CHAPTER-2
REVIEW OF LITERATURE

2.1 LITERATURE REVIEW:

The study of self-focusing of laser light in plasma is a captivating field and has become a subject of considerable interest. As these phenomena's occupy an important place in a large number of applications like x-ray lasers, harmonic generation, laser driven plasma based accelerators and inertial confinement fusion. So laser beams have always been an interesting area of research for many years.

Askaryan (1962) [1] was the first who discussed the self-focusing of laser beam in plasma by considering the energy momentum flux density of the beam. The whole plasma has been expelled and the pressure thereof was balanced by the plasma pressure profile acting against the centre of the laser beam. Askaryan was able to compare the required optical intensities for compensating the gas dynamic pressure.

Hora (1969) [28] studied the self-focusing process by a ponderomotive mechanism and treated it in terms ponderomotive acceleration. It occurs due to light intensity gradient. The radiation focusing takes place within the first minimum of diffraction. It then adjusts a lower limit for the usual lasers having laser power of the order of 1MW. But this is possible only if a temperature of about 10eV is authenticated.

Litvak (1970) [29] studied the self-focusing in magnetoactive plasma for the case of longitudinal propagation. The nonlinearity mechanisms for the magnetoactive plasma have been studied and expressions are obtained for the nonlinear corrections to the refractive index due to heating, and nonlinear motion of a single electron. They have obtained the necessary condition for self-focusing and determined the characteristic parameters for self-focusing of the beam.

Sodha et al. (1971) [30] investigated theoretically the propagation and focusing of an electromagnetic wave in inhomogeneous dielectrics. They concluded that the focusing length is enhanced in a medium where the dielectric constant is a decreasing function of axial distance of propagation and vice-versa.

Sodha et al. (1973) [31] studied the nonlinearity in the dielectric constant of strongly ionized plasma and concluded that the non-linearity comes because of the heating and redistribution of the electrons. The energy loss gained from the field is due to thermal conduction. This self-induced non-linearity causes self-focusing and oscillatory waveguide propagation of the beam even though the non-linear dielectric constant does not fall in the saturating range. In a typical case of 10^{10} W laser, the axial intensity is enhanced by a factor of 25 and has been predicted in a length scale of 0.6cm.

Sodha et al. (1974) [32] investigated the process of self-focusing/defocusing of laser beam operating in the TEM_{01} mode. They concluded that the cylindrical symmetry and the power of irradiance increases inside the medium and gets concentrated around the points of maximum irradiance in different directions. Further they observed that a figure of eight is being made by the polar representation of maximum irradiance in the transverse plane.

Sodha et al. (1974) [33] investigated the process of self-focusing of a cylindrically symmetric electromagnetic pulse in collision-less and collisional plasmas. They considered the ponderomotive force and the non-uniform heating as a source of non-linearity. They further considered that the pulse duration is larger than the characteristic time of non-linearity. They found that the beam is focused in a moving filament. But in collisional plasmas and due to relaxation effects, the peak of the pulse is shifted to higher values. However, in case of collision-less plasmas, the pulse is severely distorted because of self-focusing and the peak shift is not significant.

Sodha and Nayyar (1975) [34] observed that the electromagnetic energy gets converged in the x-direction and vice-versa as the refractive index decreases with increasing temperature in the TEM_{01} mode. They also found that in the case of geometric-optics, the thermal self-focusing / defocusing of beam occurs in TEM_{00} and TEM_{10} cylindrical modes.

Siegrist (1976) [35] used the constant shape paraxial-ray approximation to discuss the propagation of intense laser beams in plasmas. It is further found that the stationary self-focusing behavior of each mechanism is treated separately and similar with several orders of magnitude difference in critical power. Further, in stationary self-focusing due to the combined mechanisms, the occurrence of relativistic effects is prevented by the complete saturation of ponderomotive self-focusing.

Nayyar (1978) [36] studied the self focusing in strongly ionized plasma by consideration of a non-Gaussian beam that operates in TEM_{01} mode. It is found that the focusing effects are observed in Y-direction when the incident power of beam becomes larger than the critical power. Whereas, beam divergence takes place in X-direction. However, in the reverse case the normalized beam width parameter f_2 first increases in Y-direction and after penetrating a certain depth inside the medium, it attains a broadened maximum and then starts decreasing with the propagation distance. The beam continues to diverge in the X-direction and the extent of self focusing is reduced by the absorption.

Nayyar and Soni (1979) [37] reported that in collisional plasma, the non-linear dependence of the dielectric constant is due to inhomogeneous heating of energy carriers and in collision-less plasma, it is due to ponderomotive force. Further they found that the beam gets focused at different points in different planes and hence exhibits the effect of astigmatism. In certain power regions they have considered, the beam either converges or diverges in both directions. While in some other regions of the power spectrum, one dimension of the beam focuses while the other defocuses. The beam then propagates in an oscillatory waveguide mode.

