CHAPTER-8

SELF-FOCUSING OF A LASER BEAM IN THE RIPPLED DENSITY MAGNETOPLASMA

8.1 INTRODUCTION

The interaction of high intense laser beams with plasmas gives rise to various wide ranging potential applications including laser-driven acceleration, harmonic generation, x-ray lasers etc. [8,9, 128, 129]. For the success of these applications, it is to be desired that the high intensity laser beam should propagate various Rayleigh lengths without divergence. When such a laser beam interacts with a medium like plasma, it provides oscillatory velocity to the electron, which changes the dielectric constant of the medium [105]. The process of self-focusing has been examined by various authors [116, 43, 117] and is important for the above mentioned applications. The self-focusing is more at relativistic intensity as it generates a quasi-stationary magnetic field [75] and is crucial for propagation of the laser beam over extended distances. For thermal self-focusing to occur, the ion temperature is considered to be important as it seriously influences the laser pulse evolution [87]. The effect of density ramp is such as to shrink the spot size of laser beam so that it becomes more focused while propagating through the plasma. The optimized parameters are found to be crucial for self-focusing as it is enhanced with such parameters. Its ability is further increased by density transition and magnetic field. [22, 85, 97, 102, 80, 100]. Further, the linear absorption destroys the oscillatory character of beam width parameter and thereby makes the self-focusing effect weaker while as the density transition causes sooner and earlier self-focusing [118]. Wani and Kant [130] investigated the relativistic self-focusing of HcosG laser beam in collisionless plasma. They reported that the beam width parameter shows a strong oscillatory behavior and hence the laser beam becomes more focused at lower values of intensity and decentered parameters. Further, the decentered parameter is considered to be essential for self-focusing [90]. Gill *et al*. [81] investigated the relativistic self-focusing by including higher order terms of the dielectric function. They have shown that the magnetic field and hence the higher order terms of dielectric function increase the self-focusing strength.

The density ripple was used to study stimulated Raman scattering of light waves in plasma. The excitation of plasma waves took place under rippled density profile as these waves have nearly the same wavelength as those of density ripple [131]. Further, the rippled density can be used to investigate the phase-matched harmonic generation from plasmas. The efficiency of generation increases rapidly in weak and moderate relativistic regime. However, the phase-matched harmonic generation saturates in strong relativistic regime [132]. Again, Kaur *et al*. [133] found that when ripple period is large, less self-focusing is observed and vice-versa. However, the rippling effect is more pronounced at lower laser intensity. Further, the density ripple increases the length of self-focusing and thus results in decreasing the laser spot size [68]. Keeping in view the ongoing development of high intense lasers, the present paper is aimed to investigate the impact of magnetic field on the self-focusing of laser beam propagating in rippled density plasma. Effect of various optimized parameters is seen on self-focusing in rippled density magnetoplasma. There is an enhancement in self-focusing strength by increasing laser strength and magnetic field to few mega-Gausses. The paper is constituted as follows: in section 8.2, nonlinear dielectric constant is presented. In section 8.3, the basic formulation and the differential equation governing the spot size variation is presented. Section 8.4 is devoted to results and discussions. Finally, the conclusion is added in section 8.5.

8.2 NONLINEAR DIELECTRIC CONSTANT

Consider the Gaussian beam propagation along z direction in magnetoplasma with rippled density profile, $n_0 = n_0^0 + n_{0q}e^{iqz}$, where, n_{0q} is the rippled density. The electric field vector $\vec{E}$ of the laser beam in magnetoplasma can be written as

$$\vec{E} = \vec{A}\exp[i(\omega t - kz)], \tag{8.1}$$

where, $\vec{A}$ is the electric field amplitude, ω is the angular frequency and $k = (\omega / c)\sqrt{\varepsilon_0}$ is the wave number and c is the velocity of light in vacuum. The effective dielectric constant ε is of the form [113]

$$\varepsilon = \varepsilon_0 + \phi(EE^*), \tag{8.2}$$

where, $\varepsilon_0 = 1-(1-\omega_{p0}^2/\omega^2)/(1-\omega_c/\omega)^2$ represents the linear part and $\phi(EE^*) = \varepsilon_2 AA^*$, $\varepsilon_2 = (1/2)(e/mc\omega)^2(\omega_{p0}/\omega)^2/(1-\omega_c/\omega)^4$, $\omega_{p0}^2 = 4\pi n_0 e^2/m$ and $\omega_c = eB_0/mc$ represents the electron cyclotron frequency. Here, ω_{p0} is the plasma frequency, e, m and n_0 being the magnitude of the electronic charge, rest mass and electron density respectively. B_0 is the magnitude of external magnetic field.

8.3 SELF-FOCUSING EQUATIONS

The wave equation for the propagation of laser beam in magnetized plasma is of the form

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2}(\varepsilon.E)\varepsilon\vec{E} - \nabla(\nabla.\vec{E}) = 0, \tag{8.3}$$

Eq. (8.3) can be written as

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial y^2} - \frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) = -\frac{\omega^2}{c^2}(\varepsilon.E)_x \quad , \tag{8.4}$$

$$\frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_y}{\partial x^2} - \frac{\partial}{\partial y}\left(\frac{\partial E_z}{\partial z} + \frac{\partial E_x}{\partial x}\right) = -\frac{\omega^2}{c^2}(\varepsilon.E)_y \quad , \tag{8.5}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} - \frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right) = -\frac{\omega^2}{c^2}(\varepsilon.E)_z \quad . \tag{8.6}$$

