INTRODUCTION

An in-depth study of the development in the theory of point set topology from early sixties to till date reveals that one of the trends of development is the attempt of generalisations of the concepts of open and closed sets and to investigate various notions of General topology with the aid of these weakened forms of open and closed sets. As a result variants of weakened forms of open sets viz semi-open sets [40], α-sets [55], generalised closed (open) sets [41], preopen sets [47], feebly open sets [46], semi-preopen sets [1] and semi-generalised open sets [10] etc. have come into existence in the literature of point set topology. Of all the generalised open sets we pick up only the following two for our investigation in this thesis:

(i) preopen sets
(ii) semi-preopen sets.

Other weakened forms of open sets occasionally have been used in the thesis only to clarify our points in course of our investigation. The above two notions were introduced respectively by A. S. Mashhour et al. [47] and D. Andrijević [1]. After the introduction of preopen sets a large number of topologists extensively studied innumerable properties of set topology with the help of preopen sets. This may be seen in {[2], [22], [25], [62], [63], [64], [75], [82]}. On the other hand, several topologists have used semi-preopen sets as a stepping stone for their study in generalising different concepts of point set topology. This may also be noticed in {[14], [26], [29], [58], [67], [73]}. But the above two concepts are so fascinating both in the richness of the results and in the vastness of their applicability, that we have been allured to incorporate in this thesis results based mainly on these two concepts.
Though extensive works have been carried out by renowned researchers — only a few of whom we mentioned in the above references — a vast area in the theory of General topology still remains unexplored in the sense that the applicability of these concepts have not been tested in this area. We have made a modest attempt to cover a portion of this vast unexplored area in this thesis.

The thesis consists of seven chapters. Chapter 0 contains known definitions, lemmas, theorems necessary for our investigations. The contents of Chapter 0 are general in character in the sense that they have been frequently used throughout the thesis. But there are certain known definitions and results not given in Chapter 0 but are cited just before the results where they are needed.

Chapter I of this thesis is concerned with functions having closed graph. The background with a short history of this notion has been narrated at the beginning of the chapter. To avoid repetition we refrain from mentioning this here. This chapter consists of two parts. In Part I, definition of preclosed graph, its characterisation and some basic properties of this graph have been dealt with on one hand and on the other hand, relation between range space, domain space of functions with preclosed graph using the separation axioms pre-$T_i$ ($i = 1, 2$) defined by Kar et al. [36] have been studied with some detail. Preconnected mapping has been introduced to obtain preclosed graph. How under certain conditions preclosed graph generates precontinuity has been unveiled. These results contained in Part I are all rudimental in character.

T. R. Hamlett and L. L. Herrington [30] studied some deeper properties of closed graph with cluster set concept which is a very useful technique. R. V. Fuller [23] proved: Let $f : X \to Y$ be a closed function with $X$ as a regular topological space. If $f^{-1}(y)$ is closed for every $y \in Y$, then $f$ has a
closed graph. Hamlett et al. [30] gave an elegant proof of this result using the cluster set technique. Inspired by the method of Hamlett and Herrington we introduce here the notion of precluster set and prove the parallels of some results of Hamlett and Herrington along with the analogue of R. V. Fuller's Theorem.

Chapter II is concerned with function having strongly preclosed graph. In 1975, P. E. Long and L. L. Herrington [43], defined functions with strongly closed graph. In 1996, Paul and Bhattacharyya [65], introduced the notion of functions with preclosed graph. In this chapter we study this notion in considerable detail and arrive at the conclusion that functions with strongly preclosed graph is stronger than that with preclosed graph studied in Chapter I while it is weaker than strongly closed graph by Long and Herrington [43]. Definition and basic properties of this function form the earlier part of the chapter. The interrelationship among various graph conditions did not escape our notice. This interconnection has been exhibited by an arrow-headed diagram. Towards the end of this chapter some ramification of this graph has been incorporated.

In Chapter III we dwell on a weak form of continuous functions viz somewhat precontinuous function (briefly spc). A brief background of this function has been given at the beginning of this chapter and so we do not repeat it here. It has been showed that the class of spc functions contains the class of precontinuous functions introduced in 1982, by Mashhour et al. [47] and also that the class of somewhat continuous functions defined, in 1971, by Karl R. Gentry et al. [27] is contained as a proper subset of the class of spc functions. Examples have been constructed to substantiate the following : a function may be a.c.S. or w.c. without being spc; a spc function may not be w.c. or a.c.S. or a.c.H. Due attention, has been paid to the composition of two spc functions, the spc in subspaces is considered elaborately.
Projectivity and productivity of spc functions and some basic properties of spc functions have also come under the purview of our study. At the same time somewhat preopen functions along with a few of its basic properties, have received our due consideration.

