CHAPTER-3

SELF-FOCUSING OF HERMITE-COSH-GAUSSIAN LASER BEAM IN PLASMA UNDER DENSITY TRANSITION

3.1 INTRODUCTION

The process of self-focusing of laser beams in a nonlinear medium is a captivating field which has an excellence both in theoretical and experimental interests [2, 8, 12]. In magnetoplasma using variational approach, it is found that the decentered parameter b along with absorption coefficient play a key role on the self-focusing/ defocusing nature of the beam [78]. However, for a cos-Gaussian beam propagating in a kerr medium, the RMS beam width broadens, the central parts of the beam give rise to an initial radial compression and have a noticeable redistribution during propagation. The partial collapse of central part of the beam appears while the RMS beam width still increases or remains constant. It is further observed that the cos-Gaussian beam eventually converts in to a cosh-Gaussian type beam with a low and moderate power [27]. In relativistic and ponderomotive regime, it is observed that a large value of absorption level weakens the self-focusing effect in the absence of decentered parameter. However, oscillatory self-focusing takes place for a higher value of decentered parameter, b = 1, and all curves are seen as displaying the sharp self-focusing effect for b = 2 [21]. Under plasma density transition, the pulse acquires a minimum spot size very close to the axis of propagation. As the laser beam passes through the ramped density region, it detects a low pace narrowing channel. In this case the oscillation amplitude of the spot size contracts and the beam propagating under density transition tends to become more focused. In the absence of density ramp, due to the supremacy of the diffraction effect, the laser pulse is defocused. As the plasma density increases, self-focusing occurs sooner and becomes stronger. Similarly, in the absence of density transition, the beam width parameter does not increase much and after various Rayleigh lengths, it acquires a very lower value and maintains it for a large distance. Consequently, the enhancement in laser beam self-focusing is observed [77].

Nanda *et al*. [92] while studying the relativistic self-focusing of HchG beam in plasma under density transition observed that an appropriate and proper decentered parameter selection and presence of density transition results to stronger self-focusing. Further, for such beams when propagating in a magnetoplasma with a ramped density profile, the authors concluded that the presence of density transition and magnetic field enhances the self-focusing effect to a larger extent [91]. Consideration of proper and an appropriate decentered parameter selection is very much sensitive to self-focusing [90]. However in studying the self-focusing under density transition by Kant *et al*. [22], the authors found that the effects of density transition and initial intensity of the laser beam are important and have a key role in maintaining the laser plasma interaction as a captivating field of research and hence in strong self-focusing.

Recently, the HchG beam has been studied extensively and it has been found that such beams can be produced by the superposition of two decentered Hermite-Gaussian beams as cosh-Gaussian ones [23]. In this paper, we mainly study the self-focusing of HchG laser beams propagating in underdense plasma under plasma density ramp of the form $n(\xi) = n_0 \tan(\xi / d)$ by a ponderomotive mechanism. Analytical formulas for HchG beams are derived and results are discussed.

3.2 FIELD DISTRIBUTION OF HCHG LASER BEAMS

The field distribution of HchG beams propagating in the plasma medium along z-axis is of the form:

$$E(r,z) = \frac{E_0}{2f(z)}\left[H_m\left(\frac{\sqrt{2r}}{r_0 f(z)}\right)\right] Exp\left[\frac{b^2}{4}\right] \times$$

$$\left\{Exp\left[-\left(\frac{r}{r_0 f(z)} + \frac{b}{2}\right)^2\right] + Exp\left[-\left(\frac{r}{r_0 f(z)} - \frac{b}{2}\right)^2\right]\right\} \qquad (3.1)$$

Where, m represents the mode index for the Hermite polynomial of m^{th} order, r_0 is the spot size of the beam and b is the decentered parameter of the beam, r is the radial coordinate, $E(r,z)$ is the amplitude of HchG beam at $r = z = 0$. $f(z)$ is the dimensionless beam width parameter, which is a measure of both intensity along the axis and waist width of the beam.

3.3 NON-LINEAR DIELECTRIC CONSTANT

We consider propagation of HchG laser beam in a nonlinear medium characterized by dielectric constant given by Sodha *et al.* [20]:

$$\varepsilon = \varepsilon_0 + \phi(EE^*) \tag{3.2}$$

With, $\varepsilon_0 = 1 - \omega_P^2 / \omega^2$, $\omega_P^2 = 4\pi\, n(\xi) e^2 / m$, $\omega_P^2 = \omega_{p0}^2 \tan(\xi / d)$ and $\omega_{P0}^2 = 4\pi\, n_0 e^2 / m$, here '$\varepsilon_0$' represents the linear part and $\phi \approx (1/2)\varepsilon_2 A_0^2$ represents the non-linear parts of the dielectric constant respectively. Here, $\varepsilon_2 = 2(\omega_p^2 / \omega^2)\alpha$, '$\omega_{P0}$' is the plasma frequency, 'e' is the electronic charge, 'm' is the rest mass of the electron, 'ω' is the frequency of the incidents laser beam and 'n_0' is the equilibrium electron density. With, $\alpha = e^2 M / 6m^2 \omega^2 k_B T_0$, here '$M$' is the scatterer mass in the plasma, 'k_b' is the Boltzmann constant and 'T_0' is the equilibrium plasma temperature, ξ is the propagation distance and d is a constant adjustable parameter.

