Basic definitions and results in topological spaces that are used to accomplish the present study are given in this chapter.

1.1 Topological Spaces

Definition 1.1.1 Throughout the thesis (X, τ), (Y, σ) and (Z, η) denote topological spaces on which no separation axioms are mentioned unless or otherwise stated.

If A is non-empty subset of (X, τ) then the union of all open sets contained in A is called interior of A and it is denoted by int(A). The intersection of all closed sets containing A is called closure of A and it is denoted by cl(A).

Definition 1.1.2 A subset A of (X, τ) is called a

- Regular open set (Stone, 1937) if A = int(cl(A))
- Semi open set (Levine, 1963) if A ⊆ cl(int(A))
- α-open set (Njastad, 1965) if A ⊆ int(cl(int(A)))
- π-open set (Zaitsav, 1968) if it is the finite union of regular open sets.
- Pre open set (Mashhour, 1982) if A ⊆ int(cl(A))
- Semi pre open set (Andrijevic, 1986) if A ⊆ cl(int(cl(A)))
- b-open set (Andrijevic, 1996) if A ⊆ cl(int(A)) ∪ int(cl(A))

The complements of the above mentioned sets are called regular closed, semi closed, α-closed, π-closed, pre closed, semi pre closed and b-closed respectively.

The intersection of all regular closed (resp. semi closed, α-closed, π-closed, pre closed, semi pre closed and b-closed) subsets of (X, τ) containing A is called the regular closure (resp. semi closure, α-closure, π-closure, pre-closure, semi pre-closure and b-closure) of A and are denoted by rcl(A) (resp. scl(A) acl(A), πcl(A), pcl(A), spcl(A) and bcl(A)).
A subset $A$ of $(X, \tau)$ is called **clop**en if is both open and closed in $(X, \tau)$.

**Definition 1.1.3** A subset $A$ of $(X, \tau)$ is called a $\delta$-**open set** (Velicko, 1968) if $A = \text{int}_\delta(A)$ where $\text{int}_\delta(A)$ is the union of all regular open sets of $X$ contained in $A$.

That is, a set is $\delta$-open if it is the union of regular open sets.

The complement of $\delta$-open is called $\delta$-**closed**. That is, a set $A$ is called a $\delta$-**closed set** if $A = \text{cl}_\delta(A)$ where $\text{cl}_\delta(A)$ is the intersection of all regular closed sets of $(X, \tau)$ containing $A$.

**Definition 1.1.4** A subset $A$ of $(X, \tau)$ is called a $\delta$-**semi open set** (Park, 1997) if

$$A \subseteq \text{cl}(\text{int}_\delta(A)).$$

The complement of $\delta$-semi open is called $\delta$-**semi closed**. That is, a set $A$ is called a $\delta$-**semi closed set** if $A = \delta\text{-scl}(A)$ where $\delta\text{-scl}(A)$ is the intersection of all $\delta$-semi closed sets of $(X, \tau)$ containing $A$.

