CHAPTER-6

NONLINEAR PROPAGATION OF GAUSSIAN LASER BEAM IN AN INHOMOGENEOUS PLASMA UNDER PLASMA DENSITY RAMP

6.1 INTRODUCTION

The theoretical and experimental study of interaction of high intensity laser beams with plasmas is a fascinating field of research which gives rise to various important applications such as plasma based accelerators [5], inertial confinement fusion [6, 7], ionospheric modification [17, 18] etc. For the success of these applications, the laser beam propagates over distances greater than several Rayleigh lengths [64, 120, 65]. Self-focusing is a nonlinear phenomenon which is induced due to change in the refractive of the medium. It can be relativistic [121] as well as ponderomotive [42]. The former is due to relativistic mass variation of electrons and the later is due to plasma density variations produced by ponderomotive forces. The phenomenon of self-focusing has been studied by many authors [122, 35, 92, 71, 91, 22] and found that optimized parameters are important for self-focusing. Gupta *et al*. [87] found that the ion temperature causes thermal self-focusing and has a serious influence on the evolution of laser beam in plasma. However, optimum self-focusing is achieved by taking in to account the combined effect of relativistic and ponderomotive self-focusing [86].

Jafari Milani *et al*. [95] investigated the ponderomotive self-focusing of Gaussian laser beam and reported that the collision frequency at first causes self-focusing and then defocusing of laser beam takes place in warm collisional plasma. But, as collision frequency is increased, the self-focusing length becomes shorter with the result larger collision frequency prevents the longer propagation of laser beam through plasma. The higher order axial electron temperature decreases the influence of collisional nonlinearity. It changes the electron density distribution and increases the dielectric constant therefore, leads to fast divergence of the laser beam [123]. However, following higher order paraxial theory with ramped density profile enhances the focusing [99]. Again, Patil *et al*. [85] have found that the upward plasma density ramp tends to enhance the self-focusing significantly and the beam gets more focused while traversing several Rayleigh lengths as compared with uniform density relativistic plasma. Kant and Wani [181] reported that the decentered parameter and linear absorption change the self-focusing / defocusing nature of the beam. The absorption weakens the self-focusing effect and the density transition sets sooner and an earlier self-focusing.

In this paper, our purpose is to analyze the impact of upward plasma density ramp on nonlinear Gaussian propagation in an inhomogeneous plasma. The plasma density ramp profile chosen is of the form $n(\xi) = n_0 \tan(\xi / d)$. The non-linear dielectric constant of plasma is presented in ponderomotive regime. The equations governing the laser beam evolution are derived. The computational results in the context of plasma density, laser intensity and initial beam width are discussed and finally a brief conclusion is added. The importance of the present work lies in the fact that the upward plasma density ramp enhances the self-focusing to a greater extent in inhomogeneous plasma.

6.2 NONLINEAR DIELECTRIC CONSTANT

The nonlinear dielectric function ε for an isotropic inhomogeneous medium can be expressed as

$$\varepsilon = \varepsilon_r(z, EE^*) - i\varepsilon_i(z, EE^*), \qquad (6.1)$$

where, ε_r and ε_i are the functions of z and the irradiance EE^*. Further, ε_r can be expressed as:

$$\varepsilon_r(z, EE^*) = \varepsilon_0(z) + \varepsilon_s \mu(z) \frac{\varepsilon_2 EE^*}{1 + \varepsilon_2 EE^*}, \qquad (6.2)$$

where, ε_0 and μ are functions of z. The function $\mu(z)$ is identified with the plasma frequency. In this case $\varepsilon_0(z) = 1 - \left(\omega_{p0}^2 / \omega^2\right)\mu(z)$, $\varepsilon_s = \omega_{p0}^2 / \omega^2$, $\varepsilon_i(z) = \mu(z)\varepsilon_i(0)$ with, $\mu(z) = \omega_p^2 / \omega_{p0}^2 = \tan(\xi / d)$, $\omega_p^2 = \omega_{p0}^2 \tan(\xi / d)$ and $\omega_{p0}^2 = 4\pi n_0 e^2 / m$. Where, ω_{p0} is the plasma frequency, ω is the angular frequency of incident laser beam, ε_i is the characteristic of absorption in the medium, $\mu(z)$ is characteristic of the density of dipoles, m, e and n_0 are the electron's rest mass, charge on the electron and equilibrium electron density respectively, ξ is the propagation distance and d is a dimensionless parameter. In case of Gaussian beam $\varepsilon_r(z)$ can be expanded as:

