CHAPTER - 5

EFFECT OF BI-LINEAR THICKNESS VARIATIONS ON THE VIBRATIONS OF VISCO-ELASTIC RECTANGULAR PLATE WITH BI-PARABOLIC TEMPERATURE VARIATION

ABSTRACT

The Rayleigh-Ritz method has been employed to obtain the numerical solution of the vibration problem of a rectangular plate with non-uniform thickness variation and clamped boundary conditions at all four edges. The analysis is presented here to study two directional thermal effects with varying thickness in both directions. Thickness variation is considered bi-parabolic, i.e. parabolic in x-direction and parabolic in y-direction. The fourth order differential equation governing the motion of such plate has been solved by Rayleigh-Ritz method to calculate time period, deflection and logarithmic decrement for first two modes of vibration for different values of thermal gradient, taper constants and aspect ratio. All the results are presented in form of tables.

Chapter 5

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5.1 INTRODUCTION

Increasing necessities for light weight structures such as parts of airplanes, rockets, submarine etc have directed to the use of advanced materials. Use of light weight and safe structures has become a major concern for engineers and researchers. Tapering of a material plate is used to make plates light in weight and control the vibrations. In the field of aerospace technology, the analysis of plates with varying thickness variations has been of great concern due to their utility in wing of aircrafts.

Gupta and Kumar [41] gave an analysis to study the effect of linear temperature distribution in radical direction on the free vibration of a non-homogeneous visco-elastic circular plate with linear thickness variation in same direction. Filipich et al [108] used Galerkin's method to evaluate the fundamental frequency of vibration of a rectangular plate of variable thickness with different boundary conditions. Tomar and Gupta [113] presented an analysis to analyze the thermal effect on free axisymmetric vibrations of an orthotropic elastic rectangular plate with linearly varying thickness variation. Dhotarad and Ganesan [3] studied the dynamic response of thin rectangular plates subjected to one and two dimensional steady state temperature distributions satisfying Laplace's equation. The governing equations of motion had been derived by a finite difference method and solved by a simultaneous iteration technique to obtain eigenvalues and eigenvectors. Chakraverty [18] discussed basic concepts of vibration of plate using classic theory of plates. He also discussed some problems of vibration of plates for different boundary conditions and different values of parameters.

In this chapter, bi-directional variations in thickness and temperature of a rectangular plate are analyzed. Non-uniformity in thickness is considered bi-linear and temperature variation is considered parabolic in both directions. The values of deflection, time period and logarithmic decrement for the first two modes of vibration at several values of taper parameters, temperature gradient and aspect ratio for clamped boundary conditions are calculated. Results are presented in form of tables in the end of the chapter.

5.2 FORMULATION OF THE PROBLEM

The differential equation of the motion of a viscoelastic isotropic rectangular plate, taken from equation (2.2.6) and equation (2.2.7) of chapter 2, pp. 25, may be written as:

$$\Bigg[D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+2\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+2\frac{\partial^3 W}{\partial y\partial x^2}\right)$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\Bigg]-\rho p^2 hW=0$$

(5.2.1)

and $$\frac{\partial^2 T}{\partial t^2}+p^2\widetilde{D}T=0\ , \qquad (5.2.2)$$

These are the differential equations of motion (5.2.1) and time function (5.2.2) for viscoelastic isotropic rectangular plate of variable thickness respectively.

Where $w(x,y,t)=W(x,y).T(t)$ is the deflection, $W(x, y)$ is function of deflection in x and y and T(t) is time function.

- It is assumed that thickness of the rectangular plate varies linearly in both directions

  It is obtained from equation 2.3.1 of chapter 2, pp. 25 as follows:

$$h = h_0\left(1 + \beta_1 \frac{x}{a}\right)\left(1 + \beta_2 \frac{y}{b}\right) \tag{5.2.3}$$

- Here, variation in temperature is also taken bi-parabolic, i.e. parabolic in *x*-direction and parabolic in *y*-direction (from Equation 3.3.2 of chapter 3, pp. 54):

$$\tau = \tau_0\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right) \tag{5.2.4}$$

The dependency of temperature effects on modulus of elasticity can be expressed as (from Equation 3.3.4 of chapter 3, pp. 54):

$$E = E_0\left[1 - \alpha\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right] \tag{5.2.4}$$

where, $\alpha = \gamma\tau_0\,(0 \le \alpha < 1)$ is thermal gradient.

On substituting the values of *h* and *E*, the expression of flexural rigidity (D_1) becomes:

$$D_1 = \frac{E_0\left[1-\alpha\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right]{h_0}^3\left(1+\beta_1\frac{x}{a}\right)^3\left(1+\beta_2\frac{y}{b}\right)^3}{12(1-\upsilon^2)} \tag{5.2.5}$$

- Boundary condition of this chapter is considered as clamped at all the four edges. Equations satisfying the above said boundary restrictions are taken from Equation 2.3.6 of chapter 2, pp. 27 as follows:

$$W = \frac{\partial W}{\partial x} = 0 \qquad at \;\; x = 0, a \;\; \text{and}$$

$$W = \frac{\partial W}{\partial y} = 0 \qquad at \;\; y = 0,\; a \tag{5.2.6}$$

Also, two-term deflection function corresponding to the boundary conditions is taken as

$$W = \left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right]^2 \times \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right] \tag{5.2.7}$$

