1. INTRODUCTION

The Quality of a product depends upon its ability to meet the specifications over its entire life time. Quality of the product is directly related with the reliability of the product. A product with a good quality naturally have a more reliability. Reliability is one of the quality characteristics tool of the product. The warranty period of the product depends on the quality and hence reliability of the product. A product with good quality or a with very high reliability, its warranty period may be very large. The three terms Quality, reliability and warranty can be explained as follow.

1.1 QUALITY

The general meaning of quality is a product satisfying one or more desirable characteristics that a product should possess which is a useful definition of the quality. The quality of a product can be described and evaluated in several ways. Garvin (1987) provides an excellent discussion of eight components or dimension of quality which are:1. performance 2. reliability 3. durability 4. serviceability 5. aesthetics 6. features 7. perceived quality 8. conformance of standard. Thus we say that quality means fitness for use and is inversely proportional to variability.

Statistical quality control and reliability tests are performed to estimate or demonstrate quality and reliability characteristics on the basis of data collected from sampling tests. Superior productive efficiency in manufacturing can only be achieved through effective quality control reliability planning.

1.1.2 RELIABILITY FUNCTION

Life testing and reliability studies in the past decades has received a substantial amount of interest form theoreticians as well as reliability engineers. Their concern was a
product of the increased complexity and sophistication in electronic, mechanical and structural systems that came into existence very rapidly during this period.

Let \( X \) be a random variable that presents life time of the system whose value is an observed time-to-failure of the system. Then \( R(t) \) is the reliability function, also called the survivor function, can be defined as the probability that the system is still functioning at the time \( t \). In other words, reliability is the probability of no failures in the interval \([0, t]\) or equivalently, the probability of survival over the time \( t \). It can be expressed in terms of the time-to-failure density function, \( f(t) \), or the corresponding cumulative failure distribution function \( F(t) \), that describes the probability of failure up to and including time \( t \). They can be related by

\[
R(t) = P(X \geq t) = 1 - F(t) = \int_{t}^{\infty} f(t)\,dt
\]

Thus reliability means how often does the product fail for example, if an electric bulb survives up to 2000 hrs continuously without failure with probability 0.99 then we say that the reliability of the bulb for survival of 2000 hrs is 0.99. There are many industries in which the customer’s view of quality is greatly impacted by the reliability dimension of quality.

1.1.3 WARRANTY FUNCTION

A warranty is a formal commitment by a manufacturer to provide certain responsibilities for product quality after the sale of the product. Through warranties, customers are provided guarantees for failure free, acceptable service for a period of time following the purchase of a product.

1.1.2 CENSORING SCHEME

Censoring is very common in life test. It usually arises in a life test whenever the experimenter does not observe the life times of all test units.

The life tests are destructive in nature. The cost of life testing experiment directly related with the failures of the items during the test and the total testing time. So, it is advisable to stop the life testing experiment before failure of all the items on the test.
Very roughly a sample is said to be censored if out of n items placed on a life test, only m(<n) of them are actually observed to fail of course, there are many different censoring schemes. The most popular censoring schemes among the various types of censoring schemes are as follows.

1.2.1 TYPE-I CENSORING

Suppose that a life test with n items on the test is terminated as soon as the predetermined time T is observed. Such a censoring scheme is known as Type-I censoring scheme.

During the T time unit of test, we observed r failures (where r can be any number from 0 to n). The exact failure times are $x_1, x_2, \ldots, x_r$ and there are $(n-r)$ units that survived the entire T time unit without failing, where T is fixed in advance and r is random.

The Type-I censoring scheme is used when the cost of experiment heavily increases with time.

