CHAPTER 4

RELIABILITY AND PROFIT ANALYSES OF THE SYSTEMS
INCORPORATING THE POSSIBILITIES OF IMPROPER REPAIRS

INTRODUCTION

In the previous chapter, the analyses of two stochastic models for the systems have been discussed for the situations when proper repairs of the minor/major faults have been carried by the repairman. However, while collecting data on faults, maintenances, inspections, repairs etc. of the water systems working in Thermal Power Plant, Panipat (Haryana), it was also observed that some faults such as Raw Water Pump Vibration, Service Water Pump Tripping etc. and in non-redundant subsystems such as Gland Steam Cooler, Low Pressure Heater, Economizer and Boiler Drum etc. do not get repaired sometimes properly by the available repairman and then these faults lead to complete failure of the system. In those situations, an expert repairman is called provided he is available. Taking in to account this practical situation the present chapter is an attempt to analyse the systems.

In this chapter, two stochastic models for water process systems have been discussed incorporating the possibilities of improper repairs of minor faults by the available ordinary repairman in addition to the aspects taken in the Chapter 3. The models have been developed considering the observations taken that whenever a minor/major fault occurs in the system, then ordinary repairman first inspects the system to judge which subsystem has the fault and accordingly carries out the repair of the defected subsystem. But the ordinary repairman sometimes is not able to carry out the repair properly, i.e. improper repair is carried out by the ordinary repairman. In that situation and also on complete failure of the system the expert repairman is called provided he is available. During repair of a major fault of redundant subsystem if major/minor faults occur in some other subsystem then expert repairman called is assumed to repair completely all the subsystems within the redundant subsystems. Further the ordinary/ expert repairman assumed to takes negligible time to reach the system. The first model (say, Model-I) has been developed considering taking care of the
situations that improper repairs are done by the ordinary repairman when the expert repairman in not available and there is no provision of switching of the standby subsystems in the redundant subsystems on occurrence of major faults whereas the second model (say, Model-II) has been developed considering the situations that improper repairs are carried out by the ordinary repairman when the expert repairman is available and there is automatic switching of the cold standby subsystem is taken in to account on occurrence of major faults in the subsystem. Rests of the assumptions taken are as given in Chapter 3.

In addition to “State transition probabilities” and “Mean sojourn times” of the systems, the following measures of system effectiveness are obtained using “Markov processes” and “Regenerative point technique:

- Mean time to system failure
- Expected uptime of the system with full capacity
- Expected uptime of the system with reduced capacity
- Busy period of ordinary repairman for repairs only
- Busy period of ordinary repairman for inspections only
- Busy period of expert repairman for repairs only

The expected profit incurred to the system is also computed by making use of the above measures. The conclusion regarding the reliability and cost-benefit of the systems are drawn on the basis of graphical studies.

STATES OF THE SYSTEM

- $O$: Operative system.
- $O_{f}/F_{i}$: Operative/ failed system under inspection.
- $F_{r}$: Failed system under repair by the ordinary/expert repairman.
- $O_{RD_{i}}$: Operative redundant subsystem under inspection.
- $O_{RD-Ir}/O_{RD-IIr}$: Operative Type-I/ Type-II redundant subsystem under repair by the ordinary repairman.
- $O_{RD-Ir_{e}}/O_{RD-IIr_{e}}$: Operative Type-I/ Type-II redundant subsystem under repair by the expert repairman
O_{NRD}/F_{NRD} : Operative/ failed non-redundant subsystem under repair by the ordinary repairman
F_{NRDre} : Failed non-redundant subsystem under repair by the expert repairman
F_{RD-I}/F_{RD-II} : Failed Type-I/ Type-II redundant subsystem under repair.
F_{RD-Ir}/F_{RD-IIr} : Failed Type-I/ Type-II redundant subsystem under repair from the previous state.
F_{RD-Ire}/F_{RD-IIre} : Failed Type-I/ Type-II redundant subsystem under repair by the expert repairman.

