Chapter 4

Analysis & discussion of $^{28}$Si on $^{28}$Si system

This chapter is devoted to the analysis and discussion of the elastic, the inelastic angular distributions and the excitation function of the $^{28}$Si on the $^{28}$Si system. Elastic angular distributions have been studied at two different energies (63 and 72 MeV), analysed using the optical model with a phenomenological potential and is described in the first section. The coupled channels analysis with phenomenological potential has been presented in the second section. The third section describes the microscopic calculations for elastic and inelastic excitations. In the fourth section the elastic and inelastic excitation functions have been presented using both phenomenological and microscopic potentials. Finally we summarise in the last section.

4.1 Analysis of Angular distribution

$^{28}$Si on $^{28}$Si being an identical system, the identity of the particles in the entrance and exit channels imposes certain symmetry relations on the scattering amplitudes
\[ f_{IM}(\theta) = \frac{1}{2} \{f_{IM}(\theta) + f_{IM}(\pi - \theta)\}(1 + (-)^I) \]  

(4.1)

and the cross section is given by,

\[ \frac{d\sigma}{d\Omega} = \sum_M \sum_I |f_{IM}(\theta)|^2 \]  

(4.2)

The phase factor \((-)^I\) comes from the properties of the spin wave function under the exchange of particles (i.e. whether they are bosons or fermions), \(I\) is the channel spin and in the present case it is even.

If the particles in the outgoing channels are not identical i.e for the case of inelastic scattering with single excitation, the excited nucleus is distinguishable from the unexcited nucleus but in practice the detector is not able to distinguish between the scattered and recoil particles. Therefore a factor of 2 appears in the cross section. At the same time in the case of the \(^{28}\text{Si}\) on \(^{28}\text{Si}\) system, the excitation of the target as well as of the projectile are equally probable. Hence an enhanced cross section is observed in the elastic channels Therefore the explicit inclusion of both target and projectile excitations are needed to explain the inelastic cross sections.

4.1.1 Optical model analysis of elastic scattering

A consistent optical model analysis was performed for the elastic scattering of 72 MeV and 63 MeV data. Optical model parameters were obtained by using the search option of the coupled channel code ECIS [3] in the uncoupled one-channel mode. We have considered the volume Woods-Saxon form for both the real and
imaginary parts of the potential, i.e

\[ U = -V f(r, R_o, a_o) - iW f(r, R_w, a_w) \]  

(4.3)

where \( f(r, R, a) = (1 + \exp(\frac{r - R}{a}))^{-1} \)

Here \( V \) and \( W \) are the strengths of the real and imaginary potentials respectively, with radius \( R = r_o(A_t^{1/3} + A_p^{1/3}) \) (\( A_t \) and \( A_p \) being the masses of the target and projectile respectively) and diffuseness parameter \( a \). The subscripts 'o' and 'w' refer to the real and the imaginary parts respectively.

The best fit to the elastic scattering data was found by minimising the chi-squared (\( \chi^2 \)) value. It was observed that the fit was less sensitive to the variation of the optical potential parameters for 63 MeV angular distribution data as expected below the barrier. Therefore we carried out the search procedure on all the six parameters for the 72 MeV data only. The geometry parameters were kept fixed after this. For the 63 MeV data, we searched the real (\( V \)) and imaginary strengths (\( W \)) only. Final parameter sets, obtained by this procedure (Table 4.1) are consistent with previous work [3, 4].

It is known that several sets of parameters can fit the elastic data to about the same degree of accuracy especially near the Coulomb barrier. The real potential in set I could be varied \(~ \pm 13\) MeV from 77.59 MeV with a change of \( \chi^2 \) value to fit from 4.9 per degree of freedom to 5.9 per degree of freedom. The imaginary potential too was varied \(~ \pm 17\) MeV with change in \( \chi^2 \) value of about the same magnitude. Similarly for \( r_o, a_o, r_w \) and \( a_w \), the ranges were \( \pm 0.015 \) fm from 1.21 fm, \( \pm 0.014 \) fm from 0.49 fm, \( \pm 0.09 \) fm from 1.18 fm and \( \pm 0.15 \) fm from 0.4 fm, respectively without significant changes in \( \chi^2 \). In all such cases the potential
at the strong absorption radius of 9.35 fm remained practically identical (inset Fig. 4.2 and 4.4).