1.2.2 TYPE-II CENSORING

The another way of censoring the test is to terminate the test as soon as the predetermined number of failures (say r) observed. Such censoring scheme is called Type-II censoring. Here the test termination time is not fixed it is random. This censoring schemes is used when the test items are very costly. For example, you might put 100 units on test and decide to see at least half of them fail. Then r=50, but the test termination time is random, which is the failure time of the 50th failure.

we observed $x_1, x_2, \ldots, x_r$ where r is specified in advance. The test ends at time $T = x_r$, and $(n-r)$ units have survived. Again we assume it is possible to observe the exact time of failure for failed units. Type-II censoring has the significant advantage that we know in advance how many failure times your test will yield. This helps enormously when planning adequate tests. However, an open ended random test time is generally impractical from a management point of view and this type of testing is rarely seen.
1.2.3 MULTIPLY TYPE-II CENSORING

Suppose n units are put on a life test and the first r, middle l and last s observations are censored that is only ordered observations $x_{r+1}, x_{r+2}, \ldots, x_{r+k}$ and $x_{r+k+l+1}, x_{r+k+l+2}, \ldots, x_{n-s}$ are observed. The scheme is known as the multiply Type-II censoring scheme. This censoring scheme is useful in follow-up studies in epidemiology, reliability and endocrinology.

The general form of the likelihood function based on the above multiply Type-II censoring is given by

$$L(\theta, x) = \frac{n!}{r!l!m!} \left( F(x_{r+1}) \right)^r \left( \prod_{i = r+1}^{r+k} f(x_i) \right) \left[ F(x_{r+k+l+1}) - F(x_{r+k}) \right] \times \left( \prod_{i = r+k+l+1}^{n-m} f(x_i) \right)^m \left[ 1 - F(x_{n-m}) \right]^m$$

The other version of Multiply Type-II censoring has been considered by many authors such as Balakrishnan (1990), Balasubramanian and Balakrishnan (1992), Singh and Kumar (2007), Shah and Patel (2008), in which only the $r_1^{th}, r_2^{th}, \ldots, r_k^{th}$ failures are observed and then test is terminated. Here k is fixed before the experiment.

1.2.4 PROGRESSIVE CENSORING

In the above censoring schemes the test units are withdrawn only at the end of the life testing experiments. That is the withdrawn of the test units at points other than the terminal point of the experiment is not allowed. However, such withdrawal may be advantageous in some situations where an unexpected loss occurs because the test units are lost or withdrawn unintentionally from the experimentation before their failure. Moreover, there are many instance where an intentional removal of units prior to their failure is pre-planned in order to free up testing facilities for further experimentation or to reduce the experimental time and cost. For example, accidental brakeage of an
experimental unit; individual in a clinical trial drop out from the study. The withdrawal of surviving items from the life testing experiment at different stage of censoring or time is known as progressive censoring.

It is not uncommon in our life testing for items to fail for reasons quite unrelated to the normal failure mechanism. For example, consider a number of lamps placed simultaneously on life test. Suppose that some non-failure lamps are withdrawn at the time of first failure of a lamp, again some lamps are withdrawn at the time of second failure and so on and the test then terminated at pre-determined time or number of failures.

In other instances, progressively censored samples result from a compromise between the need for more rapid testing and desire to include at least some extreme life spans in the sample data when test facilities are limited and when prolonged tests are expensive.

### 1.2.5 PROGRESSIVE TYPE-II CENSORING

Cohen (1963) described progressive censoring scheme. There are mainly two type of progressive censoring schemes viz: 1. progressive Type-I censoring 2. Progressive Type-II censoring. The generalization of Type-I censoring is called progressive Type-I censoring and the generalization of Type-II censoring is known as progressive Type-II censoring. It is extremely useful in industrial life testing, clinical trials etc. In our study we have adopted progressive Type-II censoring. The progressive Type-II censoring without replacement can be described as follows.