In order to solve Eq. (8.3) we suppose the field variations in the z-direction are much larger than in the $x-y$ plane. Therefore, in the zeroth order approximation, the waves are treated as transverse and hence space charge generation is negligible. Therefore one obtains

$$\frac{\partial E_z}{\partial z} \cong -\frac{1}{\varepsilon_{zz}}\left[\varepsilon_{xx}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right) + \varepsilon_{xy}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right] , \tag{8.7}$$

on multiplying Eq. (8.5) by i, adding it to Eq. (8.4) and using Eq. (8.7), we obtain

$$\frac{\partial^2 A}{\partial z^2} + \frac{1}{2}\left(1 + \frac{\varepsilon_0}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)A + \frac{\omega^2}{c^2}(\varepsilon_0 + \varepsilon_2 AA^*)A = 0, \tag{8.8}$$

where, we have neglected the product of nonlinear part with $\partial^2 A/\partial x^2$ or $\partial^2 A/\partial y^2$. Further, $\varepsilon_{0zz} = 1-\omega_{p0}^2/\omega^2$, $\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zy} = 0$, $\varepsilon_{xx} = \varepsilon_{yy} = 1-\omega_{p0}^2/\omega^2(1-\omega_c^2/\omega^2)$ and $\varepsilon_{xy} = -\varepsilon_{yx} = -i(\omega_{p0}^2/\omega^2)(\omega_c/\omega)/(1-\omega_c^2/\omega^2)$.

Now, we assume that $A = A_0 Exp[i(\omega t - kz)]$, (8.9)

with, $k^2 = \frac{\omega^2}{c^2}\left(1-\frac{\omega_{p1}^2+\omega_{p2}^2 e^{iqz}}{(\omega-\omega_c)^2}\right)$

Differentiating Eq. (8.9) twice w. r. t x, y and z respectively, we get

$$\frac{\partial A}{\partial x} = Exp[i(\omega t - kz)]\frac{\partial A_0}{\partial x}$$

$$\frac{\partial^2 A}{\partial x^2} = Exp[i(\omega t - kz)]\frac{\partial^2 A_0}{\partial x^2}$$

$$\frac{\partial^2 A}{\partial y^2} = Exp[i(\omega t - kz)]\frac{\partial^2 A_0}{\partial y^2}$$

Similarly,

$$\frac{\partial^2 A}{\partial z^2} = -Exp[i(\omega t - kz)]\left[2ik + \frac{qz\omega^2\omega_{p2}^2 e^{iqz}}{kc^2(\omega-\omega_c)^2}\right]\frac{\partial A_0}{\partial z} - A_0 Exp[i(\omega t - kz)]\frac{qz\omega^2\omega_{p2}^2 e^{iqz}}{kc^2(\omega-\omega_c)^2}\times$$
$$\left[1-\frac{iqz}{2}-\frac{iqz\omega^2\omega_{p2}^2 e^{iqz}}{4k^2c^2(\omega-\omega_c)^2}\right] - A_0 Exp[i(\omega t - kz)]\left[k^2 - \frac{qz\omega^2\omega_{p2}^2 e^{iqz}}{c^2(\omega-\omega_c)^2}\left(i - \frac{qz\omega^2\omega_{p2}^2 e^{iqz}}{4k^2c^2(\omega-\omega_c)^2}\right)\right].$$

Now, substituting the above values in Eq. (8.8) and neglecting $\partial^2 A_0/\partial z^2$, we get

$$-k\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left(\frac{2i}{\left(\frac{\omega_{p2}}{\omega}\right)^2}\left(1-\frac{\omega_c}{\omega}\right)^2 + \frac{qze^{iqz}}{\varepsilon_0}\right)\frac{\partial A_0}{\partial z} + \frac{1}{2}\left(1+\frac{\varepsilon_0}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)A_0$$

$$-k\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left(\frac{qA_0e^{iqz}}{\varepsilon_0}\right)\left(\begin{array}{l}1+\frac{iqz}{2}+\frac{iqze^{iqz}}{4\varepsilon_0}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}-\frac{i\omega z\sqrt{\varepsilon_0}}{c}\\ -\frac{\omega qz^2e^{iqz}}{4c\sqrt{\varepsilon_0}}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\end{array}\right)$$

$$+\frac{\omega^2}{c^2}\left(\varepsilon_2A_0A_0^*\right)A_0=0, \tag{8.10}$$

where, $\omega_{p1}^2=4\pi n_0^0e^2/m$, $\omega_{p2}^2=4\pi n_{0q}e^2/m$, A_0 is the amplitude and q is the ripple wave number. Now, suggesting a two dimensional Gaussian beam for which $\partial/\partial y=0$ and using an eikonal $A_0=A_0^0\exp(ikS)$, where, k is defined above, A_0^0 and S depend on x and z. Therefore, using these values in the above equation and separating real and imaginary parts of the emerging equation, one obtains

Real part equation is

$$-\frac{\omega}{c}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left[\sin(qz)+\frac{qz}{2}\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left(\begin{array}{l}2\cos(qz)\left(1-\frac{\omega_c}{\omega}\right)^2+\left(\frac{\omega_{p1}}{\omega}\right)^2\cos(qz)+\\ \left(\frac{\omega_{p2}}{\omega}\right)^2\cos(2qz)\end{array}\right)\right]\frac{\partial A_0^0}{\partial z}$$

$$+\frac{\omega^2A_0^0}{c^2\left(1-\frac{\omega_c}{\omega}\right)^2}\left(\frac{\partial S}{\partial z}\right)\left[\begin{array}{l}2\left(1-\frac{\omega_c}{\omega}\right)^2-\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}+\frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}-2\left(\frac{\omega_{p2}}{\omega}\right)^2\times\\ \cos(qz)+qz\left(\frac{\omega_{p2}}{\omega}\right)^2\left\{\begin{array}{l}\sin(qz)-\frac{1}{4}\left(\frac{\omega_{p1}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-4}\sin(qz)+\\ \frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\sin(2qz)-\\ \frac{1}{4}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-4}\sin(2qz)\left(\cos(qz)-\sin(qz)\right)-\\ \frac{1}{4}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\sin(2qz)\end{array}\right\}\end{array}\right]$$

