CHAPTER-4

DENSITY TRANSITION BASED SELF-FOCUSING OF COSH GAUSSIAN LASER BEAM IN PLASMA WITH LINEAR ABSORPTION

4.1 INTRODUCTION

The interaction of intense laser beams with plasmas has been an important field of research due to various applications like laser electron acceleration [9-11], inertial confinement fusion [13-15] and ionospheric modification [16-19] etc. These applications require large interaction region up to several Rayleigh lengths without loss of energy. When a high power laser beam interacts with the plasma, it provides an oscillatory velocity to the electron so that the dielectric constant gets modified and leads to relativistic self-focusing [105]. The real part of dielectric constant having saturating nonlinearity characterizes the steady- state focusing or defocusing in a medium and the imaginary part being determined by multi photon absorption under paraxial approximation [58]. Takale *et al*. [71] analyzed the self-focusing and defocusing of first six TEM_{op} Hermite-Gaussian laser beams in collision-less plasma and found that modes having odd p-values defocus and those having even p-values are capable of sustaining oscillations as well as are able to have a defocusing character during propagation in collision-less plasma. However, it has been observed that a plasma density ramp of suitable length can reduce these oscillations [64].

The propagation of HchG beams in n-InSb has been studied for various mode indicies viz 0, 1 and 2 and incorporates the desirability of process of self-focusing in a particular application by taking an advantage of beams having decentered parameter [24]. The relativistic self-focusing of cosh beams illustrates that oscillatory self-focusing takes place for b = 0, 1 and sharp self-focusing effect for b = 2 [106]. Nanda *et al*. [92] have observed that decentered parameter and ramp density profile results in self-focusing of laser beam. However, by considering the magnetic field and plasma density ramp for Hermite-cosh Gaussian laser beam, it has been found that the presence of density transition and magnetic field enhance the self-focusing effect to a greater extent [91].The proper and an appropriate decentered parameter selection is very much sensitive [90]. However, Kant *et al*. [22] have observed the effect of density transition and initial intensity of the laser beam on self-focusing of laser beam. Again, the parameters like density profile, intensity parameter, and decentered parameter play a crucial role in the enhancement of laser beam focusing significantly. Further, the upward plasma density ramp in weakly relativistic and ponderomotive regime can accelerate the electron to higher energy over a long propagation distance as compared with uniform density relativistic plasma [85]. The oscillatory self-focusing takes place for different values of intensity parameter and with increase in intensity parameter, the distance between two consecutive points of intersection of two beams increases [88]. The laser beam is compressed and amplified in an enhanced manner by the combined effect of magnetic field, relativistic nonlinearity and negative initial chirp [107]. It is to be noted that strong self-focusing is obtained by optimizing wavelength and intensity parameters of beams [94]. In the investigation of self-focusing and frequency broadening of laser pulse in water medium, the laser beam initially undergoes self-focusing due to Kerr nonlinearity and then nonlinear refraction takes place which causes the laser beam to defocus [108]. Also, it has been found that with increase of power density and control parameters lead to trapping of particle in the potential and hence strong focusing [109]. For a short laser pulse undergoing self-focusing in plasma with density ramp, the pulse acquires a very low spot size and the focused pulse then diffracts and focuses in a regular and repeated manner. In this case the oscillation amplitude of the spot size contracts and results in increasing frequency. Hence, the laser propagating in plasma under plasma density ramp may likely become more focused. Further, due to the supremacy of diffraction effect and in absence of density transition, it gets defocused. As the plasma density increases, self-focusing effect becomes stronger [77]. Furthermore, the quantum effect plays a key role in laser-plasma interaction and significantly adds to self-focusing in comparison to classical relativistic case [84]. However, in addition to quantum effects, ramped density profile causes larger and higher oscillations and consequently better focusing in cold quantum plasma (CQP) [82].

In the present communication, we have studied the self-focusing of cosh-Gaussian beam in plasma by taking in to account the plasma density transition effect and linear absorption through parabolic equation approach. The second order differential equation that describes the nature of self-focusing in plasma is obtained by following paraxial approximation. The results are presented graphically and are discussed. Finally, a conclusion is drawn in the last section of this chapter.