**Progressive Type-II censoring Scheme**

<table>
<thead>
<tr>
<th>Number of removed</th>
<th>Failure time</th>
<th>Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>x₁</td>
<td>x₁</td>
<td>2</td>
</tr>
<tr>
<td>x₂</td>
<td>m⁻¹</td>
<td>m⁻¹</td>
</tr>
<tr>
<td>xₘ⁻¹</td>
<td>m₀</td>
<td>m</td>
</tr>
</tbody>
</table>

![Progressive Type-II censoring Diagram](image-url)
The scheme is described below. Let \( n \) units are placed on life test. The \( m \) and the \( r_i, i=1,2,\ldots, m-1 \) are fixed constants determined prior to the test. Here at the first failure, \( r_1 \) units are randomly removed from the remaining \( n-1 \) surviving units, at the second failure \( r_2 \) units are randomly removed from the remaining \( n-2-r_1 \) units on the test. The test continues until the \( m^{th} \) failure observed and then all remaining \( r_m = n - m - \sum_{i=1}^{m-1} r_i \) units are removed from the test and the test was terminated.

The general form of the likelihood function based on the progressive Type-II censoring is given by

\[
L(x, \theta) = c \prod_{i=1}^{m} f(x_i, \theta) \left[ 1 - F(x_i, \theta) \right]^{r_i}
\]

Where \( c = n(n-1-r_1)(n-1-r_1-r_2) \ldots (n-m+1-r_1 - \ldots - r_{m-1}) \)

**1.2.6 Generalized Progressive Type-II Censoring**

Suppose \( n \) units were placed on a life test and first \( r \) failure times \( x_1, x_2, \ldots, x_r \) are not observed. At time \( x_{r+1} \), \( R_{r+1} \) units are removed randomly from the survival units on the test, at time \( x_{r+2} \), \( R_{r+2} \) units are removed randomly from the survival units on the test and so on. Finally, experiment is terminated at the time \( x_m \) of the \( m^{th} \) failure by remaining \( R_m \) units. Therefore, \( x_{r+1} \leq \ldots \leq x_m \) are the lifetimes of the completely observed units to fail and there are \( n_i \) units on test at \( (i+1)^{th} \) failure where \( n_i = n - i - \sum_{j=r+1}^{i} R_j, i = r+1, \ldots, m-1 \). Here \( R_{r+1}, R_{r+2}, \ldots, R_m \) are fixed numbers predetermined by the experimenter.
The general form of the likelihood function based on the above described general progressive Type-II censoring is given by

\[ L(\theta, x) = \frac{n!}{r!(n-r)!} \left( \prod_{j=r}^{n} \frac{m-1}{j} \right) \left[ F(x_{r+1}) \right]^{r} \left[ \prod_{i=r+1}^{m} f(x_i, \theta) \left[ 1 - F(x_i) \right] \right]^{R_i} \]

If \( r=0 \) then the general progressive Type-II censoring scheme reduces to progressive Type-II censoring scheme.

### 1.3 Bayesian Approach

In statistical inference the two approaches viz; 1. classical (frequentist) and 2. Bayesian are generally used. In classical method the parameters of the lifetime model are considered as fixed where as in Bayesian approach the parameters are considered as random. Hence the parameters have some probability distributions. Using the prior belief of the experimenter or based on the past information the appropriate distribution of the parameters can be determined. Such distributions of the parameters are known as prior distributions. The relative influence of prior distribution and reliability data on updating beliefs depends on the importance given to the prior distribution and strengths of the data. Under the Bayesian approach the probability model and the prior distribution may be naturally combined to produce a complex model whose analysis has been often well beyond the scope of conventional statistics.
1.3.1 PRIOR DISTRIBUTION

There are different types of prior distributions like informative prior, non-informative prior etc. Informative prior contains a specific probability distribution whereas non-informative prior does not contain any parameters. The most commonly used non-informative prior distributions are Jeffrey’s prior, uniform prior, maximum entropy prior. Let us consider an example of informative prior.

