} \cdot \frac{1}{\theta^{2\alpha_i-1}} \]

Under a squared error loss function, Bayes predictive estimator of \( w \) is,

\[ w^* = E(w / x) = \left( \frac{\alpha_i}{\beta_i} \right) \frac{1}{\beta_i(s-r-n+1)} \sum_{j=0}^{r-1} (-1)^j (s-r-j)! \frac{w^2}{\theta^{2\alpha_i-1}} \int \left[ 1 + \left( \frac{n-s+j+1}{2\beta_i} \right)^2 \right] dw \]

\[ = \left( \frac{\alpha_i}{\beta_i} \right) \frac{1}{\beta_i(s-r-n+1)} \sum_{j=0}^{r-1} (-1)^j (s-r-j)! \frac{\beta_i^{2q-1} / \theta^{2\alpha_i-1}}{2} \left( \frac{n-s+j+1}{2\beta_i} \right)^{\alpha_i-1} \]
Thus the Bayes predictive estimator of $X_{(s)}$ is,

$$X_{(s)} = X_{(r)} + w^* \quad r+1 \leq S \leq n$$

Particularly, for $S = r+1$,

$$w^* = \frac{\alpha_i \beta_{\frac{3}{2}, \frac{1}{2}, \frac{1}{2}} \sqrt{2 \beta_i}}{\beta_{(1, n-r)}} \cdot \frac{1}{(n-r)^{1/2}}$$

$$= \frac{\alpha_i \beta_{\frac{3}{2}, \frac{1}{2}, \frac{1}{2}} \sqrt{2 \beta_i}}{(n-r)^{1/2}} \quad \text{(5.1)}$$

And so on $w^*$ can be obtained for $S = r+2, r+3, \ldots$.

$(1-\alpha)100\%$ equal tail credible interval $(H_1, H_2)$ for $w$ is obtained by solving the equations,

$$\int_0^{H_1} f_i(w|x)dw = \frac{\gamma}{2} = \int_{H_2}^\infty f_i(w|x)dw$$

Consider, \[ \int_0^{H_1} f_i(w|x)dw = \frac{\gamma}{2} \]

$$\Rightarrow \int_0^{H_1} \left( \frac{\alpha_i}{\beta_i} \right) \frac{1}{\beta_{(s-r,n-s+1)}} \sum_{j=0}^{s-r-1} (-1)^j(s-r-j-1)! \frac{w}{j!(s-r-j-1)!} \frac{1}{1 + \left( \frac{n-s+j+1}{2\beta_i} \right)^2} dw = \frac{\gamma}{2}$$

Let

$$u = \frac{1}{1 + \left( \frac{n-s+j+1}{2\beta_i} \right)^2} \Rightarrow -\frac{1}{u^2} du = \left( \frac{n-s+j+1}{2\beta_i} \right)^2 wdw$$
\[
\begin{align*}
\Rightarrow \frac{1}{\beta(s-r,n-s+1)} \sum_{j=0}^{s-r-1} \frac{(-1)^j (s-r-1)!}{j!(s-r-j-1)!} \left( \frac{1}{n-s+j+1} \right) u^{\alpha_j+1} du = \frac{\gamma}{2} \\
\Rightarrow \frac{1}{\beta(s-r,n-s+1)} \sum_{j=0}^{s-r-1} \frac{(-1)^j (s-r-1)!}{j!(s-r-j-1)!} \left( \frac{1}{n-s+j+1} \right) \left[ \frac{1}{1+\left( \frac{n-s+j+1}{2\beta} \right) H_1^2} \right]^{\alpha_j} - 1 = \frac{\gamma}{2}
\end{align*}
\]

..(5.2)

Similarly by solving, \( \int_{H_2} f_i(w/x) dw = \frac{\gamma}{2} \)

\[
\Rightarrow 1 - \int_{0}^{H_2} f_i(w/x) dw = \frac{\gamma}{2}
\]

\[
\Rightarrow \frac{1}{\beta(s-r,n-s+1)} \sum_{j=0}^{s-r-1} \frac{(-1)^j (s-r-1)!}{j!(s-r-j-1)!} \left( \frac{1}{n-s+j+1} \right) \left[ 1+\left( \frac{n-s+j+1}{2\beta} \right) H_2^2 \right]^{-\alpha_j} = 1- \frac{\gamma}{2}
\]