$$+\frac{1}{2}\left(1+\frac{\varepsilon_0}{\varepsilon_{0zz}}\right)\left[\begin{array}{l}\dfrac{\partial^2 A_0^0}{\partial x^2}+\dfrac{\omega}{c}\left(\dfrac{\partial A_0^0}{\partial x}\dfrac{\partial S}{\partial x}+\dfrac{A_0^0}{2}\dfrac{\partial^2 S}{\partial x^2}\right)\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\sin(qz)- \\ \dfrac{\omega^2 A_0^0}{c^2}\left\{1-\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}-\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(qz)\right\}\left(\dfrac{\partial S}{\partial x}\right)^2\end{array}\right]$$

$$+\frac{\omega^2 qSA_0^0}{2c^2\left(1-\frac{\omega_c}{\omega}\right)^2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left[\begin{array}{l}\sin(qz)\left\{\begin{array}{l}2+2\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}+\dfrac{1}{2}\left(\dfrac{\omega_{p1}}{\omega}\right)^4\left(1-\dfrac{\omega_c}{\omega}\right)^{-4}- \\ \dfrac{1}{2}\left(\dfrac{\omega_{p2}}{\omega}\right)^4\left(1-\dfrac{\omega_c}{\omega}\right)^{-4}\left(2\cos(2qz)+1\right)\end{array}\right\}- \\ q\left(\dfrac{\omega_{p2}}{\omega}\right)^4\left(1-\dfrac{\omega_c}{\omega}\right)^{-4}\cos(2qz)\cos(qz)\left\{\begin{array}{l}1+\dfrac{1}{2}\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}+ \\ \dfrac{1}{2}\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(qz)\end{array}\right\} \\ -qz\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(2qz)\left\{\begin{array}{l}1+\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2} \\ \dfrac{1}{4}\left(\dfrac{\omega_{p1}}{\omega}\right)^4\left(1-\dfrac{\omega_c}{\omega}\right)^{-4}+ \\ \dfrac{1}{2}\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(qz)\end{array}\right\}- \\ \dfrac{qz}{4}\left(\dfrac{\omega_{p2}}{\omega}\right)^6\left(1-\dfrac{\omega_c}{\omega}\right)^{-6}\left(2\cos^2(qz)-\sin^3(qz)-1\right)\end{array}\right]$$

$$-\frac{\omega qA_0^0}{2c\left(1-\frac{\omega_c}{\omega}\right)^2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left[\begin{array}{l}\left\{2+\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(qz)\right\}+\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(2qz) \\ -qz\left\{1+\dfrac{3}{4}\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\right\}\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\sin(2qz)+\dfrac{2\omega z}{c}\sin(qz) \\ -\dfrac{3qz}{4}\left(\dfrac{\omega_{p2}}{\omega}\right)^4\left(1-\dfrac{\omega_c}{\omega}\right)^{-4}\sin(3qz)-qz\sin(qz)\left\{1+\dfrac{1}{2}\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\right\} \\ -\dfrac{\omega qz^2}{2c}\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\left\{\begin{array}{l}\cos(2qz)+\left(\dfrac{\omega_{p1}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos^2(qz) \\ +\left(\dfrac{\omega_{p2}}{\omega}\right)^2\left(1-\dfrac{\omega_c}{\omega}\right)^{-2}\cos(3qz)\end{array}\right\}\end{array}\right]$$

$$+\frac{\omega^2}{c^2}\left(\varepsilon_2 A_0^0 A*_0^0\right)A_0^0 = 0 \qquad (8.11)$$

And imaginary part equation is

$$-\frac{\omega}{c}\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left[\begin{array}{l}2\left(1-\frac{\omega_c}{\omega}\right)^2+\left(\frac{\omega_{p1}}{\omega}\right)^2-\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(qz)+qz\left(\frac{\omega_{p2}}{\omega}\right)^2\sin(qz)\times\\ \left\{1+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}+\frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(qz)\right\}\end{array}\right]\frac{\partial A_0^0}{\partial z}$$

$$+\frac{\omega^2 A_0^0}{2c^2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-4}\left(\frac{\partial S}{\partial z}\right)\left[\begin{array}{l}\sin(qz)\left\{\left(\frac{\omega_{p1}}{\omega}\right)^2+qz\left(\frac{\omega_{p2}}{\omega}\right)^2\right\}-qz\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(qz)\times\\ \left\{\begin{array}{l}\left(1-\frac{\omega_c}{\omega}\right)^4-\frac{1}{4}\left(\frac{\omega_{p1}}{\omega}\right)^4-\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(qz)-\\ \frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(\cos(2qz)-\frac{1}{2}\right)\end{array}\right\}\end{array}\right]$$

$$-\frac{\omega^2 A_0^0 qS\left(\sin(qz)+\cos(qz)\right)}{2c^2\left(1-\frac{\omega_c}{\omega}\right)^4\left(\frac{\omega_{p2}}{\omega}\right)^{-2}}\left[\begin{array}{l}2\left(1-\frac{\omega_c}{\omega}\right)^2+2\left(\frac{\omega_{p1}}{\omega}\right)^2+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}\\ -\frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(2qz)+qz\left(\frac{\omega_{p2}}{\omega}\right)^2\sin(qz)+\\ \frac{qz}{2}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left\{1+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}+\\ qz\left(\frac{\omega_{p1}}{\omega}\right)^2\left(\frac{\omega_{p2}}{\omega}\right)^2\sin(qz)\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left\{1+\frac{1}{4}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}\\ +\frac{qz}{4}\left(\frac{\omega_{p2}}{\omega}\right)^6\left(1-\frac{\omega_c}{\omega}\right)^{-4}\sin(qz)\left(3-\sin^2(qz)\right)+\\ \frac{qz}{4}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}\sin(2qz)\left(2+\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right)\end{array}\right]$$