The most widely suitable prior for the parameter $\theta$, $\theta > 0$ of the exponential distribution is the gamma conjugate prior. The gamma conjugate prior for $\theta$ is given by

$$\pi(\theta) = \frac{a^b}{\Gamma(b)} \theta^{b-1}e^{-a\theta}, \theta > 0, a > 0, b > 0$$

1.3.2 POSTERIOR DISTRIBUTION

Let a random variable $X$ follows the life time model and corresponding likelihood function under the given censoring scheme is say $L(\theta, x)$. If $\pi(\theta)$ is prior distribution of parameter, $\theta$ then the joint distribution of $\theta$ and $X$ is given by

$$\phi(\theta, x) = L(\theta, x)\pi(\theta)$$

The marginal distribution of $X$ is given by

$$m(x) = \int_{\theta} \phi(\theta, x) \, d\theta$$

Then the posterior distributions of parameter $\theta$ given $x$ is defined as

$$\pi(\theta | x) = \frac{\phi(\theta, x)}{m(x)} = \frac{L(\theta, x) \pi(\theta)}{\int_{\theta} L(\theta, x) \pi(\theta) \, d\theta}$$
1.3.3 POSTERIOR PREDICTIVE DISTRIBUTION

In life testing experiment it is an interesting to know the failure times of the units which are withdrawn at the termination of the test. That is to predict failure times of the withdrawal units we have to determined predictive distribution with the help of the posterior distribution of parameter \( \theta \). Let \( T \) be the failure time of the next failure if we have continued the test instated of terminating the test, the distribution of \( T \) can be determined based on the available failure units, which is known as the posterior predictive distribution of \( T \) given \( x \). It is obtained using the following formula

\[
f(t \mid x) = \int_{\theta} f(t \mid \theta) \pi(\theta \mid x) d\theta
\]

The posterior predictive distribution function is given by \( F(t \mid x) = P(T \leq t \mid x) \).

1.4 WARRANTY POLICY

A warranty policy is the form of compensation to a buyer as a product fails during the offered warranty period. Mainly the two types of warranty policies are used. They are free replacement warranty (FRW) and pro-rata warranty (PRW).

Under FRW policy, if a product fails during the warranty period, the product is replaced by another product of the same kind free of charge.

Under PRW policy the manufacturer gives compensation to the buyer, which may be a linear function of the remaining time of the warranty linear function of the warranty period.
A combination of these two types of policies is called FRW/PRW policy.

### 1.4.1 COST FUNCTION

In our study in case of combined FRW/PRW policy we assume FRW policy during the period $[0, w_1)$, and for PRW policy during the period $[w_1, w_2)$, $w_1 \leq w_2$ are positive values. The reimbursing cost function of an item with time length $t$ for combined FRW/PRW policy is given by

$$
C_w(t) = \begin{cases} 
S, & 0 \leq t < w_1 \\
S \left( \frac{w_2 - t}{w_2 - w_1} \right), & w_1 \leq t < w_2 \\
0, & t \geq w_2
\end{cases}
$$

and in case of FRW policy ($w_1$=w_2) cost function is given by,

$$
C_w(t) = \begin{cases} 
S, & 0 \leq t < w_1 \\
0, & t \geq w_1
\end{cases}
$$

and under PRW policy ($w_1$=0) cost function is given by

$$
C_w(t) = \begin{cases} 
S \left( \frac{w_2 - t}{w_2} \right), & 0 \leq t < w_2 \\
0, & t \geq w_2
\end{cases}
$$

where $S$ is the selling price of the product which is cost to the buyer.

This cost function is also called the manufacturer loss associated with setting up a warranty.
1.4.2 ECONOMIC BENEFIT FUNCTION

The economic benefit function is proposed below as considered by Wu and Huang (2010).

\[ B(w_1, w_2) = A_1 M (1 - e^{-A_2 \frac{w_1 + w_2}{2}}) \]

where \( A_1 \) is the profit per product obtained by manufacturer and \( M \) is the potential number of products to be sold with this warranty policy. The parameter \( A_2 \) control the speed of increment in benefit. The ratio \( \frac{B(0, t_w)}{B(t_w, t_w)} = 1 - e^{-A_2 t_w} \) shows the percentage of benefit remains when the manufacturer changes the warranty from FRW to PRW.