..(5.3)

Solving the above equations (5.2) and (5.3), we get \( H_1 \) and \( H_2 \). Hence \((1-\alpha)100\%\) credible equal tail interval \( \left( H_1^*, H_2^* \right) \) for \( X_{(r)} \) becomes

\( \left( H_1 + X_{(r)}, H_2 + X_{(r)} \right) \) for \( S = r+1, r+2, \ldots \)

Particularly for \( S=r+1 \), the credible interval for \( X_{(r+1)} \) can be deduced from (5.2) and (5.3) by pitting \( S=r+1 \),

Thus we get,
\[
\frac{1}{\beta_{r(1,n-r)}} \left( \frac{1}{n-r} \right) \left[ 1 - \left( \frac{1}{1 + \left( \frac{n-r}{2\beta_i} \right) H_i^2} \right)^{\gamma_i} \right] = \frac{\gamma}{2}
\]

\[
H_i = \left( \left( 1 - \frac{\gamma_i}{2} \right)^{\frac{1}{\gamma_i}} - 1 \right) \frac{2\beta_i}{n-r}, \quad \text{..(5.4)}
\]

and similarly

\[
H_2 = \left( \left( \frac{\gamma_i}{2} \right)^{\frac{1}{\gamma_i}} - 1 \right) \frac{2\beta_i}{n-r}. \quad \text{..(5.5)}
\]

### 3.6 A Real Life Example:
In this section, we consider a real data set presented in Lawless (1982), p. 228) for illustrative purpose. This data was originally discussed by Leiblein and Zelen(1956) during the endurance test of 23 deep groove ball bearings, and the failure times (in hundreds of millions revolutions) are:

0.179, 0.289, 0.330, 0.415, 0.421, 0.456, 0.485, 0.518, 0.519, 0.541, 0.555, 0.678, 0.686, 0.686, 0.689, 0.841, 0.931, 0.986, 1.051, 1.058, 1.279, 1.280, 1.734

Her we consider \( r = 7 \) for type II censoring without replacement, so we use only first seven ordered values. The results for Bayes estimators and corresponding Bayes equal tail 95% credible intervals under different choice of parameters (\( b_i, c_i \)) for different types of double priors and only single priors are obtained as below:

In each cell the first value denotes the Bayes estimate and second denotes the credible interval.
<table>
<thead>
<tr>
<th>Double priors</th>
<th>(b_i, c_i)</th>
<th>θ</th>
<th>R(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>0.574021</td>
<td>0.963134</td>
</tr>
<tr>
<td></td>
<td>0.407154, 0.831920</td>
<td>(0.934388, 0.983876)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>0.683606</td>
<td>0.973846</td>
</tr>
<tr>
<td></td>
<td>0.484898, 0.990762</td>
<td>(0.953280, 0.988605)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>0.688808</td>
<td>0.974733</td>
</tr>
<tr>
<td></td>
<td>0.507283, 0.956837</td>
<td>(0.957225, 0.987787)</td>
<td></td>
</tr>
<tr>
<td>Jeffery’s - Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>0.674372</td>
<td>0.972221</td>
</tr>
<tr>
<td></td>
<td>0.452713, 1.041924</td>
<td>(0.946588, 0.989691)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>0.683631</td>
<td>0.973846</td>
</tr>
<tr>
<td></td>
<td>0.484898, 0.990770</td>
<td>(0.953280, 0.988605)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>0.777945</td>
<td>0.979735</td>
</tr>
<tr>
<td></td>
<td>0.551795, 1.127458</td>
<td>(0.963726, 0.991189)</td>
<td></td>
</tr>
<tr>
<td>General non informative</td>
<td>(0.5, 0.5)</td>
<td>0.618178</td>
<td>0.967667</td>
</tr>
<tr>
<td>prior - Inverted Gamma</td>
<td></td>
<td>0.427894, 0.921600</td>
<td>(0.940406, 0.986842)</td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>0.683631</td>
<td>0.973846</td>
</tr>
<tr>
<td></td>
<td>0.484898, 0.990770</td>
<td>(0.953280, 0.988605)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>0.729325</td>
<td>0.977231</td>
</tr>
<tr>
<td></td>
<td>0.527812, 1.032870</td>
<td>(0.960422, 0.989510)</td>
<td></td>
</tr>
<tr>
<td>Only Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>0.674372</td>
<td>0.972221</td>
</tr>
<tr>
<td></td>
<td>0.452713, 1.041924</td>
<td>(0.946588, 0.989691)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>0.736215</td>
<td>0.977078</td>
</tr>
<tr>
<td></td>
<td>0.509598, 1.097576</td>
<td>(0.957604, 0.990705)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>0.777945</td>
<td>0.979735</td>
</tr>
<tr>
<td></td>
<td>0.551795, 1.127458</td>
<td>(0.963726, 0.991189)</td>
<td></td>
</tr>
<tr>
<td>Double priors</td>
<td>((b_i, c_i))</td>
<td>(Z^*)</td>
<td>(X_{(r+1)})</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-------------------------</td>
<td>---------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>Hartigan – Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>1.203895 (0.608099, 2.158525)</td>
<td>0.659358 (0.514856, 0.890890)</td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>1.341164 (0.631605, 2.478076)</td>
<td>0.692650 (0.520557, 0.968392)</td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>1.347648 (0.634194, 2.464215)</td>
<td>0.694228 (0.521185, 0.965030)</td>
</tr>
<tr>
<td>Jeffery’s - Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>1.329569 (0.627181, 2.500017)</td>
<td>0.689838 (0.519484, 0.973713)</td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>1.341164 (0.631605, 2.478076)</td>
<td>0.692650 (0.520557, 0.968392)</td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>1.459281 (0.651831, 2.753043)</td>
<td>0.721298 (0.525462, 1.035081)</td>
</tr>
<tr>
<td>General non informative prior - Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>1.259192 (0.616614, 2.306238)</td>
<td>0.672769 (0.516921, 0.926715)</td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>1.341164 (0.631605, 2.478076)</td>
<td>0.692650 (0.520557, 0.968392)</td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>1.398391 (0.642276, 2.594162)</td>
<td>0.706530 (0.523145, 0.996547)</td>
</tr>
<tr>
<td>Only Inverted Gamma</td>
<td>(0.5, 0.5)</td>
<td>1.329569 (0.627181, 2.500017)</td>
<td>0.689838 (0.519484, 0.973713)</td>
</tr>
<tr>
<td></td>
<td>(1, 1)</td>
<td>1.407020 (0.641745, 2.653995)</td>
<td>0.708623 (0.523016, 1.011058)</td>
</tr>
<tr>
<td></td>
<td>(2, 2)</td>
<td>1.459281 (0.651831, 2.753043)</td>
<td>0.721298 (0.525462, 1.035081)</td>
</tr>
</tbody>
</table>
From the third and fourth column of Tables I to II it is observed that the length of the credible intervals for $\theta$, $R(t)$, $Z^*$, and $X_{(r+1)}$ are smallest in case of Hartigan – Inverted Gamma joint prior and largest for only inverted gamma prior for values of $b_i = c_i = 0.5$, 1.0 and 2.0.

3.7 Simulation Study:

A Monte Carlo simulation study is carried out to compare the performance of the Bayes estimators under different joint priors and single prior. To generate 1000 Type-II censored samples the value of the parameter $\theta$ is considered as 0.3 and the values of the hyper parameters for all joint and single priors are considered as $b_i = c_i = 0.5$, 1, 2. The reliability is calculated at time $t = 0.5$. Simulation is done for sample size $n = 20$ and of fixed censored value $r = 5$. In each case Bayes estimates of $\theta$, $R(t)$, future observation $Z^*$ and $(r+1)^{th}$ ordered failure time $X_{(r+1)}$ are obtained. Their mean square errors (MSE) and Bayes equal tail 95% credible intervals are also obtained. The first, second and third values in each cell of columns second and third of Tables III to VIII denote the Bayes estimate, MSE and credible intervals.