3.4 SELF-FOCUSING

For isotropic, non-conducting and non-absorbing medium with $\mu = 1$, Maxwell's equation are:

$$\vec{\nabla} \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \tag{3.3}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \tag{3.4}$$

$$\vec{\nabla} \cdot \vec{D} = 0 \tag{3.4a}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{3.4b}$$

Taking curl of equation (3.4),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{H}\right)$$

Substituting the value of $(\vec{\nabla}\times\vec{H})$ from equation (3.4a) and applying vector identity, $\vec{A}\times\vec{B}\times\vec{C}$, we get

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)-\vec{E}\left(\vec{\nabla}\cdot\vec{\nabla}\right)=-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial\vec{D}}{\partial t}\right)$$

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)-\nabla^2\vec{E}=-\frac{1}{c^2}\frac{\partial^2\vec{D}}{\partial t^2}$$

$$-\nabla^2\vec{E}+\frac{1}{c^2}\frac{\partial^2\vec{D}}{\partial t^2}+\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)=0 \qquad (3.5)$$

From equation (3.4a),

$$\vec{\nabla}\cdot\vec{D}=0$$

$$\vec{\nabla}\left(\varepsilon\vec{E}\right)=0$$

Here $'\varepsilon'$ is a variable quantity, thus we have,

$$\varepsilon\vec{\nabla}\cdot\vec{E}+\vec{E}\vec{\nabla}\cdot\varepsilon=0$$

$$\vec{\nabla}\cdot\vec{E}=-\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}$$

Thus equation (3.5) becomes,

$$-\nabla^2\vec{E}+\frac{1}{c^2}\frac{\partial^2\vec{D}}{\partial t^2}+\vec{\nabla}\left(-\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}\right)=0$$

$$\nabla^2\vec{E}-\frac{\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}+\vec{\nabla}\left(\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}\right)=0 \qquad (3.6)$$

For a plane polarized wave with electric field vector along y-axis, propagating in the z-direction, the solution of equation (3.6) is given by,

$$\vec{E} = \hat{j}E_0 Exp[i(\omega t - kz)] \tag{3.7}$$

where $\hat{j}$ is the unit vector along y-direction.

Differentiating equation (3.7) twice, w. r. t. 't', we get

$$\frac{\partial \vec{E}}{\partial t} = i\omega \ \hat{j}E_0 Exp[i(\omega t - kz)]$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \hat{j}E_0 Exp[i(\omega t - kz)]$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \vec{E} \tag{3.8}$$

Thus equation (3.6) becomes,

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2}\left(-\omega^2 \vec{E}\right) + \vec{\nabla}\left(\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}\right) = 0$$

$$\nabla^2 \vec{E} + \frac{\varepsilon}{c^2}\omega^2 \vec{E} + \vec{\nabla}\left(\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}\right) = 0$$

$$\nabla^2 \vec{E} + k^2 \vec{E} + \vec{\nabla}\left(\frac{\vec{E}\vec{\nabla}\cdot\varepsilon}{\varepsilon}\right) = 0 \qquad \left(\because k^2 = \varepsilon\frac{\omega^2}{c^2}\right)$$

$$\nabla^2 \vec{E} + k^2 \vec{E} + \left[-\vec{E}\left(\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right)^2 + \vec{E}\left(\frac{\nabla^2\varepsilon}{\varepsilon}\right)\right] = 0$$

$$\nabla^2 \vec{E} + \left[k^2 - \left(\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right)^2 + \left(\frac{\nabla^2\varepsilon}{\varepsilon}\right)\right]\vec{E} = 0$$

$$\nabla^2\vec{E}+\left[k^2+\vec{\nabla}\left(\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right)\right]\vec{E}=0$$

$$\nabla^2\vec{E}+\left[k^2+\nabla^2(\ln\varepsilon)\right]\vec{E}=0$$

$$\nabla^2\vec{E}+k^2\left[1+\frac{1}{k^2}\nabla^2(\ln\varepsilon)\right]\vec{E}=0 \qquad (3.9)$$

If $\left(1/k^2\right)\nabla^2(\ln\varepsilon)<<1$, then,

$$\nabla^2\vec{E}+k^2\vec{E}=0$$

Or, $\nabla^2\vec{E}+\frac{\omega^2}{c^2}\varepsilon\vec{E}=0$

In cylindrical co-ordinate system, we can write this equation as

$$\frac{\partial^2\vec{E}}{\partial z^2}+\frac{\partial^2\vec{E}}{\partial r^2}+\frac{1}{r}\frac{\partial\vec{E}}{\partial r}+\varepsilon\frac{\omega^2}{c^2}\vec{E}=0 \qquad (3.10)$$

For slowly converging or diverging cylindrically symmetric beam, the solution of equation (3.10) is of the following form,

$$\vec{E}=A(r,z)Exp[i(\omega t-kz)] \qquad (3.11)$$

With $k^2=\varepsilon_0\,\omega^2/c^2=\omega^2/c^2\left(1-\omega_{P0}^2\tan(\xi/d)/\omega^2\right)$.