Askar'yan et al. (1981) [38] studied the nonlinear defocusing of a focused beam and observed a fine beam from the focus zone. They have used a single-mode, unmodulated neodymium laser with energy of 1J and with a millisecond pulse and a YAG-Nd laser in operating single and high frequency millisecond-pulse repetition modes and experimentally explained the defocusing nature of the beam in the weakly absorbing nonlinear medium.

Mori et al. (1988) [39] found that initially relativistic self focusing occurs initially due to filamentation and then by an extreme and unexpected ponderomotive mechanism at the boundary of channel walls. It is less intense for single frequency illumination and more for resonant double frequency.

Kurki et al. (1989) [40] have obtained the steady-state asymptotic solution of beam propagation in a localized solitary waveform in slab geometry and also presented the solutions for the beam profile where it is oscillatory in nature, which correspond to the fact or condition of being present the steady-state solution of a multiple-beamlet profile.

Cicchitelli et al. (1990) [41] have shown that the electromagnetic beams in vacuum do have a longitudinal component that can be proved experimentally from the polarization independence of the energy of electrons from the focus of a laser (Lax, Louisell, and Knight (LLK)). They have developed the LLK paraxial approximation to a Maxwellian exact solution for a Gaussian beam and included the exact longitudinal field components of the laser beam.

Cohen et al. (1991) [42] investigated the dynamics of ponderomotive self-focusing in plasmas. They calculated the space-time evolution of non-linear coherent beam and the parameters under consideration have dominant non-linearity in terms of the ponderomotive force and the plasma response is quite hydrostatic. It can be important both for high-power laser applications including inertial-confinement fusion and for heating of magnetically confined plasmas with intense, pulsed free electron lasers.

Brandi et al. (1993) [43] investigated the high-irradiance propagation of a laser beam in plasma whose optical index has a non-linear relationship with light intensity through both theoretical and numerical analysis. The nonlinear effects examined include expulsion of electrons and decrease of the plasma frequency. The defocusing and focusing effects are assumed to remain cylindrical and for plasma supposed homogeneous medium are along the propagation direction.

Chen et al. (1993) [44] derived a set of three dimensional equations for the propagation of an intense laser pulse of arbitrary strength $a = eA/mc^2$ in cold underdense plasma. In different limits, these equations can be reduced to certain previous one dimensional model. Chen *et al.* found that for $|a| < 1$, an approximate set of equations from the averaged Lagrangian is obtained. They solved the axisymmetric two dimensional model equations numerically to show the effect of dispersion in the self focusing process.

Bulanov et al. (1995) [45] studied that an ultra-short, relativistically strong pulse can be self-focused in plasma with strong magnification of its amplitude and channelling in a narrow channel shaped like a "bullet". Plasma turbulence occurs in the region occupied by the pulse and behind it and leads to electron heating. It is found that a regular longitudinal electric field is produced in the wake of a wide pulse shorter than the plasma wave period and behind the shape edge of a long pulse. The transverse nonuniformity of the pulse causes the formation of horseshoe structures that is used to focus and accelerate electrons and protons. Hence fast and strong modulation of the pulse occurs by the induced focusing of the EM radiation.

Gibbon et al. (1996) [46] experimentally studied the self-channeling and relativistic self-focusing of a terawatt laser pulse in the range of $(0.7TW \le P \le 15TW)$ by using paraxial envelope model. The model described the laser propagation and the plasma response is being described by the ponderomotive force. They have shown that a laser intensity of $5-15$ times may be obtained in vacuum when P lies in the $(1.25-4)\times P_c$ range.

Asthana et al. (2000) [47] studied relativistic self-focusing when Gaussian beam is incident normally on a plane interface of a linear medium and a non-linear, non-absorbing plasma with an intensity dependent dielectric constant. They considered non-linearity to arise from the relativistic variation of mass and the Lorentz force on electrons and followed WKB and paraxial approximation to analyze the relativistic self-focusing of transmitted laser radiation for the arbitrary magnitude of non-linearity. They found that as the upper critical power increases, minimum beam radius and focal length decreases so that the refraction at the interface has an effect on self-focusing.

Belafhal and Ibnchaikh (2000) [23] have studied the relative intensity distribution of the Hermite-cosh-Gaussian beams for the propagation in the free space. They also reported the normalized intensity plots of the Hermite-cosh-Gaussian beam profiles for the propagation through an aperture lens for various values of the truncation parameter for different mode indices.