Table III.  Bayes estimates and Credible intervals for $\theta$ and $R(t)$, for $b_i = c_i = 0.5$

<table>
<thead>
<tr>
<th>Joint priors</th>
<th>$\theta$</th>
<th>$R(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – Inverted Gamma</td>
<td>0.288547</td>
<td>0.223042</td>
</tr>
<tr>
<td></td>
<td>0.002997</td>
<td>0.011128</td>
</tr>
<tr>
<td></td>
<td>(0.193705, 0.445813)</td>
<td>(0.048791, 0.514363)</td>
</tr>
<tr>
<td>Jeffery’s - Inverted Gamma</td>
<td>0.366398</td>
<td>0.358460</td>
</tr>
<tr>
<td></td>
<td>0.009029</td>
<td>0.025090</td>
</tr>
<tr>
<td></td>
<td>(0.223538, 0.633682)</td>
<td>(0.094588, 0.713631)</td>
</tr>
<tr>
<td>General non informative prior - Inverted Gamma</td>
<td>0.320605</td>
<td>0.281972</td>
</tr>
<tr>
<td></td>
<td>0.003962</td>
<td>0.013176</td>
</tr>
<tr>
<td></td>
<td>(0.206780, 0.519194)</td>
<td>(0.067207, 0.608681)</td>
</tr>
<tr>
<td>Only Inverted Gamma</td>
<td>0.366398</td>
<td>0.358460</td>
</tr>
<tr>
<td></td>
<td>0.009029</td>
<td>0.025090</td>
</tr>
<tr>
<td></td>
<td>(0.223538, 0.633683)</td>
<td>(0.094588, 0.713631)</td>
</tr>
</tbody>
</table>
### Table IV. Bayes estimates and Credible intervals for $\theta$ and R(t) for $b_1 = c_1 = 1$

<table>
<thead>
<tr>
<th>Joint priors</th>
<th>$\theta$</th>
<th>R(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – inverted Gamma</td>
<td>0.524870</td>
<td>0.606729</td>
</tr>
<tr>
<td></td>
<td>0.051460</td>
<td>0.128755</td>
</tr>
<tr>
<td></td>
<td>(0.352351, 0.810938)</td>
<td>(0.364191, 0.825597)</td>
</tr>
<tr>
<td>Jeffery’s - inverted Gamma</td>
<td>0.524870</td>
<td>0.606729</td>
</tr>
<tr>
<td></td>
<td>0.051460</td>
<td>0.128755</td>
</tr>
<tr>
<td></td>
<td>(0.352351, 0.810938)</td>
<td>(0.364191, 0.825597)</td>
</tr>
<tr>
<td>General non informative prior - inverted Gamma</td>
<td>0.524870</td>
<td>0.606729</td>
</tr>
<tr>
<td></td>
<td>0.051460</td>
<td>0.128755</td>
</tr>
<tr>
<td></td>
<td>(0.352351, 0.810938)</td>
<td>(0.364191, 0.825597)</td>
</tr>
<tr>
<td>Only inverted gamma</td>
<td>0.583183</td>
<td>0.659292</td>
</tr>
<tr>
<td></td>
<td>0.081295</td>
<td>0.168899</td>
</tr>
<tr>
<td></td>
<td>(0.376135, 0.944417)</td>
<td>(0.41170, 0.868189)</td>
</tr>
</tbody>
</table>