4.2 Coupled channels calculation for inelastic scattering

The inelastic angular distribution calculations were done using the same coupled channels code ECIS [3] with $0^+ - 2^+$ coupling. Due to the symmetry of the system, the excitation to the $2^+$ state for both the target as well as the projectile were included explicitly in the calculation. Elastic and inelastic ($2^+; 1.778$ MeV) angular distributions were fitted simultaneously to extract the real and imaginary strengths of the potential parameters. The geometry parameters were kept the same as obtained from the elastic angular distribution search procedure (Table 4.1). As expected, due to the explicit channel coupling, the imaginary potential is smaller for the coupled channels calculation. We assumed the real and imaginary deformation lengths to be equal i.e, $\delta^V = \delta^W$. The Coulomb deformation ($\beta_{0l}^c$) was calculated from the reduced matrix elements for $0 \rightarrow l$ transitions (where $l$ is the order of the multipole). This is related to the reduced electromagnetic transition rate ($B(El)$) by,

$$M(El) = \pm [(2J_i + 1)B(El; J_i \rightarrow J_f)]^{\frac{1}{2}} \quad (4.4)$$

where $J_i$ and $J_f$ are the spins of the initial and final states. For the $0^+$ to $2^+$ transition,

$$\beta_{20} = \frac{4\pi B(E2)}{3Z R_c^2} \quad (4.5)$$
Table 4.1: Optical potential parameters. The Coulomb radius parameter was 1.2 fm in each case. The sets I and II refer to potentials obtained from the elastic channel, while set CCI and CCII are obtained from the simultaneous coupled channels fitting of the elastic and inelastic cross sections.

<table>
<thead>
<tr>
<th>POTENTIAL</th>
<th>$E_{Lab}$</th>
<th>$V_0$</th>
<th>$r_o$</th>
<th>$a_o$</th>
<th>$W$</th>
<th>$r_w$</th>
<th>$a_w$</th>
<th>$\chi^2/n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>set I</td>
<td>63</td>
<td>95.41</td>
<td>1.21</td>
<td>0.49</td>
<td>12.63</td>
<td>1.18</td>
<td>0.4</td>
<td>4.5</td>
</tr>
<tr>
<td>set II</td>
<td>72</td>
<td>77.59</td>
<td>1.21</td>
<td>0.49</td>
<td>22.47</td>
<td>1.18</td>
<td>0.4</td>
<td>4.9</td>
</tr>
<tr>
<td>CCI</td>
<td>72</td>
<td>80.55</td>
<td></td>
<td></td>
<td>17.08</td>
<td></td>
<td></td>
<td>4.2</td>
</tr>
<tr>
<td>CCII</td>
<td>72</td>
<td>6.47</td>
<td>1.406</td>
<td>0.612</td>
<td>4.64</td>
<td>1.344</td>
<td>0.359</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 4.2: Deformation parameters and quadrupole moment measurements taken from the literature [5-7] (set I). Set II represents the values deduced from the experimental data.

<table>
<thead>
<tr>
<th>$J^*$</th>
<th>$E^*$</th>
<th>$\beta^N$</th>
<th>$M(El)$</th>
<th>$Q_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$MeV$</td>
<td>$e.fm^1$</td>
<td>$e.fm^2$</td>
<td></td>
</tr>
<tr>
<td>set I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>1.778</td>
<td>0.33</td>
<td>18.3</td>
<td>16.3</td>
</tr>
<tr>
<td>$3^-$</td>
<td>6.879</td>
<td>0.23</td>
<td>59.9</td>
<td>....</td>
</tr>
<tr>
<td>$4^+$</td>
<td>4.618</td>
<td>0.12</td>
<td>159.6</td>
<td>....</td>
</tr>
<tr>
<td>set II</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>1.778</td>
<td>0.386</td>
<td>18.3</td>
<td>16.3</td>
</tr>
</tbody>
</table>
with \( Z = 14 \) for \( ^{28}\text{Si} \) and the conventional target charge radius, \( R_c = 1.2 A^{1/3} \). We have taken the value of the reduced matrix element \( M(E2) \) equal to 18.3 efm\(^2\) [5] and the nuclear deformation length as 1.22 fm from the work of Zalmstra et al [6]. Assuming a rotational model, the quadrupole moment for the excited 1.78 MeV, \( 2^+ \) state was also taken into account through the reorientation coupling term \( \beta_{22} \). \( \beta_{22} \) was obtained from the static quadrupole moment value of the \( 2^+ \) state [7]. All the deformation parameters are shown in Table 4.2.