**Definition 1.1.5** A subset $A$ of a topological space $(X, \tau)$ is called

1. **generalized closed** (briefly **g-closed**) (Levine, 1970) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
2. **semi generalized closed** (briefly **sg-closed**) (Bhattacharya et.al., 1987) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $(X, \tau)$.
3. **generalized semi-closed** (briefly **gs-closed**) (Arya, 1990) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
4. **generalized $\alpha$-closed** (briefly **g$\alpha$-closed**) (Maki et.al., 1993) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.
5. **$\alpha$-generalized closed** (briefly **ag-closed**) (Maki, 1994) if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
6. **generalized semi pre-closed** (briefly **gsp-closed**) (Dontchev, 1995) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
7. **$\delta$-generalized closed** (briefly **$\delta$g-closed**) (Dontchev, 1996) if $\text{cl}_\delta(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
8. **generalized pre-closed** (briefly **gp-closed**) (Maki, 1996) if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
9. **generalized δ-closed** (briefly **gδ-closed**) (Dontchev, 2004) if cl(A) ⊆ U whenever A ⊆ U and U is δ-open in (X, τ).
10. **δg #-closed** (Dontchev, 2004) if clδ(A) ⊆ U whenever A ⊆ U and U is δ-open in (X, τ).
11. **π-generalized closed** (briefly **πg-closed**) (Dontchev et al., 2000) if cl(A) ⊆ U whenever A ⊆ U and U is π-open in (X, τ).
12. **g *-closed** (Murugalingam, 2005) if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
13. **g *p -closed** (Veera Kumar, 2002) if pcl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
14. **g -closed** (Veera Kumar, 2003) if cl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
15. **g #s -closed** (Veera Kumar, 2003) if scl(A) ⊆ U whenever A ⊆ U and U is αg-open in (X, τ).
16. **δg # -closed** (Veera Kumar, 2003) if clδ(A) ⊆ U whenever A ⊆ U and U is δ-open in (X, τ).
17. **π-generalized pre-closed** (briefly **πgp-closed**) (Park, 2006) if pcl(A) ⊆ U whenever A ⊆ U and U is π-open in (X, τ).
18. **sag * -closed** (Maragathavali et al., 2005) if acl(A) ⊆ U whenever A ⊆ U and U is g *-open in (X, τ).
19. **ag * -closed** (Maragathavali et al., 2005) if cl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ).
20. **#gs -closed** (Veera Kumar, 2005) if scl(A) ⊆ U whenever A ⊆ U and U is *g-open in (X, τ).
21. **π-generalized semi closed** (briefly **πgs-closed**) (Aslim, 2006) if scl(A) ⊆ U whenever A ⊆ U and U is π-open in (X, τ).
22. **ag -closed** (Abd El-Monsef, 2007) if acl(A) ⊆ U whenever A ⊆ U and U is g *-open in (X, τ).
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23. $\delta$-gs-closed (Park et al., 2007) if $\delta$-scl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\delta$-open in $(X, \tau)$.

24. g$\delta$s-closed (Park et al., 2007) if $\delta$-scl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

25. $\tilde{g}$s-closed (Sundaram et al., 2007) if scl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^#$gs-open in $(X, \tau)$.

26. $^*$-g-closed (Jafari, 2008) if cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$.

27. $\tilde{g}$-closed (Jafari et al., 2008) if cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^#$gs-open in $(X, \tau)$.

28. $\pi$-generalized $\alpha$-closed (briefly $\pi$ga-closed) (Janaki, 2009) if acl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$.

29. $\tilde{g}$$\alpha$-closed (Jafari et al., 2010) if acl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^#$gs-open in $(X, \tau)$.

30. $\delta$$\tilde{g}$-closed (Lellis Thivagar, 2010) if cl$_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\tilde{g}$-open in $(X, \tau)$.

31. $\pi$-generalized semi pre-closed (briefly $\pi$gsp-closed) (Sarsak, 2010) if spcl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$.

32. generalized semi pre regular-closed (briefly gspr-closed) (Navalagi, 2010) if spcl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X, \tau)$.

33. g$\tilde{g}$p-closed (Ganesan et al., 2011) if pcl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^#$gs-open in $(X, \tau)$.

34. g$^*$-s-closed (Pushpalatha, 2011) if scl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is gs-open in $(X, \tau)$.

35. $\pi$-generalized b-closed (briefly $\pi$gb-closed) (Sreeja et al., 2011) if bcl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$.

36. $\delta$g$^*$-closed (Sudha, 2012) if cl$_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$.

37. w$\delta$g$^*$-closed (Sudha, 2014) if cl$_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.

38. $\Delta^*$-closed (Meena, 2014) if cl$_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\delta$g-open in $(X, \tau)$.

39. (gs)$^*$-closed (Elvina Mary, 2014) if cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is gs-open in $(X, \tau)$.