Hafizi et al. (2000) [48] found that the ponderomotive force associated with an intense laser beam expels electrons radially and can lead to cavitations in plasma. Relativistic effects as well as ponderomotive force which acts in such a way that it expels electrons and hence modifies the refractive index. They derived an equation for the laser spot size, using the source-dependent expansion method with Laguerre-Guassian Eigen functions, and reduced to quadrature. The envelop equation is valid for arbitrary laser intensity within the long pulse, quasi-static approximation and neglects instabilities. The significant contraction of the spot size and an increase in intensity is possible when the laser power surpasses the critical power.

Osman et al. (2000) [49] proposed an investigation of the behavior of a laser beam having finite diameter in plasma. They studied it with respect to the forces and optical properties leading to the self-focusing in non-relativistic regime. The fugacious setting of ponderomotive nonlinearity in a collision less plasma results in the focusing of plasma wave at high laser intensities. Further, they considered the relativistic effects to compute an expression for the relativistic self-focusing for Nd glass radiation, at different plasma densities. Furthermore, a numerical program in c^{++} has been developed to examine or to investigate the deepness of self-focusing.

Liu and Tripathi (2000) [50] investigated the laser frequency up-shift, ring formation in tunnel ionizing gases and self-defocusing in plasmas. In their work a high-intensity laser produces rapid tunnel ionization of gas and the refractive index decreases by increasing plasma density which in turn causes frequency up-shift and super-continuum generation. However, refractive index decreases due to tunnel ionization when the laser intensity profile peak is on the axis and thus causes defocusing of the laser. This defocusing reduces the rate of ionization and frequency up-shift.

Feit et al. (2001) [51] studied the description of powerful beam self focusing in plasma. They emphasized on the total electron evacuation under the effect of ponderomotive forces. They displayed a method which showed that a laser beam can be self channeled in underdense plasma if the laser intensity is high enough to yield cavitations. It is studied that cavitation results in suppression of filamentation and the possibility to channel power well above the nominal critical power of self focusing for a distance of many Rayleigh lengths.

Faure et al. (2002) [52] discussed experimentally pulse duration effects on self-focusing of lasers in underdense plasmas. It was shown by them that the nominal critical power *Pc* for relativistic self-focusing in particular is not the only parameter that describes the pulse duration in comparison to plasma particle motion. However, using time resolved shadowgraphs, it has been displayed by them that a pulse does not self-focus relativistically if its duration is excessively short. This is due to divergence by the longitudinal wake that has been generated by the laser pulse itself. However, the phenomenon of self-focusing can occur for powers much less than the critical power. This is because of the radial expansion of ions that creates a channel which combines with relativistic effect and makes the laser pulse to focus.

Nitikant and Sharma **(2004)** [53] have seen the pulse slippage effect on resonant second harmonic generation**.** In their work they found that process of second harmonic generation is enhanced resonantly by the application of a magnetic wiggler. The laser gives an oscillatory velocity to electrons at $(2\omega_1, 2k_1)$ and exerts a longitudinal ponderomotive force. The electrons at the second harmonic acquire an oscillatory velocity and the wiggler magnetic field beats to produce a transverse second harmonic current at $(2\omega_1, 2k_1 + k_0)$. It then drives the second harmonic electromagnetic radiation whose amplitude saturates. The so created beam then stumbles out of the main beam.

Sharma et al. (2004) [54] extended the formalism of self-focusing of electromagnetic waves to include nonlinear absorption by the medium. Further they employed a complex eikonal which does not need any approximation about the relative magnitudes of the real and imaginary parts of the dielectric constant or their dependence on the irradiance of the beam. They found that the nonlinearity in absorption tends to cancel the effect of divergence on account of diffraction. The beam-width and attenuation depends on distance of propagation

Jha et al. (2004) [55] studied the propagation characteristics and modulation instability of a laser beam propagating through partially stripped plasma. They found that the ponderomotive non linearity tends to defocus the laser beam as against the nonlinear relativistic self-focusing phenomenon. Also the current density perturbation arising due to ponderomotive nonlinearity, when combined with relativistic nonlinearity tends to increase the modulation instability of the laser beam. However the peak growth rate is enhanced and also increases the range of unstable wave numbers in comparison to the case where ponderomotive nonlinearity is neglected.

Kant and Sharma (2004) [56] studied that second harmonic in plasma is generated by a Gaussian beam when magnetic wiggler of suitable period is present. For a particular value of the Wiggler period, the phase matching conditions are satisfied. The intensity of the second harmonic pulse is enhanced by self-focusing of the primary pulse. It then experiences a repeated and regular focusing in plasma channel which is formed by the primary wave.

Kant and Sharma (2005) [57] reported that a laser pulse focuses near the axis when incident on a cylindrical dielectric fibre perpendicular to the axis of the fibre. The focusing effect is enhanced for a given radius of the fibre and for a laser of specific intensity. However, in the axial region the tunnel ionization is produced due to high intensity of the laser. Further, the optical breakdown of the dielectric results in electron-hole pair production and plasma formation in the form of capillary and the plasma tends to self-defocus the laser.

