

## CHAPTER-7

### SELF-FOCUSING / DEFOCUSING OF CHIRPED GAUSSIAN LASER BEAM IN COLLISIONAL PLASMA WITH LINEAR ABSORPTION

#### 7.1 INTRODUCTION

The interaction of high power laser beams with plasmas has known to be an important and captivating field of research because of its applications in charged particle accelerators, x-ray lasers etc. [8, 115, 4]. In all these applications, it is necessary to know the characteristics of a high power beam that propagates over extended distances with no loss of energy. When such a beam interacts with the plasma, various nonlinear phenomena's (self-focusing, harmonic generation, electron acceleration in vacuum etc.) are likely to occur. Among, these phenomena's self-focusing is very important nonlinear phenomenon in which the wave front of laser acquires a curvature and laser tends to focus. In general, there are two types of self-focusing viz., relativistic self-focusing [121] and ponderomotive self-focusing [42] and many papers have been published in achieving the self-focusing of laser beams in plasmas [28, 113, 77, 64, 85]. The self-focusing decreases with rising intensity of the beam due to supremacy of diffraction effect at higher intensity [84]. Gill *et al.* [81] used the higher order paraxial theory to study the relativistic self-focusing of super Gaussian laser beam in plasma and reported that the inclusion of higher order terms of dielectric function affects the behavior of beam width parameter significantly and the magnetic field improves the self-focusing of laser beam in plasma [101, 124]. Recently, Habibi and Ghamari [98] have extended the same theory for cosh-Gaussian beam in quantum plasma. By using more effective decentered parameter, better self-focusing is observed for cosh-Gaussian beams in comparison to Gaussian beams in quantum plasma.

The density transition is considered to be important in laser and plasma interactions, particularly for the self-focusing in plasma medium. Increase in initial density and ramp slope lowers the minimum spot size of the laser beam. These parameters play a key role and are crucial for self-focusing as it is enhanced with optimized laser and plasma parameters [22, 92, 97, 102, 91]. Kant and Wani [118] studied the self-focusing under density transition with linear absorption. Due to absorption, the self-focusing effect weakens and density transition sets sooner and an earlier self-

focusing. Gupta *et al.* [87] considered the relativistic ponderomotive nonlinearity and their results have shown that the ion temperature causes thermal self-focusing and has a serious influence on laser beam evolution in collisional plasma. Taking in to account the higher order axial electron temperature, it has been reported that it decreases the influence of collisional nonlinearity. It further changes the electron density distribution and increases the dielectric constant, with the result, it leads to fast divergence of the beam [123]. The chirp was used to study the electron acceleration in vacuum. It increases the electron energy and hence momentum so that the electron escapes from the laser beam. The value of chirp parameter decreases with laser intensity and initial electron energy. It further, increases the amplitude of wake wave that has been generated in the plasma by an electromagnetic beam [125, 126]. Ghotra and Kant [127] used the chirped laser pulse to study the electron acceleration in vacuum in presence of azimuthal magnetic field. The chirp increases the duration of interaction of laser beam with electron and strengths the resonance for longer duration. Further, the magnetic field improves the electron acceleration to high energy of the order of GeV.

In the present communication, we analyzed the effect of chirp on the self-focusing / defocusing of Gaussian beam propagating in collisional plasma with linear absorption. Effects of chirp parameter, collision frequency and other laser plasma parameters are seen on the self-focusing / defocusing in plasma. Although, without chirp, the laser beam shows self-focusing but, as the propagation distance increases, it starts to experience defocusing. To reduce this defocusing, the effect of chirp is considered. The chirp parameter minimizes the defocusing and increases the ability of self-focusing. Further, the amplitude of oscillations decreases with the propagation distance so that sooner and stronger self-focusing is achieved. This paper is constituted as follows: in section 7.2 the nonlinear dielectric constant and the equation that governs the behavior of beam width parameter with the propagation distance is presented. Section 7.3 is devoted to results and discussions. Finally, the conclusion is considered in last section 7.4.

## **7.2 SELF FOCUSING OF CHIRPED GAUSSIAN LASER BEAM**

Consider the propagation of a Gaussian beam in plasma along the  $z$  - axis. Its initial intensity distribution is given by

$$EE^* = E_0^2 \exp\left(-\frac{r^2}{r_0^2}\right), \quad (7.1)$$

where,  $\vec{E}$  is the electric vector and  $r_0$  is the waist width of the beam. The wave equation that governs the laser beam propagation may be written as

$$\nabla^2 \vec{E} + \left(\frac{\omega^2}{c^2}\right) \varepsilon \vec{E} + \nabla \left(\frac{\vec{E} \cdot \nabla \varepsilon}{\varepsilon}\right) = 0. \quad (7.2)$$

The last term on the left hand side of above equation (7.2) is neglected under the condition  $k^{-2} \nabla^2 (\ln \varepsilon) \ll 1$ , where,  $k$  represents the wave number of the laser beam. Thus,

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2} (\varepsilon \cdot \vec{E}) = 0.$$