### Table V. Bayes estimates and Credible intervals for $\theta$ and R(t) for $b_1 = c_1 = 2$

<table>
<thead>
<tr>
<th>Joint priors</th>
<th>$\theta$</th>
<th>R(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – inverted Gamma</td>
<td>0.581127</td>
<td>0.670960</td>
</tr>
<tr>
<td></td>
<td>0.079420</td>
<td>0.178053</td>
</tr>
<tr>
<td></td>
<td>(0.412192, 0.842214)</td>
<td>(0.478547, 0.837991)</td>
</tr>
<tr>
<td>Jeffery’s - inverted Gamma</td>
<td>0.682717</td>
<td>0.741303</td>
</tr>
<tr>
<td></td>
<td>0.147007</td>
<td>0.242221</td>
</tr>
<tr>
<td></td>
<td>(0.458315, 1.054816)</td>
<td>(0.550841, 0.893423)</td>
</tr>
<tr>
<td>General non informative prior - inverted Gamma</td>
<td>0.625827</td>
<td>0.705252</td>
</tr>
<tr>
<td></td>
<td>0.106613</td>
<td>0.208097</td>
</tr>
<tr>
<td></td>
<td>(0.433189, 0.933005)</td>
<td>(0.513047, 0.865859)</td>
</tr>
<tr>
<td>Only inverted gamma</td>
<td>0.682717</td>
<td>0.741303</td>
</tr>
<tr>
<td></td>
<td>0.147007</td>
<td>0.242221</td>
</tr>
<tr>
<td></td>
<td>(0.458315, 1.054816)</td>
<td>(0.550841, 0.893423)</td>
</tr>
</tbody>
</table>
Table VI. Bayes estimates and Credible intervals for $Z^*$ and $x_{(r+1)}$ for $b_i = c_i = 0.5$

<table>
<thead>
<tr>
<th>Joint priors</th>
<th>$Z^*$</th>
<th>$x_{(r+1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – inverted Gamma</td>
<td>0.581969</td>
<td>0.3109480.738994</td>
</tr>
<tr>
<td></td>
<td>0.011617</td>
<td>0.003312</td>
</tr>
<tr>
<td></td>
<td>(0.281434, 1.082774)</td>
<td>(0.235807, 436142)</td>
</tr>
<tr>
<td>Jeffery’s - inverted Gamma</td>
<td>0.679467</td>
<td>0.335316</td>
</tr>
<tr>
<td></td>
<td>0.015842</td>
<td>0.003851</td>
</tr>
<tr>
<td></td>
<td>(0.295146, 1.372061)</td>
<td>(0.239235, 0.508464)</td>
</tr>
<tr>
<td>General non informative prior - inverted Gamma</td>
<td>0.622117</td>
<td>0.320978</td>
</tr>
<tr>
<td></td>
<td>0.013277</td>
<td>0.003529</td>
</tr>
<tr>
<td></td>
<td>(0.287255, 1.197904)</td>
<td>(0.237262, 464925)</td>
</tr>
<tr>
<td>Only inverted gamma</td>
<td>0.679467</td>
<td>0.335316</td>
</tr>
<tr>
<td></td>
<td>0.015842</td>
<td>0.003851</td>
</tr>
<tr>
<td></td>
<td>(0.295146, 1.372061)</td>
<td>(0.239235, 0.508464)</td>
</tr>
</tbody>
</table>

Table VII. Bayes estimates and Credible intervals for $Z^*$ and $x_{(r+1)}$ for $b_i = c_i = 1$

<table>
<thead>
<tr>
<th>Joint priors</th>
<th>$Z^*$</th>
<th>$x_{(r+1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – inverted Gamma</td>
<td>0.877933</td>
<td>0.384932</td>
</tr>
<tr>
<td></td>
<td>0.006102</td>
<td>0.002513</td>
</tr>
<tr>
<td></td>
<td>(0.331259, 1.788903)</td>
<td>(0.248263, 612674)</td>
</tr>
<tr>
<td>Jeffery’s - inverted Gamma</td>
<td>0.877933</td>
<td>0.384932</td>
</tr>
<tr>
<td></td>
<td>0.006102</td>
<td>0.002513</td>
</tr>
<tr>
<td></td>
<td>(0.331259, 1.788903)</td>
<td>(0.248263, 612674)</td>
</tr>
<tr>
<td>General non informative prior - inverted Gamma</td>
<td>0.877933</td>
<td>0.384932</td>
</tr>
<tr>
<td></td>
<td>0.006102</td>
<td>0.002513</td>
</tr>
<tr>
<td></td>
<td>(0.331259, 1.788903)</td>
<td>(0.248263, 612674)</td>
</tr>
<tr>
<td>Only inverted gamma</td>
<td>0.950964</td>
<td>0.403190</td>
</tr>
<tr>
<td></td>
<td>0.006768</td>
<td>0.002617</td>
</tr>
<tr>
<td></td>
<td>(0.341846, 1.998324)</td>
<td>(0.250910, 0.665030)</td>
</tr>
</tbody>
</table>
Table VIII. Bayes estimates and Credible intervals for $Z^r$ and $x_{(r+1)}$ for $b_i = c_i = 2$