$$+\frac{\omega}{2c}\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left(1+\frac{\varepsilon_0}{\varepsilon_{0zz}}\right)\left[\begin{array}{l}\left\{\left(1-\frac{\omega_c}{\omega}\right)^2-\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2-\frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(qz)\right\}\left(2\frac{\partial A_0^0}{\partial x}\frac{\partial S}{\partial x}+A_0^0\frac{\partial^2 S}{\partial x^2}\right)\\ +\frac{\omega A_0^0}{c}\left(\frac{\omega_{p2}}{\omega}\right)^2\sin(qz)\left(\frac{\partial S}{\partial x}\right)^2\end{array}\right]$$

$$-\frac{\omega A_0^0 q}{c\left(1-\frac{\omega_c}{\omega}\right)^4}\left(\frac{\omega_{p2}}{\omega}\right)^2\left[\begin{array}{l}\sin(qz)\left\{\left(1-\frac{\omega_c}{\omega}\right)^2+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2+\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(qz)\right\}-\\ \frac{z\omega}{c}\left(1-\frac{\omega_c}{\omega}\right)^2\cos(qz)+qz\cos(qz)\left\{\left(1-\frac{\omega_c}{\omega}\right)^2+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2\right\}+\\ qz\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(2qz)\left\{1+\frac{3}{4}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}\\ -\frac{qz^2\omega}{2c}\left(\frac{\omega_{p2}}{\omega}\right)^2\times\left\{\begin{array}{l}\sin(2qz)+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\sin(2qz)\\ +\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\sin(3qz)\end{array}\right\}\\ +\frac{3qz}{4}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(3qz)\end{array}\right]=0$$

(8.12)

Further, $\left(A_0^0\right)^2=(E_{00}^2/f^2)\exp(-x^2/r_0^2 f^2)$, r_0 is the spot size of laser beam and $S=\left(x^2/2\right)\beta(z)+\varphi(z)$, where, $\beta(z)=2\left(1+\varepsilon_0/\varepsilon_{0zz}\right)^{-1}\left(1/f(z)\right)\left(\partial f/\partial z\right)$ and β^{-1} may be regarded as curvature radius of laser beam, φ is a constant, independent of x and $f(z)$ represents the beam width parameter. On substituting these values in Eq. (8.11) and after equating the coefficients of x^2 on both sides of the resulting equation, the expression for beam width parameter is obtained as follows:

$$-\left(\frac{\omega_{p2}}{\omega}\right)^2\left(\frac{1}{f}\right)^2\sqrt{1-\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}-\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(\xi d)}\times$$

$$\left[2\sin(\xi d)+\xi d\left\{\cos(\xi d)+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}+\frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(\xi d)\right\}\right]\left(\frac{\partial f}{\partial \xi}\right)$$

$$+\frac{\rho_0^2\alpha_0^2}{2f^3}\left(1-\frac{\omega_c}{\omega}\right)^{-2}\left[\left(\frac{\omega_{p1}}{\omega}\right)^2+\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(\xi d)\right]\left[\begin{array}{l}1-\left(\frac{\omega_{p1}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}- \\ \left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(\xi d)\end{array}\right]$$

$$+\left[\frac{\partial^2 f}{\partial \xi^2}-\frac{1}{f}\left(\frac{\partial f}{\partial \xi}\right)^2\right]\left[\frac{1-\left(\frac{\omega_{p1}}{\omega}\right)^2-\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(\xi d)}{2-\left(\frac{\omega_{p1}}{\omega}\right)^2\left\{\left(1-\frac{\omega_c}{\omega}\right)^{-2}+1\right\}-\left(\frac{\omega_{p2}}{\omega}\right)^2\left\{1+\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}\cos(\xi d)}\right]\times$$

$$\left[\begin{array}{l}2\left(1-\frac{\omega_c}{\omega}\right)^2-\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}-2\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(\xi d)-\left(\frac{\omega_{p1}}{\omega}\right)^2\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(\xi d)- \\ \frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(2\xi d)+\frac{\xi d\sin(\xi d)}{4}\left(\frac{\omega_{p2}}{\omega}\right)^2\left\{\begin{array}{l}4+4\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-2}\cos(\xi d)- \\ \left(\frac{\omega_{p2}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-4}-\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^4\left(1-\frac{\omega_c}{\omega}\right)^{-4} \\ -2\left(\frac{\omega_{p1}}{\omega}\right)^2\left(\frac{\omega_{p2}}{\omega}\right)^2\left(1-\frac{\omega_c}{\omega}\right)^{-4}\cos(\xi d)\end{array}\right\}\end{array}\right]$$

$$+\left(\frac{4}{f}\right)\left(\frac{\partial f}{\partial \xi}\right)^2\left[\left(1-\frac{\omega_c}{\omega}\right)^{-2}-\left(\frac{\omega_{p1}}{\omega}\right)^2-\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(\xi d)\right]\times$$

$$\left[\frac{1-\left(\frac{\omega_{p1}}{\omega}\right)^2-\left(\frac{\omega_{p2}}{\omega}\right)^2\cos(\xi d)}{2-\left(\frac{\omega_{p1}}{\omega}\right)^2\left\{\left(1-\frac{\omega_c}{\omega}\right)^{-2}+1\right\}-\left(\frac{\omega_{p2}}{\omega}\right)^2\left\{1+\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}\cos(\xi d)}\right]$$