Or, in cylindrical co-ordinate system, we can write this equation as

$$\frac{\partial^2 \vec{E}}{\partial z^2} + \frac{\partial^2 \vec{E}}{\partial r^2} + \frac{1}{r} \frac{\partial \vec{E}}{\partial r} + \varepsilon \frac{\omega^2}{c^2} \vec{E} = 0 \quad (7.3)$$

The effective dielectric constant of the plasma can be expressed as

$$\varepsilon = \varepsilon_0 + \phi(EE^*) - i\varepsilon_i, \quad (7.4)$$

where,  $\varepsilon_0 = 1 - \omega_p^2 / \omega^2$  is a linear part and  $\phi$  is a nonlinear part of dielectric constant,  $\varepsilon_i = (\omega_p^2 / \omega^2)(\nu / \omega)$  takes care of linear absorption ( $\varepsilon_i \ll \varepsilon_0$ ),  $\nu$  is the collision frequency,  $\omega = \omega_0(1 + b(\omega_0 t - \omega_0 z / c))$  is the angular frequency of chirped Gaussian laser beam,  $\omega_0$  is the angular frequency of incident laser beam,  $b$  is the chirp parameter,  $c$  is the velocity of light,  $\omega_p$  is the plasma frequency given by  $\omega_p^2 = 4\pi n_0 e^2 / m$ , where,  $m$  is the rest mass of electron,  $e$  is the charge of electron and  $n_0$  is the equilibrium electron density. Following [114],  $\phi(EE^*)$  can be expressed as:

$$\phi(EE^*) = \frac{\omega_p^2}{\omega^2} \left[ 1 - \left( 1 + \frac{\alpha}{2} EE^* \right)^{(s_0/2)-1} \right], \quad (7.5)$$

where,  $s_0$  is a parameter characterizing the nature of collisions,  $\alpha = e^2 M / 6m^2 \omega^2 k_B T$ ,  $M$  is the mass of scatterer in the plasma,  $T$  is the equilibrium plasma temperature and  $k_B$  is the Boltzmann constant. Now, the solution of equation (7.3) is of the following form,

$$\bar{E} = A(r, z, t) \text{Exp}[i(\omega t - kz)] \quad (7.6)$$

With  $k^2 = \epsilon_0 \omega^2 / c^2 = \omega_0^2 / c^2 (1 + b(\omega_0 t - \omega_0 z / c))^2 [1 - (\omega_p^2 / \omega_0^2)(1 + b(\omega_0 t - \omega_0 z / c))^{-2}]$ , where,  $A(r, z)$  is the complex amplitude. Differentiating equation (7.6) twice w. r. t. 'r' and 'z', we get

$$\frac{\partial \bar{E}}{\partial r} = \text{Exp}[i(\omega t - kz)] \frac{\partial A(r, z, t)}{\partial r}$$

$$\frac{\partial^2 \bar{E}}{\partial r^2} = \text{Exp}[i(\omega t - kz)] \frac{\partial^2 A(r, z, t)}{\partial r^2}$$

And

$$\frac{\partial \bar{E}}{\partial z} = \text{Exp}[i(\omega t - kz)] \left[ \frac{\partial A}{\partial z} + \frac{iAz b \omega_p^2}{c^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{-1/2} \right]$$

$$- \frac{iA\omega_0}{c} \text{Exp}[i(\omega t - kz)] \left[ b\omega_0 t + \left(1 + b\omega_0 t - \frac{2b\omega_0 z}{c}\right) \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} \right]$$

$$\frac{\partial^2 \bar{E}}{\partial z^2} = \frac{2iAb\omega_0^2}{c^2} \text{Exp}[i(\omega t - kz)] \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} + \frac{iAb\omega_0^2}{c^2} \frac{\omega_p^2}{\omega_0^2} \text{Exp}[i(\omega t - kz)] \times$$

$$\left[ \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{-1/2} \right] \left[ \begin{array}{l} 2 + \frac{b\omega_0 z}{c} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-1} \\ + \frac{b\omega_0 z}{c} \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-3} \times \\ \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{-1} \end{array} \right]$$

$$\begin{aligned}
& -\frac{2ib\omega_0}{c} \frac{\partial A}{\partial z} \text{Exp}[i(\omega t - kz)] \left[ \omega_0 t + \frac{z\omega_p^2}{c\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{-1/2} \right] \\
& -\frac{2i\omega_0}{c} \frac{\partial A}{\partial z} \text{Exp}[i(\omega t - kz)] \left(1 + b\omega_0 t - \frac{2b\omega_0 z}{c}\right) \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} \\
& -A \text{Exp}[i(\omega t - kz)] \left[ \frac{b^2\omega_0^4 t^2}{c^2} + \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\} \left\{ \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^2 \right. \\
& \quad \left. - \frac{b\omega_0^2 z}{c^2} \right] \\
& -\frac{Ab^2\omega_0^4 z^2}{c^4} \frac{\omega_p^4}{\omega_0^4} \text{Exp}[i(\omega t - kz)] \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-4} \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{-1} \\
& -\frac{2Ab\omega_0^2 t}{c} \text{Exp}[i(\omega t - kz)] \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} \left\{ \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right) - \frac{b\omega_0^2 z}{c^2} \right\} \\
& +\frac{2Abz\omega_0^2}{c^2} \frac{\omega_p^2}{\omega_0^2} \text{Exp}[i(\omega t - kz)] \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \left\{ \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right) - \frac{b\omega_0^2 z}{c^2} \right\} \\
& +\frac{2Ab^2 tz\omega_0^4}{c^3} \frac{\omega_p^2}{\omega_0^2} \text{Exp}[i(\omega t - kz)] \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{-1/2}
\end{aligned}$$