The elastic and inelastic cross sections from the coupled channels calculations along with the data are shown in Fig. 4.1(a). The solid line represents the calculation with reorientation coupling and the dotted line is without reorientation. In the present work we have adopted the normally accepted oblate shape for the \( ^{28}\text{Si} \) (i.e. a negative deformation was used). However, the interference pattern is not reproduced correctly with the potential set CCI and deformation set I. A variety of options were tried to fit the inelastic \( 2^+ \)-state data. By allowing both the potential strengths and the geometry parameters to vary, a marginal improvement in the quality of the fit was observed. But the imaginary diffuseness and the real radius came out to be very low (\( a_w \sim 0.09 \) fm and \( r_o \sim 0.95 \) fm). In the next step we varied the nuclear deformation length with potential strengths and diffuseness parameters. We found a reasonable fit for the inelastic data with a large nuclear deformation length (\( R_o \beta_{02}^w = 1.65 \), and \( R_w \beta_{02}^w = 1.58 \)), and a shallow potential depth (see CCII in Table 4.1). The resulting fit is shown in Fig. 4.1(b). The chi-squared improved for the fit to the inelastic angular distribution from 6.8 per degree of freedom to 4.3 per degree of freedom, while the elastic fit remained practically identical between the parameter sets CCI and CCII. We also varied the Coulomb
Figure 4.1: Elastic and inelastic ($2^+$, 1.778 MeV) angular distributions for $^{28}$Si + $^{28}$Si. (a) Solid curves result from the coupled channels calculations with potential CCI (Table 4.1), including target and projectile excitations of first $2^+$ state and quadrupole reorientation coupling (Table 4.2). Dotted line is without reorientation. The elastic curves for these cases are indistinguishable. (b) With the potential set CCII and the deformation parameter set II (see text).
deformation length \( (R_c\beta_{22}) \) and the reorientation coupling strength \( (\beta_{22}) \), one after another, but no further improvement could be obtained.

### 4.3 Microscopic calculation

While the fit using phenomenological optical potentials in a coupled channels description is good, our main interest was to obtain a description in a microscopic model with as few free parameters as possible. We proceeded in the following manner.

The real part of optical potential was calculated from a double folding between an effective (M3Y) two-body interaction and the density of the two interacting nuclei. The absorptive part of the potential was derived from the Feshbach formalism using the approach of Vinh Mau et al [24, 9] and Pacheco et al [5] (see chapter 3). The transition potential for the \( 2^+ \) excitation of the target and projectile nuclei was calculated by using the deformed folding model with a shell model transition density [11].

#### 4.3.1 Real Folded Potential

The calculation is based on the generalised version of the folding model [12, 13] as described in chapter 3. The widely used M3Y effective nucleon-nucleon (NN) interaction was employed in our calculation which is based on the G-matrix elements of the Reid soft-core interaction in an oscillator basis [23] with explicit density dependence to account for the medium dependent effects (DDM3Y). Here we used the zero-range pseudopotential for calculating the \( V_{EX} \) term. It was observed that at low energies, the effect of finite range exchange was very negligible for \( ^{28}\text{Si} \).
Table 4.3: Parameters of ground state density obtained from literature [15].