$$
+\left(\frac{d}{2}\right)\left(\frac{\omega_{p2}}{\omega}\right)^{2}\left(\frac{\partial f}{\partial \xi}\right)\left[\frac{1-\left(\frac{\omega_{p1}}{\omega}\right)^{2}-\left(\frac{\omega_{p2}}{\omega}\right)^{2}\cos(\xi d)}{2-\left(\frac{\omega_{p1}}{\omega}\right)^{2}\left\{\left(1-\frac{\omega_c}{\omega}\right)^{-2}+1\right\}-\left(\frac{\omega_{p2}}{\omega}\right)^{2}\left\{1+\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}\cos(\xi d)}\right]\times
$$

$$
\left[\begin{array}{l}
\sin(\xi d)\left\{\begin{array}{l}2+2\left(\frac{\omega_{p1}}{\omega}\right)^{2}\left(1-\frac{\omega_c}{\omega}\right)^{-2}+\frac{1}{2}\left(\frac{\omega_{p1}}{\omega}\right)^{4}\left(1-\frac{\omega_c}{\omega}\right)^{-4}-\frac{1}{2}\left(\frac{\omega_{p2}}{\omega}\right)^{-4}\left(1-\frac{\omega_c}{\omega}\right)^{-4}+\\ \left(\frac{\omega_{p2}}{\omega}\right)^{-4}\left(1-\frac{\omega_c}{\omega}\right)^{-4}\sin^{2}(\xi d)\end{array}\right\}\\
-\left(\xi d\cos(2\xi d)\right)\left(\frac{\omega_{p2}}{\omega}\right)^{2}\left(1-\frac{\omega_c}{\omega}\right)^{-6}\left\{\begin{array}{l}\left(1-\frac{\omega_c}{\omega}\right)^{4}+\left(\frac{\omega_{p1}}{\omega}\right)^{2}\left(1-\frac{\omega_c}{\omega}\right)^{2}+\frac{1}{4}\left(\frac{\omega_{p1}}{\omega}\right)^{4}+\\ \frac{\cos(\xi d)}{2}\left(\frac{\omega_{p2}}{\omega}\right)^{2}\left(1-\frac{\omega_c}{\omega}\right)^{2}+\frac{\cos(\xi d)}{4}\left(\frac{\omega_{p2}}{\omega}\right)^{2}\left(\frac{\omega_{p1}}{\omega}\right)^{2}\\ +\frac{\sec(2\xi d)}{2}\left(\frac{\omega_{p2}}{\omega}\right)^{4}-\frac{3\cos^{2}(\xi d)}{4\sec(2\xi d)}\left(\frac{\omega_{p2}}{\omega}\right)^{4}+\frac{\cos^{4}(\xi d)}{2\sec(2\xi d)}\left(\frac{\omega_{p2}}{\omega}\right)^{4}\end{array}\right\}
\end{array}\right]
$$

$$
+\left(\frac{1}{2f^{3}}\right)\left[\left(1-\frac{\omega_c}{\omega}\right)^{-2}-\left(\frac{\omega_{p1}}{\omega}\right)^{2}-\left(\frac{\omega_{p2}}{\omega}\right)^{2}\cos(\xi d)\right]\times
$$

$$
\left[\frac{2-\left(\frac{\omega_{p1}}{\omega}\right)^{2}\left\{1+\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}-\left(\frac{\omega_{p2}}{\omega}\right)^{2}\left\{1+\left(1-\frac{\omega_c}{\omega}\right)^{-2}\right\}\cos(\xi d)}{1-\left(\frac{\omega_{p1}}{\omega}\right)^{2}-\left(\frac{\omega_{p2}}{\omega}\right)^{2}\cos(\xi d)}\right]=0, \tag{8.13}
$$

where, $\rho_0 = r_0\omega/c$ is the equilibrium beam radius, $\alpha_0^2 = e^2E_{00}^2/m^2c^2\omega^2$ represents the initial laser beam intensity (laser strength) parameter, $\xi = z/R_d$ is propagation distance, $R_d = kr_0^2$ represents the diffraction length and $d = qR_d$ represents the normalized ripple wave number. Eq. (8.13) represents the spot size variation of laser beam with the propagation distance.

8.4 RESULTS AND DISCUSSION

Eq. (8.13) represents a nonlinear differential equation which governs the behavior of f with distance ξ in rippled density magnetoplasma. We solved Eq. (8.13) numerically by applying the initial condition at $\xi = 0$, $f = 1$, $(\partial f / \partial \xi) = 0$ and $(\partial^2 f / \partial \xi^2) = 0$ with the typical parameters given as [94]; angular frequency of laser $\omega = 1.778 \times 10^{14} rad/\sec$, laser beam spot size $r_0 = 40 \mu m$ and equilibrium plasma density $n_0^0 = 2.55 \times 10^{18}$ cm^{-3}. By optimizing laser and plasma parameters, we have investigated laser beam dynamics in rippled density magnetoplasma.