Now, substituting the above values in Eq. (7.3) and employing the WKB approximation, we get

$$-2ik \frac{\partial A}{\partial z} \left[ b\tau(1+b\tau)\sqrt{\varepsilon_0} + (1+b\tau)^2 \varepsilon_0 + \frac{bzk(\sqrt{\varepsilon_0}-1)}{\sqrt{\varepsilon_0}} \right] + \left( \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} \right) (1+b\tau)^2 \varepsilon_0 +$$

$$k^2(1+b\tau)^2 [\varepsilon_0 + \phi(EE^*) - i\varepsilon_i] A + ik^2 A \left( 2b\sqrt{\varepsilon_0} + \frac{b^2 kz(1-\varepsilon_0)(1+b\tau)^{-2}}{\varepsilon_0} \right) -$$

$$A \left[ b\tau + \frac{\varepsilon_0}{k} \frac{\omega_p^2}{\omega_0^2} (1+b\tau)^2 + \frac{bzk(\varepsilon_0 - \sqrt{\varepsilon_0})}{(1+b\tau)\varepsilon_0^{3/2}} \right]^2 = 0. \quad (7.7)$$

To solve Eq. (7.7), we express  $A$  as

$$A = A_0(r, z) \exp(-ikS), \quad (7.8)$$

where,  $A_0$  and  $S$  depend on  $r$  and  $z$ . Now, differentiating Eq. (7.8) twice w. r. t  $r$  and  $z$ , we get

$$\begin{aligned} \frac{\partial A}{\partial r} &= \text{Exp}[-ikS] \left[ \frac{\partial A_0}{\partial r} - \frac{i\omega_0 A_0}{c} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right) \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} \left(\frac{\partial S}{\partial r}\right) \right] \\ \frac{\partial^2 A}{\partial r^2} &= \text{Exp}[-ikS] \left[ \frac{\partial^2 A_0}{\partial r^2} - \frac{\omega_0^2 A_0}{c^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^2 \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right) \right\} \left(\frac{\partial S}{\partial r}\right)^2 \right] \\ &\quad - \frac{i\omega_0}{c} \text{Exp}[-ikS] \left[ \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right) \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} \left(\frac{\partial S}{\partial r}\right)^2 \right] \times \\ &\quad \left[ A_0 \frac{\partial^2 S}{\partial r^2} + 2 \frac{\partial S}{\partial r} \frac{\partial A_0}{\partial r} \right] \\ \frac{\partial A}{\partial z} &= \text{Exp}[-ikS] \left[ \frac{\partial A_0}{\partial z} + \frac{i\omega_0 A_0}{c} \left\{ \frac{bS\omega_0}{c} + \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right) \frac{\partial S}{\partial z} \right\} \left\{ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right\}^{1/2} \right] \\ &\quad + \frac{ibS\omega_0^2}{c^2} \frac{\omega_p^2}{\omega_0^2} \text{Exp}[-ikS] \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \left[ 1 - \frac{\omega_p^2}{\omega_0^2} \left(1 + b\omega_0 t - \frac{b\omega_0 z}{c}\right)^{-2} \right]^{-1/2}, \end{aligned}$$

Substituting Eq. (7.8) and all the above values in Eq. (7.7) and after separating real and imaginary parts, one can obtain

Real part equation is

$$\begin{aligned} \frac{\varepsilon_0}{k^2} \left( \frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r} \frac{\partial A_0}{\partial r} \right) - A_0 \left[ 1 - \frac{\omega_p^2}{\omega_0^2} (1 + b\tau) \right] \left( \frac{\partial S}{\partial r} \right)^2 - A_0 (1 - \varepsilon_0) \left[ 1 - \left( 1 + \frac{\alpha}{2} EE^* \right)^{(s_0/2)-1} \right] + \\ \frac{2A_0 bS}{(1 + b\tau)} \left[ \frac{\{k + (1 + b\tau)\sqrt{\varepsilon_0} - (1 + b\tau)\varepsilon_0^{3/2}\}}{(1 + b\tau)\sqrt{\varepsilon_0}} + b \left( \tau + \frac{zk}{(1 + b\tau)\sqrt{\varepsilon_0}} \right) \left( \frac{k}{(1 + b\tau)^2 \varepsilon_0} + \frac{1 - \varepsilon_0}{\sqrt{(1 + b\tau)^2 + \varepsilon_0 - 1}} \right) \right] \end{aligned}$$