The complements of the above mentioned closed sets are called their respective open sets.
Remark 1.1.6  regular closed (open) $\rightarrow$ $\delta$-closed (open) $\rightarrow$ $\delta g^*$-closed (open) $\rightarrow$ $\delta g$-closed (open) $\rightarrow$ g-closed (open)

Remark 1.1.7  For every subset A of X, $\text{spcl}(A) \subseteq \text{scl}(A) \subseteq \delta \text{-scl}(A) \subseteq \text{cl}_\delta(A)$.

Definition 1.1.8  A topological space $(X, \tau)$ is said to be a

1. $T_{1/2}$-space (Dunham, 1977) if every g-closed subset of $(X, \tau)$ is closed in $(X, \tau)$.
2. semi-$T_{1/2}$-space (Bhattacharya, 1987) if every sg-closed subset of $(X, \tau)$ is semi closed in $(X, \tau)$.
3. $T_b$-space (Devi, 1993) if every gs-closed subset of $(X, \tau)$ is closed in $(X, \tau)$.
4. $T_d$-space (Devi, 1993) if every gs-closed subset of $(X, \tau)$ is g-closed in $(X, \tau)$.
5. Door space (Dontchev, 1995) if every subset is either open or closed.
6. Submaximal space (Dontchev, 1995) if every dense subset of $(X, \tau)$ is open.
7. $T_{3/4}$-space (Dontchev, 1996) if every $\delta g$-closed subset of $(X, \tau)$ is $\delta$-closed in $(X, \tau)$.
8. $\alpha T_b$-space (Devi, 1998) if every $\alpha g$-closed subset of $(X, \tau)$ is closed in $(X, \tau)$.
9. $\alpha T_d$-space (Devi, 1998) if every $\alpha g$-closed subset of $(X, \tau)$ is g-closed in $(X, \tau)$.
10. $T_c$-space (Veera Kumar, 2000) if every gs-closed subset of $(X, \tau)$ is $g^*$-closed in $(X, \tau)$.
11. $\alpha T_c$-space (Veera Kumar, 2000) if every $\alpha g$-closed subset of $(X, \tau)$ is $g^*$-closed in $(X, \tau)$.
12. $T_{1/2}^*$-space (Veera Kumar, 2000) if every $^*g$-closed subset of $(X, \tau)$ is closed in $(X, \tau)$.
13. $\text{gs} T_{\alpha g^*}$-space (Sudha, 2014) if every gs-closed subset of $(X, \tau)$ is $\delta g^*$-closed in $(X, \tau)$.

Results 1.1.9  Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a function. If $A_1$ and $A_2$ are subsets of X and Y respectively then the following results are true.

1. If $A_1 \subseteq A_2$ then $f(A_1) \subseteq f(A_2)$.
2. If $A_1 \subseteq A_2$ then $f^I(A_1) \subseteq f^I(A_2)$.
3. In general, $A \subseteq f^I[f(A)]$. If $f$ is injective then $A = f^I[f(A)]$.
4. In general, $f[f^I(A)] \subseteq A$. If $f$ is injective then $A = f[f^I(A)]$.
5. If $f$ is surjective then $[f(A)]^c \subseteq f(A^c)$.
6. If $f$ is bijective then $[f(A)]^c = f(A^c)$. 

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**Definition 1.1.10** A function $f: (X, \tau) \to (Y, \sigma)$ is called

1. **Semi continuous** (Levine, 1963) if $f^{-1}(V)$ is a semi closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

2. **Continuous** (Levine, 1970) if $f^{-1}(V)$ is a closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

3. **Super continuous** (Munshi et al., 1982) if $f^{-1}(V)$ is a $\delta$-closed set of $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

4. **sg-continuous** (Bhattacharya et al., 1987) if $f^{-1}(V)$ is a sg-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

5. **g-continuous** (Balachandran, 1991) if $f^{-1}(V)$ is a g-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

6. **gs-continuous** (Devi et al., 1995) if $f^{-1}(V)$ is a gs-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

7. **gsp-continuous** (Dontchev, 1995) if $f^{-1}(V)$ is a gsp-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