$$\varepsilon_r(z) = \varepsilon_{r0}(z) - r^2 \varepsilon_{r_2}(z). \qquad (6.3)$$

6.3 SELF-FOCUSING EQUATIONS

Consider the Gaussian laser beam propagating along the z - direction with electric vector $\vec{E}$ satisfies the scalar wave equation of the form

$$\frac{\partial^2 E}{\partial z^2} + \left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right\}E + \frac{\omega^2}{c^2}\varepsilon(r,z)E = 0 \ , \tag{6.4}$$

where, c is the speed of light in vacuum. Eq. (6.4) can be solved in the paraxial approximation by adopting the analysis of Akhmanov *et al*. [113] and Sodha *et al*. [20, 114]. The solution of equation (6.4) is of the form

$$E(r,z) = A(r,z)\exp\left[-\int_0^z ik(z)dz\right], \tag{6.5}$$

where, $A(r,z)$ is the slowly varying envelope of the beam and k is the wave propagation constant which is given by $k^2(z) = (\omega^2/c^2)\varepsilon_{r0}(z)$.

Differentiating equation (6.5) twice w. r. t. $'r'$ and $'z'$ and neglecting $(\partial^2 A/\partial z^2)$, we get

$$\frac{\partial \vec{E}}{\partial r} = Exp\left[-\int_0^z ik(z)dz\right]\frac{\partial A(r,z)}{\partial r}$$

$$\frac{\partial^2 \vec{E}}{\partial r^2} = Exp\left[-\int_0^z ik(z)dz\right]\frac{\partial^2 A(r,z)}{\partial r^2}$$

And

$$\frac{\partial \vec{E}}{\partial z} = Exp\left[-\int_0^z ik(z)dz\right]\left[\frac{\partial A(r,z)}{\partial z} - iA(r,z)k(z)\right]$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} = Exp\left[-\int_0^z ik(z)dz\right]\left[-2ik(z)\frac{\partial A(r,z)}{\partial z} - iA(r,z)\frac{\partial k(z)}{\partial z} - k^2(z)A(r,z)\right]$$

Substituting the above values in Eq. (6.4), under WKB approximation, one obtains

$$-2ik\frac{\partial A}{\partial z} - iA\frac{\partial k}{\partial z} - k^2A + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\omega^2}{c^2}\varepsilon(r,z)A = 0, \tag{6.6}$$

To solve Eq. (6.6) in the paraxial approximation, the complex amplitude $A(r,z)=A_0(r,z)\exp[-ik(z)S(r,z)]$ is considered. Here, A_0 and S depend on r and z. Therefore, from Eq. (6.6), we get

$$-2k^2A_0\frac{\partial S}{\partial z}-2kA_0S\frac{\partial k}{\partial z}-k^2A_0+\frac{\partial^2 A_0}{\partial r^2}-k^2A_0\left(\frac{\partial S}{\partial r}\right)^2+\frac{1}{r}\frac{\partial A_0}{\partial r}+\frac{\omega^2A_0}{c^2}\left(\varepsilon_{r0}(z)-r^2\varepsilon_{r2}(z)\right)-$$

$$\left(2k\frac{\partial A_0}{\partial z}+A_0\frac{\partial k}{\partial z}+kA_0\frac{\partial^2 S}{\partial r^2}+2k\frac{\partial S}{\partial r}\frac{\partial A_0}{\partial r}+\frac{kA_0}{r}\frac{\partial S}{\partial r}+\frac{\omega^2\varepsilon_i(0)A_0}{c^2}\tan(z/dR_d)\right)=0 \quad (6.7)$$