$$\begin{aligned}
& -A_0 \left[ \frac{b^2 \tau^2}{(1+b\tau)^2} - \frac{\omega_p^6 \varepsilon_0}{\omega_0^6 k^2} + \frac{b^2 z^2 k^2 (3\sqrt{\varepsilon_0} - \varepsilon_0^{3/2} - 2)}{(1+b\tau)^4 \varepsilon_0^{3/2}} + \frac{2b^2 z \tau k (\sqrt{\varepsilon_0} - 1)}{(1+b\tau)^3 \varepsilon_0} + \frac{2b\varepsilon_0 \omega_p^2}{k \omega_0^2} \left( \tau + \frac{zk}{(1+b\tau)\sqrt{\varepsilon_0}} \right) \right] + \\
& \left[ \frac{(1+b\tau)\varepsilon_0}{k^2} \left( \frac{\omega_p^4}{\omega_0^4} (1+b\tau) - \frac{2bz\omega_p^2}{\omega_0^2} - \frac{k^2}{(1+b\tau)\varepsilon_0} \right) + \frac{b^2 z^2 (1-\varepsilon_0)^2}{(1+b\tau)^2 \varepsilon_0} \right] \\
& + 2A_0 \frac{\partial S}{\partial z} \left[ \varepsilon_0 + \frac{b\tau\sqrt{\varepsilon_0}}{1+b\tau} - \frac{bz(1-\varepsilon_0)(1-\varepsilon_0^3)}{(1+b\tau)\varepsilon_0^3} - \frac{bzk(1-\sqrt{\varepsilon_0})}{(1+b\tau)^2 \sqrt{\varepsilon_0}} \right] - \frac{2A_0 b^2 S z k^2 (2-\varepsilon_0)^2}{(1+b\tau)^4 \varepsilon_0^2} = 0
\end{aligned} \tag{7.9}$$

Imaginary part equation is

$$\begin{aligned}
& A_0^2 \left( \frac{\partial^2 S}{\partial r^2} + \frac{1}{r} \frac{\partial S}{\partial r} \right) + \frac{\partial S}{\partial r} \frac{\partial A_0^2}{\partial r} + \frac{\partial A_0^2}{\partial z} \left[ 1 + \frac{b\tau}{(1+b\tau)\sqrt{\varepsilon_0}} + \frac{bzk(1-\sqrt{\varepsilon_0})}{(1+b\tau)^2 \varepsilon_0} - \frac{bz(1-\varepsilon_0)}{k(1+b\tau)^2 \sqrt{\varepsilon_0}} \right] \\
& - \frac{bkA_0^2}{\varepsilon_0^{3/2} (1+b\tau)^2} \left[ 2 + \frac{bz(1-\varepsilon_0)}{(1+b\tau)\varepsilon_0} \right] - \frac{kA_0^2 \varepsilon_i}{\varepsilon_0} = 0.
\end{aligned} \tag{7.10}$$

Where,  $\tau$  is the dimensionless retarded time. Following Akhmanov et al. [113] and Sodha et al. [20, 114], we can write as follows

$$A_0^2 = \frac{E_0^2}{f^2} \exp \left[ -\frac{r^2}{r_0^2 f^2} \right] \exp(-2k_i z), \tag{7.11}$$

$$S = \frac{r^2}{2} \beta(z) + \varphi(z), \tag{7.12}$$

where,  $k_i = k\varepsilon_i / 2\varepsilon_0$  is the absorption coefficient with  $k = \omega\varepsilon_0^{1/2} / c$  and  $\beta(z) = (1/f)(\partial f / \partial z)$ ,  $\beta^{-1}$  is interrupted as curvature radius and  $f(z)$  represents the beam width parameter. Now, using paraxial approximation and substituting Eq. (7.11) and Eq. (7.12) in Eq. (7.9) and then equate the coefficients of  $r^2$  on both sides of the emerging equation, the differential equation for  $f(z)$  is obtained as:

$$\begin{aligned}
& \frac{\partial^2 f}{\partial \xi^2} \left[ 1 + \frac{b\tau}{(1+b\tau)\sqrt{\varepsilon_0}} + \frac{b\xi\rho_0^2}{\varepsilon_0^{7/2}} (\varepsilon_0^{7/2} - \varepsilon_0^4 - \varepsilon_0 - 1) \right] + \frac{b\rho_0^2}{\varepsilon_0} \left( \frac{\partial f}{\partial \xi} \right) \times \\
& \left[ \frac{\sqrt{\varepsilon_0}(1+b\tau) + b\tau}{1+b\tau} + (\sqrt{\varepsilon_0} + b\tau)(1 - \varepsilon_0) + b\xi\rho_0^2 (4\varepsilon_0 - \varepsilon_0^{3/2}(1+b\tau) + (2+b\tau)\sqrt{\varepsilon_0} - \varepsilon_0^2 - 4) \right] \\
& - \frac{1}{f} \left( \frac{\partial f}{\partial \xi} \right)^2 \left[ \frac{1}{\varepsilon_0} + (1+b\tau) \left( 1 - \frac{\omega_p^2}{\omega_0^2 \varepsilon_0} \right) + \frac{b\xi\rho_0^2}{\varepsilon_0^{7/2}} (\varepsilon_0^{7/2} - \varepsilon_0^4 (1+b\tau)^2 - \varepsilon_0 - 1) \right] \\
& - \frac{\omega_p^2 \alpha E_0^2 \rho_0^2}{2\omega_0^2 f^3} \left( \frac{s_0}{2} - 1 \right) \exp(-2k'_i \xi) \left( 1 + \frac{\alpha E_0^2}{2f^2} \right)^{(s_0/2)-1} - \frac{1}{f^3} = 0, \tag{7.13}
\end{aligned}$$

where,  $\xi = z/R_d$  is propagation distance,  $R_d = kr_0^2$  represents diffraction length,  $\rho_0 = r_0\omega_0/c$  represents equilibrium beam radius,  $k'_i$  is the normalized absorption coefficient. Eq. (7.13) represents the spot size variation of laser beam with the propagation distance.