By fixing the value of percentage of benefit given by \( \frac{B(0, t_w)}{B(t_w, t_w)} \), the value of the parameter \( A_2 \) can be derived by solving the above equation.

1.4.3 WARRANTY COST FUNCTION

The warranty cost function \( W(t, w_1, w_2) \) is an item \( C_w(t) \) times the expected number of items that fail under the warranty period. The expected number of failures can be determine based on the posterior predictive cumulative distribution function \( F(\cdot | x) \) as shown below,

\[ \text{The expected number of failures} = \begin{cases} M \ P(0 < t < w_1), & \text{if } 0 \leq t < w_1 \\ M \ P(w_1 < t < w_2), & \text{if } w_1 \leq t < w_2 \\ M \ P(t \geq w_2), & \text{if } t \geq w_2 \end{cases} \]

Hence, the warranty cost function can be written as
\[ W(t, w_1, w_2) = \]

\[ MF(w_1 | x)[0, w_1](t) + M[F(w_2 | x) - F(w_1 | x)]S(\frac{w_2 - t}{w_2 - w_1})I_{[w_1, w_2]}(t) \]

where

\[ I_{(a, b)}(t) \]

is an indicator function defined as

\[ I_{(a, b)}(t) = \begin{cases} 
1, & \text{if } a \leq t \leq b \\
0, & \text{e.w} 
\end{cases} \]

### 1.4.4 Dissatisfaction Cost Function

The dissatisfaction cost is the manufacturer’s indirect cost, when the product fails during the warranty period, or fails during the time just after warranty, such cost function is used by Patankar and Mitra (1996) and Kelly (1996). Under the combined FRW/PRW policy the dissatisfaction cost function considered by Wu and Huang (2010) is given as

\[ D(t, w_1, w_2) = D_1(t, w_1) + D_2(t, w_1, w_2) + D_3(t, w_2) \]

In case of FRW policy, when product fails in the time period \([0, w_1]\), the dissatisfaction cost is a proportion \(q_1(0 < q_1 < 1)\) of the sales price \(S\), multiplied by the expected number of failures,

\[ D_1(t, w_1) = M F(w_1 | x)S q_1 I_{[0, w_1]}(t) \]

The second component is for the product fails during the time interval \([w_1, w_2]\). Here it is assumed that the dissatisfaction cost of an item linearly decreased with time with maximum \(S q_1\) and minimum \(S q_2\), \(0 < q_2 < q_1 < 1\). Hence
And the third component $D_3(t, w_2)$ is for the product fails after the expiration of warranty, but the customer may still be unsatisfied with the product unless its lifetime exceeds a specified value $L, L > w_2$ here $D_3(t, w_2)$ decreases linearly with time $t$, reaching to zero when lifetime is $L$ and given by

$$D_3(t, w_2) = M \left[ F(w_2|x) - F(w_1|x) \right] S q_2 - (S q_1 - S q_2) \left( \frac{t - w_1}{w_2 - w_1} \right) I_{[w_1, w_2]}(t)$$

The value of $L$ may be considered as the mean or median or percentile of the posterior predictive distribution.

### 1.4.5 Expected Utility

We consider the utility function used by Wu and Huang (2010) based on the economic benefit function $B(w_1, w_2)$, the warranty cost function $W(t, w_1, w_2)$ and the dissatisfaction cost function $D(t, w_1, w_2)$ as

$$U(t, w_1, w_2) = B(w_1, w_2) - W(t, w_1, w_2) - D(t, w_1, w_2)$$

The optimal warranty $(w_1^*, w_2^*)$ is that which maximize the expected value of the utility function $E(U(t, w_1, w_2))$ with expectation over the posterior predictive distribution, that is

$$EU = E(U(T, w_1, w_2)) = \int_0^\infty [U(t, w_1, w_2) f(t|x) dt$$
Using economic benefit function, warranty cost function, dissatisfaction cost function and posterior predictive distribution, we get the expression for the expected utility function as

\[
E(U(T, w_1, w_2)) = \int_0^\infty \left[ \mathcal{B}(t, w_1, w_2) - W(t, w_1, w_2) - \mathcal{D}(t, w_1, w_2) \right] f(t \mid x)dt
\]