Aggarwal *et al*. [94] claimed that the self-focusing of laser beam in rippled density plasma occurs at large ξ values ($\xi = 2.25$ and $\xi = 1.75$). Further, their results reveal that there is a direct dependence of decentered parameter on self-focusing. But, in the present communication, the magnetic field of a few MG gives rise to strong self-focusing in rippled density plasma. Further, the self-focusing occurs earlier at $\xi = 0.12$ as depicted in figure 8.1 which shows the behavior of f with ξ for different values of laser strength parameter. The other parameters are: $\omega_{p1} / \omega = 0.4$, $\omega_{p2} / \omega = 0.15$, $\omega_c / \omega = 0.12$ ($B_0 = 12.19 MG$) and $d = 59$ as taken by Lin *et al*. [134]. It is observed that as the laser strength parameter is increased, the nonlinear term dominates the diffraction term and self-focusing occurs earlier at $\xi = 0.12$. The laser spot size gets reduced which in turn decreases the beam width parameter and hence laser beam is self-focused. Figure 8.2 illustrates the variation of f with ξ for different values of λ; $\lambda = 0.5 \mu m$, $1.06 \mu m$ and $1.5 \mu m$. The laser strength parameter is kept fixed at $\alpha_0 = 0.3$ and the other parameters are same as taken in Figure 8.1. It can be seen from figure 8.2 that the beam width parameter acquires minimum at a very short propagation distance (ξ = 0.24) corresponding to $\lambda = 0.5 \mu m$. Therefore, stronger and earlier self-focusing is achieved for $\lambda = 0.5 \mu m$ than $1.06 \mu m$ and $1.5 \mu m$. Hence, the selection of laser wavelength is also important in achieving earlier focusing of the laser beam. Figure 8.3 depicts the behavior of f with ξ for different values of ω_{p1} / ω. The laser strength parameter is fixed at $\alpha_0 = 0.3$ and the other parameters are same as taken in figure 8.1. It is found that as the electron density increases, the plasma dielectric constant decreases with the result, the amplitude of laser spot size gets reduced close to the axis of propagation. Consequently, the minimum value of f shifts towards lower value of ξ and the beam focuses in high plasma density region.

Figure 8.4 depicts the behavior of f with ξ for various values of ω_{p2}/ω = 0.15, 0.25 & 0.3 with $\alpha_0 = 0.2$. The other parameters are same as has been taken in Figure 8.1. It is clear from figure 8.4 that as ω_{p2}/ω increases, self-focusing becomes stronger. It is due to the fact that as ω_{p2}/ω increases, the medium acquires a saturating nonlinearity which limits the energy associated with the electrons of the laser beam. Therefore, the beam width parameter reaches to its minimum value for ω_{p2}/ω = 0.3 at ξ = 0.18. Hence, one would expect the efficient self-focusing because of suitable wavelength of electron density ripple is present in the plasma. Therefore, the outcomes of present analysis are in a good agreement with those of Kaur and Sharma [68]. Where in the effect of density ripple is to decrease the minimum spot size of the laser beam. Figure 8.5 depicts the behavior of f with ξ for various values of magnetic field; $\omega_c/\omega = 0.01$, 0.04, 0.08 & 0.12 (corresponding values of magnetic field are $B_0 = 1.01MG$, $B_0 = 4.06MG$, $B_0 = 8.12MG$ and $B_0 = 12.19MG$ respectively). The other remaining parameters have been taken same as in Figure 8.1. The figure shows that it is the magnetic field which has a serious influence on the behavior of beam width parameter. As the magnetic field increases, the nonlinear term begins to control over the diffractional divergence term. It further, changes the propagation characteristics of the medium. Due, to strong interaction between the laser field and magnetic field, the laser beam is more focused. Thus, magnetic field has a significant role in enhancing the self-focusing of laser beam in rippled density plasma. Figure 8.6 illustrates the behavior of f with ξ for various values of normalized ripple wave number; d = 50, 65 & 75 with $\alpha_0 = 0.3$. The other parameters are: $d = 59$, $\omega_{p1}/\omega = 0.4$, $\omega_{p2}/\omega = 0.15$ and $\omega_c/\omega = 0.12$. It is interesting to note that the beam width parameter shows a different kind of behavior. As we increase the normalized ripple wave number, the beam width parameter decreases gradually at ξ = 0.12. Further, due to large ripple wave number, the wavefront curvature continues to focus the laser beam inside the magnetoplasma. Thus, one can say that the self-focusing strength of laser beam increases in rippled density magnetoplasma. The present analysis results may be useful in laser driven fusion and plasma based accelerators.

8.5 CONCLUSION

In the present communication, we investigated the self-focusing of laser beam in rippled density magnetoplasma. Effect of magnetic field and normalized ripple wave number on self-focusing of a laser beam has been analyzed at various optimized parameters. The differential equation for the beam width parameter has been obtained and the results have been plotted and have been discussed. The results revealed that the magnetic field of a few *MG* increases the self-focusing capacity of laser beam strongly in rippled density plasma. Further, there is a strong coupling between the magnetic field and laser field. Due to the presence of suitable wavelength of density ripple in plasma, stronger and earlier self-focusing is achieved. The outcomes obtained are expected to be useful in laser driven fusion and plasma based accelerators.