### 7.3 RESULTS AND DISCUSSION

Eq. (7.13) is the second order differential equation that governs the behavior of  $f(z)$  of chirped Gaussian beam in collisional plasma with linear absorption. We have solved Eq. (7.13) numerically by applying the initial condition at  $\xi = 0$ ,  $f = 1$ ,  $(\partial f / \partial \xi) = 0$  and  $(\partial^2 f / \partial \xi^2) = 0$  with the following set of typical parameters[125];  $\omega_0 = 1.778 \times 10^{14}$  rad/sec, laser beam radius  $20 \mu m$  and equilibrium plasma density  $n_0 = 4 \times 10^{19} cm^{-3}$ . By optimizing suitable laser and plasma parameters, we have investigated the self-focusing / defocusing of chirped Gaussian beam in collisional plasma.

Figure 7.1 shows the dependence of  $f$  on  $\xi$  for various values of  $\nu/\omega_0$ . The other parameters are:  $\omega_p/\omega_0 = 0.4$ ,  $\alpha E_0^2 = 0.4$  and  $b = 0$ . It is observed that while, neglecting the effect of chirp, the laser beam shows defocusing character. The defocusing of laser beam increases with increase in the values of  $\nu/\omega_0$ . It is due to the fact that the absorption (corresponding to collision frequency term  $\nu/\omega_0$ ) becomes significant and the laser beam shows fast divergence. The



amplitude of oscillations of beam width parameter becomes too large, there by the beam width parameter diverges continuously. In other words, the beam width parameter increases on account of collision frequency and steady divergence occurs due to strong energy attenuation. So, one can say that the laser beam becomes more defocused due to diffraction and absorption effects at higher oscillation frequencies. The outcomes so obtained in this analysis can be compared with those of Navare *et al.* [80], wherein increase in collision frequency is subjected to increase in oscillation amplitude of beam width parameter. Moreover, for higher values of  $\nu/\omega_0$ , the absorption is more significant and overcomes the self-focusing effect. Again, Jafari Milani *et al.* [95] investigated the ponderomotive self-focusing in warm collisional plasma and reported that firstly self-focusing is caused by collision frequency and secondly it defocuses the laser beam for longer propagation.

Now, in order to account for the defocusing, the effect of chirp is considered. For investigating the effect of chirp parameter ( $b$ ) on the laser beam propagation in collisional plasma, various values of  $b$  are considered. Figure 7.2 (a) illustrates the behavior of  $f$  with  $\xi$  for various values of chirp parameter  $b$  and the other parameters are same as taken in figure 7.1. It is observed from figure 7.2 (a) that in the absence of collision frequency, the beam width parameter initially decreases and then increases showing that the laser beam gets defocused. However, this defocusing can be minimized by increasing the chirp parameter. As soon as the chirp parameter is increased, the amplitude of oscillations of the beam decreases with the distance of propagation. Further, with its passage in the plasma, the angular frequency increases with the result, the dielectric constant of the plasma decreases. The decrease in dielectric constant reduces the spot size amplitude of laser beam close to the axis of propagation. Consequently,  $f(z)$  attains a minimum value for further distance of propagation. The effect of negative chirp on the self-focusing or defocusing is shown in the figure 7.2 (b) which represents the variation of  $f$  with  $\xi$  for various values of negative chirp. From the figure 7.2 (b), it is clear that on increasing the values of negative chirp, the self-focusing at first is strengthened and after attaining a critical value, it gets defocused. This is because the frequency of a linear and negative chirped laser beam changes during the propagation in the plasma. Therefore, the spot size of laser beam depends on  $\xi$  and at propagation distances much greater than the Rayleigh length the temporal shape of the

chirped laser beam will be changed. Therefore, the defocusing of laser beam is weakened and there by the self-focusing effect is strengthened by using chirp. Hence, the chirp parameter is important for minimizing the defocusing and increasing the ability of self-focusing in collisional plasma