Now, equating real and imaginary parts of Eq. (6.7), we get

Real part equation is

$$2\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial r}\right)^2+\left(\frac{2S}{k}\right)\frac{\partial k}{\partial z}=\frac{1}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right)-\frac{r^2\varepsilon_{r_2}(z)}{\varepsilon_{r_0}(z)}, \quad (6.8)$$

Imaginary part equation is

$$\frac{\partial A_0^2}{\partial z}+A_0^2\left(\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)+\frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r}=A_0^2\left(\frac{-\tan\left(z/d\mathrm{R}_d\right)k\varepsilon_i(0)}{\varepsilon_{r_0}(z)}-\frac{1}{k}\frac{\partial k}{\partial z}\right). \quad (6.9)$$

The solution of Eq. (6.9) can be written as

$$A_0^2(r,z)=\left(\frac{E_0^2(z)}{f^2(z)}\right)\Im\left(\frac{r}{r_0 f(z)}\right), \quad (6.10)$$

where, $\Im$ is any arbitrary parameter and $E_0^2(z)$ is the axial irradiance. For an initially Gaussian beam at $z=0$, $EE^*=A_0^2=E_{00}^2\exp(-r^2/r_0^2)$ and the eikonal $S(r,z)=(r^2/2)\beta(z)+\phi(z)$, where, $\beta(z)=(1/f(z))(\partial f/\partial z)$ represents the curvature of the wavefront. Substituting for A_0^2 and using the above values in Eq. (6.8), we get

$$r^2\left[\frac{1}{r_0^4k^2f}\frac{\partial^2 f}{\partial\xi^2}+\frac{1}{2k^2fr_0^4\varepsilon_{r0}}\left(\frac{\partial f}{\partial\xi}\right)\left(\frac{\partial\varepsilon_{r0}}{\partial\xi}\right)\right]+2\frac{\partial\phi}{\partial z}+\frac{\phi(z)}{kr_0^2\varepsilon_{r0}}\left(\frac{\partial\varepsilon_{r0}}{\partial\xi}\right)=-\frac{2c^2}{\omega^2r_0^2\varepsilon_{r0}f^2}$$

$$+r^2\left[\frac{c^2}{\omega^2r_0^4\varepsilon_{r0}f^4}-\frac{\varepsilon_{r2}(z)}{\varepsilon_{r0}}\right]. \quad (6.11)$$

Now, equating the coefficients of r^2 on both sides of Eq. (6.11), we get

the resulting equation. One obtains

$$\frac{\partial^2 f}{\partial \xi^2} = -\frac{\rho^2}{f}\left[\varepsilon'_{r_2} - \frac{1}{\rho^2 f^2}\right] - \frac{1}{2\varepsilon_{r_0}}\left(\frac{\partial f}{\partial \xi}\right)\left(\frac{\partial \varepsilon_{r_0}}{\partial \xi}\right), \tag{6.12}$$

where, $\varepsilon'_{r2} = r_0^2 f^2 \varepsilon_{r2}(E_0^2 / f^2)$, $\rho_0 = r_0 \omega / c$ is the equilibrium beam radius, $\xi = z / R_d$ is the propagation distance, $R_d = kr_0^2$ represents the diffraction length. Thus, with the dependence of ε'_{r2} on f and ξ, Eq. (6.12) can be solved for f as a function of ξ. Further, expanding $A_0^2(r,z)$ from Eq. (6.10) in a series of r^2 and retaining terms up to r^2 (in the paraxial approximation). We have

$$EE^* = \frac{E_0^2}{f^2}\left(1 - \frac{r^2}{r_0^2 f^2}\right), \tag{6.13}$$

Using Eq. (6.13) in Eq. (6.2) one obtains

$$\varepsilon_{r_0} = \varepsilon_0(z) + \varepsilon_s \tan\left(z / d\mathrm{R}_d\right) \frac{\left(\varepsilon_2 E_0^2 / f^2\right)}{1 + \left(\varepsilon_2 E_0^2 / f^2\right)} \tag{6.14}$$

and

$$\varepsilon'_{r_2} = \frac{\varepsilon_s \varepsilon_2 E_0^2 \tan\left(z / d\mathrm{R}_d\right)}{1 + \varepsilon_2 E_0^2 / f^2}. \tag{6.15}$$