5.3 TIME FUNCTION, DEFLECTION AND LOGARITHMIC DECREMENT

Rayleigh-Ritz technique has been used to solve deflection, time function and logarithmic decrement. As one can see from the earlier chapters the procedure of finding frequency equation is almost same. Therefore, the quite convenient frequency equation of the motion can be directly taken from equation 2.4.7 of chapter 2, pp. 28 as follows:

$$S_E^{**} - \lambda^2 K_E^{**} = 0 \tag{5.3.1}$$

where $K_E^{**} = \int_0^1 \int_0^{b/a} \left[(1+\beta_1 X)\left(1+\beta_2 \frac{a}{b} Y\right) \overline{W}^2\right] dY\, dX$ (5.3.2)

& $S_E^{**} = \int_0^1 \int_0^{b/a} \left\{1-\alpha(1-X^2)\left(1-\frac{a^2}{b^2}Y^2\right)\right\}(1+\beta_1 X)^3 \left(1+\beta_2 \frac{a}{b} Y\right)^3 \times$

$$\left[\left(\frac{\partial^2 \overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2 \overline{W}}{\partial Y^2}\right)^2 + 2\upsilon \frac{\partial^2 \overline{W}}{\partial X^2}\frac{\partial^2 \overline{W}}{\partial Y^2} + 2(1-\upsilon)\left(\frac{\partial^2 \overline{W}}{\partial X\, \partial Y}\right)^2\right] dY\, dX \tag{5.3.3}$$

Here, $\lambda^2 = \frac{12\rho p^2 a^2 (1-\upsilon^2)}{E_0 \overline{h_0}^2}$ is a parameter of frequency.

Equation (5.3.1) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of $\overline{W}$. These two constants are to be determined as follows:

$$\frac{\partial}{\partial A_n}\left(S_E^{**} - \lambda^2 K_E^{**}\right) = 0\,, \quad n = 1, 2 \tag{5.3.4}$$

On simplifying equation (5.3.4), one gets

$$b_{n1} A_1 + b_{n2} A_2 = 0, \quad n = 1,\ 2 \tag{5.3.5}$$

where b_{n1} and b_{n2} include parametric constants i.e. taper parameters, temperature gradient, aspect ratio and frequency parameter.

For a non-trivial solution, determinant of the coefficient must be zero. So, one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \tag{5.3.6}$$

With the help of equation (5.3.6), one can obtain a quadratic equation in λ^2 from which the two values of λ^2 can found.

Here, $b_{11} = 2(L - \lambda^2 l)$, $b_{12} = b_{21} = (M - \lambda^2 m)$, and $b_{22} = 2(N - \lambda^2 n)$

where,

$\boldsymbol{L} = -\frac{1}{24012450b(1+\nu)}a(99\beta_1(88(-3+\alpha)+99(-4+\alpha)\beta_2+6(-44+9\alpha)\beta_2^2+6(-11+2\alpha)\beta_2^3)+44(44(-9+4\alpha)+198(-3+\alpha)\beta_2+36(-11+3\alpha)\beta_2^2+3(-33+8\alpha)\beta_2^3)+18\beta_1^2(88(-11+3\alpha)+33(-44+9\alpha)\beta_2+2(-484+81\alpha)\beta_2^2+(-242+36\alpha)\beta_2^3)+3\beta_1^3(44(-33+8\alpha)+198(-11+2\alpha)\beta_2+12(-121+18\alpha)\beta_2^2+(-363+48\alpha)\beta_2^3))-\frac{1}{37837800b^3(1-\nu^2)}a^3(117\beta_1(-616+260\alpha+3(-308+75\alpha)\beta_2+8(-88+15\alpha)\beta_2^2+(-198+25\alpha)\beta_2^3)+52(-924+416\alpha+18(-77+20\alpha)\beta_2+96(-11+2\alpha)\beta_2^2+(-297+40\alpha)\beta_2^3)+12\beta_1^2(52(-63+25\alpha)+9(-546+125\alpha)\beta_2+24(-156+25\alpha)\beta_2^2+(-1053+125\alpha)\beta_2^3)+\beta_1^3(52(-147+55\alpha)+9(-1274+275\alpha)\beta_2+24(-364+55\alpha)\beta_2^2+(-2457+275\alpha)\beta_2^3))-\frac{1}{37837800(a-a\nu^2)}b(24\beta_1^2(208(-11+2\alpha)+39(-88+15\alpha)\beta_2+12(-156+25\alpha)\beta_2^2+(-364+55\alpha)\beta_2^3)+52(-924+416\alpha+9(-154+65\alpha)\beta_2+12(-63+25\alpha)\beta_2^2+(-147+55\alpha)\beta_2^3)+\beta_1^3(52(-297+40\alpha)+117(-198+25\alpha)\beta_2+12(-1053+125\alpha)\beta_2^2+(-2457+275\alpha)\beta_2^3)+9\beta_1(104(-77+20\alpha)+39(-308+75\alpha)\beta_2+12(-546+125\alpha)\beta_2^2+(-1274+275\alpha)\beta_2^3))-\frac{1}{96049800(b-b\nu^2)}a\nu(198(22(-16+9\alpha)+33(-16+9\alpha)\beta_2+6(-44+25\alpha)\beta_2^2+(-44+26\alpha)\beta_2^3)+297\beta_1(22(-16+9\alpha)+33(-16+9\alpha)\beta_2+6(-44+25\alpha)\beta_2^2+(-44+26\alpha)\beta_2^3)+6\beta_1^2(198(-44+25\alpha)+297(-44+25\alpha)\beta_2+6(-1089+625\alpha)\beta_2^2+$