Differentiating equation (3.11) twice w. r. t. $'r'$ and $'z'$, we get

$$\frac{\partial\vec{E}}{\partial r}=Exp[i(\omega t-kz)]\frac{\partial A(r,z)}{\partial r}$$

$$\frac{\partial^2\vec{E}}{\partial r^2}=Exp[i(\omega t-kz)]\frac{\partial^2 A(r,z)}{\partial r^2}$$

And

$$\frac{\partial \vec{E}}{\partial z} = Exp[i(\omega t - kz)]\left[-\frac{i\omega A}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)} + \left(\frac{i\omega}{2cdR_d}\right)\frac{\omega_{p0}^2}{\omega^2}\frac{zASec^2(z/dR_d)}{\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}} + \frac{\partial A}{\partial z}\right]$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} = Exp[i(\omega t - kz)]\left[\frac{\partial^2 A(r,z)}{\partial z^2} + \left(\frac{i\omega}{c}\right)\frac{\partial A(r,z)}{\partial z}\left(\frac{\omega_{p0}^2 zSec^2(z/dR_d)}{\omega^2 dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}} - 2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\right)\right]$$

$$+\frac{\omega A}{c}Exp[i(\omega t - kz)]\left[\frac{i\omega_{p0}^2 zSec^2(z/dR_d)}{\omega^2 dR_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}} + \frac{\omega z\omega_{p0}^2 zSec^2(z/dR_d)}{c\omega^2 dR_d}\right]$$

$$+\frac{\omega}{c}Exp[i(\omega t - kz)]\frac{iA\omega_{p0}^2 zSec^2(z/dR_d)}{\omega^2 d^2 R_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\left[\tan(z/dR_d) + \frac{\omega_{p0}^2 Sec^2(z/dR_d)}{4\omega^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\right]$$

$$-\frac{\omega^2 A}{c^2}Exp[i(\omega t - kz)]\left[\frac{\omega_{p0}^4 z^2 Sec^4(z/dR_d)}{4\omega^4 d^2 R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)} + \left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)\right]$$

Substituting these values in equation (3.10), and neglecting $\partial^2 A/\partial z^2$ we get

$$\frac{i\omega}{c}\left(2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}-\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}}\right)\left(\frac{\partial A}{\partial z}\right)-\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{ASec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}}$$

$$-\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{AzSec}^2\left(z/d\mathrm{R}_d\right)}{d^2\mathrm{R}_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}}\left(\mathrm{Tan}\left(z/d\mathrm{R}_d\right)+\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{Sec}^2\left(z/d\mathrm{R}_d\right)}{4\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)$$

$$=\frac{\omega^2}{c^2}\left(\frac{\frac{\omega_{p0}^2}{\omega^2}\mathrm{AzSec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d}-\frac{\left(\frac{\omega_{p0}^2}{\omega^2}\right)^2\mathrm{Az}^2\mathrm{Sec}^4\left(z/d\mathrm{R}_d\right)}{4d^2\mathrm{R}_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{\omega^2}{\mathrm{c}^2}\Phi\left(AA^*\right)A$$

(3.12)

To solve equation (3.12), we express

$$A(r,z)=A_0(r,z)Exp[-ikS(r,z)] \tag{3.13}$$

Where, k has been defined above and A_0 and S are the real functions of $'r'$ and 'z'.

Differentiating equation (3.13) twice, w. r. t. $'r'$, we get

$$\frac{\partial A(r,z)}{\partial r}=Exp[-ikS(r,z)]\left[\frac{\partial A_0}{\partial r}-\frac{i\omega A_0}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S(r,z)}{\partial r}\right)\right]$$

$$\frac{\partial^2 A(r,z)}{\partial r^2}=Exp[-ikS(r,z)]\left[\frac{\partial^2 A_0}{\partial r^2}-\frac{2i\omega}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S}{\partial r}\right)\left(\frac{\partial A_0}{\partial r}\right)\right]-$$

$$\frac{\omega A_0}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left[i\left(\frac{\partial^2 S}{\partial r^2}\right)+\frac{\omega}{c}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S}{\partial r}\right)^2\right]Exp[-ikS(r,z)]$$

Now differentiating equation (3.13) w. r. t. $'z'$,

$$\frac{\partial A(r,z)}{\partial z} = Exp\left[-ikS(r,z)\right]\left[\frac{\partial A_0}{\partial z} - \frac{i\omega A_0}{c}\left\{\begin{array}{l}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left(\frac{\partial S}{\partial z}\right) - \\ \frac{S(r,z)\omega_{p0}^2}{2d\omega^2 R_d}\frac{Sec^2(z/dR_d)}{\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\end{array}\right\}\right]$$