NOTATIONS

\[ \lambda_1 / \lambda_2 : \text{Rate of major/minor faults.} \]
\[ x / y : \text{Probability of fault in non-redundant/redundant subsystem.} \]
\[ a / b : \text{Probability of fault in Type-I/Type-II redundant subsystem,} \]
\[ p_1 / p_2 / p_3 : \text{Probability of proper repair of non-redundant subsystem/ Type-I redundant subsystem/ Type-II redundant subsystem.} \]
\[ q_1 / q_2 / q_3 : \text{Probability of improper repair in non-redundant subsystem/ Type-I redundant subsystem/ Type-II redundant subsystem.} \]
\[ i_1(t) / I_1(t) : \text{P.d.f./c.d.f. of time to inspect the system.} \]
\[ i_2(t) / I_2(t) : \text{P.d.f./c.d.f. of time to inspect the redundant subsystem.} \]
\[ g(t) / G(t) : \text{P.d.f./c.d.f. of time to repair a major fault.} \]
\[ g_1(t) / G_1(t) : \text{P.d.f./c.d.f. of time to repair the non-redundant subsystem.} \]
\[ g_2(t) / G_2(t) : \text{P.d.f./c.d.f. of time to repair the Type-I redundant subsystem.} \]
\[ g_3(t) / G_3(t) : \text{P.d.f./c.d.f. of time to repair the Type-II redundant subsystem.} \]
\[ g_e(t)/G_e(t) : \text{P.d.f./c.d.f. of time to repair the system by the expert repairman.} \]

MODEL-I

The state transition diagram depicting the various states of transition is shown in fig.4.1. The epochs of entry in to state 0, 1, 2, 3, 4, 5, 6 are regenerative points, i.e. the states are regenerative states while the state 2 is a failed state.
TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The state transition probabilities are obtained as under:

\[ dQ_{01} (t) = \lambda_2 e^{-(\lambda_1 + \lambda_2)t} dt \]
\[ dQ_{13} (t) = x_i (t) dt \]
\[ dQ_{20} (t) = g(t) dt \]
\[ dQ_{32} (t) = q_1 \lambda_2 e^{-(\lambda_2)t} dt \]
\[ dQ_{46} (t) = b_i (t) dt \]
\[ dQ_{52} (t) = q_2 \lambda_2 e^{-(\lambda_2)t} dt \]
\[ dQ_{62} (t) = q_3 \lambda_2 e^{-(\lambda_2)t} dt \]
\[ dQ_{02} (t) = \lambda_1 e^{-(\lambda_1 + \lambda_2)t} dt \]
\[ dQ_{14} (t) = y_i (t) dt \]
\[ dQ_{45} (t) = a_i (t) dt \]
\[ dQ_{50} (t) = p_i g(t) (t) dt \]
\[ dQ_{56} (t) = p_2 g(t) (t) dt \]
\[ dQ_{60} (t) = p_3 g(t) (t) dt \]

**Fig. 4.1 State Transition Diagram**
Then the non-zero elements $p_{ij}$ are given by

$$p_{ij} = \lim_{s \to 0} Q_{ij}^*(s),$$

where $Q_{ij}^*(s)$ is Laplace Stieltjes transformation of $Q_{ij}(t)$.

Thus, we have

$$p_{01} = \frac{\lambda_2}{\lambda_1 + \lambda_2}, \quad p_{02} = \frac{\lambda_1}{\lambda_1 + \lambda_2},$$

$$p_{13} = x_{i1}(0), \quad p_{14} = y_{i1}(0),$$

$$p_{20} = g^*(0), \quad p_{30} = p_{1}z_1^*(0),$$

$$p_{32} = q_1, \quad p_{45} = a_i^2(0),$$

$$p_{46} = b_i^2(0), \quad p_{50} = p_2g_2^*(0),$$

$$p_{52} = q_2, \quad p_{60} = p_3g_3^*(0),$$

$$p_{62} = q_3$$

By these transition probabilities, it can be verified that

$$p_{01} + p_{02} = 1,$$  \hspace{1cm}  $$p_{13} + p_{14} = 1,$$

$$p_{30} + p_{32} = 1,$$  \hspace{1cm}  $$p_{45} + p_{46} = 1,$$

$$p_{50} + p_{52} = 1,$$  \hspace{1cm}  $$p_{60} + p_{62} = 1,$$

$$p_{20} = 1$$

“The unconditional mean time taken by the system to transit for any regenerative state ‘$j$’, when it is counted from epoch of entrance into that state ‘$i$’, is mathematically stated as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^*(0),$$