Chapter IV deals with the development of the separation axiom called Urysohn space. s-Urysohn space with the aid of semi-open sets of Levine (Definition 0.1) exists in the literature and has been mentioned by Arya et al. [3]. In 1996, M. Pal and P. Bhattacharyya [63] defined pre-Urysohn space with the help of preopen sets in a different context. They did not carry out their investigation concerning this new concept except one result. But in 1999, R. Paul and P. Bhattacharyya [66] took up this notion and studied this space afresh and obtained some properties of this space. Prompted by their works we, in this chapter, introduce a new separation axiom known as sp-Urysohn space using semi-preopen sets defined by Andrijević [1] and obtain some properties of this space which are interesting. A portion of this chapter has been engrossed in investigation of subspace, transformation of sp-Urysohn spaces and product of sp-Urysohn spaces.

Chapter V is the continuation of our study on separation axioms and we have designated this chapter as “More on separation axioms”. In 1943, N. Shanin [76] introduced $R_0$ separation axiom while in 1961, Davis [17] introduced the $R_1$-axiom. After the introduction of these notions a large number of topologists extensively studied myriads of properties of these two spaces $R_0$ and $R_1$. Some topologists studied various existing properties of these spaces using generalised open sets like semi-open and preopen sets. As for instance Maheshwari et al. [45] introduced $(R_0)_s$ spaces with the aid of semi-open sets while Caldas et al. [15] defined $R_0$ and $R_1$ spaces utilising preopen sets. Chapter V of this thesis has been devoted to the study of generalised $R_0$ and $R_1$ spaces with the help of semi-preopen sets of Andrijević
and they have been termed as sp-R₀ and sp-R₁ spaces respectively. The concepts semi-Kernel (briefly skernel) [45], pre-Kernel (briefly pker) [37] are current in the literature. Analogous notion sp-kernel (See Definition 5.1) has been defined in this chapter and with the help of this notion characterisation of sp-R₀ and some basic properties of this space are given. In like manner sp-R₁ space is introduced. Its characterisation, relation with sp-R₀ space have been studied. Weakly semi-R₀, defined by Arya et al. [4] attracted our attention and we analogously have introduced weakly semi-pre R₀ (briefly wsp-R₀) spaces and studied some basic and interesting properties along with the productivity of wsp-R₀. All the material of these three spaces are included in a portion of Chapter V and this has been marked as Part I.

Part II of this chapter is concerned with another type of separation axiom. Kelley [38] defined door space and gave some properties of this space. In 1995, Dontchev [18] studied this space in great detail. J. P. Thomas [83] generalised, in 1968, this door space to semi-door space. We could not resist ourselves from dwelling on this space in brief. We introduced sp-door space with the aid of semi-preopen sets and touched upon a few basic properties of this space.

Chapter VI, the concluding chapter of the thesis, is concerned with multifunctions. The results of multifunctions are nothing but the extensions of those in single-valued functions. The setting of General topology is a function f : X → Y where X and Y are arbitrary topological spaces. The main consideration in this structure is to study the variant of conditions which make f continuous. Through the sustained endeavour of a large number of renowned topologists for the last four decades the continuity conditions have been weakened yielding a variety of weak forms of continuity. In recent years attempts have been made to extend these weak continuity conditions for single-valued functions to multi-valued functions. The term “multifunction”
which is an abbreviation of the word “multi-valued function” is now prevalent in the current literature. Throughout the thesis this abbreviated term “multifunction” has been used.

In 1932, Kuratowski (Fund. Math. 18 (1932), p-148) and Bouligand (Ens. Math. (1932), p-14) independently introduced two kinds of semi-continuity of a multifunction which are termed as upper semi-continuous and lower semi-continuous (briefly u.s.c. and l.s.c.) functions (the author had no access to these articles of Kuratowski and Bouligand). Though the concept of multifunctions is an active field of study, this has not found a place in most text books till date. Only in 1963, C. Berge [8] discussed this in his book and very recently multifunction has also been included in the book of Aubin et al. [5]. But this book suits more to the need of a person interested in Analysis than to that of a Topologist. Our interest lies in multifunction from functional point of view such as continuity, graph etc. R. E. Smithson in [79] has published a survey of some results on multifunctions. He has elegantly pointed out in this paper the difference between the theory of single-valued functions and that of multifunctions. Recently R. E. Smithson [(78), [79], [80], [81]}, V. Popa [(69), [70], [71]}, T. Noiri et al. [(59), [60]}, J. E. Joseph [35], Neubrunn [54], P. Martiz [52], S. Ganguly et al. [24] and many others have investigated multifunctions extensively. They have extended and generalised many results of single-valued functions to multifunctions. In many cases these extensions and generalisations give new and stirring results while in other cases these generalisations are such that corresponding results of single valued functions become particular cases of these results.

In 1986, Andrijević introduced the notion of semi-preopen sets and in 1992, Przemski defined semi-precontinuity as single-valued function utilising semi-preopen sets defined by Andrijević. In the concluding chapter of this
thesis we take up only the case of single-valued semi-precontinuous functions for its generalisation to multifunction.

Attempts have been made to keep each individual chapter more or less self-contained. When we write Theorem a.b where a and b are positive integers, we mean the b-th Theorem of a-th chapter. The same convention has been followed in case of Definitions, Corollaries and Lemmas.

During the preparation of the thesis a number of books and journals have been consulted. The references to these books and journals have been appended at the end of the thesis in the form of a “bibliography” and the numerals corresponding to the references in the bibliography have been inserted at places where they are needed.

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