<table>
<thead>
<tr>
<th>Shape</th>
<th>$c$ (fm)</th>
<th>$z$ (fm)</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3pG</td>
<td>1.954</td>
<td>2.076</td>
<td>0.286</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters of ground state density obtained from literature [15].

on $^{28}$Si system. The ground state density $\rho_0(r)$ was taken from the experimental charge density [15]. This charge density was parametrised as a three-parameter Gaussian (3pG) distribution (Table 4.3) which is of the form,

$$\rho_0(r) \propto \frac{(1 + wr^2)}{1 + \exp\left(\frac{(r - c)^2}{z^2}\right)}$$

where $c$ is the half-density radius, $z$ is the skin thickness and $w$ is generally the influence of the tail of $\rho(r)$. Fig. 4.2 shows the calculated potential with a zero range pseudopotential for 72 MeV incident energy. In our calculation we used corrections for the finite charge distribution of the proton and took the shape of the neutron and proton distributions to be the same, the normalisation being in the $\frac{N}{Z}$ ratio.

4.3.2 Real Transition Potential

The transition potential was calculated for the $2^+$ excited state by using the one-body density matrices (OBDM) taken from ref. [11] to generate the quadrupole transition density $\rho_2 = \rho_2^{(p)} + \rho_2^{(n)}$ (for protons and neutrons). They derived the
Figure 4.2: (a) Real folded potential for $^{28}\text{Si}$ on $^{28}\text{Si}$ system at $E_{\text{lab.}} = 72$ MeV, calculated with energy and density dependent M3Y interaction (DDM3Y). Exchange part (dash-dot curve) was calculated by taking DDM3Y interaction with zero range pseudopotential. Solid curve in the figure is the result of the contributions of direct (dotted curve) and exchange parts (top). The comparison between phenomenological and folded potentials are shown in Fig. (b) at the bottom. Where curves A and B are phenomenological potential from Table 4.1, set CCI and CCII respectively. The Curve C is the double folded potential.
OBDM values for the $^{28}$Si ($0^+ - 2^+$) transition using the universal sd-shell effective interaction of Wildenthal [16] in the model space of sd shell. The transition density $\rho_i^j (i=p,n)$ is written in terms of a valence part $A^{(i)}(r)$ and a core part $C^{(i)}(r)$. The former was explicitly calculated from the relation,

$$A^{(i)}(r) = \sum_{jj'} D_{jj'}^{(i)} R(j, r) R(j', r) \langle j|| \sum_j Y_{\lambda \mu} || j' \rangle$$

where $D_{jj'}$ are the one-body density matrices (OBDM) and the $R(j, r), R(j', r)$ are the radial single-particle wave functions. We used a Woods-Saxon potential for the central part ($V_0 = 51.41$ MeV, $r_o = 1.277$ fm, $a_o = 0.362$ fm), a standard spin-orbit potential ($V_{so} = 24.0$ MeV, $r_{so} = 1.1$ fm, $a_{so} = 0.65$ fm) and a Coulomb potential for a proton in an orbit-independent method (the same potential was used for all levels in the sd-shell) to generate the single-particle radial wave function. These were extracted using the BOUND subroutine of the DWBA code DWUCK4 [17]. The value of the reduced matrix elements of $Y_{\lambda, \mu}$ were taken from standard texts [18]. With these wave functions, the valence neutron and proton transition matrix elements were calculated and found to be $M_n = 10.1572$ efm$^2$ and $M_p = 10.499$ efm$^2$. The effect of core polarisation was taken into account through the effective charges $e_p = 1.35$ and $e_n = 0.35$. The resulting total transition matrix elements (including valence and core) for protons and neutrons are $M_p = 17.7286$ efm$^2$ and $M_n = 17.3868$ efm$^2$ respectively.

A double-folding procedure was used for the calculation with the energy dependent DDM3Y type of interaction. In the case of single excitation of spin zero particles, the folded potential can be written (in momentum space) as,

$$V_{\lambda}(k) = \frac{1}{2\pi^2} \int k^2 dk j_{\lambda}(kR) \tilde{\nu}(k) \tilde{\rho}_{\lambda}(k)$$

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Figure 4.3: Solid curve (curve A) represents the transition potential for $^{28}\text{Si}$ on $^{28}\text{Si}$ system at $E_{\text{lab.}} = 45^\circ$ from folding model calculation using shell model transition density (with normalisation factor $\frac{r^{-\frac{1}{2}}}{2}$). Dotted line (curve B) is from the deformed optical model potential ($= \frac{r^{-\frac{1}{2}}}{2} \delta V \frac{\partial V}{\partial r}$). Logarithmic plot of the potential in the surface region to which the scattering is sensitive is shown in the inset.
The calculated transition potential is shown in Fig. 4.3. Around the strong absorption region (9.35 fm), both the phenomenologically-calculated transition potential and microscopically-derived potential show similar behaviour.