$(-1089+650\alpha)\beta_2^3)+\beta_1^3(396(-22+13\alpha)+594(-22+13\alpha)\beta_2+6(-1089+650\alpha)\beta_2^2+(-1089+676\alpha)\beta_2^3))$

$\boldsymbol{M} = -\frac{1}{832431600(a-a\nu^2)}b(297\beta_1(-520+190\alpha+30(-26+9\alpha)\beta_2+6(-70+23\alpha)\beta_2^2+5(-16+5\alpha)\beta_2^3)+264(-390+190\alpha+45(-13+6\alpha)\beta_2+3(-105+46\alpha)\beta_2^2+5(-12+5\alpha)\beta_2^3)+12\beta_1^2(130(-66+19\alpha)+1170(-11+3\alpha)\beta_2+6(-1155+299\alpha)\beta_2^2+5(-264+65\alpha)\beta_2^3)+\beta_1^3(-25740+5890\alpha+270(-143+31\alpha)\beta_2+6(-3465+713\alpha)\beta_2^2+5(-792+155\alpha)\beta_2^3))-\frac{1}{832431600b^3(1-\nu^2)}a^3(270\beta_1(44(-13+6\alpha)+33(-26+9\alpha)\beta_2+52(-11+3\alpha)\beta_2^2+(-143+31\alpha)\beta_2^3)+5\beta_1^3(264(-12+5\alpha)+297(-16+5\alpha)\beta_2+12(-264+65\alpha)\beta_2^2+(-792+155\alpha)\beta_2^3)+10(264(-39+19\alpha)+297(-52+19\alpha)\beta_2+156(-66+19\alpha)\beta_2^2+(-2574+589\alpha)\beta_2^3)+6\beta_1^2(132(-105+46\alpha)+297(-70+23\alpha)\beta_2+12(-1155+299\alpha)\beta_2^2+(-3465+713\alpha)\beta_2^3))-\frac{1}{901800900b(1+\nu)}a(13(-6292+2925\alpha+78(-121+45\alpha)\beta_2+42(-143+45\alpha)\beta_2^2+5(-286+81\alpha)\beta_2^3)+21\beta_1^2(26(-143+45\alpha)+39(-143+36\alpha)\beta_2+21(-169+36\alpha)\beta_2^2+(-845+162\alpha)\beta_2^3)+39\beta_1(26(-121+45\alpha)+39(-121+36\alpha)\beta_2+21(-143+36\alpha)\beta_2^2+(-715+162\alpha)\beta_2^3)+\beta_1^3(65(-286+81\alpha)+39(-715+162\alpha)\beta_2+21(-845+162\alpha)\beta_2^2+(-4225+729\alpha)\beta_2^3))-\frac{1}{5410805400(b-b\nu^2)}a\nu(117\beta_1(-6292+3679\alpha+78(-121+72\alpha)\beta_2+(-4576+2814\alpha)\beta_2^2+11(-65+43\alpha)\beta_2^3)+13(26(-1452+833\alpha)+117(-484+283\alpha)\beta_2+24(-1144+691\alpha)\beta_2^2+11(-390+253\alpha)\beta_2^3)+\beta_1^3(143(-390+253\alpha)+1287(-65+43\alpha)\beta_2+3(-13351+9238\alpha)\beta_2^2+(-6084+4648\alpha)\beta_2^3)+3\beta_1^2(104(-1144+691\alpha)+78(-2288+1407\alpha)\beta_2+6(-14365+9164\alpha)\beta_2^2+(-13351+9238\alpha)\beta_2^3))$

$\boldsymbol{N} = -\frac{1}{801600800b(1+\nu)}a(24\beta_1^2(4(-26+9\alpha)+3(-52+15\alpha)\beta_2+24(-4+\alpha)\beta_2^2+(-22+5\alpha)\beta_2^3)+\beta_1^3(-572+180\alpha+3(-286+75\alpha)\beta_2+24(-22+5\alpha)\beta_2^2+(-121+25\alpha)\beta_2^3)+4(-676+324\alpha+3(-338+135\alpha)\beta_2+24(-26+9\alpha)\beta_2^2+(-143+45\alpha)\beta_2^3)+3\beta_1(-1352+540\alpha+3(-676+225\alpha)\beta_2+24(-52+15\alpha)\beta_2^2+(-286+$

$75\alpha)\beta_2^3)) - \frac{1}{7214407200(b-b\nu^2)}a\nu(36\beta_1(-1014 + 590\alpha + 9(-169 + 100\alpha)\beta_2 + (-741 +$

$450\alpha)\beta_2^2 + 3(-39 + 25\alpha)\beta_2^3) + 18\beta_1^2(-988 + 590\alpha + 6(-247 + 150\alpha)\beta_2 + (-722 +$