Prakash et al. (2005) [58] investigated the focusing and defocusing of a beam in a medium which is depicted by built in radial inhomogeniety. They used the paraxial approximation and nonlinearity having saturating nature. Using an eikonal and parabolic equation for wave propagation, they found that the beam width and an axis irradiance depends on the distance of propagation. Further they concluded that the media in which an electromagnetic beam is guided by inhomogeneity in refractive index with a small cross section over long path lengths are ideal for achieving highly efficient nonlinear interactions.

Varsheney et al. (2005) [59] presented an investigation of relativistic self-focusing of laser radiation in inhomogeneous plasma by using paraxial approximation. The nonlinearity in the dielectric constant appears on account of the relativistic mass variation for an arbitrary magnitude of intensity. For a circularly polarized wave, the nonlinear dielectric constant has been used in analyzing the laser-beam propagation. The dependence of the beam width parameter variation, the self-trapping condition and the critical power have been examined. Depending on the plasma inhomogeneity and the initial intensity, the laser beam behaves in such a way that it likely acquires a constant value whose numerical estimation is done for typical values of the laser–plasma interaction.

Saini and Gill (2006) [60] presented the dynamics of combined effects of nonlinearity and spatial diffraction. They used the variation approach and observed the phenomenon of cross-focusing where focusing of one beam width parameter results in defocusing of another beam width parameter and vice-versa. However no stationary self-trapping is observed but oscillatory self-trapping occurs far below the threshold and the regularized phase is always negative in collision-less magneto-plasma.

Kumar et al. (2006) [61] discussed the nonlinear effects due to relativistic decrease of the plasma frequency and the ponderomotive expulsion of electrons. From the fluid equations they obtained have been used to study the amplitude variation of the excited electron plasma wave. It is observed that the inclusion of ponderomotive nonlinearity is significant on the excitation of plasma wave. This affects the number of energetic electrons and their energy ranges on account of wave particle interaction.

Sodha and Sharma (2006) [62] investigated the mutual focusing/defocusing of Gaussian electromagnetic beamsin collisional plasma. In their work, they have considered the mutual focusing/defocusing in singly ionized collisional plasma which is initially in thermal equilibrium and the ionosphere with singly charged ions. They started from the expression of the electron temperature in terms of the irradiance of the waves and derived expressions for the electron density and the dielectric function. The power loss by electrons to heavy particles is supposed to be more larger than that due to thermal conduction. The dominant nonlinearity considered here is the radial redistribution of the electron density on account of the radial dependence of the electric field of the waves and consequently of the electron temperature. Using this expression for the dielectric function, the coupled wave equations corresponding to different beams have been solved in the paraxial approximation, yielding a system of coupled second-order differential equations for the beam-widths. They also solved coupled equations for the widths of two beams numerically for some typical cases and correspondingly the critical curves for the two beams have also been obtained. They have also considered specifically effect of one beam on the critical curve of the other beam. They presented their results in the form of graphs for plasmas in thermal equilibrium and also for day-time mid-latitude ionosphere at a height of 150 km.

Gupta and Suk (2007) [63] studied the self-focusing and spot size behavior in semiconductor plasma. They have shown that enhancement in focusing is possible by beating of two co-propagating laser beams that can promote a large amplitude plasma wave in a resonant manner inside a narrow gap semiconductor. The medium is made highly nonlinear by the ponderomotive force of the electrons due to the plasma beat wave. As a result, the incident laser beam becomes self-focused.

Gupta et al. (2007) [64] found that a high- power laser beam propagating through underdense plasma under plasma density ramp acquires a very lower spot size due to relativistic self-focusing. Further away from the focus, it is the nonlinear refraction that weakens and the waist size of the laser increases. The density transition is introduced in order to abstain the laser from defocusing. This causes the reduction in the laser spot size close to the axis of propagation. In the absence density ramp, the laser is de-focused beyond the Rayleigh length due to the supremacy of the diffraction effect and in the presence of an upward plasma density ramp, as the plasma density increases, it occurs sooner and becomes stronger.

Gupta et al. (2007) [65] observed the focusing by taking in to account both density transition and magnetic field. They found that the magnetic field acts in such a way so as to increase the rate of self-focusing. This scheme forms a basis for various laser-driven applications as the laser beam not only is focused but propagates over a long distance without divergence.