4.2 FIELD DISTRIBUTIO OF COSH – GAUSSIAN BEAMS

The field distribution of cosh-Gaussian laser beam at z = 0 is characterized by [110-112]

$$E(r,0)=E_0\exp\left(-\frac{r^2}{r_0^2}\right)\cosh(\Omega_0 r) \tag{4.1}$$

Where, r_0 is the waist width, r is the radial coordinate, E_0 is the amplitude of the electric field at centre position and Ω_0 is called the cosh factor. On the other hand Eq. (4.1) can be expressed as follows:

$$E(r,0)=\frac{E_0}{2}\exp\left(\frac{b^2}{4}\right)\left\{\exp\left[-\left(\frac{r}{r_0}+\frac{b}{2}\right)^2\right]+\exp\left[-\left(\frac{r}{r_0}-\frac{b}{2}\right)^2\right]\right\} \tag{4.2}$$

Where, $b = r_0\,\Omega_0$ is called the decentered parameter.

Now, corresponding to absorption alone, the concentration of energy of the laser beam decreases by a factor of exp(-2∫ $k_i$dz) which weakens the nonlinearity effect. Therefore, in accordance with Eq. (4.2), we can construct the following ansatz for the field distribution along the z-axis.

$$E(r,0)=\frac{E_0}{2f}\exp\left(\frac{b^2}{4}\right)\exp(-2k_i z)\left\{\exp\left[-\left(\frac{r}{r_0 f}+\frac{b}{2}\right)^2\right]+\exp\left[-\left(\frac{r}{r_0 f}-\frac{b}{2}\right)^2\right]\right\} \tag{4.3}$$

Where k_i is the absorption coefficient and f = f (r, z) is the dimensionless beam-width parameter.

4.3 NONLINEAR DIELECTRIC CONSTANT

The cosh-Gaussian beam propagation in plasma is being characterized by a dielectric constant in the following form

$$\varepsilon=\varepsilon_0+\phi(EE^*) \tag{4.4}$$

With, $\varepsilon_0=1-\omega_P^2/\omega^2$, $\omega_P^2=4\pi\, n(\xi)e^2/m$, $\omega_P^2=\omega_{p0}^2\tan(\xi/d)$ and $\omega_{P0}^2=4\pi\, n_0 e^2/m$, here '$\varepsilon_0$' represents the linear part and ϕ represents the non-linear parts of the dielectric constant respectively. Here, 'ω_{P0}' the plasma frequency, 'e' the electronic charge, 'm' the rest mass of the electron, 'ω' the frequency of the incidents laser beam and 'n_0' the equilibrium electron density, ξ the normalized propagation distance and d is a dimensionless adjustable parameter.

4.4 SELF-FOCUSING EQUATIONS

The wave equation that describes the laser beam propagation in plasma may be written as

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2}\left(-\omega^2 \vec{E}\right) + \vec{\nabla}\left(\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}\right) = 0 \tag{4.5}$$

The last term of Eq. (4.5) on left hand side is neglected under the condition $k^{-2}\nabla^2(\ln\varepsilon) << 1$, where 'k'represents the wave number the laser beam. Thus,

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2}\varepsilon\vec{E} = 0$$

This equation is solved by employing WKB approximation. In cylindrical co-ordinate system, we can write this equation as

$$\frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial^2 \vec{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{E}}{\partial r} + \varepsilon\frac{\omega^2}{c^2}\vec{E} = 0 \tag{4.6}$$

For slowly converging or diverging cylindrically symmetric beam, the solution of equation (4.6) is of the following form,

$$\vec{E} = A(r,z)Exp[i(\omega t - kz)] \tag{4.7}$$

With $k^2 = \varepsilon_0\,\omega^2/c^2 = \omega^2/c^2\left(1-\omega_{P0}^2\tan(\xi/d)/\omega^2\right)$.

Differentiating equation (4.7) twice w. r. t. 'r' and 'z', we get

$$\frac{\partial \vec{E}}{\partial r} = Exp[i(\omega t - kz)]\frac{\partial A(r,z)}{\partial r}$$

$$\frac{\partial^2 \vec{E}}{\partial r^2} = Exp[i(\omega t - kz)]\frac{\partial^2 A(r,z)}{\partial r^2}$$