Thus, we have

$$m_{01} + m_{02} = \mu_0$$  \hspace{1cm}  $$m_{13} + m_{14} = \mu_1,$$

$$m_{20} = \mu_2$$  \hspace{1cm}  $$m_{30} + m_{32} = \mu_3,$$

$$m_{45} + m_{46} = \mu_4$$  \hspace{1cm}  $$m_{50} + m_{52} = \mu_5,$$

$$m_{60} + m_{62} = \mu_6.$$
“The mean sojourn time ($\mu_i$) in the regenerative state ‘i’ is defined as the time of stay in that state before transition to any other state”. If ‘$T_i$’ denotes the sojourn time in regenerative state ‘i’, then

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2} \quad \mu_1 = -i'_i(0)$$

$$\mu_2 = -g''(0) \quad \mu_3 = \frac{q_i}{\lambda_2} - p_i g'(0)$$

$$\mu_4 = -i''_i(0) \quad \mu_5 = \frac{q_i}{\lambda_2} - p_i g'(0)$$

$$\mu_6 = \frac{q_i}{\lambda_2} - p_i g'(0)$$

**MEAN TIME TO SYSTEM FAILURE**

The mean time to system failure (MTSF) has been obtained by considering the failed states of the system as absorbing states. The following recursive relations for $\phi_i(t)$, c.d.f of the first passage time from regenerative state ‘i’ to failed state are obtained using concepts of probability:

$$\phi_0(t) = Q_{01}(t) \otimes \phi_0(t) + Q_{02}(t)$$

$$\phi_i(t) = Q_{i1}(t) \otimes \phi_3(t) + Q_{i4}(t) \otimes \phi_4(t)$$

$$\phi_3(t) = Q_{31}(t) \otimes \phi_0(t) + Q_{32}(t)$$

$$\phi_4(t) = Q_{45}(t) \otimes \phi_3(t) + Q_{46}(t) \otimes \phi_6(t)$$

$$\phi_5(t) = Q_{50}(t) \otimes \phi_0(t) + Q_{52}(t)$$

$$\phi_6(t) = Q_{60}(t) \otimes \phi_0(t) + Q_{62}(t)$$

These equations are solved using Laplace Stieltjes transformation and $\phi^{**}(s)$, is obtained as under:

$$\phi^{**}(s) = \frac{N(s)}{D(s)}$$

where

$$N(s) = Q^{**}_{02}(s) + Q^{**}_{01}(s)\left[ Q^{**}_{13}(s)Q^{**}_{32}(s) + Q^{**}_{14}(s)\left\{ Q^{**}_{45}(s)Q^{**}_{52}(s) + Q^{**}_{46}(s)Q^{**}_{62}(s) \right\} \right]$$

and

$$D(s) = 1 - Q^{**}_{01}(s)\left[ Q^{**}_{13}(s)Q^{**}_{36}(s) + Q^{**}_{14}(s)\left\{ Q^{**}_{45}(s)Q^{**}_{50}(s) + Q^{**}_{46}(s)Q^{**}_{60}(s) \right\} \right]$$
The mean time to system failure when the system starts from the state 0, is defined by

\[ T_0 = \lim_{s \to 0} \frac{1 - \phi_0^*(s)}{s} \]

Substituting the value of \( \phi_0^*(s) \) and using L' Hospital Rule, we get

\[ T_0 = \frac{N}{D} \]

where

\[ N = \mu_0 + p_{01} \left[ \mu_1 + p_{13} \mu_3 + p_{14} (\mu_4 + p_{45} \mu_5 + p_{46} \mu_6) \right] \]

and

\[ D = 1 - p_{01} \left[ p_{13} p_{30} + p_{14} (p_{45} p_{50} + p_{46} p_{60}) \right] \]