$450\alpha)\beta_2^2 + 3(-38 + 25\alpha)\beta_2^3) + 3\beta_1^3(-936 + 590\alpha + 36(-39 + 25\alpha)\beta_2 + 18(-38 +$

$25\alpha)\beta_2^2 + 3(-36 + 25\alpha)\beta_2^3) + 2(-12168 + 6962\alpha + 36(-507 + 295\alpha)\beta_2 + 18(-494 +$

$295\alpha)\beta_2^2 + 3(-468 + 295\alpha)\beta_2^3)) - \frac{1}{122644922400b^3(1-\nu^2)}a^3(78\beta_1^2(-5984 + 2730\alpha +$

$12(-748 + 273\alpha)\beta_2 + 42(-136 + 41\alpha)\beta_2^2 + (-1360 + 343\alpha)\beta_2^3) + 153\beta_1(130(-44 +$

$21\alpha) + 156(-55 + 21\alpha)\beta_2 + 42(-130 + 41\alpha)\beta_2^2 + (-1300 + 343\alpha)\beta_2^3) +$

$68(4290(-2 + \alpha) + 2574(-5 + 2\alpha)\beta_2 + 6(-1365 + 451\alpha)\beta_2^2 + (-1950 + 539\alpha)\beta_2^3) +$

$2\beta_1^3(78(-561 + 245\alpha) + 117(-561 + 196\alpha)\beta_2 + 21(-1989 + 574\alpha)\beta_2^2 + (-9945 +$

$2401\alpha)\beta_2^3)) - \frac{1}{122644922400(a-a\nu^2)}b(234\beta_1(748(-5 + 2\alpha) + 102(-55 + 21\alpha)\beta_2 +$

$4(-748 + 273\alpha)\beta_2^2 + (-561 + 196\alpha)\beta_2^3) + 78(3740(-2 + \alpha) + 255(-44 + 21\alpha)\beta_2 +$

$(-5984 + 2730\alpha)\beta_2^2 + 2(-561 + 245\alpha)\beta_2^3) + 6\beta_1^2(68(-1365 + 451\alpha) + 1071(-130 +$

$41\alpha)\beta_2 + 546(-136 + 41\alpha)\beta_2^2 + 7(-1989 + 574\alpha)\beta_2^3) + \beta_1^3(68(-1950 + 539\alpha) +$

$153(-1300 + 343\alpha)\beta_2 + 78(-1360 + 343\alpha)\beta_2^2 + 2(-9945 + 2401\alpha)\beta_2^3))$

$\boldsymbol{l} = \frac{b(2+\beta_1)(2+\beta_2)}{1587600a}$; $\boldsymbol{m} = \frac{b(2+\beta_1)(2+\beta_2)}{15367968a}$; $\boldsymbol{n} = \frac{b(2+\beta_1)(2+\beta_2)}{577152576a}$

Taking A_1=1, one gets $A_2 = -\frac{b_{11}}{b_{12}}$ for n = 1 and then W comes out as

$$W = \left[X\left(\frac{a}{b}Y\right)(1-X)\left(1-\frac{a}{b}Y\right)\right]^2 \times \left[1+\left(-\frac{b_{11}}{b_{12}}\right)(X)\left(\frac{a}{b}Y\right)(1-X)\left(1-\frac{a}{b}Y\right)\right] \quad (5.3.7)$$

The time function for free vibrations of viscoelastic plate is well defined by the general differential equation (5.2.2). The solution of time function can be obtained easily from Equation 2.5.5 of chapter 2, pp. 32 as follows:

$$T = e^{a_1 t}[C_1 \cos b_1 t + C_2 \sin b_1 t], \quad (5.3.8)$$

where C_1 = 1 and $C_2 = \frac{p^2\eta}{2\,G} = -\frac{a_1}{b_1}$

Thus, using Equation (5.3.7) and (5.3.8), deflection w may be presented as:

$$w = \left[X\left(\frac{a}{b}Y\right)(1-X)\left(1-\frac{a}{b}Y\right)\right]^2 \times \left[1+\left(-\frac{b_{11}}{b_{12}}\right)(X)\left(\frac{a}{b}Y\right)(1-X)\left(1-\frac{a}{b}Y\right)\right]$$

$$\times e^{a_1 t}\left[\cos b_1 t + \left(-\frac{a_1}{b_1}\right)\sin b_1 t\right] \qquad (5.3.9)$$

Time period and logarithmic decrement of the vibration of the plate are as same as in Equations 2.5.7 and 2.5.8 respectively, *i.e.*

$$K = \frac{2\pi}{p} \qquad (5.3.10)$$

and

$$\Lambda = \log_e\left(\frac{w_2}{w_1}\right) \qquad (5.3.11)$$

5.4 RESULTS AND DISCUSSION

All the results are calculated for same parameters which are used in previous chapters. The numerical values of deflection, time period and logarithmic decrement are calculated for several combinations of parameters of plates i.e. aspect ratio, taper parameters and temperature gradient. All the results are obtained for first two modes of vibrations and well tabulated as follows:

Table5.4.1 : Time Period Vs Taper Constants at $\alpha = 0.0$, $a/b = 1.5$

Table5.4.2 : Time Period Vs Taper Constants at $\alpha = 0.4$, $a/b = 1.5$

Table5.4.3 : Time Period Vs Thermal Gradient at $a/b = 1.5$

Table5.4.4 : Time Period Vs Aspect Ratio

Table5.4.5 : Deflection($\times 10^{-5}$) Vs Thermal Gradient at $T = 0.K$, $\beta_1 = \beta_2 = 0.0$ & $a/b = 1.5$

Table5.4.6 : Deflection($\times 10^{-5}$) Vs Thermal Gradient at $T = 1.K$, $\beta_1 = \beta_2 = 0.0$ & $a/b = 1.5$

Table5.4.7 : Deflection($\times 10^{-5}$) Vs Thermal Gradient at $T = 0.K$, $\beta_1 = \beta_2 = 0.6$ & $a/b = 1.5$

Table5.4.8 : Deflection($\times$ 10^{-5}) Vs Thermal Gradient at T = 1.K, $\beta_1 = \beta_2 = 0.6$ & $a/b = 1.5$

Table5.4.9 : Deflection($\times$ 10^{-5}) Vs Aspect Ratio at T = 0.K, $\alpha = \beta_1 = \beta_2 = 0.6$

Table5.4.10 : Deflection($\times$ 10^{-5}) Vs Aspect Ratio at T = 1.K, $\alpha = \beta_1 = \beta_2 = 0.6$

Table5.4.11 : Logarithmic Decrement Vs Thermal Gradient at $\beta_1 = \beta_2 = 0.0$, $a/b = 1.5$

Table5.4.12 : Logarithmic Decrement Vs Thermal Gradient at $\beta_1 = \beta_2 = 0.6$, $a/b = 1.5$

Table5.4.13 : Logarithmic Decrement Vs Aspect Ratio at $\alpha = \beta_1 = \beta_2 = 0.6$

Table5.4.1 to 5.4.4

Tables 5.4.1 to 5.4.4 present the mathematical values of time period for initial two modes of vibration for several combinations of plate's parameters. Tables 5.4.1 and 5.4.1 show the influence of thickness variation on the vibration of plate. As taper parameters β_1 and β_2 rise from 0.0 to 0.8, time period drops continuously as same as the results of previous chapters.

From the table 5.4.3, one can easily observed the effect of increasing thermal gradient that the time period raises as temperature gradient rises from 0.0 to 0.8 at several combinations of combined values of taper constants with fixed value of aspect ratio of 1.5. The results of table 4.5.4 indicate the influence of varying aspect ratio i.e. size of plate on the vibration of plate. Time period decreases fast as aspect ratio increase from 0.5 to 2.5 for various combinations of combined values of taper constants and thermal gradient.

Tables from 5.4.5 to 5.4.10 show the effect on the deflection of the plate for initial two modes of vibration for many arrangements of taper parameters, temperature gradient and aspect ratio at two different values of time period i.e. T = 0.K & T = 1.K.

From tables 5.4.5 and 5.4.6, one can easily notice that variations in the values of deflection in these tables are as same as variations in deflection in tables of previous chapter, i.e. table 4.5.5 & 4.5.6, but values of deflection in present chapter is slightly less as compared to previous chapter.

At $\beta_1 = \beta_2 = 0.6$, authors again analyzed tables 5.4.7 & 5.4.8 and found that deflection for both the modes of vibration vary in the same way as it varies in tables 4.5.7 & 4.5.8of previous chapter. Again deflection for both the modes of vibrations is found quite less in this chapter as compared to chapter 4. Also, it is noticed that values of deflection at $\beta_1 = \beta_2 = 0.0$ (in tables 5.4.5 & 5.4.6) are greater than deflection at $\beta_1 = \beta_2 = 0.6$ (in tables 5.4.7 & 5.4.8).

At $\alpha = \beta_1 = \beta_2 = 0.6$, tables 5.4.9 & 5.4.10show the variations of deflection for both the modes of vibration with respect to the values of aspect ratio at $T = 0.K$ & $T = 1.K$ respectively.

In tables 5.4.11 to 5.4.13, Logarithmic decrement for both the modes of vibration is tabulated at various parameters of plate. Here, authors conclude that variations in Logarithmic decrement are similar to variations in Logarithmic decrement in table 4.5.11 to 4.5.13. Further Logarithmic decrement is found less in present chapter as compared to previous chapter i.e. chapter 4.

Table 5.4.1 : Time Period Vs Taper Constants at $\alpha = 0.0$, $a / b = 1.5$

$\beta_2 \rightarrow$ / $\beta_1 \downarrow$	0		0.4		0.8	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	667.93	169.01	547.54	138.39	455.59	114.83
0.4	552.71	139.94	453.15	114.59	377.15	95.09
0.8	467.50	118.48	383.39	97.03	319.23	80.53

Table 5.4.2 : Time Period Vs Taper Constants at $\alpha = 0.4$, $a / b = 1.5$

$\beta_2 \rightarrow$ / $\beta_1 \downarrow$	0		0.4		0.8	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	739.25	186.91	596.24	150.26	491.55	123.19
0.4	607.59	153.76	490.72	123.72	404.97	101.54
0.8	511.59	129.59	413.69	104.37	341.75	85.72

