CHAPTER 4

A DECISION-MAKING MODEL FOR OPTIMUM UTILIZATION
OF A FLEET OF BUSES

4.1. Introduction

The Head Office and its different branches, factories, and the staff-colony of a giant company are located at distant places around a big city. With the company's existing buses, the management faces a great difficulty in carrying the staff from various bus stops in the colony at different times depending upon the timings of the offices/factories, because during the peak hours the existing buses usually get overcrowded. The same difficulty exists at the time of carrying them back to the colony at the end of the duty hours at different points of time.

In the above situation, the question of increasing the carrying capacity of a bus does not arise. Also, additional new buses involve a considerable amount of capital expenditure which may not be easy to approve by the company.

The content of this chapter is based on one of our published papers (Pal and Basu [73]).
In such a case, if it is not possible to change the timings of the offices/factories, the only possible solution is to increase a number of economic trips.

Several management science and computer based models have been suggested for an equitable busing program (1, 23, 29, 34, 68). Each of the studies described has a single objective, either cost or distance minimization. However, the management problems inherent in busing issue do not lend themselves to a model centering on one criterion. Implementation of a busing program must consider multiple objective criteria. In this regard, an attempt has been made by Lee and Moore (59) for school busing program with certain social and racial constraints in view of the privilege in a country.

In this chapter, a priority based GP model for equitable busing program is developed where the problem of deciding as to the number of staff to be picked up, per trip, from different bus stops in the colony for different offices/factories is considered for most satisfactory utilization of the existing buses.
4.2. The Problem and Model Formulation

The general GP model with preemptive priority structure can be mathematically expressed as:

\[
\begin{align*}
\text{minimize} : & \quad p_1(w_{1i}^-d_{1i}^- + w_{1i}^+d_{1i}^+), \\
\text{minimize} : & \quad p_2(w_{12}^-d_{12}^- + w_{12}^+d_{12}^+), \\
& \quad \vdots \\
\text{minimize} : & \quad p_k(w_{1k}^-d_{1k}^- + w_{1k}^+d_{1k}^+), \\
& \quad \vdots \\
\text{minimize} : & \quad p_K(w_{iK}^-d_{iK}^- + w_{iK}^+d_{iK}^+); \quad i = 1, 2, \ldots, m; \\
& \quad K = 1, 2, \ldots, m \\
\text{subject to} : \\
\sum_{j=1}^{n} a_{ij}x_j + d_{i}^- - d_{i}^+ &= b_i; \quad i = 1, 2, \ldots, m \\
x_j, d_{i}^-, d_{i}^+ &> 0; \quad j = 1, 2, \ldots, n \\
i = 1, 2, \ldots, m \\
\end{align*}
\]

(4-1)
All the notations in (4-1) are defined in (1-1).

To formulate the model of the problem of transporting the staff from various bus stops (sources) in the colony to the offices/factories (destinations), the groups are formed with the destinations with some specified radius which can reduce the number of destinations. Also, suitable bus stops are chosen within the colony.

Now, to design the goals of the problem, the following notations are introduced.

Let:

\[ x_{ijt} = \text{number of staff picked up from source } i \text{ to destination } j \text{ in trip } t; \]
\[ X_{ij} = \text{total number of staff is to be transported from source } i \text{ to destination } j; \]
\[ x_i = \text{proportion of the staff at source } i; \]
\[ c_{ij} = \text{variable time per staff from source } i \text{ to destination } j \text{ (where the variable time implies the time to check the bus pass, allowing a staff to go in a bus);} \]
where $x_{ijt}$, $X_{ij}$, $x_{i}$, $c_{ij} > 0$;

\[ i = 1, 2, \ldots, m; \]

\[ j = 1, 2, \ldots, n; \]

\[ t = 1, 2, \ldots, p. \]

Also, to present the goals in the model of the problem, $d_{1}^{-}( \geq 0)$ and $d_{1}^{+}( \geq 0)$ are used to represent the under- and over-deviational variables, respectively, for goal 1.

In this busing program, the flexible goals and a system of rigid constraints (i.e., inflexible goals) are generally arisen. The flexible goals include the objectives of the problem, and the rigid constraints must be adhered to before an optimal solution could be considered.