Thus equation (3.12) becomes,

$$\frac{i\omega}{c}\left(\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{d\omega^2 R_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}} - 2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\right)\frac{\partial A_0}{\partial z} + \frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r} +$$

$$\frac{\omega^2 A_0}{c^2}\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}\left[\begin{array}{l}\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{d\omega^2 R_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)} - \frac{S\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)} \\ +\frac{\partial S}{\partial z} - 2\end{array}\right]$$

$$+\frac{i\omega A_0\omega_{p0}^2 Sec^2(z/dR_d)}{cd\omega^2 R_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)}}\left[1+\frac{z}{dR_d}\left(\tan(z/dR_d) + \frac{\omega_{p0}^2 Sec^2(z/dR_d)}{4\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\right)\right]$$

$$= \frac{\omega^2 A_0}{c^2}\left(\frac{z^2\omega_{p0}^4 Sec^4(z/dR_d)}{4d^2\omega^4 R_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)} - \frac{\omega_{p0}^2 z Sec^2(z/dR_d)}{dR_d} + \left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)\left(\frac{\partial s}{\partial r}\right)^2\right)$$

$$+\frac{i\omega}{c}\sqrt{\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\left(2\frac{\partial S}{\partial r}\frac{\partial A_0}{\partial r} + A_0\frac{\partial^2 S}{\partial r^2} + A_0\frac{\partial S}{\partial r}\right) - \frac{\omega^2 A_0}{c^2}\phi(A_0^2) \qquad (3.14)$$

Comparing real and imaginary parts of equation (3.14), we get

Real part equation is

$$\frac{c^2}{\omega^2 A_0}\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right)+\Phi\left(A_0^2\right)=\left(2\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)-\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d}\right)\frac{\partial S}{\partial z}$$

$$+\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)\left(\frac{\partial S}{\partial r}\right)^2-\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{Sec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d}\left(S+z-\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)(S-z/2)}{2d\mathrm{R}_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)$$

(3.15)

Imaginary part equation is

$$\left(1-\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)}{2d\mathrm{R}_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)\frac{\partial A_0^2}{\partial z}+\frac{\partial S}{\partial r}\frac{\partial A_0^2}{\partial r}+\left(\frac{\partial^2 S}{\partial r^2}+\frac{1}{r}\frac{\partial S}{\partial r}\right)A_0^2$$

$$-\left(\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{Sec}^2\left(z/d\mathrm{R}_d\right)}{d\mathrm{R}_d\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}}+\frac{\omega_{p0}^2}{\omega^2}\frac{\mathrm{zSec}^2\left(z/d\mathrm{R}_d\right)\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}{d^2\mathrm{R}_d^2\sqrt{1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)}}\right)A_0^2$$

$$-\left(\frac{\omega_{p0}^2}{\omega^2}\right)^2\left(\frac{\mathrm{zSec}^4\left(z/d\mathrm{R}_d\right)}{4d^2\mathrm{R}_d^2\left(1-\frac{\omega_{p0}^2}{\omega^2}\mathrm{Tan}\left(z/d\mathrm{R}_d\right)\right)}\right)A_0^2=0 \qquad (3.16)$$

For initially Hermite-cosh-Gaussian beam, the solution of equation (3.15) and (3.16) are of the form

$$A_0^2 = \frac{E_0^2}{4f^2(z)}\left[H_m\left(\frac{\sqrt{2}r}{r_0 f(z)}\right)\right]^2 Exp\left[\frac{b^2}{2}\right]$$

$$\left\{Exp\left[-2\left(\frac{r}{r_0 f(z)}+\frac{b}{2}\right)\right]+Exp\left[-2\left(\frac{r}{r_0 f(z)}-\frac{b}{2}\right)\right]+2Exp\left[-\left(\frac{2r^2}{r_0^2 f^2(z)}+\frac{b^2}{2}\right)\right]\right\}$$

(3.17)

And

$$S(r,z) = \frac{r^2}{2}\beta(z) + \varphi(z) \qquad (3.18)$$

with, $\beta(z) = (1/f(z))\partial f/\partial z.$ where $'\varphi(z)'$ is an arbitrary function of $'z'$.