Using Eq. (6.14) and (6.15) in Eq. (6.12), we get

$$\frac{\partial^2 f}{\partial \xi^2} = -\frac{\rho^2}{f}\left[\frac{\varepsilon_s \varepsilon_2 E_0^2 \tan\left(\xi / d\right)}{1 + \varepsilon_2 E_0^2 / f^2} - \frac{1}{\rho^2 f^2}\right] + \left[\frac{1}{1 - \varepsilon_s \tan\left(\xi / d\right) + \dfrac{\varepsilon_s \varepsilon_2 E_0^2 \tan\left(\xi / d\right)}{f^2 + \varepsilon_2 E_0^2}}\right] \times$$

$$\left[\frac{f^2 \mathrm{Sec}^2\left(\xi / d\right)}{2d} + \frac{\varepsilon_2 E_0^2 f \tan\left(\xi / d\right)}{f^2 + \varepsilon_2 E_0^2}\left(\frac{\partial f}{\partial \xi}\right)\right] \frac{\varepsilon_s}{f^2 + \varepsilon_2 E_0^2}\left(\frac{\partial f}{\partial \xi}\right). \tag{6.16}$$

Equation (6.16) is the required equation for the beam width parameter f, which could be solved by applying the initial condition at $\xi = 0$, $f = 1$, $(\partial f / \partial \xi) = 0$ and $\left(\partial^2 f / \partial \xi^2\right) = 0$

6.4 RESULTS AND DISCUSSION

In the present communication, we have seen the effect of ramped density profile on the propagation of Gaussian laser beam in inhomogeneous plasma. We have solved Eq. (6.16) numerically and the various parameters taken for numerical calculation are: $\omega = 1.778\times 10^{14}\, rad/\sec$, $r_0 = 20\mu m$ and $\lambda = 1.06\mu m$ [101]. The value of intensity of laser beam is $I_0 = 1.21\times 10^{18}\, W/cm^2$. Fig. 6.1 represents the dependence f on ξ for various values of ω_{p0}/ω. The other parameters are $d = 10$, $\varepsilon_2 E_0^2 = 10$, $\rho^{-2} = 0.04$. It is observed that by increasing the values of ω_{p0}/ω, f decreases strongly with ξ and reaches its minimum at $\omega_{p0}/\omega = 0.8$. The laser beam then undergoes oscillatory behavior and the frequency of oscillation increases, while amplitude decreases gradually close to the propagation axis. Therefore, the plasma dielectric constant decreases rapidly as initial electron density depends on ξ with the result, self-focusing is observed at large ξ values. The results of present analysis can be compared with those of Kaur *et al.* [109]. Wherein, the introduction of two scale length leads the laser beam to oscillate periodically for a long propagation distance with constant amplitude. Fig. 6.2 illustrates the behavior of f with ξ for various values of $\varepsilon_2 E_0^2$. The other parameters are $\omega_{p0}/\omega = 0.4$, $d = 15$, $\rho^{-2} = 0.08$. From figure 6.2 it is clear that while propagating the laser beam through the plasma, the diffraction of beam starts earlier with increase in $\varepsilon_2 E_0^2$ and hence controls the behavior of beam width parameter. Due to increase in the value of intensity, highly energetic electrons will continue to move forward without loss of energy. Further, the beam width parameter is a function of laser spot size and depends on intensity of laser beam. Therefore, the intensity rise results in the reduction of spot size of laser beam with the result, self-focusing of laser beam slows down and becomes stronger. Figure 6.3 shows the dependence of f on ξ for different ρ^{-2} values. The plasma density is fixed at $\omega_{p0}/\omega = 0.8$ and the other parameters are same as taken in figure 6.1. From figure 6.3, it is clear that the oscillatory character of the beam width parameter is observed for a chosen set of parameters. Remarkably, the amplitude of oscillations decreases gradually with the distance of propagation on account of effect of nonlinearity. Further, with increase in the values of $1/\rho^{-2}$, the beam width parameter first increases, attains a maximum and then decreases thereby, exhibits oscillatory character. Therefore, the behavior of beam width parameter is highly affected by the beam radius under density transition. Again, Navare *et al*. [80] concluded that by taking in to account the collisional nonlinearity and linear absorption, the oscillatory behavior of beam width parameter weakens with the distance of propagation. The self-focusing of laser beam takes place only for a short propagation distance and the beam then defocuses. However, in the present work, introduction of density ramp leads the beam width parameter to decrease with a higher rate. Consequently, the self-focusing of laser beam is enhanced to a greater extent by exploiting the density transition in an inhomogeneous plasma. Above all, the density transition plays a vital role in laser plasma interaction and is important for the injection of plasma electrons to acceleration stage.