8. **$g^\#s$-continuous** (Veera Kumar, 2003) if $f^{-1}(V)$ is a $g^\#s$-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

9. **$\delta$-semi continuous** (Ekici, 2005) if $f^{-1}(V)$ is a $\delta$-semi closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

10. **$\pi gs$-continuous** (Aslim, 2006) if $f^{-1}(V)$ is a $\pi gs$-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

11. **$g_{gs}$-continuous** (Sundaram et al., 2007) if $f^{-1}(V)$ is a $g_{gs}$-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

12. **$\delta gs$-continuous** (Park et al., 2007) if $f^{-1}(V)$ is a $\delta gs$-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

13. **$g\delta s$-continuous** (Park et al., 2007) if $f^{-1}(V)$ is a $g\delta s$-closed set in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.
14. \( \pi \text{gsp-continuous} \) (Sarsak, 2010) if \( f^{-1}(V) \) is a \( \pi \text{gsp-closed set} \) in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).

15. \( \text{gspr-continuous} \) (Navalagi, 2010) if \( f^{-1}(V) \) is a gspr-closed set in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).

16. \( \pi \text{gb-continuous} \) (Sreeja et.al., 2011) if \( f^{-1}(V) \) is a \( \pi \text{gb-closed set} \) in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).

17. \( g^* \text{s-continuous} \) (Pushpalatha et.al., 2011) if \( f^{-1}(V) \) is a \( g^* \text{s-closed set} \) in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).

18. \( \delta \text{g}^* \text{-continuous} \) (Sudha, 2013) if \( f^{-1}(V) \) is a \( \delta \text{g}^* \text{-closed set} \) in \((X, \tau)\) for every closed set \( V \) in \((Y, \sigma)\).

**Definition 1.1.11** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called

1. **Strongly continuous** (Levine, 1960) if the inverse image of every subset of \((Y, \sigma)\) is clopen in \((X, \tau)\).

2. **Perfectly continuous** (Noiri, 1978) if the inverse image of every closed set of \((Y, \sigma)\) is clopen in \((X, \tau)\).

3. **Totally continuous** (Jain, 1980) if the inverse image of every open set of \((Y, \sigma)\) is clopen in \((X, \tau)\).

4. **Locally continuous** (briefly \( \text{LC-continuous} \)) (Ganster, 1989) if the inverse image of every open set of \((Y, \sigma)\) is a locally closed set (briefly lc set) in \((X, \tau)\).

5. **Generalized Locally continuous** (briefly \( \text{GLC continuous} \)) (Balachandran, 1996) if the inverse image of every open set of \((Y, \sigma)\) is a generalized locally closed set (briefly glc set) in \((X, \tau)\).

6. **Contra continuous** (Dontchev, 1996) if the inverse image of every open set of \((Y, \sigma)\) is a closed set in \((X, \tau)\).

7. **Somewhat continuous** (Gentry et.al., 1971) if for \( U \in (Y, \sigma) \) and \( f^{-1}(U) \neq \emptyset \), there exists \( V \in (X, \tau) \) such that \( V \neq \emptyset \) and \( V \subset f^{-1}(U) \).

**Definition 1.1.12** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called
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1. **irresolute** (Crossley, 1972) if $f^{-1}(V)$ is a semi open set of $(X, \tau)$ for every semi open set $V$ of $(Y, \sigma)$.

2. **sg-irresolute** (Bhattacharya et al., 1987) if $f^{-1}(V)$ is a sg-open set of $(X, \tau)$ for every sg-open set $V$ of $(Y, \sigma)$.

3. **δg-irresolute** (Dontchev, 1996) if $f^{-1}(V)$ is a δg-open set of $(X, \tau)$ for every δg-open set $V$ of $(Y, \sigma)$.

4. **gs-irresolute** (Dontchev, 1996) if $f^{-1}(V)$ is a gs-open set of $(X, \tau)$ for every gs-open set $V$ of $(Y, \sigma)$.