Figure 7.3 presents the dependence of beam  $f$  on  $\xi$  for various values of  $\omega_p / \omega_0$ . The other parameters are:  $\nu / \omega_0 = 0.002$ ,  $\alpha E_0^2 = 0.4$  and  $b = 0.002$ . It is evident from figure 7.3 that with increase in  $\omega_p / \omega_0$ , the nonlinearity of plasma medium increases, with the result, the amplitude of oscillations decreases further close to the propagation axis. Consequently,  $f_{\min}$  shifts towards lower value of  $\xi = 0.4$ . Therefore, the beam self-focusing occurs earlier and thus supports the results [80, 86]. Figure 7.4 illustrates the behavior  $f$  with distance  $\xi$  for various values of  $\alpha E_0^2$ . The relative plasma density is fixed at  $\omega_p / \omega_0 = 0.4$  and the other parameters are same as taken in figure 7.3. The curves demonstrate that with increase in  $\alpha E_0^2$  of the beam, the spot size and hence the self-focusing length decreases. Again, increase in laser intensity results in increasing the nonlinearity which is responsible for the self-focusing of laser beam in plasma. Consequently, the laser beam bends more towards the focusing mode for higher values of intensity of laser beam. Furthermore, at higher intensity and for higher plasma density, a beam having more electrons travels with the laser beam and generates a higher current. Consequently, a higher quasi-stationary magnetic field is generated, which reduces the focusing length and hence adds to self-focusing.

Again, taking in to account the laser intensities ( $10^{20}\text{W/cm}^2$ ) closer the realistic values, the changing behavior of  $f$  with  $\xi$  is shown in figure 7. 5. The other parameters are:  $b = 0.002$ ,  $\nu / \omega_0 = 0.002$  and  $\omega_p / \omega_0 = 0.6$ . From the figure 7.5, it is observed that at higher intensities, the oscillating behavior of beam width parameter is destroyed during propagation in plasma and the laser beam undergoes defocusing. In other words, the self-focusing of laser beam disappears with very high intensity and ponderomotive defocusing occurs. This is because of the supremacy of the diffraction effect at high intensity. Further, the frequency of a chirped laser beam changes during the propagation in plasma. As the beam waist depends on  $\xi$  and at larger propagation distances, the temporal shape of the chirped laser beam will be changed. However, for shorter

propagation distance less than the Rayleigh length, the change in laser pulse shape is not considerable.

#### **7.4 CONCLUSION**

In the present communication, we have investigated the self-focusing / defocusing of chirped Gaussian beam in collisional plasma with linear absorption. We derived the required differential equation for  $f(z)$  by using the WKB and paraxial ray approximations and investigated the impression of optimized parameters on the self-focusing / defocusing in collisional plasma. From the results, one can conclude that the chirp parameter is important for the self-focusing / defocusing and maintains the necessary importance of laser-plasma interaction. The laser beam is defocused due to strong diffraction and absorption effects at higher oscillation frequencies. It is further, revealed that initially the amplitude of beam width parameter is too large and continuously diverges in the collisional plasma. The chirp parameter minimizes the divergence and consequently, sooner and an earlier self-focusing is observed. Thus, apart from electron acceleration, the chirp can also be used to analyze the self-focusing / defocusing of laser beam in plasma. The results of present research may be useful in laser – driven fusion and laser plasma based accelerators.

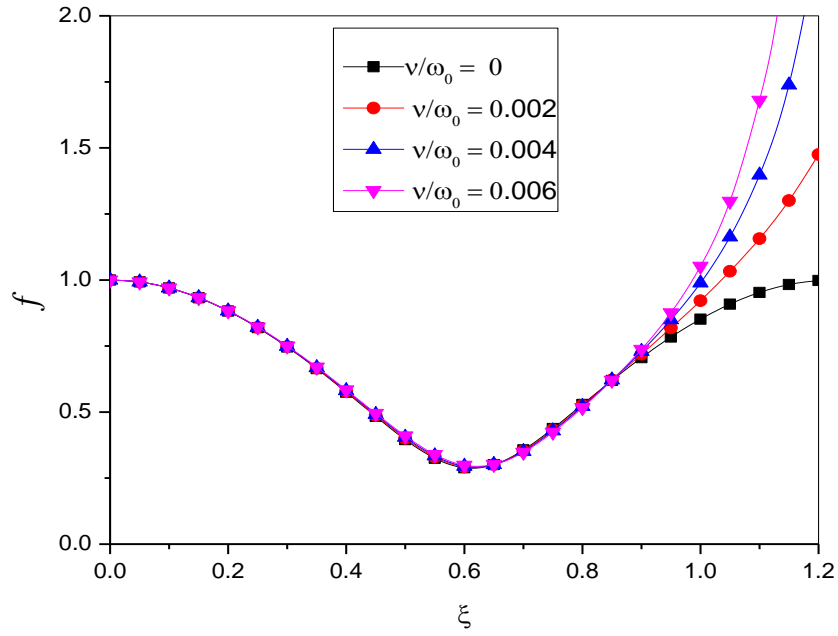


Figure 7.1: Dependence of  $f$  on  $\xi$  for various values of  $\nu/\omega_0$ . The other parameters are:  $\omega_p/\omega_0 = 0.4$ ,  $\alpha E_0^2 = 0.4$  and  $b = 0$