6.5 CONCLUSION

In the present investigation, we have investigated the Gaussian beam propagation in inhomogeneous plasma under plasma density ramp. The differential equation for beam width parameter is established under paraxial approximation. The effect of plasma density, laser intensity and initial beam width on self-focusing has been discussed. By optimizing laser and plasma parameters, the effect of density transition on the behavior of beam width parameter with the propagation distance has been analyzed and plotted. The results reveal that the amplitude of oscillation decreases considerably with the distance. The oscillatory behavior of beam width parameter becomes slow with increase in relative plasma density and intensity of laser beam. The saturation behavior of the beam width parameter shows that the laser beam evolves differently when propagates through underdense plasma. Further, after initial laser focusing, the relativistic mass effect is more declared in high plasma density region. Therefore, the plasma density ramp enhances the self-focusing effect to a greater extent. The outcomes obtained in the present analysis may be useful in understanding the physics of plasma based accelerators.

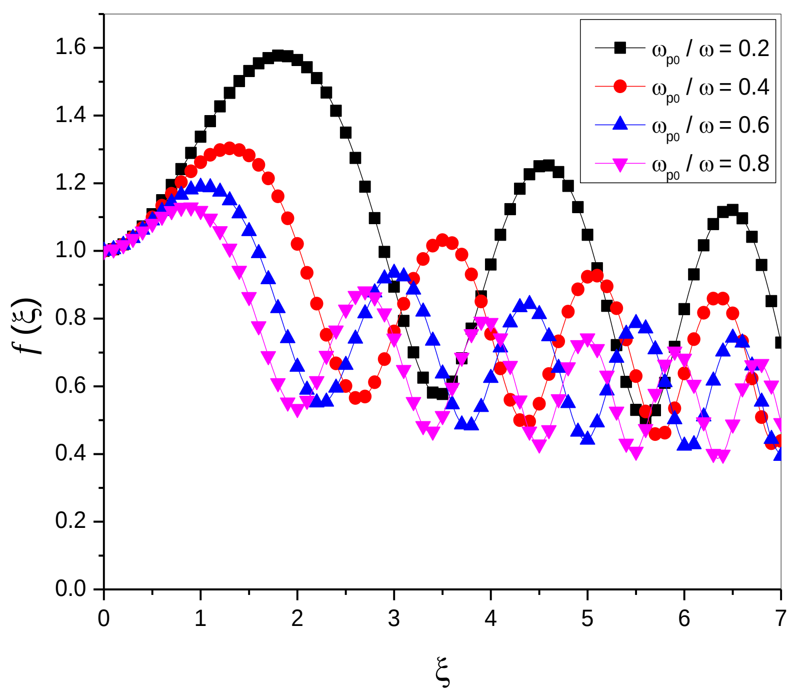

Figure 6.1: Dependence f on ξ for various values of ω_{p0}/ω. The other parameters are $d = 10$, $\varepsilon_2 E_0^2 = 10$, $\rho^{-2} = 0.04$.