And

$$\frac{\partial \vec{E}}{\partial z} = Exp[i(\omega t - kz)]\left[\frac{\partial A}{\partial z} - \frac{i\omega A}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)} + \left(\frac{i\omega}{2cdR_d}\right)\frac{\omega_{p0}^2}{\omega^2}\frac{zASec^2(z/dR_d)}{\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\right]$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} = Exp[i(\omega t - kz)]\left[\frac{\partial^2 A(r,z)}{\partial z^2} + \left(\frac{i\omega}{c}\right)\frac{\partial A(r,z)}{\partial z}\left(\frac{\omega_{p0}^2 zSec^2(z/dR_d)}{\omega^2 dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}} - 2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\right)\right]$$

$$+\frac{\omega A}{c}Exp[i(\omega t - kz)]\left[\frac{i\omega_{p0}^2 zSec^2(z/dR_d)}{\omega^2 dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}} + \frac{\omega z\omega_{p0}^2 zSec^2(z/dR_d)}{c\omega^2 dR_d}\right]$$

$$+\frac{\omega}{c}Exp[i(\omega t - kz)]\frac{iA\omega_{p0}^2 zSec^2(z/dR_d)}{\omega^2 d^2 R_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\left[\tan(z/dR_d) + \frac{\omega_{p0}^2 Sec^2(z/dR_d)}{4\omega^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\right]$$

$$-\frac{\omega^2 A}{c^2}Exp[i(\omega t - kz)]\left[\frac{\omega_{p0}^4 z^2 Sec^4(z/dR_d)}{4\omega^4 d^2 R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)} + \left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)\right]$$

Substituting these values in equation (4.6), and neglecting $\partial^2 A/\partial z^2$ we get

$$\frac{i\omega}{c}\left(2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}(z/d\mathrm{R}_d)} - \frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{zSec}^2(z/d\mathrm{R}_d)}{d\mathrm{R}_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}(z/d\mathrm{R}_d)}}\right)\left(\frac{\partial A}{\partial z}\right) - \frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{ASec}^2(z/d\mathrm{R}_d)}{d\mathrm{R}_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}(z/d\mathrm{R}_d)}}$$

$$-\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{AzSec}^2(z/d\mathrm{R}_d)}{d^2\mathrm{R}_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}(z/d\mathrm{R}_d)}}\left(\mathrm{Tan}(z/d\mathrm{R}_d) + \frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{Sec}^2(z/d\mathrm{R}_d)}{4\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}(z/d\mathrm{R}_d)\right)}\right)$$

$$=\frac{\omega^2}{c^2}\left(\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{AzSec}^2(z/d\mathrm{R}_d)}{d\mathrm{R}_d} - \frac{\left(\frac{\omega_{p0}^2}{\omega^2}\right)^2\mathrm{Az}^2\mathrm{Sec}^4(z/d\mathrm{R}_d)}{4d^2\mathrm{R}_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}(z/d\mathrm{R}_d)\right)}\right)$$

$$+\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{\omega^2}{c^2}\Phi\left(AA^*\right)A \tag{4.8}$$

To solve equation (4.8), we express

$$A(r,z)=A_0(r,z)Exp[-ikS(r,z)] \tag{4.9}$$

Where, k has been defined above and A_0 and S depend on $'r'$ and 'z'.

Differentiating equation (4.9) twice, w. r. t. $'r'$, we get

$$\frac{\partial A(r,z)}{\partial r}=Exp[-ikS(r,z)]\left[\frac{\partial A_0}{\partial r}-\frac{i\omega A_0}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S(r,z)}{\partial r}\right)\right]$$

$$\frac{\partial^2 A(r,z)}{\partial r^2}=Exp[-ikS(r,z)]\left[\frac{\partial^2 A_0}{\partial r^2}-\frac{2i\omega}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S}{\partial r}\right)\left(\frac{\partial A_0}{\partial r}\right)\right]-$$

$$\frac{\omega A_0}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left[i\left(\frac{\partial^2 S}{\partial r^2}\right)+\frac{\omega}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S}{\partial r}\right)^2\right]Exp[-ikS(r,z)]$$

Now differentiating equation (4.9) w. r. t. $'z'$,

$$\frac{\partial A(r,z)}{\partial z}=Exp[-ikS(r,z)]\left[\frac{\partial A_0}{\partial z}-\frac{i\omega A_0}{c}\left\{\begin{array}{l}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S}{\partial z}\right)- \\ \frac{S(r,z)\omega_{p0}^2}{2d\omega^2 R_d}\frac{Sec^2(z/dR_d)}{\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\end{array}\right\}\right]$$