Agarwal and Sodha (2007) [66] analyzed the linear absorption effect and initial curvature on focusing/defocusing in an inhomogeneous nonlinear medium by following paraxial approximation. It has been found that the lower and higher values of the beam width go on decreasing with increase in absorption along with propagation distance. This continues till the beam becomes very weak and diverges in a steep manner. Its penetrating power also decreases with increasing absorption in an overdense medium. Depending up on the initial values of beam width and axial irradiance, the beam initially converges and then goes in the oscillatory divergence, self-focusing or smooth divergence mode. The greatest value of the penetrating power occurs in the range -0.7 < $(\partial f/\partial z)_{\text{at } z=0}$ < 0.4 and outside these limits, it falls in a sharp manner.

Faisal et al. (2008) [67] developed the energy balance equation for electrons and equations which express the pressure gradient balance of electrons and ions to the force produced by space charge field. They also solved equation for initial time profile at z=0 of the pulse to obtain f as a function of normalized distance and time profile. It is seen by them that in the initial period the beam suffers steady divergence because of the nonlinearity that does not build up to sufficient extent. Later, the behavior changes to oscillatory divergence, then oscillatory convergence, and again oscillating divergence and finally smooth divergence. This is explained by the fact that focusing is dependent on the rate of change of nonlinearity with the irradiance, rather than on the magnitude of the nonlinearity. They used both Gaussian as well as sine time profile of the pulse for investigation.

Patil et al. (2008) [25] studied the HchG laser beam propagation by considering a non-degenerate germanium containing neutrality of space charge. They applied the parabolic wave equation approach and used the paraxial approximation to obtain the analytical solutions by following the inequality $R_n < R_d$, where, R_n and R_d are the self-focusing length and diffraction length respectively. The so examined behavior of beam width parameter showed that the process of self-focusing occurs for various decentered parameter values.

Kaur and Sharma (2009) [68] observed that a laser beam propagates in a regularly and repeatedly focused manner above the critical power of the laser. However, the beam follows divergence below this power. At significantly greater powers, the laser beam converges till the saturation effect of nonlinearity puts an end to self-focusing and diffraction succeeds. The density ripple causes self-focusing length to rise and the spot size variation depends on the wave number of the ripple.

Verma and Sharma (2009) [69] developed a theoretical model to study the self-focusing effect. They found that after the passage of laser beam, the plasma expands and creates a channel with minimum electron density on the axis. The second pulse of lower duration and much lower intensity is capable of heating the electrons, hence raising the ionization rate and suppressing the recombination ratio. This leads to significant enhancement of the plasma channel lifetime. The formation of the channel by the first pulse requires a time delay of ~ 5-10 ns between the two pulses to allow radial ambipolar diffusion of the plasma. The second laser pulse undergoes periodic focusing in such a channel leading to a strong heating rate of the electrons. The second pulse then self-focuses, enhances the heating rate and lengths the lifetime of the plasma channel.

Parashar (2009) [70] investigated the self- focusing effect on third harmonic generation in a gas embedded with atomic clusters. The results obtained reveal that the effectiveness of third harmonic is sensitive to the ratio of electron density inside the clusters to critical density. As the clusters expand under hydrodynamic pressure caused by the laser, efficiency is maximum at an instant when this ratio is three for clusters on the laser axis as these clusters contribute maximum to harmonic generation. The efficiency is also greatly enhanced by self-focusing. Since laser spot size varies in a periodic manner with the propagation distance. Due to competition between self-focusing and diffraction effects, it was found that the efficiency of harmonic generation also shows a similar behavior.

Takale et al. (2009) [71] used the parabolic wave equation approach for entire theoretical formulation and completed the numerical computation of TEM_{op} Hermite-Gaussian laser beam by following Runge-Kutta method. They obtained the differential equations for beam width parameters under paraxial approximation and observed perfectly synchronized periodic oscillations of beam width parameters in transverse directions in small scale spatial manipulations in optical trap.

Bonabi et al. (2009) [72] analyzed the Gaussian beam propagation in underdense plasma by considering density transition. The outcomes they have obtained indicate that the laser beam becomes highly focused and penetrates deep into the plasma medium by reduction of diffraction effect. In their work, they introduced a unique ramped density profile which increased self-focusing effect for intense laser systems and puts an end to the transverse variation of the wave packet. They established an equation for laser spot size and presented the computational curves for self-focusing. The outcomes they have obtained showed that the intense laser beams can be focused down to diameters comparable to the Nd-glass laser wavelength. The effects of laser intensity on the self-focusing parameters were also investigated. Based on their reliable derived equations and introduction of more effective ramped density profile, a much sooner and stronger focusing is observed.

Patil et al. (2009) [73] established the differential equation under WKB and paraxial approximations and analytical solution is obtained for the same. By considering various absorption levels in the medium, the behavior of beam width parameter is studied at various decentered parameter b values. Their results show that the sharp self-focusing occurs on the grounds of absorption. Further they suggested that depending up on the state of being desirable self-focusing in a particular application, the decentered parameter of beams can have an extraordinary deed with particular absorption level in the medium.