Table 5.4.3 : Time Period Vs Thermal Gradient at *a* / *b* = 1.5

α ↓	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = \beta_2 = 0.4$		$\beta_1 = \beta_2 = 0.8$	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	667.93	169.01	453.15	114.59	319.23	80.53
0.2	700.83	177.29	470.70	118.89	329.80	83.01
0.4	739.25	186.91	490.72	123.72	341.75	85.72
0.6	784.98	198.29	513.89	129.19	355.41	88.71
0.8	840.74	212.08	541.24	135.44	371.32	92.02

Table 5.4.4 : Time Period Vs Aspect Ratio

a / *b* ↓	$\alpha = \beta_1 = \beta_2 = 0.0$		$\alpha = \beta_1 = \beta_2 = 0.4$		$\alpha = \beta_1 = \beta_2 = 0.8$	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.5	1650.14	412.60	1212.49	301.66	917.59	223.88
1.0	1128.99	288.48	829.33	211.42	627.45	157.56
1.5	667.93	169.01	490.72	123.72	371.32	92.02
2.0	412.53	103.15	303.12	75.41	229.39	55.97
2.5	274.43	68.08	201.66	49.73	152.62	36.86

Table 5.4.5: Deflection (x 10^{-5}) Vs Thermal Gradient at $T = 0.K$, $\beta_1 = \beta_2 = 0.0$ & $a/b = 1.5$

α ↓	Y → / X ↓	**0.2 × (*b*/*a*)**		**0.4 × (*b*/*a*)**		**0.6 × (*b*/*a*)**		**0.8 × (*b*/*a*)**	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	**0.2**	66.29	33.07	150.03	37.91	150.03	37.91	66.29	33.07
	0.4	150.03	37.91	340.46	-37.93	340.46	-37.93	150.03	37.91
	0.6	150.03	37.91	340.46	-37.93	340.46	-37.93	150.03	37.91
	0.8	66.29	33.07	150.03	37.91	150.03	37.91	66.29	33.07
0.4	**0.2**	67.58	33.15	154.37	38.03	154.37	38.03	67.58	33.15
	0.4	154.37	38.03	355.13	-37.52	355.13	-37.52	154.37	38.03
	0.6	154.37	38.03	355.13	-37.52	355.13	-37.52	154.37	38.03
	0.8	67.58	33.15	154.37	38.03	154.37	38.03	67.58	33.15
0.8	**0.2**	69.82	33.17	161.94	38.22	161.94	38.22	69.82	33.17
	0.4	161.94	38.22	380.66	-36.86	380.66	-36.86	161.94	38.22
	0.6	161.94	38.22	380.66	-36.86	380.66	-36.86	161.94	38.22
	0.8	69.82	33.17	161.94	38.22	161.94	38.22	69.82	33.17

Table 5.4.6: Deflection (x 10^{-5}) Vs Thermal Gradient at $T = 1.K$, $\beta_1 = \beta_2 = 0.0$ & $a/b = 1.5$

$\alpha \downarrow$	Y → / X ↓	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.0	0.2	56.26	17.22	127.31	19.74	127.31	19.74	56.26	17.22
	0.4	127.31	19.74	288.91	-19.75	288.91	-19.75	127.31	19.74
	0.6	127.31	19.74	288.91	-19.75	288.91	-19.75	127.31	19.74
	0.8	56.26	17.22	127.31	19.74	127.31	19.74	56.26	17.22
0.4	0.2	58.26	18.36	133.09	21.09	133.09	21.09	58.26	18.36
	0.4	133.09	21.09	306.17	-20.81	306.17	-20.81	133.09	21.09
	0.6	133.09	21.09	306.17	-20.81	306.17	-20.81	133.09	21.09
	0.8	58.26	18.36	133.09	21.09	133.09	21.09	58.26	18.36
0.8	0.2	61.29	19.74	142.14	22.75	142.14	22.75	61.29	19.74
	0.4	142.14	22.75	334.11	-21.94	334.11	-21.94	142.14	22.75
	0.6	142.14	22.75	334.11	-21.94	334.11	-21.94	142.14	22.75
	0.8	61.29	19.74	142.14	22.75	142.14	22.75	61.29	19.74

Table 5.4.7: Deflection (x 10^{-5}) Vs Thermal Gradient at $T = 0.K$, $\beta_1 = \beta_2 = 0.6$ & $a/b = 1.5$

$\alpha \downarrow$	$Y \rightarrow$ / $X \downarrow$	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	**0.2**	68.52	33.14	157.55	38.12	157.55	38.12	68.52	33.14
	0.4	157.55	38.12	365.84	-37.23	365.84	-37.23	157.55	38.12
	0.6	157.55	38.12	365.84	-37.23	365.84	-37.23	157.55	38.12
	0.8	68.52	33.14	157.55	38.12	157.55	38.12	68.52	33.14
0.4	**0.2**	71.53	33.21	167.70	38.36	167.70	38.36	71.53	33.21
	0.4	167.70	38.36	400.11	-36.42	400.11	-36.42	167.70	38.36
	0.6	167.70	38.36	400.11	-36.42	400.11	-36.42	167.70	38.36
	0.8	71.53	33.21	167.70	38.36	167.70	38.36	71.53	33.21
0.8	**0.2**	76.48	33.30	184.40	38.68	184.40	38.68	76.48	33.30
	0.4	184.40	38.68	456.46	-35.33	456.46	-35.33	184.40	38.68
	0.6	184.40	38.68	456.46	-35.33	456.46	-35.33	184.40	38.68
	0.8	76.48	33.30	184.40	38.68	184.40	38.68	76.48	33.30