Thus equation (4.8) becomes,

$$\frac{i\omega}{c}\left(\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{d\omega^2 R_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}-2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\right)\frac{\partial A_0}{\partial z}+\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}+$$

$$\frac{\omega^2 A_0}{c^2}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left[\begin{array}{l}\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{d\omega^2 R_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}-\frac{S\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\\+\frac{\partial S}{\partial z}-2\end{array}\right]$$

$$+\frac{i\omega A_0\omega_{p0}^2 Sec^2(z/dR_d)}{cd\omega^2 R_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\left[1+\frac{z}{dR_d}\left(\tan(z/dR_d)+\frac{\omega_{p0}^2 Sec^2(z/dR_d)}{4\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\right)\right]$$

$$=\frac{\omega^2 A_0}{c^2}\left(\frac{z^2\omega_{p0}^4 Sec^4(z/dR_d)}{4d^2\omega^4 R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}-\frac{\omega_{p0}^2 zSec^2(z/dR_d)}{dR_d}+\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)\left(\frac{\partial s}{\partial r}\right)^2\right)$$

$$+\frac{i\omega}{c}\sqrt{\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\left(2\frac{\partial S}{\partial r}\frac{\partial A_0}{\partial r}+A_0\frac{\partial^2 S}{\partial r^2}+A_0\frac{\partial S}{\partial r}\right)-\frac{\omega^2 A_0}{c^2}\phi(A_0^2) \qquad (4.10)$$

Comparing real and imaginary parts of equation (4.10), we get

Real part equation is

$$\frac{1}{\omega^2c^2}\left[\omega^2-\omega_{p_0}^2\tan(z/dR_d)-\frac{\omega_{p_0}^2 z\sec^2(z/dR_d)}{2dR_d}\right]\left(\frac{\partial s}{\partial z}\right)+\frac{1}{\omega^2c^2}\left[\omega^2-\omega_{p_0}^2\tan(z/dR_d)\right]\left(\frac{\partial s}{\partial r}\right)^2+$$

$$\frac{\omega_{p_0}^2\sec^2(z/dR_d)}{4\omega^2c^2d^2R_d^2\left(\omega^2-\omega_{p_0}^2\tan(z/dR_d)\right)}\left[\omega_{p_0}^2 z(z+s)\sec^2(z/dR_d)-2dR_d(s+2z)\left(\omega^2-\omega_{p_0}^2\tan(z/dR_d)\right)\right]$$

$$= \frac{1}{\omega^2 A_0}\left[\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right] + \frac{\Phi\left(A_0^2\right)}{c^2} \tag{4.11}$$

Imaginary part equation is

$$\left(1 - \frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)}{2d\mathrm{R}_d\left(1 - \frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)\frac{\partial A_0^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r} + \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right)A_0^2$$

$$-\left(\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{Sec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d\sqrt{1 - \frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}} + \frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}{d^2\mathrm{R}_d^2\sqrt{1 - \frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}}\right)A_0^2$$

$$-\left(\frac{\omega_{p0}^2}{\omega^2}\right)^2\left(\frac{\mathrm{zSec}^4\left(z/d\mathrm{R}_d\right)}{4d^2\mathrm{R}_d^2\left(1 - \frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)A_0^2 = 0 \tag{4.12}$$

For cosh-Gaussian beam, the solution of equation (4.11) and (4.12) are of the form

$$A_0^2 = \frac{E_0^2}{4f^2(z)}Exp\left[\frac{b^2}{2}\right]Exp\left[-k_i z\right]$$
$$\left\{Exp\left[-2\left(\frac{r}{r_0 f(z)} + \frac{b}{2}\right)\right] + Exp\left[-2\left(\frac{r}{r_0 f(z)} - \frac{b}{2}\right)\right] + 2Exp\left[-\left(\frac{2r^2}{r_0^2 f^2(z)} + \frac{b^2}{2}\right)\right]\right\} \tag{4.13}$$

And

$$S(r,z) = \frac{r^2}{2}\beta(z) + \varphi(z) \tag{4.14}$$

The eikonal $S(r,z)$ determines the convergence or divergence of the beam. with, $\beta(z) = (1/f(z))\partial f/\partial z$ is regarded as the wavefront curvature, '$\varphi(z)$' is an arbitrary function of 'z' and is called phase factor. Also,

$$\Phi\left(A_0^2\right)=\frac{1}{2}\varepsilon_2 A_0^2 \tag{4.15}$$

Where ε_2 is the nonlinear coefficient.