5. **g’s-irresolute** (Veera Kumar, 2003) if $f^{-1}(V)$ is a $g^s$-open set of $(X, \tau)$ for every $g^s$-open set $V$ of $(Y, \sigma)$.

6. **g * s-irresolute** (Pushpalatha, 2011) if $f^{-1}(V)$ is a $g^*s$-open set of $(X, \tau)$ for every $g^*s$-open set $V$ of $(Y, \sigma)$.

**Definition 1.1.13** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a

1. **Semi closed** (resp. semi open) **function** (Noiri, 1973) if $f(V)$ is semi closed (resp. semi open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

2. **δ-closed** (resp. δ-open) **function** (Noiri, 1978) if $f(V)$ is δ-closed (resp. δ-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

3. **g-closed** (resp. g-open) **function** (Malghen, 1982) if $f(V)$ is g-closed (resp. g-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

4. **gs-closed** (resp. gs-open) **function** (Arya, 1990) if $f(V)$ is gs-closed (resp. gs-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

5. **gsp-closed** (resp. gsp-open) **function** (Dontchev, 1995) if $f(V)$ is gsp-closed (resp. gsp-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

6. **δ-semi closed** (resp. δ-semi open) **function** (Park, 1997) if $f(V)$ is δ-semi closed (resp. δ-semi open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

7. **πgs-closed** (resp. πgs-open) **function** (Aslim, 2006) if $f(V)$ is πgs-closed (resp. πgs-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.
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8. $\delta g s$-closed (resp. $\delta g s$-open) function (Park et.al., 2007) if $f(V)$ is $\delta g s$-closed (resp. $\delta g s$-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

9. $g \delta s$-closed (resp. $g \delta s$-open) function (Park et.al., 2007) if $f(V)$ is $g \delta s$-closed (resp. $g \delta s$-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

10. $\pi g s p$-closed (resp. $\pi g s p$-open) function (Sarsak, 2010) if $f(V)$ is $\pi g s p$-closed (resp. $\pi g s p$-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

11. $\delta g^*$-closed (resp. $\delta g^*$-open) function (Sudha, 2012) if $f(V)$ is $\delta g^*$-closed (resp. $\delta g^*$-open) in $(Y, \sigma)$ for every closed (resp. open) set $V$ of $(X, \tau)$.

12. Somewhat open (Gentry et.al., 1971) if for $U \in (X, \tau)$ and $U \neq \emptyset$, there exists $V \in (Y, \sigma)$ such that $V \neq \emptyset$ and $V \subset f(U)$.

Definition 1.1.14 A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

1. g-homeomorphism (Maki, 1991) if $f$ is bijective, g-open and g-continuous.

2. gs-homeomorphism (Devi, 1995) if $f$ is bijective, gs-open and gs-continuous.

3. sg-homeomorphism (Devi, 1995) if $f$ is bijective, sg-open and sg-continuous.

4. gsp-homeomorphism (Dontchev, 1995) if $f$ is bijective, gsp-open and gsp-continuous.

5. $\pi g s p$-homeomorphism (Aslim, 2006) if $f$ is bijective, $\pi g s p$-open and $\pi g s p$-continuous.

6. $\delta g s$-homeomorphism (Park et.al., 2007) if $f$ is bijective, $\delta g s$-open and $\delta g s$-continuous.

7. $g \delta s$-homeomorphism (Park et.al., 2007) if $f$ is bijective, $g \delta s$-open and $g \delta s$-continuous.

8. $\pi g s p$-homeomorphism (Sarsak, 2010) if $f$ is bijective, $\pi g s p$-open and $\pi g s p$-continuous.

Definition 1.1.15 A subset $A$ of a topological space $(X, \tau)$ is called

1. Locally closed (briefly lc) (Ganster, 1989) if $A = U \cap F$ where $U$ is open and $F$ is closed in $(X, \tau)$. The class of all locally closed sets are denoted by LC$(X, \tau)$.