For, $m = 0$, mode (chG beam) $\left[H_m\left(\frac{\sqrt{2}r}{r_0 f(z)}\right)\right]^2 = 1$

$$\therefore\ A_0^2 = \frac{E_0^2}{4f^2(z)} Exp\left[\frac{b^2}{2}\right]$$

$$\left\{Exp\left[-2\left(\frac{r}{r_0 f(z)}+\frac{b}{2}\right)^2\right]+Exp\left[-2\left(\frac{r}{r_0 f(z)}-\frac{b}{2}\right)^2\right]+2Exp\left[-\left(\frac{2r^2}{r_0^2 f^2(z)}+\frac{b^2}{2}\right)\right]\right\} \qquad (3.19)$$

Differentiating equation (3.18) w. r. t. $'z'$ and $'r'$ respectively,

$$\frac{\partial S(r,z)}{\partial z} = \frac{r^2}{2}\frac{\partial \beta}{\partial z} + \frac{\partial \varphi}{\partial z}$$

$$\frac{\partial S(r,z)}{\partial z} = \frac{r^2}{2f(z)}\frac{\partial^2 \beta}{\partial z^2} - \frac{r^2}{2f^2(z)}\left(\frac{\partial f(z)}{\partial z}\right)^2 + \frac{\partial \varphi}{\partial z}$$

$$\frac{\partial S(r,z)}{\partial r} = 2r\frac{\beta}{2} = r\beta = \frac{r}{f(z)}\frac{\partial f(z)}{\partial z}$$

Differentiating equation (3.19) twice w. r. t. $'r'$, we get

$$\frac{\partial A_0}{\partial r} = \frac{E_0}{2f(z)}\left\{\left(\frac{2rb^2}{r_0^2 f^2(z)}\right) - \frac{4r}{r_0^2 f^2(z)} - \left(\frac{4r^3 b^2}{r_0^4 f^4(z)}\right) + \frac{4r^3}{r_0^4 f^4(z)}\right\}$$

$$\frac{\partial^2 A_0}{\partial r^2} = \frac{E_0}{2f(z)}\left\{\begin{array}{l}\left(\frac{2rb^3}{r_0^3 f^3(z)}\right) - \frac{2r^2 b^2}{r_0^4 f^4(z)} - \left(\frac{4r^3 b}{r_0^5 f^5(z)}\right) - \left(\frac{12rb}{r_0^3 f^3(z)}\right) - \left(\frac{4}{r_0^2 f^2(z)}\right) \\ - \frac{4r^2}{r_0^4 f^4(z)} - \left(\frac{6r^3 b^3}{r_0^5 f^5(z)}\right)\end{array}\right\}$$

Substituting the values of $\partial S(r,z)/\partial z$, $\partial S(r,z)/\partial r$, $\partial A_0/\partial r$, $\partial^2 A_0/\partial r^2$ and A_0^2 in equation (3.15) and solving, we get

$$\frac{c^2 E_0}{2\omega^2 A_0 f(z)}\left(\frac{2rb^3}{r_0^3 f^3(z)} - \frac{2r^2 b^2}{r_0^4 f^4(z)} - \frac{4r^3 b}{r_0^5 f^5(z)} - \frac{12rb}{r_0^3 f^3(z)} - \frac{6r^3 b^3}{r_0^5 f^5(z)} + \frac{2b^2}{r_0^2 f^2(z)} - \frac{4r^2}{r_0^4 f^4(z)}\right)$$
$$- \frac{E_0^2 r^2}{2r_0^4 f^4(z)}\left(\frac{\alpha\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right) = \left(1 - \frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right) r^2\beta^2 + \varphi(z) + z$$
$$+\left(2 - \frac{2\omega_{p0}^2}{\omega^2}\tan(z/dR_d) - \frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{d\omega^2 R_d}\right)\left(\frac{r^2\partial\beta}{2\partial z} + \frac{\partial\varphi}{\partial z}\right) + \frac{r^2\beta z\omega_{p0}^4 Sec^4(z/dR_d)}{4d^2\omega^4 R_d^2\left(1 - \frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}$$
$$- \frac{r^2\beta\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d} - \frac{\omega_{p0}^2 z Sec^2(z/dR_d)\varphi(z)}{2d\omega^2 R_d\left(1 - \frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}$$

$$+ \frac{\omega_{p0}^2 z^2 Sec^2(z/dR_d)\varphi(z)}{4d\omega^2 R_d\left(1 - \frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)} \qquad (3.20)$$

Now, equating coefficients of r^2 on both sides of Eq. (3.20), one obtains

$$\frac{6c^2b^2}{(2-b^2)\omega^2 r_0^2 f(z)}+\frac{\alpha E_0^2\omega_{p0}^2}{2r_0^2 f^3(z)\omega^2}\tan(z/dR_d)=\left(\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d}+\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)-1\right)\frac{\partial^2 f}{\partial z^2}$$

$$-\frac{1}{f(z)}\left(\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d}\right)\left(\frac{\partial f}{\partial z}\right)^2+\frac{\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d}\left(1-\frac{z\omega_{p0}^2 Sec^2(z/dR_d)}{2d\omega^2 R_d\left(1-\frac{\omega_{p0}^2}{\omega^2}\tan(z/dR_d)\right)}\right)\frac{\partial f}{\partial z}$$

(3.21)