Differentiating equation (4.14) w. r. t. $'z'$ and $'r'$ respectively,

$$\frac{\partial S(r,z)}{\partial z}=\frac{r^2}{2}\frac{\partial \beta}{\partial z}+\frac{\partial \varphi}{\partial z}$$

$$\frac{\partial S(r,z)}{\partial z}=\frac{r^2}{2f(z)}\frac{\partial^2 \beta}{\partial z^2}-\frac{r^2}{2f^2(z)}\left(\frac{\partial f(z)}{\partial z}\right)^2+\frac{\partial \varphi}{\partial z}$$

$$\frac{\partial S(r,z)}{\partial r}=2r\frac{\beta}{2}=r\beta=\frac{r}{f(z)}\frac{\partial f(z)}{\partial z}$$

Now, using paraxial approximation and differentiating equation (4.13) twice w. r. t. $'r'$, we get

$$\frac{\partial A_0}{\partial r}=\frac{r^2 b E_0}{2r_0^3 f^4(z)}Exp\left[\frac{b^2}{4}-2k_i z\right]$$

$$\frac{\partial^2 A_0}{\partial r^2}=-\frac{r^2 E_0}{r_0^4 f^5(z)}Exp\left[\frac{b^2}{4}-2k_i z\right]\left(5+\frac{3}{2}b^2\right)$$

Substituting the values of $\partial S(r,z)/\partial z$, $\partial S(r,z)/\partial r$, $\partial A_0/\partial r$, $\partial^2 A_0/\partial r^2$ and A_0^2 in equation (4.11) and solving, we get

$$\frac{1}{\omega^2}\left[-\frac{r^2}{r_0^4 f^4(z)}\left(5+\frac{3}{2}b^2\right)-\frac{r^2 b^2}{4r_0^4 f^4(z)}\left(5+\frac{3}{2}b^2\right)\right]-\frac{r^2 E_0 \varepsilon_2}{c^2 r_0^2 f^4(z)}Exp\left[\frac{b^2}{2}-k_i \xi k r_0^2\right]$$

$$=\frac{1}{\omega^2 c^2}\left[\omega^2-\omega_{p0}^2\tan(\xi/d)-\frac{\omega_{p0}^2 \xi Sec^2(\xi/d)}{d}\right]\left[\frac{r^2}{2r_0^4 k^2 f}\left\{\frac{\partial^2 f}{\partial \xi^2}-\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2\right\}+\frac{\partial \varphi}{\partial z}\right]$$

$$+\frac{\omega_{p0}^2 (S+\xi k r_0^2) Sec^2(\xi/d)}{4\omega^2 c^2 d^2 k r_0^2\left(\omega^2-\omega_{p0}^2\tan(\xi/d)\right)}\left[\omega_{p0}^2 \xi Sec^2(\xi/d)-2d(\omega^2-\omega_{p0}^2\tan(\xi/d))\right]$$

$$+\frac{r^2}{\omega^2 c^2 r_0^4 k^2 f^2}\left(\omega^2-\omega_{p0}^2\tan(\xi/d)\right)\left(\frac{\partial f}{\partial \xi}\right)^2 \tag{4.16}$$

Now, equating coefficients of r^2 on both sides of Eq. (4.16) and adopting the procedure of Akhmanov *et al*. [113] and Sodha *et al*. [20], the differential equation for the cosh-Gaussian propagation with linear absorption is written as:

$$\left[1-\frac{\omega_{p_0}^2}{\omega^2}\tan(\xi/d)-\frac{\omega_{p_0}^2}{\omega^2}\left(\frac{\xi}{d}\right)\sec^2(\xi/d)\right]\frac{\partial^2 f}{\partial \xi^2}+\left[1-\frac{\omega_{p_0}^2}{\omega^2}\tan(\xi/d)+\frac{\omega_{p_0}^2}{\omega^2}\left(\frac{\xi}{d}\right)\sec^2(\xi/d)\right]\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2$$

$$=\frac{2}{f^3}\left(1-\frac{\omega_{p_0}^2}{\omega^2}\tan(\xi/d)\right)\left[\left(5+\frac{3b^2}{2}\right)\left(1+\frac{b^2}{4}\right)-\left(\frac{r_0\omega}{c}\right)^2\varepsilon_2 E_0^2\exp\left(\frac{b^2}{2}-k_i^{/}\xi\right)\right] \qquad (4.17)$$

Where, $k_i^{/}=k_i r_0\left(\frac{r_0\omega}{c}\right)\sqrt{1-\frac{\omega_{p_0}^2}{\omega^2}\tan(\xi/d)}$ is the normalized absorption coefficient.

Equation (4.17) is the required equation for the beam width parameter and can be solved with f depending on ξ for various $k_i^{/}$ levels.