### 1.5 Life Time Models

In life testing experiment the life time of failure units follow different life time distributions. The appropriate life time distribution for a given data of observed failure time may be determine based on the failure rate. If failure rate is constant, then appropriate model is an exponential distribution. If failure rate is increasing function of time, then Rayleigh life time model is appropriate. There are different life time models like Weibull, Gamma which possess increasing, constant and decreasing failure rate. In our study we have consider \textbf{Exponential, Kumaraswamy, Power function and Pareto life time models.}

#### 1.5.1 Exponential Life Time Model

The exponential distribution has its own importance in the life testing experiments due to its important properties, and its applicability in the area of life testing experiments. It possesses constant failure rate and many interesting properties, See Epstein and Sobel (1953, 1954).

The probability density function of exponential distribution is given by

\[
f(x \mid \theta) = \theta e^{-\theta x}, \quad 0 < x < \infty, \quad \theta > 0
\]

Its cumulative distribution function is given by

\[
F(x \mid \theta) = 1 - e^{-\theta x}, \quad 0 < x < \infty, \quad \theta > 0
\]

The mean, variance and failure rate are respectively \(1/\theta\), \(1/\theta^2\) and \(\theta\).
1.5.2 KUMARASWAMY LIFE TIME MODEL

The Kumaraswamy distribution is very similar to the Beta distribution but it has advantages of closed form of its cumulative distribution function. Kumaraswamy (1980) developed a more general distribution known as Kumaraswamy’s distribution. Now a day Kumaraswamy distribution is also used as a life time model for the product possessing increasing failure rate. Many authors like Eldin et al. (2014), Gholizadeh et al. (2011) have used Kumaraswamy distribution as a life time model.

The probability density function(pdf) of Kumaraswamy distribution is given by

\[ f_T(t) = \theta \lambda^{\lambda-1} t^{\lambda-1} (1-t^\lambda)^{\theta-1}, 0 < t < 1, \theta, \lambda > 0 \]

where \( \theta \) and \( \lambda \) are shape parameters.

Here we have considered the shape parameter \( \lambda \) as known and equal to 1. hence the pdf becomes

\[ f_T(t) = \theta (1-t)^{\theta-1}, 0 < t < 1, \theta > 0 \]

and its cumulative distribution function is given by

\[ F_T(t, \theta) = 1 - (1-t)^\theta, 0 < t < 1, \theta > 0 \]

The failure rate of the distribution is given by

\[ H(t) = \frac{\theta}{1-t}, 0 < t < 1 \]

1.5.3 POWER FUNCTION LIFE TIME MODEL

The power function distribution has an increasing or decreasing failure rate it possesses also many important properties, so it is also a useful life time model in the area of life
testing experiments. The power function distributed life time model has been considered for estimating the parameter and reliability characteristics by Zaka et al. (2013), Zarrin et al. (2013), Sultan et al. (2014) and many more. The probability density function and cumulative distribution function of the power function distribution are given by respectively as

\[
f(x | \theta) = \theta x^{\theta-1} \quad \text{and} \quad F(x | \theta) = x^\theta, \quad 0 < x < 1, \quad \theta > 0
\]

The failure rate of the distribution is given as

\[
h(t) = \frac{\theta t^{\theta-1}}{1 - t^\theta}, \quad 0 < t < 1
\]

1.5.4 Pareto Life Time Model

The Pareto distribution is mostly useful in business and economics but now a day it is also used as a model in insurance, engineering, medical etc. This distribution is used as a life time model by many authors like Aggarwala & Childs (1999), Arnold (1983), Chareti (1988), Hossain and Zimmer (2000), Mahmmad et al. (2013), Podder et al. (2004), Shah & Patel (2007).