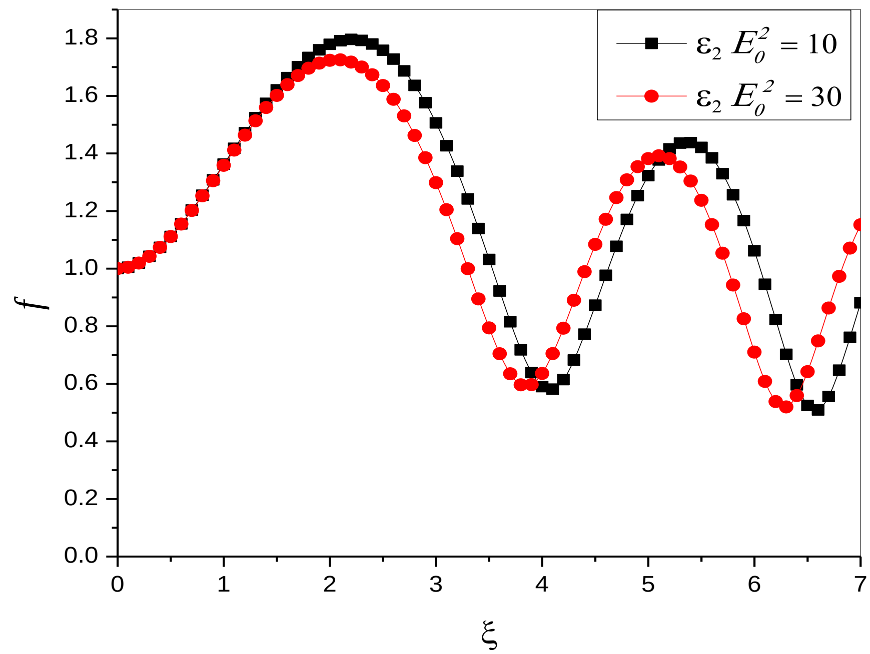

Figure 6.2: Dependence f on ξ for various values of $\varepsilon_2 E_0^2$. The other parameters are $\omega_{p0}/\omega = 0.4$, $d = 15$, $\rho^{-2} = 0.08$

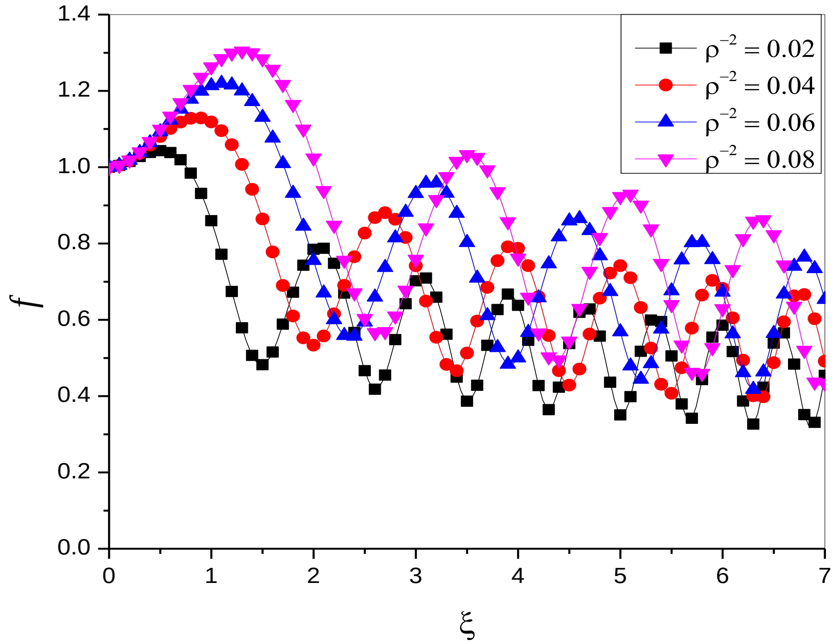

Figure 6.3: Dependence f on ξ for various values of ρ^{-2}. The other parameters are $\varepsilon_2 E_0^2 = 10$, $\omega_{p0}/\omega = 0.8$, $d = 10$