Since, $\xi = z/R_d$, $z=\xi R_d$ and $z/dR_d = \xi/d$ .Therefore, we can write as:

$$\frac{\partial f}{\partial z}=\frac{\partial f}{\partial \xi}\times\frac{\partial \xi}{\partial z}=\frac{1}{R_d}\frac{\partial f}{\partial \xi} \quad \text{and} \quad \frac{\partial^2 f}{\partial z^2}=\frac{1}{R_d^2}\frac{\partial^2 f}{\partial \xi^2}$$

Substituting the above values in Eq. (3.21), we get

$$\left(1-\frac{\omega_{p0}^2}{\omega^2}Tan(\xi/d)\right)\left(\frac{6b^2}{(2-b^2)f}\right)+\frac{\alpha E_0^2}{2}\left(\frac{r_0\omega}{c}\right)^2\left(\frac{\omega_{p0}^2}{\omega^2}\right)\left(1-\frac{\omega_{p0}^2}{\omega^2}Tan(\xi/d)\right)\frac{Tan(\xi/d)}{f^3}$$

$$=\left[\left(\frac{\omega_{p0}^2}{\omega^2}\right)\mathrm{Tan}(\xi/d)+\left(\frac{\omega_{p0}^2}{\omega^2}\right)\frac{\xi\mathrm{Sec}^2(\xi/d)}{2d}-1\right]\frac{\partial^2 f}{\partial \xi^2}-\left[\left(\frac{\omega_{p0}^2}{\omega^2}\right)\frac{\xi\mathrm{Sec}^2(\xi/d)}{2d}\right]\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2$$

$$+\left(\frac{\omega_{p0}^2}{\omega^2}\right)\frac{\mathrm{Sec}^2(\xi/d)}{2d}\left[1-\left(\frac{\omega_{p0}^2}{\omega^2}\right)\frac{\xi\mathrm{Sec}^2(\xi/d)}{2d\left(1-\left(\frac{\omega_{p0}^2}{\omega^2}\right)\mathrm{Tan}(\xi/d)\right)}\right] \qquad (3.22)$$

Equations (3.22) is the required expression for beam width parameter f .

3.5 RESULTS AND DISCUSSION

For an initially plane wave front of the beam, we follow the boundary condition $f = 1$ and $\partial f / \partial \xi = 0$ at $\xi = 0.$ For the analysis done above (Eq. 3.22), the following parameters are chosen for the purpose of numerical calculations: $\omega = 3\times 10^{14} rad/\sec$, $r_0 = 3\times 10^{-4} cm$ and $n_0 = 9.983\times 10^{17} cm^{-3}$ [90]. Figures 3.1 (a) and 3.1 (b) show the dependence of f on ξ with upward density transition for different values of ω_{p0}/ω. The decentered parameter is fixed at $b = 0$ and 1 respectively. From these figures, it is clear that with increase in the values of relative plasma density, the beam width parameter decreases sharply. The plots reveal that due to supremacy of nonlinear term, the laser beam gets more focused. This is due to the fact that in the low plasma density region, the electrons are forced to move away from the region having high intensity by a ponderomotive mechanism. The nonlinearity in the plasma comes because of mass variation of electron, which is also due to high intensity of the laser beam. Figures 3.2 (a) and 3.2 (b) represent the dependence of f on ξ for various decentered parameter values. Keeping ω_{p0}/ω fixed at 0.02 in figure 3.2 (a) and at 0.03 in figure 3.2 (b) respectively. From these figures it is clear that on increasing decentered parameter *b*, the beam width parameter *f* decreases on a large scale. Hence self-focusing occurs sooner and becomes further strong. Thus, it is obvious from the figures that the decentered parameter affects the behavior of beam width parameter to greater extent. Moreover, the appropriate selection and sensitivity of decentered parameter is very important in deciding the focusing of laser beam.

Figure 3.3 represents the dependence of f on ξ for various decentered parameter values. Keeping ω_{p0}/ω fixed at 0.04 and the other parameters are d =5 and decentered parameter b = 0 (Red curve), b = 1 (Black curve). The figure 3.3 reveals that as we increase the decentered parameter, the beam width parameter decreases greatly. It is because of the fact that the decentered parameter is sensitive to self-focusing. Hence, one can say that the laser beam gives a self-focusing effect for $b \leq 1$. It is further observed that as the plasma density increases self-focusing becomes much stronger. Combining the results of this chapter with the previous studies on Gaussian beams [20, 25], we see that HchG beams give freedom to mode index (m) and decentered parameter (b) in changing the self-focusing nature more accurately. However, in the absence of plasma density ramp, the beam-width parameter decreases on a large scale because of nonlinear effects. As the diffraction effects become prevalent, the beam-width parameter increases and after acquiring a very lower value, the laser beam starts to diverge due to nonlinearity saturation. To surmount the defocusing, introduction of plasma density transition is necessary and it is obvious that by applying such a transition, the self-focusing effect is enhanced and the laser is more focused i. e. self-focusing becomes much stronger. Hence, the upward plasma density ramp or density transition has a key role in laser focusing enhancement.