2. $\delta$ Locally closed (briefly $\delta$lc) (Sudha, 2014) if $A = U \cap F$ where $U$ is $\delta$-open and $F$ is $\delta$-closed in $(X, \tau)$. The class of all $\delta$ locally closed sets are denoted by $\delta$LC$(X, \tau)$.

3. Generalized locally closed (briefly glc) (Balachandran, 1996) if $A = U \cap F$ where $U$ is g-open and $F$ is g-closed in $(X, \tau)$. The class of all g-locally closed sets are denoted by GLC$(X, \tau)$.
4. **δ-semi Locally closed** (briefly δslc) (Park, 1997) if $A = U \cap F$ where $U$ is δ-semi open and $F$ is δ-semi closed in $(X, \tau)$. The class of all δ-semi locally closed sets are denoted by $\delta\text{SLC}(X, \tau)$.

5. **Generalized locally semi closed** (briefly glsc) (Gnanambal, 1998) if $A = U \cap F$ where $U$ is g-open and $F$ is semi closed in $(X, \tau)$. The class of all glsc sets are denoted by $\text{GLSC}(X, \tau)$.

**Definition 1.1.16** A topological space $(X, \tau)$ is said to be **Resolvable** (Ganster, 1987) if there exists a dense set $A$ in $(X, \tau)$ such that $X \setminus A$ is also dense in $(X, \tau)$. Otherwise, $(X, \tau)$ is called irresolvable.

**Definition 1.1.17** If $X$ is a set, $\tau$ and $\sigma$ are topologies on $X$, then $\tau$ is said to be **Equivalent** (Gentry et.al., 1971) to $\sigma$ provided if $U \in \tau$ and $U \neq \emptyset$, then there is an open set $V$ in $X$ such that $V \neq \emptyset$ and $V \subseteq U$ and if $U \in \sigma$ and $U \neq \emptyset$, then there is an open set $V$ in $(X, \tau)$ such that $V \neq \emptyset$ and $U \supset V$. 

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“(X, τ), (Y, σ) and (Z, η) denote non-empty topological spaces on which no separation axioms are mentioned, unless it is stated specifically”

\[ \text{G}(X, \tau) \] - The class of all g-closed sets of \((X, \tau)\).

\[ \text{SG}(X, \tau) \] - The class of all sg-closed sets of \((X, \tau)\).

\[ \text{GSC}(X, \tau) \] - The class of all gs-closed sets of \((X, \tau)\).

\[ \text{GaC}(X, \tau) \] - The class of all ga-closed sets of \((X, \tau)\).

\[ \text{αG}(X, \tau) \] - The class of all αg-closed sets of \((X, \tau)\).

\[ \text{GSPC}(X, \tau) \] - The class of all gsp-closed sets of \((X, \tau)\).

\[ \text{δC}(X, \tau) \] - The class of all δ-closed sets of \((X, \tau)\).

\[ \text{δSC}(X, \tau) \] - The class of all δ-semi closed sets of \((X, \tau)\).

\[ \text{δG}(X, \tau) \] - The class of all δg-closed sets of \((X, \tau)\).

\[ \text{GPC}(X, \tau) \] - The class of all gp-closed sets of \((X, \tau)\).

\[ \text{GδC}(X, \tau) \] - The class of all gδ-closed sets of \((X, \tau)\).

\[ \text{δG}(X, \tau) \] - The class of all δg-closed sets of \((X, \tau)\).

\[ \text{G}^*\text{C}(X, \tau) \] - The class of all g*-closed sets of \((X, \tau)\).

\[ \text{GP}(X, \tau) \] - The class of all gp*-closed sets of \((X, \tau)\).

\[ \text{G}(X, \tau) \] - The class of all g-closed sets of \((X, \tau)\).

\[ \text{G}^\#\text{S}(X, \tau) \] - The class of all g"s-closed sets of \((X, \tau)\).

\[ \text{Gδ}(X, \tau) \] - The class of all gδ-closed sets of \((X, \tau)\).