4.2.1. the flexible goals

The flexible goals are:

(i) **Trip-time**: The first objective of the management is to arrange the total trips among the available buses. Let the total variable time for each trip of each bus can not be less than $X$ minutes and not greater than $Y$ minutes.
The goals can be expressed as:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ijt} + d_{1}^{-} - d_{1}^{+} = x
\]

\[\ldots \quad (4-2)\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ijt} + d_{1}^{-} - d_{1}^{+} = y
\]

\[\ldots \quad (4-3)\]

where:

- \(t\) = numbers of the trips of the buses;

- \(l\) = total number of trips, where
  
  \[l = 1, 2, \ldots, p \text{ for (4-2), and} \]
  
  \[l = p+1, p+2, \ldots, 2p \text{ for (4-3).}\]

(ii) **Trip-restriction**: The goals of the form:

\[
\sum_{i=1}^{m} x_{ijt} + d_{1}^{-} - d_{1}^{+} = 0
\]

\[\ldots \quad (4-4)\]

imply that the trip \(t\) can not run through the restricted destination \(j\).

where:

- \(t\) = numbers of restricted trips;
(iii) Proportion of the staff: It is desired that the number of staff to be picked up at a source must be in the same proportion to the total number of staff who have to go in that trip. So, the inconvenience of early arrival or waiting is to be shared equally by the staff of all places in the colony.

Therefore, the goals take the form:

\[
\sum_{j=1}^{n} x_{ij} - x_1 \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} + d_i - d_1 = 0
\]

... (4-5)

where \( i = 1, 2, \ldots, m-1; \)
\( t = 1, 2, \ldots, p-1; \)
\( l = 2p+g+1, 2p+g+2, \ldots, 2p+g+pm-p-m+1. \)
(iv) **Bus capacity**: Let $C^t$ be the capacity of a bus. So, in any trip, the total number of staff cannot be more than $C^t$. Also, it is desired that a bus may not be ready for a trip if the total number of staff is less than $C^t/4$ at that time.

The goals can be presented as:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijt} d_i^r - d_i^f = C^t$$  \hspace{1cm} \ldots \hspace{1cm} (4-6)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijt} d_i^r - d_i^f = C^t/4$$ \hspace{1cm} \ldots \hspace{1cm} (4-7)$$

where:

\begin{align*}
l & = 2p + g + pm - m + 2, 2p + g + pm - p - m + 5, \ldots, \\
& \quad 2p + g + pm - m + 1 \text{ in (4-6)}; \\
l & = 2p + g + pm - m + 2, 2p + g + pm - m + 2, \ldots, \\
& \quad 2p + g + pm - m + p + 1 \text{ in (4-7)}; \\
t & = 1, 2, \ldots, p.
\end{align*}
4.2.2. the rigid constraints

The rigid constraints can be defined as follows:

The total number of staff that can be picked up in different trips must be equal to the total number of staff from source $i$ to destination $j$.

The rigid constraints appear as:

$$\sum_{t=1}^{P} x_{ijt} = X_{ij}; \quad i = 1, 2, \ldots, m$$
$$\quad j = 1, 2, \ldots, n$$

... (4-8)

4.2.3. the objective function

In a real-world problem, all the goals are not equally important. To avoid the solution regret, the following preemptive priorities are considered to pursue the various stated goals.

$P_1$: The first priority is assigned to 'trip-time' and 'trip-restriction'.
P₂ : The second priority is assigned to 'proportion of the staff'.

P₃ : The last priority is assigned to 'bus capacity'.

Considering all the goals with priorities and their relative importance at the same priority level, the objective function can be designed as:

minimize : \( P_1(d^-_1 + d^-_2 + \ldots + d^-_p + d^+_p + d^+_p+1 + d^+_p+2 + \ldots + d^+_2p + \ldots + d^+_f) \),

\[ \text{minimize : } P_2 (d^-_{r+1} + d^-_{r+2} + \ldots + d^-_h + d^+_h + d^+_h+1 + d^+_h+2 + \ldots + d^+_h) \],

\[ \text{minimize : } P_3 (d^+_h+1 + d^+_h+2 + \ldots + d^+_r + d^+_r+1 + d^-_r+1 + d^-_r+2 + \ldots + d^-_s) \]

\[ \ldots \quad (4-9) \]
where:
\[ f = 2p + g, \]
\[ h = 2p + g + pm - p - m + 1, \]
\[ r = 2p + g + pm - m + 1, \]
\[ s = 2p + g + pm - m + p + 1. \]

Now, the GP model formed with the objective function in (4-9) and the associated goals in (4-2) through (4-7) can be used for making decision of the problem under consideration.

4.3. A Numerical Example

To illustrate the proposed GP model, the following simplified numerical example is considered.

Let there be 2 bus stops (sources) in the colony and 20 Offices/factories (destinations) to which the buses of the company have to ply. By grouping together the destinations within a radius of 0.5 km, let the number of destination can be reduced from 20 to 3.

At a particular peak hour, the total number of staff and variable time are displayed in Table 4-1 and 4-2, respectively.
Now, the following restrictions are considered to design the model of the problem.