### 4.3.3 Absorptive Potential

The absorptive part of the potential was calculated following the approach of Vinh Mau and Pacheco et al [24, 5] as described in the chapter 3. The calculated potentials are shown in the Fig. 4.4. Here we have taken only limited number of channels for such calculations. The strength of these transition (channel) used in our calculation are given in Table 4.2.

### 4.3.4 Results and discussions

In the present calculation the imaginary potentials included only the contribution from the $2^+$ inelastic states for target, projectile excitations (both single and the mutual) (Fig 4.4(a)). The real part of the polarisation potential which includes all the open and closed channel contribution, was then renormalised by a factor $\lambda$ (to incorporate the effects of those channels which are not taken into account explicitly in the calculation). So our total potential for the elastic scattering analysis was,

$$V = V_0(R) + \lambda \Delta V_L(R) + W_L(R) + V_e$$

Using this microscopic potential ($\lambda = 1.$), we performed the elastic scattering calculation at 72 MeV and 63 MeV. The nuclear deformation length $\delta = \beta_{20}R$ used was 1.22 fm [6]. The calculated cross section along with the elastic data are shown in Fig. 4.5. For both the energies, the theoretical calculations agreed...
Figure 4.4: Imaginary potentials of $^{28}\text{Si} + ^{28}\text{Si}$ at $E_{\text{lab.}} = 72$ MeV. Solid curve (curve C) in the figure is the total imaginary potential (see text) including the contribution of $2^+$ state of target and projectile (dash-dot curve, A) and their mutual contribution (dotted curve, B).
excellently with the experiment. The introduction of other inelastic channels (4+, 3-) in the calculation produce negligible improvement to the fit.

In the next step, we performed a coupled channel calculation using the 0+ - 2+ coupling with both target and projectile excitations. A real double-folding potential, normalised real polarisation potential and a microscopically-calculated, imaginary potential, which included the 3- and 4+ states only (Fig. 4.4(b)), were used as the diagonal terms of the potential. The real transition potential was derived from the shell model transition density as mentioned earlier and the imaginary part of the transition potential was calculated by taking the derivative of the imaginary potential used in the diagonal term of the potential multiplied by the deformation length (\(\delta_t\)). Using this prescription, the coupled channels calculations were performed with the coupled channels code FRESCO [6] which can include both inelastic and transfer channels. In our calculation we have included all the partial waves up to \(l=250\hbar\) (checking for convergence of result). The matching radius was set at 30 fm by the same criterion and the integration step size was chosen as 0.05 fm.

The experimental data for the elastic channel are well reproduced by the theoretical calculation with or without the 2+ - 2+ reorientation coupling. In the case of inelastic scattering, the theoretically predicted cross section without reorientation coupling exhibit the interference pattern but does not describe the experimental data very well. Introduction of reorientation coupling terms in the calculation reproduces the oscillation pattern of the experiment at larger angles. As already noted, the effect of the reorientation term in the elastic channel is negligible. By changing the sign of the quadrupole moment i.e, the shape of the nucleus from...
Figure 4.5: Elastic angular distribution for $^{28}\text{Si} + ^{28}\text{Si}$ system. Solid curve shows the theoretical predictions with microscopically calculated potential (see text).
Figure 4.6: Solid curve represents the coupled channels calculation with microscopically derived potential (see text) including 0+ - 2+ coupling of target and projectile excitations and quadrupole reorientation coupling (oblate shape). Dotted line with prolate shape.
oblate to prolate, a distinctive effect on the inelastic channel was seen but nothing significant in the elastic channel. There is better agreement between calculation and experimental data with oblate deformation (Fig. 4.6).