Xiong et al. (2010) [74] developed a method to study the self-focusing effect in cold plasma using the power of arbitrary magnetic field. They set the magnetic field in the plane which includes y and z-axis. Their results show that there is a different effect on self-focusing corresponding to different angle and intensity. The larger angle between the y axis and outside magnetic field weakens the self-focusing effect and is strengthened by increasing outside magnetic field.

Singh and Walia (2010) [75] established the differential equation under moment theory approach and analytical solution is obtained for the same. They used the Runge-Kutta method to solve it numerically. The results they have obtained are in agreement with the findings of the simulation (3D PIC) and observed a new stable form of self-channelling propagation. Further they reported that the self-focusing length decreases with increase in intensity of the beam. They also found that due to dynamic balance between two competing nonlinear effects i.e. diffraction and non-linear refraction, periodic self-focusing occurs.

Patil et al. (2010) [76] investigated the focusing of Hermit-cosh-Gaussian (HchG) laser beams in collision-less magneto-plasma by using WKB and paraxial approximations. They presented the dynamics of the combined effects of nonlinearity and spatial diffraction and highlighted the nature of focusing. They found that the self-focusing/defocusing of HChG beams is dependent on the mode index and decentered parameter.

Kant et al. (2011) [77] analyzed the self-focusing and found that ponderomotive self-focusing becomes stronger when density transition is taken into account. To reduce the oscillation amplitude, plasma density is increased slowly. Further, in the ramped density region, the laser can perceive a narrowing channel at a slow pace due to which the oscillation amplitude of the spot size contracts. Therefore, the laser beam undergoing plasma density ramp is expected to become more focused.

Gill et al. (2011) [78] investigated the characteristics of cosh-Gaussian laser beam propagation in magnetoplasma using variational approach. They derived nonlinear Schrodinger equation in an appropriate way and discussed the necessary phase modulation. They found that the decentered parameter b along with magnetic field play a key role in self-focusing/ defocusing enhancement of the beam.

Kim et al. (2011) [79] investigated the effect of the density ramp structure on the electron energy in laser wake field acceleration. They have reported that with a downward density ramp, the electron energy decreased due to a lag in the acceleration region and to the acceleration field strength being lower than that with a uniform density, but with an upward ramp, the energy increased because of the higher acceleration field and the position of the acceleration region. These effects were studied by using simulations having a 2-dimensional particle-in-cell code and by experiments using a 20TW laser.

Gill et al. (2011) [21] have taken in to account the combined effect of relativistic and ponderomotive nonlinearities to analyze the self-focusing with linear absorption. At various b values with various absorption levels, the self-phase modulation, self-focusing and self-trapping of beam have been studied. They observed that in the absence of decentered parameter, the self-focusing effect becomes weaker for a large value of absorption coefficient. However, an oscillatory self-focusing takes place for a higher value of decentered parameter b = 1 and for b = 2, self-focusing effect is observed in a sharp manner.

Kant et al. (2012) [22] reported the self-focusing of Hermite-Gaussian beam propagation in plasma under the application of plasma density ramp. They derived the differential equation by using WKB approximation and paraxial approximation. The density ramp causes reduction in the spot size of laser beam close to the axis of propagation. They found that by choosing appropriate and optimized parameters, self-focusing effect is enhanced.

Navare et al. (2012) [80] investigated theoretically the self-focusing of laser in collisional plasma and considered the impact of linear absorption. They used the parabolic equation approach in order to obtain the differential equation through WKB and paraxial approximations. While, considering the collisional nonlinearity and linear absorption, it has been found that the absorption plays a crucial role in self-focusing effect. It destroys the oscillatory character of laser beam during propagation. Further, with increase in initial irradiance, the laser beam bends towards the focusing mode.

Gill et al. (2012) [81] established the differential equation for super-Gaussian beam propagation and analytical solution is obtained for the same. They considered the magnetic field and the condition for the formation of a dark and bright ring. Their work involves the higher order terms of the dielectric function and reported that the inclusion of such terms affects the beam width parameter. Consequently a substantial increase in self-focusing is observed only in case of a dark ring. However, the results contradict for a bright ring.

Habibi and Ghamari (2012) [82] used the density ramp profile to investigate the process of self-focusing in cold quantum plasma (CQP). They established the differential equation and analytical solution is obtained for the same. Their results reveal that the quantum effect significantly adds to self-focusing effect in comparison to classical relativistic effect. However, apart from quantum effects, the ramped density profile gives rise to higher oscillations and enhanced focusing of laser beam in cold quantum plasma.