Table 5.4.8: Deflection (x 10^{-5}) Vs Thermal Gradient at $T = 1.K$, $\beta_1 = \beta_2 = 0.6$ & $a/b = 1.5$

$\alpha \downarrow$	$Y \rightarrow$ / $X \downarrow$	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	**0.2**	51.27	10.26	117.88	11.80	117.88	11.80	51.27	10.26
	0.4	117.88	11.80	273.72	-11.53	273.72	-11.53	117.88	11.80
	0.6	117.88	11.80	273.72	-11.53	273.72	-11.53	117.88	11.80
	0.8	51.27	10.26	117.88	11.80	117.88	11.80	51.27	10.26
0.4	**0.2**	54.63	11.15	128.07	12.88	128.07	12.88	54.63	11.15
	0.4	128.07	12.88	305.55	-12.23	305.55	-12.23	128.07	12.88
	0.6	128.07	12.88	305.55	-12.23	305.55	-12.23	128.07	12.88
	0.8	54.63	11.15	128.07	12.88	128.07	12.88	54.63	11.15
0.8	**0.2**	59.77	12.19	144.10	14.16	144.10	14.16	59.77	12.19
	0.4	144.10	14.16	356.72	-12.93	356.72	-12.93	144.1	14.16
	0.6	144.10	14.16	356.72	-12.93	356.72	-12.93	144.1	14.16
	0.8	59.77	12.19	144.10	14.16	144.10	14.16	59.77	12.19

Table 5.4.9: Deflection (x 10^{-5}) Vs Aspect Ratio at $T = 0.K$, $\alpha = \beta_1 = \beta_2 = 0.6$

a/b ↓	Y → X ↓	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.5	**0.2**	75.19	33.28	180.05	38.60	180.05	38.60	75.19	33.28
	0.4	180.05	38.60	441.80	-35.58	441.80	-35.58	180.05	38.60
	0.6	180.05	38.60	441.80	-35.58	441.80	-35.58	180.05	38.60
	0.8	75.19	33.28	180.05	38.60	180.05	38.60	75.19	33.28
1.0	**0.2**	72.40	33.23	170.63	38.42	170.63	38.42	72.40	33.23
	0.4	170.63	38.42	410.01	-36.20	410.01	-36.20	170.63	38.42
	0.6	170.63	38.42	410.01	-36.20	410.01	-36.20	170.63	38.42
	0.8	72.40	33.23	170.63	38.42	170.63	38.42	72.40	33.23
1.5	**0.2**	73.66	33.25	174.89	38.51	174.89	38.51	73.66	33.25
	0.4	174.89	38.51	424.38	-35.91	424.38	-35.91	174.89	38.51
	0.6	174.89	38.51	424.38	-35.91	424.38	-35.91	174.89	38.51
	0.8	73.66	33.25	174.89	38.51	174.89	38.51	73.66	33.25

Table 5.4.10: Deflection (x 10^{-5}) Vs Aspect Ratio at $T = 1.K$, $\alpha = \beta_1 = \beta_2 = 0.6$

a / *b* ↓	*Y* → *X* ↓	0.2 × (*b*/*a*)		0.4 × (*b*/*a*)		0.6 × (*b*/*a*)		0.8 × (*b*/*a*)	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.5	**0.2**	67.73	21.77	162.18	25.25	162.18	25.25	67.73	21.77
	0.4	162.18	25.25	397.95	-23.28	397.95	-23.28	162.18	25.25
	0.6	162.18	25.25	397.95	-23.28	397.95	-23.28	162.18	25.25
	0.8	67.73	21.77	162.18	25.25	162.18	25.25	67.73	21.77
1.0	**0.2**	62.14	18.13	146.44	20.96	146.44	20.96	62.14	18.13
	0.4	146.44	20.96	351.88	-19.75	351.88	-19.75	146.44	20.96
	0.6	146.44	20.96	351.88	-19.75	351.88	-19.75	146.44	20.96
	0.8	62.14	18.13	146.44	20.96	146.44	20.96	62.14	18.13
1.5	**0.2**	56.88	11.65	135.06	13.49	135.06	13.49	56.88	11.65
	0.4	135.06	13.49	327.72	-12.58	327.72	-12.58	135.06	13.49
	0.6	135.06	13.49	327.72	-12.58	327.72	-12.58	135.06	13.49
	0.8	56.88	11.65	135.06	13.49	135.06	13.49	56.88	11.65

Table 5.4.11: Logarithmic Decrement Vs Thermal Gradient at $\beta_1 = \beta_2 = 0.0$, $a/b = 1.5$