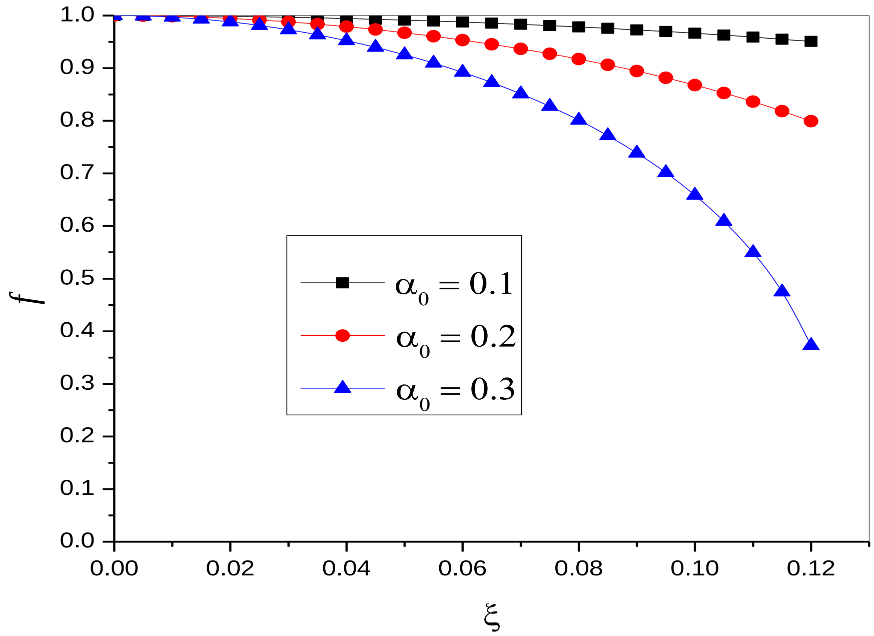

Figure 8.1: Dependence of f on ξ for various values of α_0. The other parameters are: $d = 59$, $\omega_{p1}/\omega = 0.4$, $\omega_{p2}/\omega = 0.15$ and $\omega_c/\omega = 0.12$.

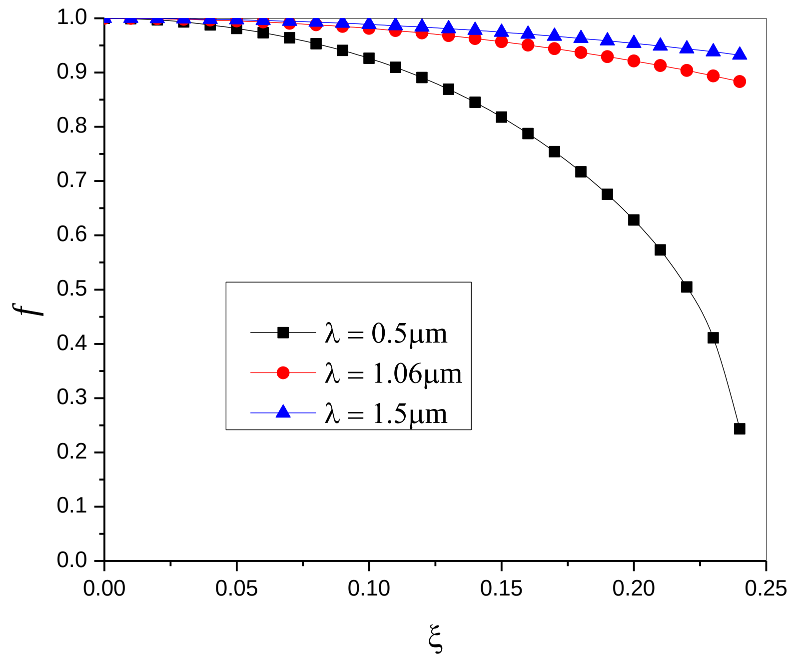

Figure 8.2: Dependence of f on ξ for various values of λ. The other parameters are: $d = 59$, $\omega_{p1}/\omega = 0.4$, $\omega_{p2}/\omega = 0.15$, $\omega_c/\omega = 0.12$ and $\alpha_0 = 0.3$.

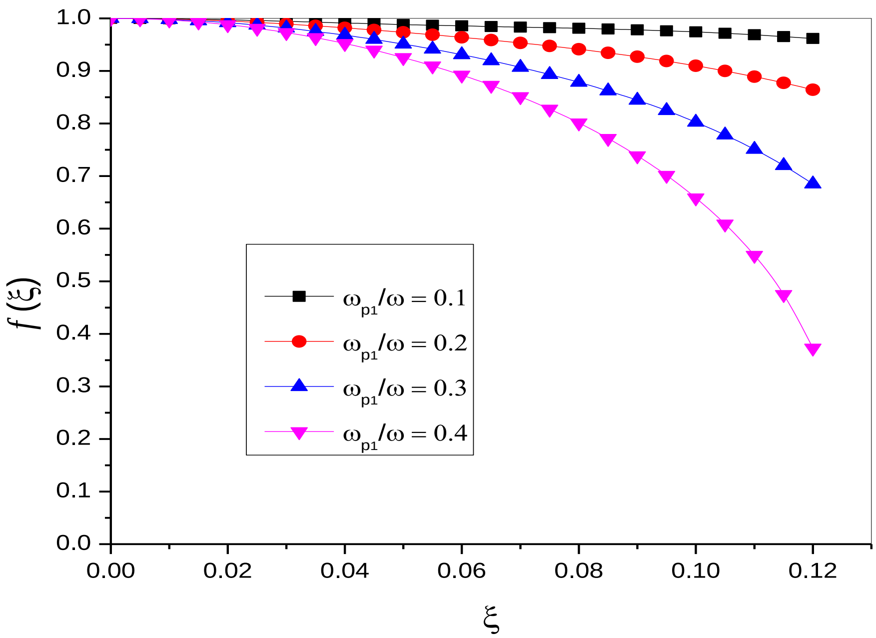

Figure 8.3: Dependence of f on ξ for various values of ω_{p1}/ω. The other parameters are: d = 59, $\omega_c/\omega = 0.12$, $\omega_{p2}/\omega = 0.15$ and $\alpha_0 = 0.3$