Abari and Shokri (2012) [83] investigated the process of self-focusing and defocusing in underdense plasma by considering the nonlinear ohmic heating and ponderomotive force effects. They reported that it is the ion temperature which strongly influences the laser spot size. Further, in the self-focusing regime, the perturbed electrons oscillate continuously in between the initial and a minimum value due to high ion temperature. However, reverse is true for defocusing.

Patil et al. (2013) [84] found that with increase in intensity, there is a faster decrease in initial beam width for cold quantum plasma than classical relativistic case. Further, the beam is weaker at high intensity for classical relativistic plasma than cold quantum case. Moreover, the quantum effect plays an important and a captivating role in making the self-focusing effect stronger.

Patil and Takale (2013) [85] reported that the upward plasma density ramp in weakly relativistic and ponderomotive regime can accelerate the electron to higher energy over a long propagation distance as compared with uniform density relativistic plasma. Further, apart from density profile and intensity parameter, the electronic temperature plays a captivating role in self-focusing of laser and hence gives reasonably interesting results.

Patil et al. (2013) [86] explored the impact of electron plasma temperature, relative density plasma and intensity parameter on the laser beam evolution in plasma. They established the differential equation for beam propagation and analytical solution is obtained for the same. Their results reveal that as relative plasma density grows, the self-focusing of laser beam takes place for earlier values of propagation distance and then becomes stronger. Hence, the optimum self-focusing is achieved.

Gupta et al. (2013) [87] found that the ion temperature causes thermal self-focusing and has a serious influence on the evolution of laser beam in plasma. Further, by modifying the plasma density resulting in the generation of the nonlinearity, their obtained outcomes show a noticeable nonlinearity in laser self-focusing.

Mahajan et al. (2013) [88] reported that the oscillatory self-focusing takes place for different intensity parameter values. They established the differential equation for beam propagation and analytical solution is obtained for the same. As soon as intensity parameter is increased, the distance between the points having a logical sequence at intersecting point of two beams increases. Again, with increase in intensity parameter, a substantial increase in self-focusing is observed.

Kaur et al. (2013) [89] investigated the interaction between parallel Gaussian electromagnetic beams in relativistic magnetoplasma. They found that in relativistic magnetoplasma, self-focusing occurs at lower values of distance of propagation. It is further observed that for higher values of magnetic field, an earlier self-focusing is observed and the beam shows oscillatory behavior with increasing intensity.

Nanda et al. (2013) [90] laid an emphasis on the decentered parameter sensitivity for relativistic self-focusing. They used the WKB and paraxial approximation to derive the nonlinear differential equation for three mode indicies 0, 1 and 2. In their work, the emphasis was laid on the selection of decentered parameter. The results, they have obtained indicate that the proper and appropriate decentered parameter selection is enough important for self-focusing of HchG beam.

Nanda et al. (2013) [91] found that it is the decentered parameter and ramped density profile that results in self-focusing of laser beam. They used the WKB and paraxial approximation to derive the nonlinear differential equation. By considering the ramped density profile and magnetic field for HchG laser beam, it has been found that the presence of density transition and magnetic field enhances the required effect to a larger extent.

Nanda and Kant (2014) [92] investigated the relativistic self-focusing in an enhanced manner under density transition. They used the WKB and paraxial approximation to derive the nonlinear differential equation for three mode indicies 0, 1 and 2. For decentered parameter b = 1.8 and for m = 0 and 1 modes, early and enhanced self-focusing is seen. However, for b = 1.8 and for m = 2 mode, diffraction is seen. They also observed that decentered parameter and ramp density transition enhances the required self-focusing effect.

Nanda and Kant (2014) [93] studied strongly the process of self-focusing in collisionless magnetoplasma under ramped density profile. They used the WKB and paraxial approximation to derive the nonlinear differential equation by considering ponderomotive nonlinearity. They analyzed the density transition effect and magnetic field on the propagation of cosh-Gaussian beam. The focusing and defocusing nature of beam has been studied at various optimized parameters. They observed that for decentered parameter b =2.12, sooner and better self-focusing. Further, their results reveal that the optimized laser and plasma parameters are important in making the effect.

Aggarwal et al. (2014) [94] derived the nonlinear partial differential equation which governs the laser spot size by using paraxial approximation and slowly varying approximation. They found that due to the prime importance of self-focusing effect over diffraction effect, the laser beam converges in high plasma density region and diverges in low plasma density region. Further, the required effect is obtained by optimizing wavelength and intensity parameters of beams in rippled density plasma.

Milani et al. (2014) [95] derived the coupled differential equations by using WKB and paraxial approximations. Effects of collision frequency, axis laser intensity distribution and initial laser intensity are analyzed in warm collisional plasma. Their results reveal that firstly, the collision frequency causes self-focusing and secondly, it defocuses the laser. However, as soon as it is increased, the self-focusing length becomes shorter with the result the larger collision frequency prevents the longer beam propagation through the plasma.