α ↓	Y → X ↓	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	**0.2**	-0.164055	-0.652555	-0.164210	-0.652568	-0.164210	-0.652568	-0.164055	-0.652555
	0.4	-0.164210	-0.652568	-0.164182	-0.652589	-0.164182	-0.652589	-0.164210	-0.652568
	0.6	-0.164210	-0.652568	-0.164182	-0.652589	-0.164182	-0.652589	-0.164210	-0.652568
	0.8	-0.164055	-0.652555	-0.164210	-0.652568	-0.164210	-0.652568	-0.164055	-0.652555
0.4	**0.2**	-0.148396	-0.590868	-0.148327	-0.589576	-0.148327	-0.589576	-0.148396	-0.590868
	0.4	-0.148327	-0.589576	-0.148343	-0.589440	-0.148343	-0.589440	-0.148327	-0.589576
	0.6	-0.148327	-0.589576	-0.148343	-0.589440	-0.148343	-0.589440	-0.148327	-0.589576
	0.8	-0.148396	-0.590868	-0.148327	-0.589576	-0.148327	-0.589576	-0.148396	-0.590868
0.8	**0.2**	-0.130304	-0.518999	-0.130413	-0.518794	-0.130413	-0.518794	-0.130304	-0.518999
	0.4	-0.130413	-0.518794	-0.130436	-0.518815	-0.130436	-0.518815	-0.130413	-0.518794
	0.6	-0.130413	-0.518794	-0.130436	-0.518815	-0.130436	-0.518815	-0.130413	-0.518794
	0.8	-0.130304	-0.518999	-0.130413	-0.518794	-0.130413	-0.518794	-0.130304	-0.518999

Table 5.4.12: Logarithmic Decrement Vs Thermal Gradient at $\beta_1 = \beta_2 = 0.6$, $a/b = 1.5$

α ↓	Y → / X ↓	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.0	**0.2**	-0.290020	-1.172488	-0.290076	-1.172640	-0.290076	-1.172640	-0.290020	-1.172488
	0.4	-0.290076	-1.172640	-0.290090	-1.172163	-0.290090	-1.172163	-0.290076	-1.172640
	0.6	-0.290076	-1.172640	-0.290090	-1.172163	-0.290090	-1.172163	-0.290076	-1.172640
	0.8	-0.290020	-1.172488	-0.290076	-1.172640	-0.290076	-1.172640	-0.290020	-1.172488
0.4	**0.2**	-0.269534	-1.091412	-0.269600	-1.091340	-0.269600	-1.091340	-0.269534	-1.091412
	0.4	-0.269600	-1.091340	-0.269626	-1.091226	-0.269626	-1.091226	-0.269600	-1.091340
	0.6	-0.269600	-1.091340	-0.269626	-1.091226	-0.269626	-1.091226	-0.269600	-1.091340
	0.8	-0.269534	-1.091412	-0.269600	-1.091340	-0.269600	-1.091340	-0.269534	-1.091412
0.8	**0.2**	-0.246525	-1.004941	-0.246600	-1.004902	-0.246600	-1.004902	-0.246525	-1.004941
	0.4	-0.246600	-1.004902	-0.246550	-1.005182	-0.246550	-1.005182	-0.246600	-1.004902
	0.6	-0.246600	-1.004902	-0.246550	-1.005182	-0.246550	-1.005182	-0.246600	-1.004902
	0.8	-0.246525	-1.004941	-0.246600	-1.004902	-0.246600	-1.004902	-0.246525	-1.004941

Table 5.4.13: Logarithmic Decrement Vs Aspect Ratio at $\alpha = \beta_1 = \beta_2 = 0.6$

a/b ↓	Y → X ↓	$0.2 \times (b/a)$		$0.4 \times (b/a)$		$0.6 \times (b/a)$		$0.8 \times (b/a)$	
		Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.5	**0.2**	-0.104489	-0.424424	-0.104528	-0.424426	-0.104528	-0.424426	-0.104489	-0.424424
	0.4	-0.104528	-0.424426	-0.104531	-0.424189	-0.104531	-0.424189	-0.104528	-0.424426
	0.6	-0.104528	-0.424426	-0.104531	-0.424189	-0.104531	-0.424189	-0.104528	-0.424426
	0.8	-0.104489	-0.424424	-0.104528	-0.424426	-0.104528	-0.424426	-0.104489	-0.424424
1.0	**0.2**	-0.152816	-0.605885	-0.152882	-0.605962	-0.152882	-0.605962	-0.152816	-0.605885
	0.4	-0.152882	-0.605962	-0.152891	-0.605906	-0.152891	-0.605906	-0.152882	-0.605962
	0.6	-0.152882	-0.605962	-0.152891	-0.605906	-0.152891	-0.605906	-0.152882	-0.605962
	0.8	-0.152816	-0.605885	-0.152882	-0.605962	-0.152882	-0.605962	-0.152816	-0.605885
1.5	**0.2**	-0.258516	-1.048749	-0.258438	-1.048969	-0.258438	-1.048969	-0.258516	-1.048749
	0.4	-0.258438	-1.048969	-0.258470	-1.048908	-0.258470	-1.048908	-0.258438	-1.048969
	0.6	-0.258438	-1.048969	-0.258470	-1.048908	-0.258470	-1.048908	-0.258438	-1.048969
	0.8	-0.258516	-1.048749	-0.258438	-1.048969	-0.258438	-1.048969	-0.258516	-1.048749