The probability density function of Pareto distribution is given by

\[
f(x | \theta) = \theta x^{-\theta-1}, \quad x \geq 1, \quad \theta > 0
\]

Its cumulative distribution function is given by

\[
F(x | \theta) = 1 - x^{-\theta}, \quad x \geq 1, \quad \theta > 0
\]
1.6 PRE WORK

In the literature very few references have been found related to the work on determination of warranty length based on different censoring schemes. Agrawal et al (1996), Menezes and Currim (1992) have done a work on determination of warranty and product reliability but they have not used any censoring scheme or Bayesian approach. Wu et al (2006) also have considered normal distribution as a product life time model which is suitable when the product possesses increasing failure rate. Wu and Huang (2010) have considered optimal warranty length in case of the product having Rayleigh distributed life time. The Rayleigh distribution has an increasing failure rate over a time. In the existing literature the work related to optimal warranty has not been done on the basis of life time models other than Rayleigh and Normal.

1.7 CHAPTER WISE SUMMERY

In this thesis we have developed optimal warranty length of the product having different life time distributions like Exponential, Kumaraswamy, Power function, Pareto based on multiply Type-II censoring and general progressive Type-II censoring schemes. For each of these models a simulation study is carried out to check the effect of changing the values of the parameters of prior distribution and also shown out of the three policies FRW, PRW and Combined which one has maximum expected utility. The detail chapter wise summery are given below.

Chapter 1 consists of literature review of important background knowledge for a Bayesian approach to optimal warranty length for developing the different life time models under multiply Type-II and General Progressive Type-II censoring scheme.

Chapter 2 deals with the determination of optimal warranty period using a Bayesian approach when product life time falls exponential distribution to determine reliability of the product, general progressive Type-II censoring and multiply Type-II censoring schemes are used. A likelihood function and posterior distribution are constructed under multiply Type-II censoring scheme. Hence posterior predictive distribution for the life of the product is derived. The warranty policy and expected
utility function is obtained in this chapter. A real life example is considered to exemplify the theory. To check the performance of warranty length and expected utility under different choice of prior parameters sensitivity analysis is done and then in subsequent sections the above all the things are considered in case of general progressive Type-II censoring scheme.

In Chapter-3 we investigate a decision problem under the warranty which is a combination of free-replacement, and pro-rata policies. We have used a Bayesian approach to determine the optimal warranty lengths. The Kumaraswamy distribution is employed to describe the product life time. To check the performance of warranty length and expected utility under different choice of prior parameters sensitivity analysis is done. Above all the things are considered in case of multiply Type-II and general progressive Type-II censoring scheme.

Chapter-4 covers the determination of optimal warranty period using a Bayesian approach when product life time follows power function distribution. To determine reliability of the product, multiply Type-II censoring scheme and general progressive Type-II censoring scheme are used. A likelihood function and posterior distribution are constructed under multiply Type-II censoring scheme and general progressive Type-II censoring scheme. Hence posterior predictive distribution for the life of the product is derived. The warranty policy and expected utility functions are also obtained. A real life example is considered to exemplify the theory.

Chapter-5 focus on the study of optimal warranty of Pareto distributed product based on multiply Type-II censoring scheme and general progressive Type-II censoring scheme. we consider free-replacement, Pro-rata and combination of the two policies under Bayesian set up. Optimal warranty lengths under the above policies are determined by maximizing expected utility function. A numerical data is used to exemplify the theory. A simulation study is also carried out to check the effect of hyper parameters on optimal warranty length under the above three warranty policies.

Chapter-6 Covers over all conclusion observed from the five chapters.