3.6 CONCLUSION

In the present investigation, we have studied the self-focusing of Hermite-cosh-Gaussian (HchG) laser beam in plasma by considering plasma density ramp in a parabolic medium under paraxial approximation. The field distribution of the laser beam is expressed in terms of beam width parameter and decentered parameter. The differential equation for the beam width parameter is derived by using parabolic wave equation and paraxial approximation. To keep away the laser from defocusing, upward density ramp or transition based density is considered and hence, the beam is focused to a small spot size. Such a density transition reduces the defocusing effect and maintains the focal spot size up to several Rayleigh lengths. To discuss the self-focusing nature, the behavior of beam width parameter with the dimensionless distance of propagation for various decentered parameter values has been examined by numerical calculations. Our simulation results show that as the plasma density and decentered parameter increases, the self-focusing effect occurs sooner and becomes stronger. However, sharp self-focusing of such beams occurs for $b \leq 1$. Hence, by introducing such a density profile, a much stronger self-focusing is observed which can be used for various interesting applications.

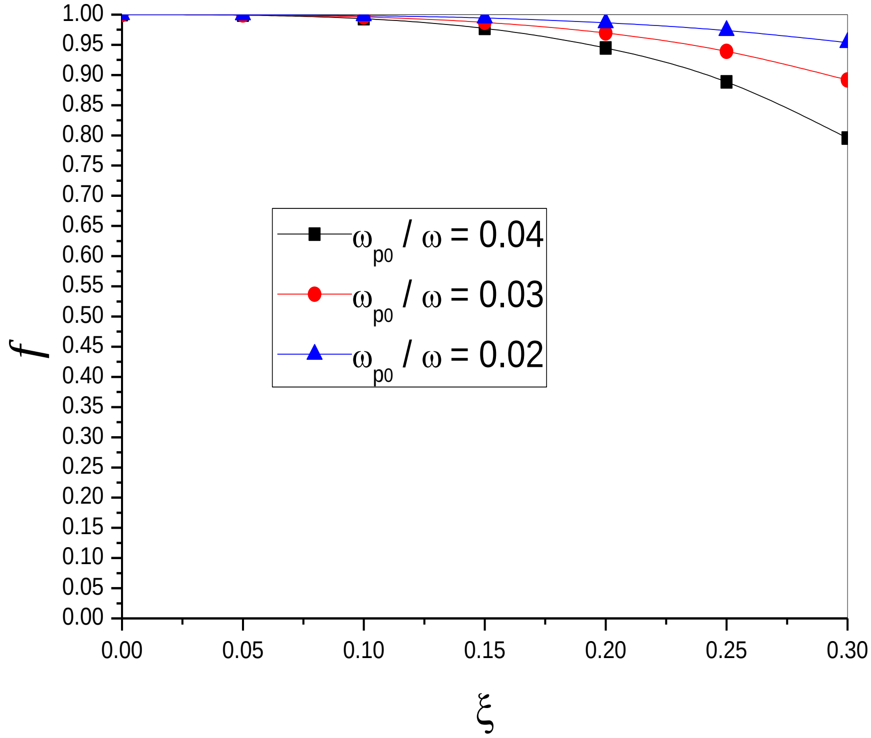

Figure 3.1 (a): Dependence of f on ξ for various values of ω_{p0}/ω. The other parameters are b = 0 and d = 5.

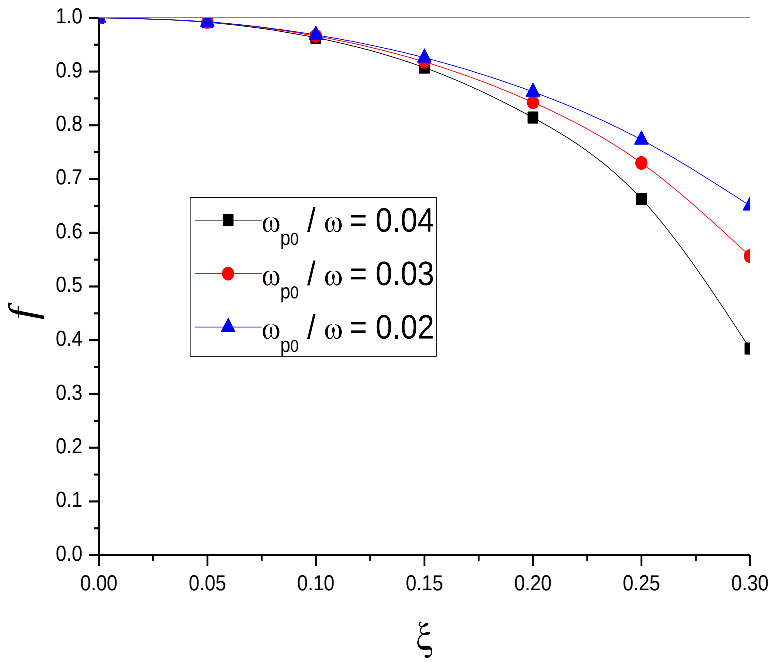

Figure 3.1 (b): Dependence of f on ξ for various values of ω_{p0}/ω. The other parameters are b = 1 and d = 5.