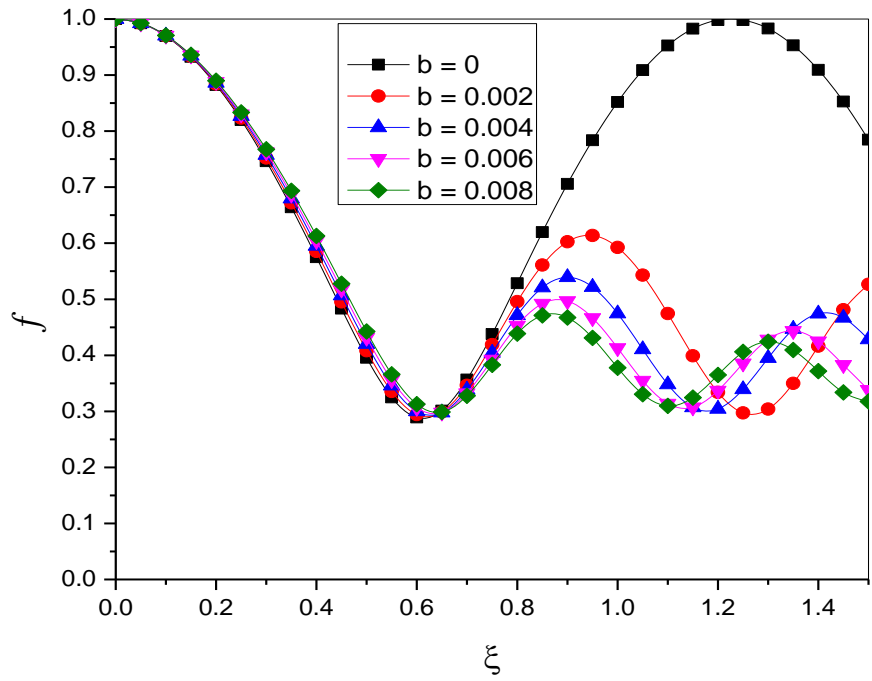


Figure 7.2 (a): Dependence of  $f$  on  $\xi$  for various values of  $b$ . The other parameters are:  $\omega_p / \omega_0 = 0.4$ ,  $\alpha E_0^2 = 0.4$  and  $\nu / \omega_0 = 0$

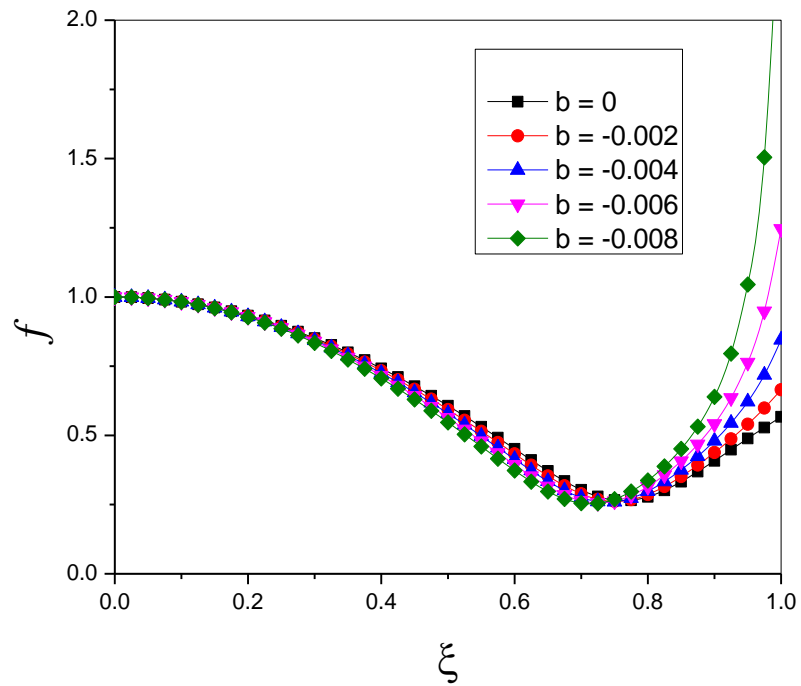


Figure 7.2 (b): Dependence of  $f$  on  $\xi$  for various values of negative chirp. The other parameters are same as taken in the figure 7.2 (a).