4.5 RESULTS AND DISCUSSION

Eq. (4.17) is the second order nonlinear differential equation governing beam width parameter of beam in plasma with density ramp and linear absorption. The self-focusing (convergence) or defocusing (divergence) of the laser beam is estimated by the relative magnitude of nonlinear and diffraction terms of Eq. (4.17). The numerical solution of this equation is possible by using Runge–Kutta method with the set of following parameters for the purpose of numerical calculation [73]:

$\omega=1.778\times10^{14}\,rad/\sec$, $r_0=253\mu m$, $n_0=10^{17}\,cm^{-3}$ and the value of intensity is $I_0=10^{19}\,W/cm^2$. Fig. 4.1 shows the dependence of beam-width parameter f with propagation distance ξ for decentered parameter b = 0 with ω_{p0}/ω = 0.2, 0.3, 0.4, 0.5 for different absorption levels $k_i^{/}$ = 0.5, 0.6, 0.7, 0.8. These curves illustrate that beam-width parameter first decreases, attains a minimum value and then increases, with the result sharp self-focusing occurs up to $\omega_{p0}/\omega=0.5$, $k_i^{/}$ = 0.8 and then defocusing takes place as absorption weakens self-focusing effect. In fig. 4.2, the dependence of f on ξ for b = 1 is shown. It is clear from the figure that sharp self-focusing is observed for $\omega_{p0}/\omega=0.5$, $k_i^{/}$ =1.3. So, with further increase in absorption level, the laser beam is further enhanced. This is because, the parameters like decentered parameter, plasma density ramp and absorption coefficient are such that they change the self-focusing / defocusing nature of the beam in a significant manner. Figure 4.3 shows the dependence of f on ξ for various combinations of ω_{p0}/ω and k'_i. Beam width parameter attains a minimum value at $\omega_{p0}/\omega = 0.2$ with k'_i = 2 and $\omega_{p0}/\omega = 0.3$ with k'_i = 3 for decentered parameter b = 2 which leads to strong self-focusing in plasma. Thereafter as soon as values of ω_{p0}/ω and k'_i are increased, defocusing of laser beam takes place. But, the self-focusing length increases with absorption level. However, in fig. 4.4, the dependence of f on ξ is shown for decentered parameter b = 0, 1, 2 and keeping ω_{p0}/ω and absorption coefficient k'_i constant at 0.2 and 2 respectively. It is clear from Fig. 4.4 that sharp self-focusing occurs for b = 2 and for b = 0 and 1, f first decreases and then increases very slowly for lower values of "b". Our results support the results obtained with different approach by Gill *et al*. [21] Figure 4.5 shows the dependence of f on ξ for $\omega_{p0}/\omega = 0.5$ and b = 1 with different values of absorption level k'_i. It is important to notice here that early and strong self-focusing occurs for $k'_i < 2$. After $k'_i \geq 2$, the beam width parameter decreases slowly and defocusing takes place.

However, Patil *et al*. [73] have reported the self-focusing of cosh beams in a parabolic medium at various values of linear absorption (k'_i) and decentered parameter (b) and concluded that for b=0, the self-focusing occurs only up to $k'_i < 2$. But, firstly f decreases and secondly it increases slowly for $k'_i \geq 2$ corresponding to ξ = 0.3. However, for decentered parameter b =1, the self-focusing occurs only up to $k'_i < 3$ corresponding to ξ = 0.2. Finally for decentered parameter b =2, the beam width parameter attains a minimum value at ξ = 0.06 showing that the self-focusing length increases with absorption level. Further, in the work of Navare *et al*. [80], while considering the collisional nonlinearity, they found that the absorption plays a vital role in the self-focusing effect and destroys the oscillatory self-focusing character of laser beam during propagation. Hence, in comparison to ref. [73] and ref. [80], by applying the density ramp and taking in to account the effect of linear absorption, we observe that self-focusing occurs even at ξ = 0.02. Further, we found that study of cosh beams can be analyzed in a medium like plasma, but the important thing is that the decentered parameter, absorption coefficient and plasma density ramp are found to behave in such a way that they change the self-focusing / defocusing nature of the laser beam in a significant manner.

4.6 CONCLUSION

This communication provides us an analysis of the evolution of cosh beams in plasma with density ramp and linear absorption using paraxial approximation. The effect of density transition on self-focusing has been studied at various values of absorption levels and decentered parameter. By choosing appropriate and optimized parameters, the combined effect of density ramp, decentered parameter and linear absorption on beam width parameter variation has been investigated and hence plotted. The results show that self-focusing occurs earlier and then defocusing takes place. However, it would be quite interesting to compare the investigated results for non-paraxial region of the beam, which so far has not been studied as per the literature available at present.