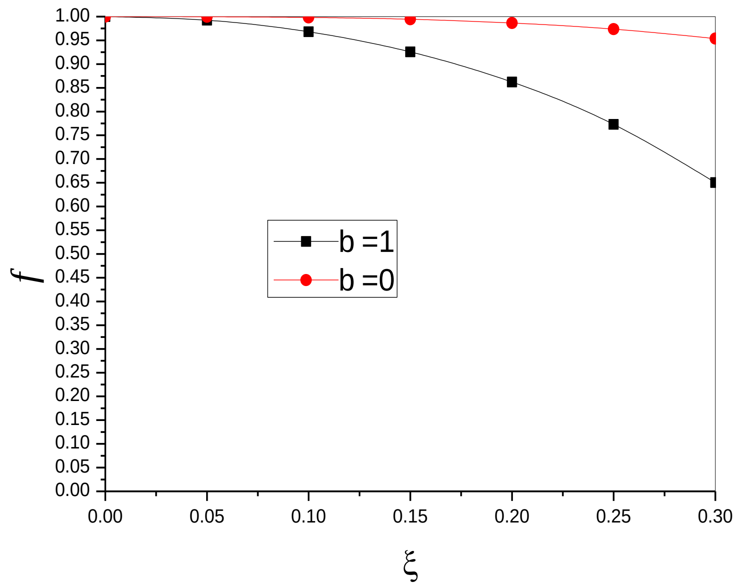

Figure 3.2 (a): Dependence of f on ξ for various values of b. The other parameters are ω_{p0}/ω = 0.02 and d = 5.

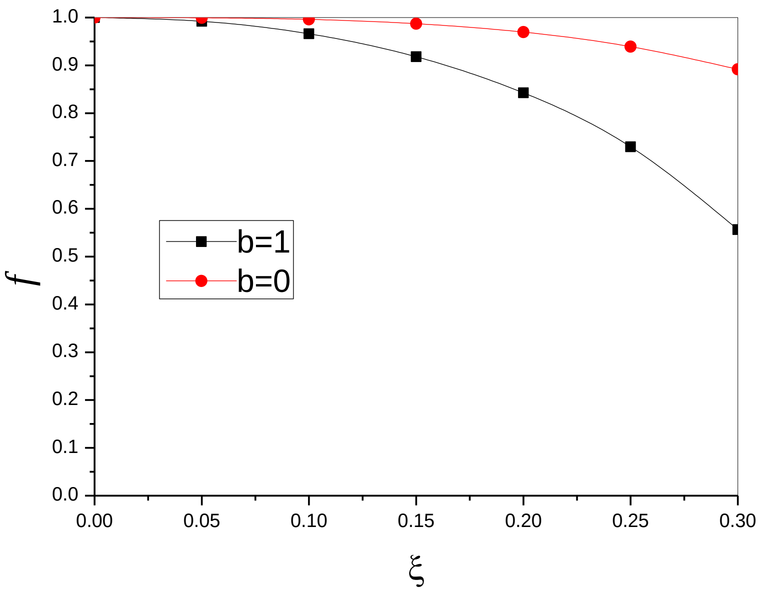

Figure 3.2 (b): Dependence of f on ξ for various values of b. The other parameters are ω_{p0}/ω = 0.03 and d = 5.

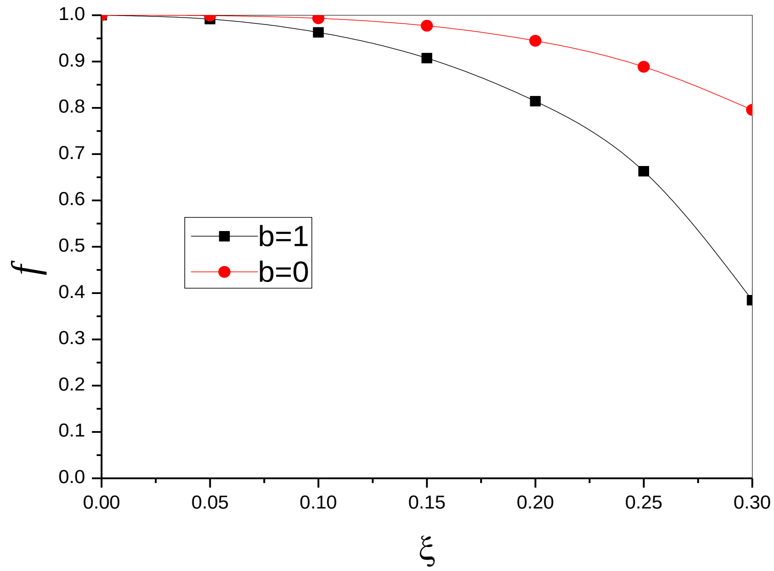

Figure 3.3: Dependence of f on ξ for various values of b. The other parameters are ω_{p0}/ω = 0.04 and d = 5.