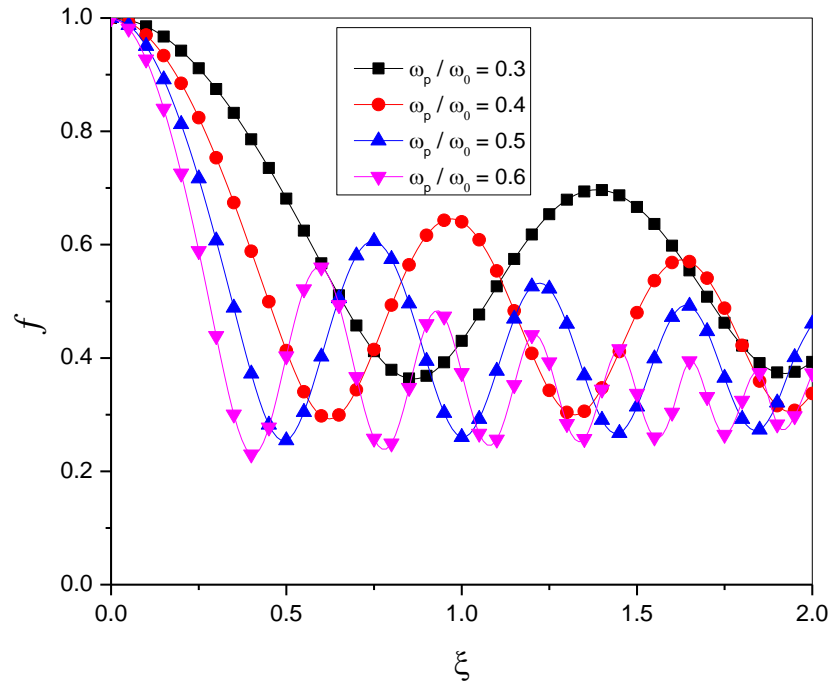


Figure 7.3: Dependence of  $f$  on  $\xi$  for various values of  $\omega_p/\omega_0$ . The other parameters are:

$\nu/\omega_0 = 0.002$ ,  $\alpha E_0^2 = 0.4$  and  $b = 0.002$

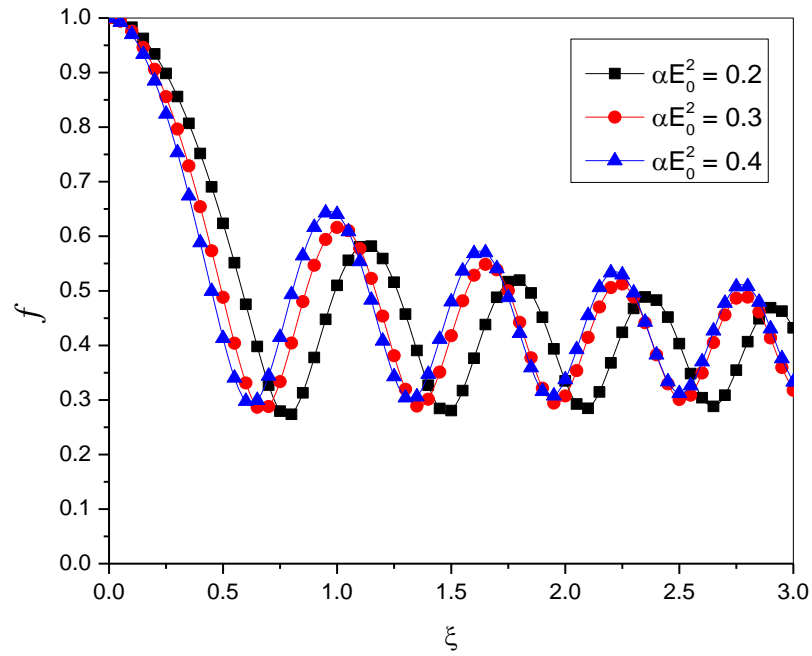


Figure 7.4: Dependence of  $f$  on  $\xi$  for various values of  $\alpha E_0^2$ . The other parameters are:  $\nu/\omega_0 = 0.002$ ,  $\omega_p/\omega_0 = 0.4$ , and  $b = 0.002$



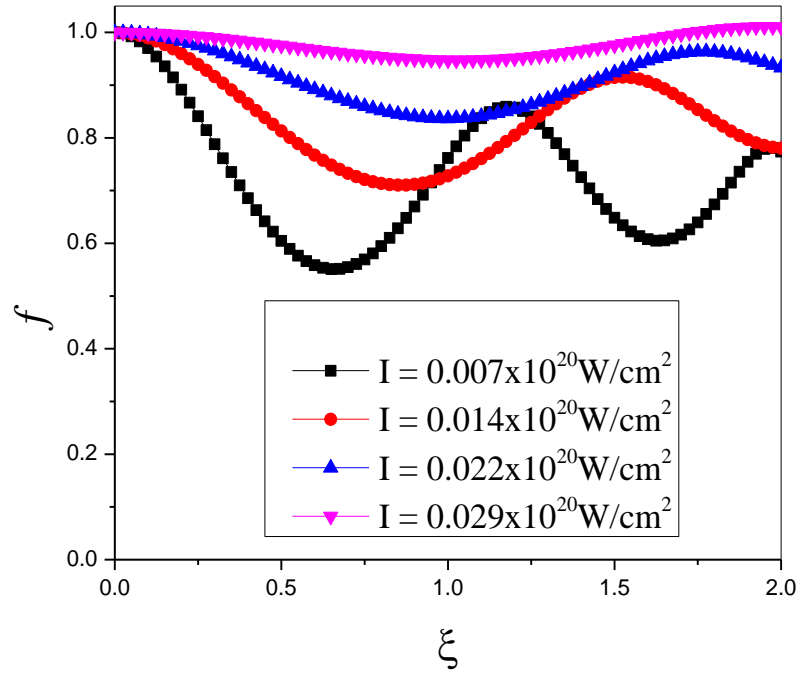


Figure 7.5: Dependence of  $f$  on  $\xi$  for various values of intensity. The other parameters are:  $\nu/\omega_0 = 0.002$ ,  $\omega_p/\omega_0 = 0.6$ , and  $b = 0.002$