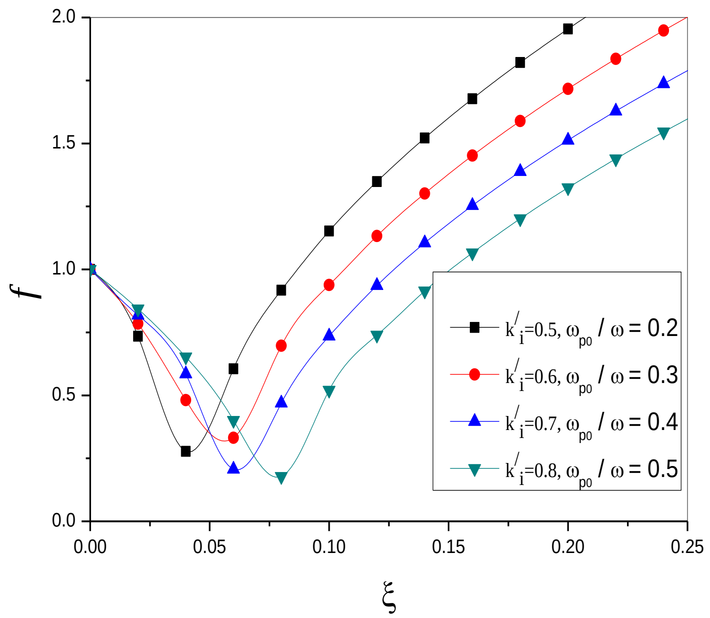

Figure 4.1: Dependence of f on ξ for various values of k'_i and ω_{p0}/ω at b = 0.

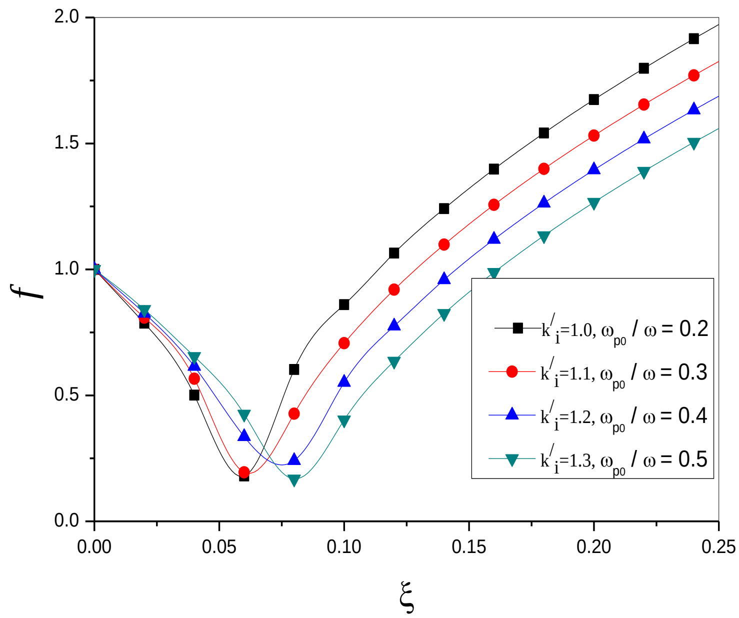

Figure 4.2: Dependence of f on ξ for various values of k'_i and ω_{p0}/ω at b = 1

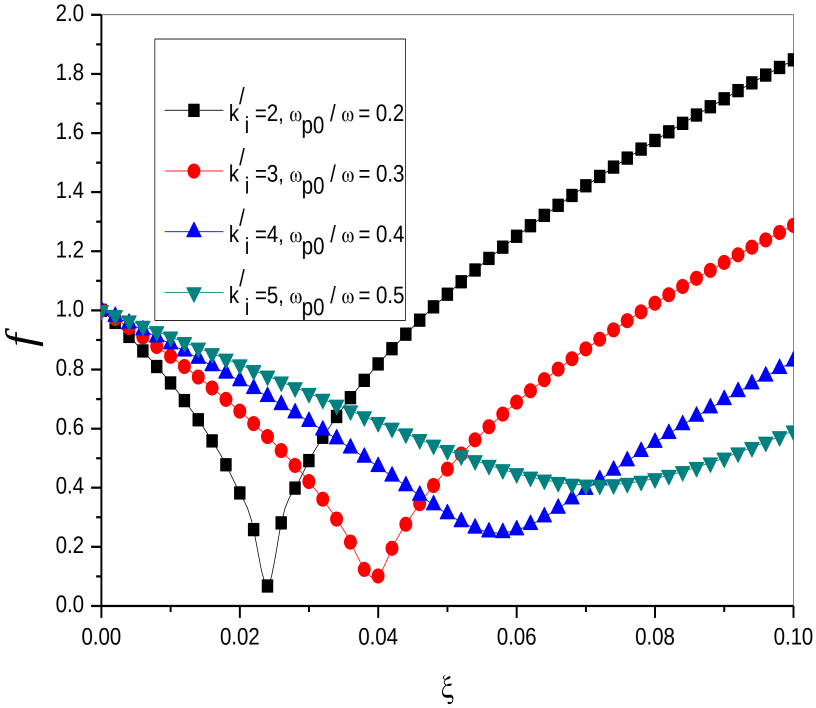

Figure 4.3: Dependence of f on ξ for various values of k'_i and ω_{p0}/ω at b = 2.