In the low energy around the barrier the absorption is mostly concentrated in a small region of the nuclear surface [5]. On the other hand, fusion which is also one of the important channels around the barrier should be represented by a volume type of potential which is sensitive to the inner radial domain. In our calculation a Woods-Saxon squared imaginary potential \( V_I = 10 \text{ MeV}, r_I = 1 \text{ fm}, a_I = 0.4 \text{ fm} \) was added to the central part of the potential to incorporate the fusion contribution with in a small radial region [20]. The effect of this potential on the elastic and the inelastic \( (2^+) \) channel cross sections was found to be negligible.

### 4.4 Excitation function

Excitation functions for elastic and inelastic \( (2^+) \) scattering were also analysed using both the phenomenological optical model and the microscopic model.

#### 4.4.1 Elastic excitation function

In the present study, we have seen that the microscopic potentials are reproducing the elastic scattering data well. With these potentials we performed a detailed analysis of the elastic excitation function by taking into account the coupling of the first excited state \( (2^+) \) to the ground state using the coupled channels code FRESCO. For each energy point (at steps of 0.5 MeV), the microscopic imaginary potential was calculated with contributions from channels \( 4^+ \) and \( 3^- \) only (since the \( 2^+ \) is already explicitly coupled). The calculated imaginary potential at and a
few MeV above the Coulomb barrier becomes zero at some points in radial space. This reflects the very small diffusivity of the nuclear potentials near barrier. The energy and density-dependent real folded potential was calculated for different energy points. We observed a weak energy dependence in the real part at the strong absorption radius also.

Fig. 4.7(a) shows the elastic excitation function at $\theta_{\text{c.m.}} = 90^\circ$. For an identical particle system, the cross section is symmetric around this point. The solid curve in the figure represents the coupled channels calculation with both $0^+ - 2^+$ coupling and also $(2^+ - 2^+)$ reorientation coupling, the quality of the fit is satisfactory.

We also performed the coupled channels calculation using the phenomenological potential described by parameter set CCI of Table 4.1. The calculated cross-sections at the higher energy (a few MeV above the Coulomb barrier) slightly overpredict the experimental cross sections, but generally the fit is a satisfactory one.

### 4.4.2 Inelastic excitation function and the Coulomb - nuclear interference

Inelastic cross sections were also obtained as the offshoot of the same calculation. Fig.4.7(b) shows the prominent Coulomb nuclear dip in the experimental excitation function. The theoretical prediction with the microscopically calculated potential including $0^+ - 2^+$ coupling and quadrupole reorientation coupling does not reproduce the experimental data very well. One possible reason for this discrepancy might be that in the way we derive the coupling potential, the imaginary part may not be appropriate at and around the barrier energies. It was observed that at low energy, absorption was vanishing at a few radial points. As the effect of the
Coulomb excitation of other channels also comes through the coupling potential, so the absence of that absorption might be the reason for the low cross sections in the theoretical calculation at the low energy.

Coupled channels calculations (0\(^+\) - 2\(^+\) coupling) with a phenomenological potential (Table 4.1) and reorientation coupling (oblate deformation) reproduce the cross sections above the barrier energy within experimental error (see Fig. 4.8(a) also). At the low energy side the magnitude of the predicted cross-section is about a factor of two less than the experiment. This discrepancy has also been observed by Emiling et al. [4] in their inelastic scattering data at 66 MeV and 68 MeV. Below and at the barrier, the inelastic cross section is mostly dominated by the Coulomb excitation. The effect of nuclear reorientation and absorption due to the nuclear process is negligible. The calculated values of the inelastic cross-section can be increased by the introduction of a large Coulomb reorientation term.