**EXPECTED UPTIME OF THE SYSTEM WITH FULL CAPACITY**

The expected uptime of the system with full capacity \( \text{AF}_i(t) \), the probability that the system is up at instant ‘t’ with full capacity given that it entered regenerative state ‘i’ at t = 0, by making use of the concepts of probability in theory of regenerative processes, gives the following recursive relations:

\[ \text{AF}_0(t) = M_0(t) + q_{01}(t) \odot \text{AF}_1(t) + q_{02}(t) \odot \text{AF}_2(t) \]

\[ \text{AF}_1(t) = q_{13}(t) \odot \text{AF}_3(t) + q_{14}(t) \odot \text{AF}_4(t) \]

\[ \text{AF}_2(t) = q_{20}(t) \odot \text{AF}_5(t) \]

\[ \text{AF}_3(t) = q_{30}(t) \odot \text{AF}_6(t) + q_{32}(t) \odot \text{AF}_2(t) \]

\[ \text{AF}_4(t) = q_{45}(t) \odot \text{AF}_5(t) + q_{46}(t) \odot \text{AF}_6(t) \]

\[ \text{AF}_5(t) = q_{50}(t) \odot \text{AF}_0(t) + q_{52}(t) \odot \text{AF}_2(t) \]

\[ \text{AF}_6(t) = q_{60}(t) \odot \text{AF}_0(t) + q_{62}(t) \odot \text{AF}_2(t) \]

where

\[ M_0(t) = e^{-(\lambda_1 + \lambda_2)t} \]

These equations are solved using Laplace transform and \( \text{AF}_0^*(s) \), is obtained as under:

\[ \text{AF}_0^*(s) = \frac{N_1(s)}{D_1(s)} \]

where

\[ N_1(s) = M_0^*(s) \]
and
\[
D_1(s) = 1 - q_{0^*}(s)q_{20^*}(s) - q_{01^*}(s) \left[ q_{13^*}(s) \left( q_{32^*}(s)q_{20^*}(s) + q_{30^*}(s) \right) + q_{14^*}(s) \left( q_{42^*}(s)q_{20^*}(s) + q_{40^*}(s) \right) \right] \\
+ q_{14^*}(s) \left( q_{42^*}(s) \left( q_{32^*}(s)q_{20^*}(s) + q_{30^*}(s) \right) \right)
\]

“In steady state the expected uptime of the system with full capacity is defined by
\[
AF_0 = \lim_{s \to 0} \left( sAF_0^*(s) \right).
\]

Thus, we have \( AF_0 = \frac{N}{D_1} \),

where
\[
N_i = \mu_i
\]

and
\[
D_i = \mu_i + p_{02} \mu_2 + p_{01} \left[ \mu_1 + \mu_3 + p_{33} \mu_2 \right] + p_{14} \left[ \mu_4 + p_{45} \left( \mu_5 + p_{53} \mu_2 \right) + p_{46} \left( \mu_6 + p_{63} \mu_2 \right) \right]
\]

EXPECTED UPTIME OF THE SYSTEM WITH REDUCED CAPACITY

In the similar way, the expected uptime of the system with reduced capacity \( AR_i(t) \), the probability that the system is up at instant ‘t’ with reduced capacity given that it entered regenerative state ‘i’ at \( t = 0 \), can be obtained by using the following recursive relations:

\[
AR_0(t) = q_{01}(t) \odot AR_1(t) + q_{02}(t) \odot AR_2(t)
\]

\[
AR_1(t) = M_1(t) + q_{13}(t) \odot AR_2(t) + q_{14}(t) \odot AR_3(t)
\]

\[
AR_2(t) = q_{20}(t) \odot AR_0(t)
\]

\[
AR_3(t) = M_3(t) + q_{30}(t) \odot AR_0(t) + q_{32}(t) \odot AR_2(t)
\]

\[
AR_4(t) = M_4(t) + q_{45}(t) \odot AR_5(t) + q_{46}(t) \odot AR_6(t)
\]

\[
AR_5(t) = M_5(t) + q_{50}(t) \odot AR_0(t) + q_{52}(t) \odot AR_2(t)
\]

\[
AR_6(t) = M_6(t) + q_{60}(t) \odot AR_0(t) + q_{62}(t) \odot AR_2(t)
\]
where
\[ M_1(t) = \overline{I}_1(t); \quad M_3(t) = \overline{G}_1(t)e^{-2_3(t)}; \]
\[ M_4(t) = \overline{I}_2(t); \quad M_5(t) = \overline{G}_2(t); \]
\[ M_6(t) = \overline{G}_3(t) \]