Malekshahi et al. (2014) [96] investigated the self-focusing of the high intensity ultra short laser pulse propagating through relativistic magnetized plasma. They have taken in to account the nonlinearity up to third order and external magnetic field and studied the relativistic effect under paraxial approach. Their results reveal that imposing the external magnetic field enhances the capacity of self-focusing. However, the self-focusing property decreases by increasing the angle between the laser field and external magnetic field.

Zare et al. (2015) [97] have considered the density ramped profile to study the propagation of Gaussian x – ray laser beam by using WKB and paraxial approximation in thermal collisionless quantum plasma. A mathematical formulation is obtained by following parabolic approach. They found that increase in plasma density leads to stronger self-focusing effect i. e the beam having less oscillation amplitude and smaller spot size, focuses faster.

Further, it has been found that the laser and plasma parameters are crucial for self-focusing as it is enhanced with optimized laser and plasma parameters.

Habibi and Ghamari (2015) [98] used the higher order paraxial theory (up to r^4) to investigate the focusing of a laser beam in quantum plasma. The eikonal have been taken in to account and extended paraxial theory to investigate the preliminary study of ChG beams. The paraxial theory allows the adjustment in the shape of radial intensity distribution and affects the beam spot size. Further, the inclusion of higher order terms of dielectric function affects the behavior of beam width parameter significantly. By using more effective decentered parameter, better self-focusing is observed for chG beams in comparison to Gaussian laser beams in cold quantum plasma (CQP).

Habibi and Ghamari (2015) [99] studied the propagation of high power laser beam entering in to the high density plasmas by considering ramped density profile and quantum correction in relativistic regime. They followed the higher order paraxial theory and derived the governing equations in cylindrical coordinate system. By utilizing higher order paraxial theory and the sensitivity of decentered parameter b, a significant enhancement in self-focusing is reported. Further, their results reveal that a stronger self-focusing effect is observed in inhomogeneous cold quantum plasma (ICQP) under plasma density ramp.

Aggarwal et al. (2015) [100] investigated the effect of self-focusing in an inhomogeneous magnetized plasma with ponderomotive nonlinearity. They have taken in to account the paraxial approximation and developed desired relation for dielectric constant. They have derived an appropriate expression for the nonlinear differential equation in presence of external magnetic field and linear absorption. They predicted that initially converging beams show oscillatory convergence while as initially diverging beams show oscillatory divergence. Further, the laser beam is more focused at lower intensity in extraordinary as well as in ordinary mode.

Aggarwal et al. (2015) [101] studied the circularly polarized quadruple Gaussian beam propagation in magnetoplasma. They considered the nonlinearity due to relativistic mass increase of electrons which changes the refractive index. They used the WKB approximation for the derivation of nonlinear differential equation and self-trapped condition. On the application of external magnetic field, the quadruple Gaussian beam can be studied in three different regions especially self-focusing, oscillatory and smooth divergence. They reported that the magnetic field improves the self-focusing effect in extraordinary mode and worsen it in ordinary mode.

Aggarwal et al. (2015) [102] paid an attention to the investigation of beam propagation in cold magnetized plasma in presence of density transition. They used the Maxwell's equations and derived the differential equation by using WKB and paraxial approximation. They found that the laser and plasma parameters are crucial for self-focusing as it is enhanced with such optimized parameters. The strong self-focusing is obtained at optimized intensity $\alpha_0 E_{00}^2 = 0.6$ for extra ordinary mode and a comparison have been made in presence and absence of transition based density at $\omega_c / \omega_0 = 0.3$. The plasma density ramp and the magnetic field are found to increase ability of self-focusing in cold plasma.

Varshney et al. (2016) [103] analyzed the relativistic nonlinear propagation of rippled Gaussian beam by following WKB and paraxial-ray approximations for arbitrary magnitude of nonlinearity. At relativistic intensities, the nonlinearity allows the refractive index to have slower radial dependence in the paraxial regime. They found that a small ripple grows rapidly on the axis of the main beam. Further, the nonlinear refractive index has a slower radial dependence in the paraxial regime. It therefore results in extraction of less energy from its vicinity.

Eslami and Nami (2016) [104] investigated the self-focusing characteristics of a laser pulse by considering lateral and axial plasma density variations. They have taken ponderomotive and relativistic effects to derive nonlinear dielectric permittivity of plasma and to develop a complete analytical model to study the phenomenon of self-focusing. With increase in channel width, the elf-focusing length increases. Further, they reported that the self-focusing effect is enhanced and shifts towards lower values of distance of propagation due to increase in intensity of the laser beam.