To understand this we decompose the contribution of the Coulomb excitation from the nuclear coupling. Fig. 4.8(b) shows the Coulomb and the nuclear contribution in the inelastic cross-section by dotted and dash-dot-dot curve respectively. It should be noted that the observed dip cannot possibly be explained by changing the magnitude of the nuclear amplitude. A change in phase is also needed. It is clear that just the inclusion of the 0\(^+\) - 2\(^+\) coupling along with the change in real and imaginary potentials and nuclear strength parameter (\(\beta_{02}\)) will not suffice to reproduce the observed dip. Changing these parameters simply changes the overall magnitude leaving the shape of the excitation function unchanged. Therefore the only possible way to bring about a more satisfactory fit is by changing the reorientation coupling.

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Figure 4.7: Excitation function for elastic and inelastic ($2^+$) scattering of $^{28}\text{Si}$ on $^{28}\text{Si}$ system at $\theta_{\text{lab.}} = 45^\circ$. Continuous curves are the results of coupled channels calculation including $0^+ - 2^+$ coupling of both target and projectile excitation and quadrupole reorientation. Solid curve represents the microscopic calculation and dotted curve represents the phenomenological (see text).
Figure 4.8: Calculations with prolate and oblate shapes are shown in Fig. (a). The contribution of the Coulomb excitation and the nuclear coupling are shown in Fig. (b).
Figure 4.9: Excitation function fit with an energy dependent reorientation coupling (see text).
As an alternative scheme to fit the Coulomb-nuclear interference region in the excitation function, the following procedure was adopted. The matrix element associated with the reorientation term \( \beta_{22} \) is given an energy dependence of the form,

\[
\langle 2^+|E_2|2^+ \rangle = \left[ \sqrt{2} A \sigma^{-\frac{3}{2}} \exp\left(-\frac{2(E - E_0)^2}{\sigma^2}\right) + 1 \right] \langle 2^+|E_2|2^+ \rangle_0
\]

which is a Gaussian form, \( \langle 2^+|E_2|2^+ \rangle_0 \) being the energy independent term. The expression is similar to the energy dependent real potential \( U(r, E) = V(r) + \Delta V(r, E) \), \( \Delta V \) being the polarisation potential. The parameters were obtained through a fitting procedure and the values arrived at are,

- \( \sigma \): width of the distribution : 11.53 MeV
- \( A \): Area under the Gaussian : 71.37 MeV^2
- \( E_0 \): Centroid : 55.61 MeV

The resulting fit is shown in the Fig 4.9. The quality of the fit is fairly satisfactory, except the low energy peak where it underpredicts the data by about 30%.

The physical significance of the energy dependence is as follows. The static quadrupole moment being a structure dependent quantity cannot possibly have any dependence on the projectile energy. However, the corresponding matrix element denoting off-diagonal channel coupling potential may have an energy dependence due to the same reason which causes the energy dependence in the diagonal optical potential. Indeed we have tried to keep a similar Gaussian form for this transition potential as in the real diagonal optical potential. Energy dependence in the reorientation coupling has been previously suggested \[21\] though a detailed fitting
as in our work, has never been attempted.

4.5 Summary

The elastic scattering angular distribution for $^{28}\text{Si} + ^{28}\text{Si}$ was measured at 63 and 72 MeV energies. The inelastic angular distribution was measured at 72 MeV. Around the Coulomb barrier from 58 MeV to 84 MeV the excitation function for elastic and inelastic ($2^+$) were also measured. The data were analysed in a systematic way by using phenomenological potentials as well as microscopically derived potential from basic nucleus-nucleus interaction. Calculated cross sections obtained from these approaches show a very good agreement with the experimental angular distribution of elastic scattering with reasonable fits to inelastic scattering. But the Coulomb-nuclear interference dip observed in the excitation function of inelastic ($2^+$) state is not described well. The observed enhancement in the inelastic excitation function at and below the Coulomb barrier remained unexplained in our calculation. At the higher energies, (78 MeV and above, a few MeV above the Coulomb barrier) the theoretically derived behaviour of the excitation functions from the microscopic potential show a different pattern from the experimental behaviour. It is likely that above the Coulomb barrier more number of channels are contributing to the absorption which we have not taken into account. A more elaborate calculation of the imaginary potential including explicit coupling of those channels is probably required.

We observed that changing the strength of the associated matrix element in the reorientation coupling with energy leads to a better fit to the excitation function for the inelastic $2^+$ state.
Bibliography