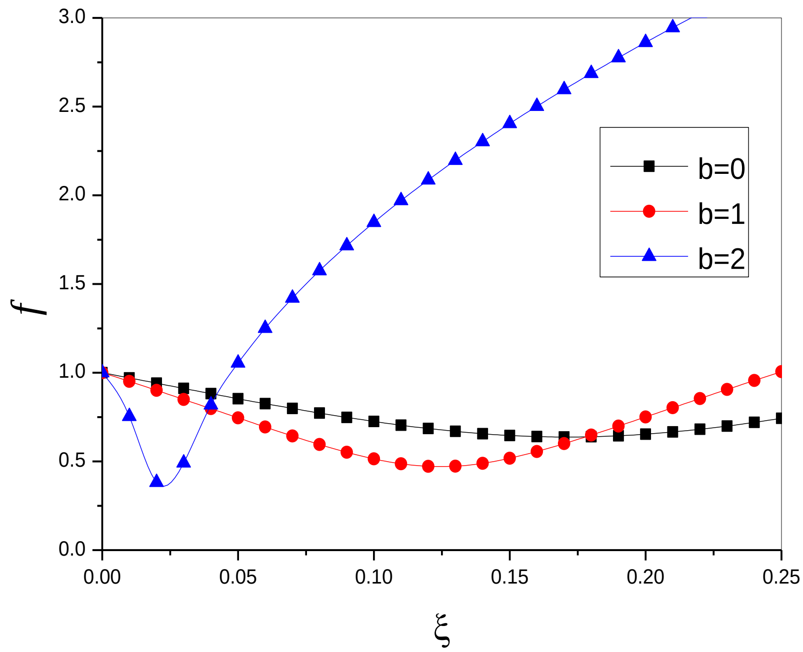

Figure 4.4: Dependence of f on ξ for $\omega_{p0}/\omega = 0.2$, $k'_i = 2$ and for various decentered parameter (b) values.

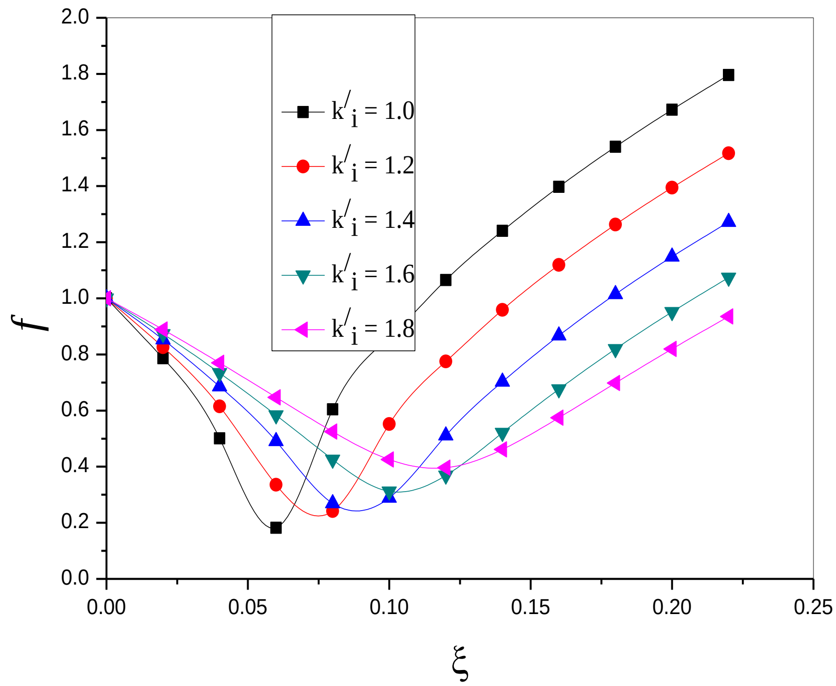

Figure 4.5: Dependence of f on ξ for $\omega_{p0} / \omega = 0.5$, b = 1 and for various values of $k^{/}_{i}$