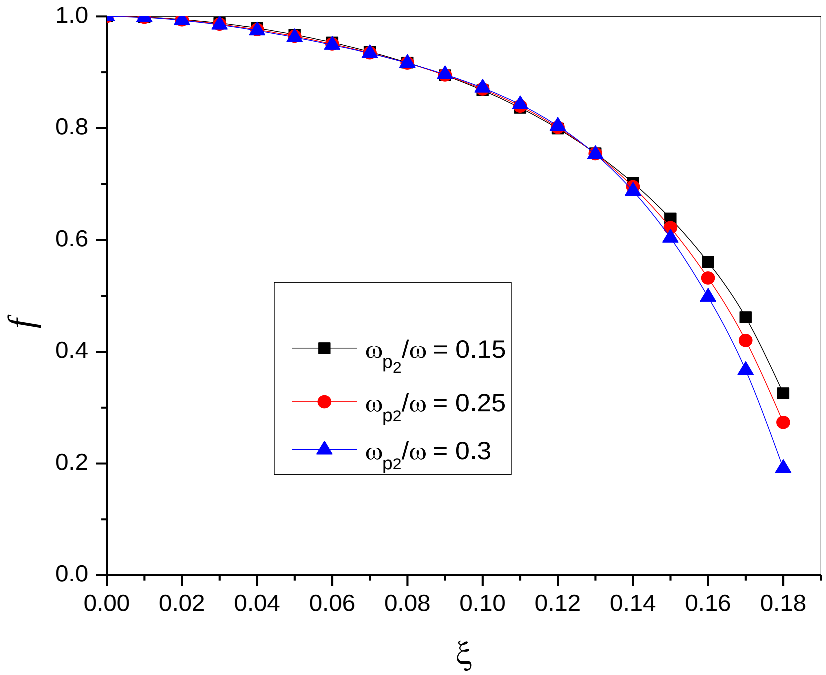

Figure 8.4: Dependence of f on ξ for various values of ω_{p2}/ω. The other parameters are: d = 59, $\omega_c/\omega = 0.12$, $\omega_{p1}/\omega = 0.4$ and $\alpha_0 = 0.2$

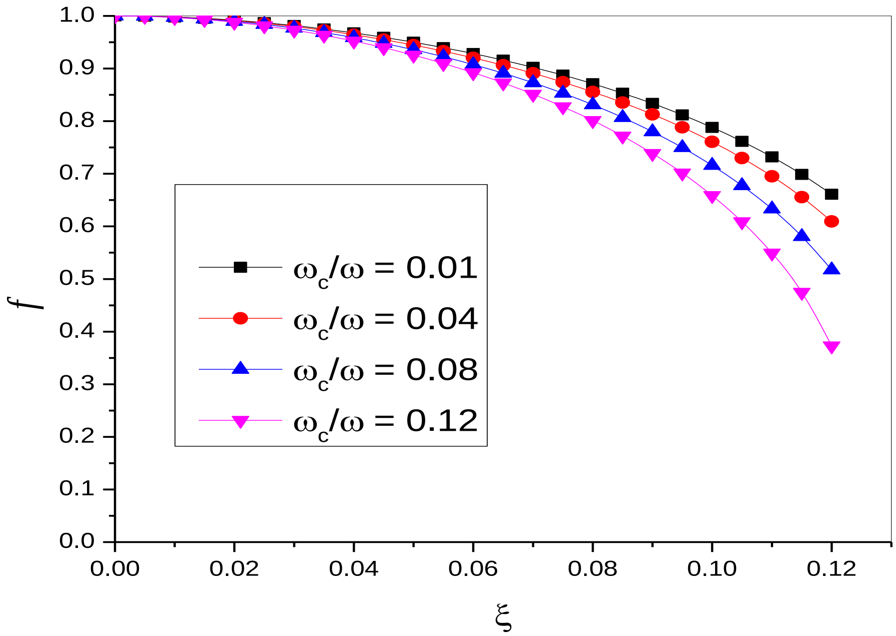

Figure 8.5: Dependence of f on ξ for various values of ω_c/ω. The other parameters are: d = 59, $\omega_{p1}/\omega = 0.4$, $\omega_{p2}/\omega = 0.15$ and $\alpha_0 = 0.3$

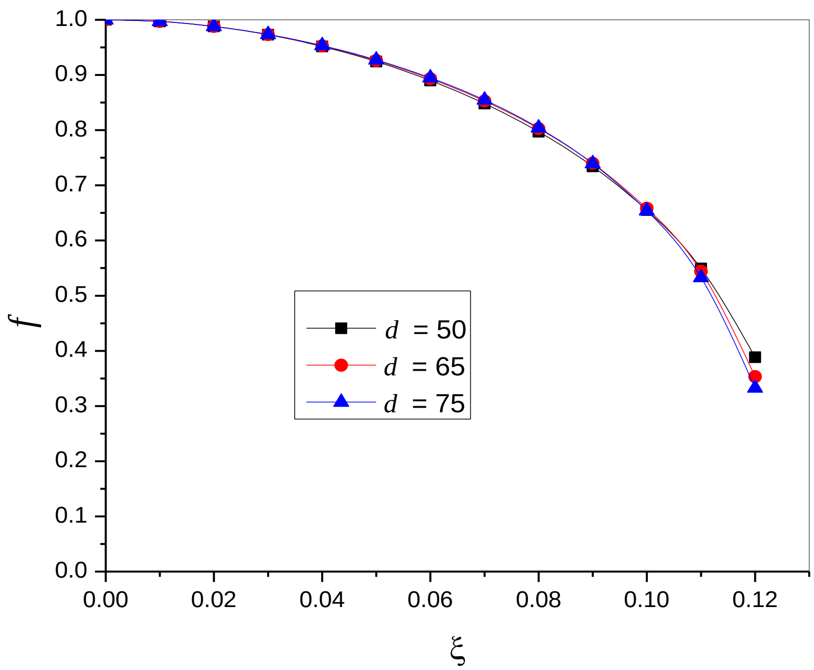

Figure 8.6: Dependence of f on ξ for various values of d. The other parameters are: $\omega_{p1}/\omega =$ 0.4, $\omega_{p2}/\omega = 0.15$, $\alpha_0 = 0.3$ and $\omega_c/\omega = 0.12$