A. In each trip, the total variable trip-time is within 20 and 30 minutes.

B. The destination 1 can not be covered by the first trip.

C. For each trip, the proportion of the staff is 0.6 for source 1.

D. The carrying capacity of a bus to 120.

Table 4-1: Number of Staff

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
<th>Staff</th>
<th>Total</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>10</td>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>40</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>
Table 4-2: Variable Time

<table>
<thead>
<tr>
<th>Source</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.45</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.4</td>
<td>0.45</td>
</tr>
</tbody>
</table>

It is clear from Table 4-1 and the stated bus capacity that two trips are required to transport the staff to the destinations.

By virtue of the stated restrictions and Table 4-1, the goals of the model can be expressed as:

\[
\sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij} x_{ij1} + d_{1j}^*-d_{1j}^+ = 20 \quad \ldots \quad (4-10)
\]

\[
\sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij} x_{ij2} + d_{2j}^*-d_{2j}^+ = 20 \quad \ldots \quad (4-11)
\]

\[
\sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij} x_{ij3} + d_{3j}^*-d_{3j}^+ = 30 \quad \ldots \quad (4-12)
\]
$$\sum_{i=1}^{2} \sum_{j=1}^{3} c_{ij} x_{ij} + d_{i} - d_{i}^{+} = 30 \quad \ldots (4-13)$$

$$\sum_{i=1}^{2} x_{i11} + d_{5} - d_{5}^{+} = 0 \quad \ldots (4-14)$$

$$\sum_{j=1}^{3} x_{1j1} - 0.6 \quad \sum_{i=1}^{2} \sum_{j=1}^{3} x_{ij1} + d_{6} - d_{6}^{+} = 0 \quad \ldots (4-15)$$

$$\sum_{i=1}^{2} \sum_{j=1}^{3} x_{ij1} + d_{7} - d_{7}^{+} = 120 \quad \ldots (4-16)$$

$$\sum_{i=1}^{2} \sum_{j=1}^{3} x_{ij2} + d_{8} - d_{8}^{+} = 120 \quad \ldots (4-17)$$

$$\sum_{i=1}^{2} \sum_{j=1}^{3} x_{ij1} + d_{9} - d_{9}^{+} = 30 \quad \ldots (4-18)$$

$$\sum_{i=1}^{2} \sum_{j=1}^{3} x_{ij2} + d_{10} - d_{10}^{+} = 30 \quad \ldots (4-19)$$

Also, from Table 4-1, the rigid constraints can be presented as:

$$\sum_{t=1}^{9} x_{11t} = 10, \quad \sum_{t=1}^{9} x_{12t} = 70,$$

$$\sum_{t=1}^{9} x_{13t} = 40, \quad \sum_{t=1}^{9} x_{21t} = 40,$$
The objective function of the model takes the form:

\[ \text{minimize : } P_1(d_1^- + d_2^- + d_3^+ + d_4^+ + d_5^+), \]

\[ \text{minimize : } P_2(d_6^- + d_6^+), \]

\[ \text{minimize : } P_3(d_7^+ + d_8^+ + d_9^- + d_{10}^-) \]

Now, introducing the values of \( c_{ij} \) (i = 1, 2; j = 1, 2, 3) from Table 4-2 to the goals in (4-10) through (4-19), and applying the solution procedure, presented by Schniederjans and Kwak, to the model formed with the stated goals and the objective function in (4-21), the positive integral values of the decision variables and the deviational variables are obtained as follows:
<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Deviational Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{112} = 10$</td>
<td>$d^+_1 = 10$</td>
</tr>
<tr>
<td>$x_{121} = 11$</td>
<td>$d^+_2 = 43$</td>
</tr>
<tr>
<td>$x_{122} = 59$</td>
<td>$d^+_4 = 53$</td>
</tr>
<tr>
<td>$x_{131} = 27$</td>
<td>$d^-_7 = 56$</td>
</tr>
<tr>
<td>$x_{132} = 13$</td>
<td>$d^+_8 = 16$</td>
</tr>
<tr>
<td>$x_{212} = 40$</td>
<td>$d^+_9 = 34$</td>
</tr>
<tr>
<td>$x_{222} = 10$</td>
<td>$d^+_{10} = 106$</td>
</tr>
<tr>
<td>$x_{231} = 26$</td>
<td></td>
</tr>
<tr>
<td>$x_{232} = 4$</td>
<td></td>
</tr>
</tbody>
</table>

All other variables are zeroes.

It is apparent from the result above that the goals are achieved by satisfying the system of rigid constraints in (4-20).
4.4 Conclusion

In this chapter, the solution of a busing problem is presented within a preemptive priority structure of the goals. If the result is not met to the satisfaction of the management, then the priorities are to be changed to make the desired decision of the problem. Also, under the flexible nature of the GP approach, several different objectives of the management may also be included in a specific situation. However, the general problem of the management is modelled in this chapter for making decision in busing program.