\[ \text{δG}^\#\text{C}(X, \tau) \] - The class of all δg"-closed sets of \((X, \tau)\).

\[ \text{πG}(X, \tau) \] - The class of all πg-closed sets of \((X, \tau)\).

\[ \text{δG}^\dagger\text{C}(X, \tau) \] - The class of all δg"'-closed sets of \((X, \tau)\).

\[ \text{αG}^*\text{C}(X, \tau) \] - The class of all αg*-closed sets of \((X, \tau)\).
Chapter 1

# $\text{GSC}(X, \tau)$ - The class of all $^g_{\text{gs}}$-closed sets of $(X, \tau)$.

$\pi\text{GSC}(X, \tau)$ - The class of all $\pi_{\text{gs}}$-closed sets of $(X, \tau)$.

$\alpha\text{G}(X, \tau)$ - The class of all $\alpha_{\hat{g}}$-closed sets of $(X, \tau)$.

$\delta\text{GSC}(X, \tau)$ - The class of all $\delta_{\text{gs}}$-closed sets of $(X, \tau)$.

$G\delta\text{SC}(X, \tau)$ - The class of all $g\delta_{\text{s}}$-closed sets of $(X, \tau)$.

$\tilde{G}_{s}(X, \tau)$ - The class of all $\tilde{g}_{s}$-closed sets of $(X, \tau)$.

$^*G\text{C}(X, \tau)$ - The class of all $^*g$-closed sets of $(X, \tau)$.

$\tilde{G}\text{C}(X, \tau)$ - The class of all $\tilde{g}$-closed sets of $(X, \tau)$.

$\pi\text{G}_{\text{a}}\text{C}(X, \tau)$ - The class of all $\pi_{g_{\alpha}}$-closed sets of $(X, \tau)$.

$\tilde{G}_{\alpha}(X, \tau)$ - The class of all $\tilde{g}_{\alpha}$-closed sets of $(X, \tau)$.

$\delta\tilde{G}\text{C}(X, \tau)$ - The class of all $\delta\tilde{g}$-closed sets of $(X, \tau)$.

$\pi\text{GSP}\text{C}(X, \tau)$ - The class of all $\pi_{g_{\text{sp}}}$-closed sets of $(X, \tau)$.

$\text{GSPr}(X, \tau)$ - The class of all $g_{\text{sp}}$-closed sets of $(X, \tau)$.

$\tilde{G}_{p}\text{C}(X, \tau)$ - The class of all $\tilde{g}_{p}$-closed sets of $(X, \tau)$.

$G^*\text{SC}(X, \tau)$ - The class of all $g^*_{\text{s}}$-closed sets of $(X, \tau)$.

$\pi\text{G}_{\text{b}}\text{C}(X, \tau)$ - The class of all $\pi_{g_{b}}$-closed sets of $(X, \tau)$.

$\delta G^*\text{C}(X, \tau)$ - The class of all $\delta g^*$-closed sets of $(X, \tau)$.

$W\delta G^*\text{C}(X, \tau)$ - The class of all $W\delta g^*$-closed sets of $(X, \tau)$.

$\Delta^*\text{C}(X, \tau)$ - The class of all $\Delta^*$-closed sets of $(X, \tau)$.

$(\text{GS})^*\text{C}(X, \tau)$ - The class of all $(gs)^*$-closed sets of $(X, \tau)$.

$L\text{C}(X, \tau)$ - The class of all locally closed sets of $(X, \tau)$.

$\delta L\text{C}(X, \tau)$ - The class of all $\delta$-locally closed sets of $(X, \tau)$.

$\text{GLC}(X, \tau)$ - The class of all generalized locally closed sets of $(X, \tau)$.

$\delta\text{SLC}(X, \tau)$ - The class of all $\delta$-semi locally closed sets of $(X, \tau)$.

$\text{GLSC}(X, \tau)$ - The class of all generalized locally semi closed sets of $(X, \tau)$. 