<table>
<thead>
<tr>
<th>Joint priors</th>
<th>$Z^r$</th>
<th>$x_{(r+1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hartigan – inverted Gamma</td>
<td>0.948389</td>
<td>0.402546</td>
</tr>
<tr>
<td></td>
<td>0.004271</td>
<td>0.002204</td>
</tr>
<tr>
<td></td>
<td>(0.345221, 1.914832)</td>
<td>(0.251754, 0.644157)</td>
</tr>
<tr>
<td>Jeffery’s - inverted Gamma</td>
<td>1.075618</td>
<td>0.434353</td>
</tr>
<tr>
<td></td>
<td>0.004851</td>
<td>0.002305</td>
</tr>
<tr>
<td></td>
<td>(0.364538, 2.260550)</td>
<td>(0.256583, 0.730586)</td>
</tr>
<tr>
<td>General non informative prior -</td>
<td>1.004371</td>
<td>0.416542</td>
</tr>
<tr>
<td>inverted Gamma</td>
<td>0.004521</td>
<td>0.002248</td>
</tr>
<tr>
<td></td>
<td>(0.353841, 2.064373)</td>
<td>(0.253909, 0.681542)</td>
</tr>
<tr>
<td>Only inverted gamma</td>
<td>1.075618</td>
<td>0.434353</td>
</tr>
<tr>
<td></td>
<td>0.004851</td>
<td>0.002305</td>
</tr>
<tr>
<td></td>
<td>(0.364538, 2.260550)</td>
<td>(0.256583, 0.730586)</td>
</tr>
</tbody>
</table>

3.8 Conclusions:

8-A. Comparison of priors based on the MSE and credible interval of $\theta$.
From the second column of Tables III to V it is observed that the values of the MSE of the Bayes estimator of parameter $\theta$ and length of its credible intervals are smallest in case of Hartigan – Inverted Gamma joint prior and largest for only inverted gamma prior for values of $b_i = c_i = 0.5, 1.0$ and $2.0$.

8-B. Comparison of priors based on the MSE and credible interval of $R(t)$.
From the third column of Tables III to V it is observed that the values of the MSE of the Bayes estimator of R(t) and length of its credible intervals are smallest in case of Hartigan – Inverted Gamma joint prior and largest for only inverted gamma prior for values of $b_i = c_i = 0.5, 1.0$ and $2.0$.

8-C. Comparison of priors based on MSE and credible interval of future predicted value.
From the third column of Tables VI to VIII it is observed that the values of the MSE of the Bayes estimator of future predicted value and length of its credible intervals are smallest in case of Hartigan – Inverted Gamma joint prior and largest for only inverted gamma prior for values of $b_i = c_i = 0.5, 1.0$ and $2.0$. 
8-D. Comparison based on the MSE and credible interval of next ordered failure time $X_{(r+1)}$.

From the fourth column of Tables VI to VIII it is observed that the values of the MSE of the Bayes estimator of future predicted value and length of its credible intervals are smallest in case of Hartigan – Inverted Gamma joint prior and largest for only inverted gamma prior for values of $b_i = c_i = 0.5, 1.0$ and 2.0. Thus we observed that Hartigan – Inverted Gamma joint prior performs well compared to the joint priors Jeffery’s - inverted Gamma, General non informative prior - inverted Gamma and single prior inverted gamma for different choice of the prior parameters considered in this study.