These equations are solved using Laplace transform and \( \text{AR}_0^*(s) \), is obtained as under:
\[ \text{AR}_0^*(s) = \frac{N_2(s)}{D_1(s)}, \]
where
\[ N_2(s) = q_{01}^*(s) \left[ M_1^*(s) + q_{13}^*(s) M_3^*(s) \right] \]
\[ + q_{14}^*(s) \left[ M_4^*(s) + q_{45}^*(s) M_5^*(s) + q_{46}^*(s) M_6^*(s) \right] \]
and
\[ D_1(s) \text{ is as defined earlier.} \]

“In steady state the expected uptime of the system with reduced capacity is defined by
\[ \text{AR}_0 = \lim_{s \to 0} \left( s \text{AR}_0^*(s) \right). \]
Thus, we have \( \text{AR}_0 = \frac{N_2}{D_1}, \)
where
\[ N_2 = p_{01} \left[ \mu_1 + p_{13} \mu_3 + p_{14} \left( \mu_4 + p_{45} \mu_5 + p_{46} \mu_6 \right) \right] \]
and
\[ D_1 \text{ is as defined earlier.} \]

**BUSY PERIOD OF ORDINARY REPAIR MAN (INSPECTION TIME ONLY)**

The following recursive relations for \( B_{i0}(t) \) are obtained using the concepts of probability in regenerative process:
\[ B_{i0}(t) = q_{i0}(t) \odot B_{i1}(t) + q_{i2}(t) \odot B_{i2}(t) \]
\[ B_{i1}(t) = W_1(t) + q_{13}(t) \odot B_{i3}(t) + q_{14} \odot B_{i4}(t) \]
\[ B_{i2}(t) = q_{20}(t) \odot B_{i0}(t) \]
\[ B_{i3}(t) = q_{30}(t) \odot B_{i0}(t) + q_{32}(t) \odot B_{i2}(t) \]
\[ B_{i4}(t) = W_4(t) + q_{45}(t) \odot B_{i5}(t) + q_{46}(t) \odot B_{i6}(t) \]
\[ B_{i5}(t) = q_{50}(t) \odot B_{i0}(t) + q_{52}(t) \odot B_{r2}(t) \]
\[ B_{i6}(t) = q_{60}(t) \odot B_{i0}(t) + q_{62}(t) \odot B_{r2}(t), \]

where
\[ W_i(t) = \overline{I}_1(t); \quad W_4(t) = \overline{I}_2(t) \]

These equations are solved using Laplace transform and \( B_{i0}^*(s) \), is obtained as under:
\[ B_{i0}^*(s) = \frac{N_3(s)}{D_1(s)}, \]

where
\[ N_3(s) = q_{i0}(s) \left[ W_1^*(s) + q_{i4}(s) W_4^*(s) \right] \]

and
\[ D_1(s) \text{ is as defined earlier.} \]

“In steady state busy period of the ordinary repairman for inspection of the system is defined by
\[ B_{i0} = \lim_{s \to 0} \left( s B_{i0}^*(s) \right) \]." Thus, we have \( B_{i0} = \frac{N_3}{D_1}, \)

where
\[ N_3 = p_{01}(\mu_1 + p_{i4}\mu_i) \]

and \( D_1 \) is as defined earlier

**BUSY PERIOD OF ORDINARY REPAIR MAN (REPAIR TIME ONLY)**

The following recursive relations for \( B_{i0}(t) \) are obtained using the concepts of probability in regenerative process:
\[ B_{r0}(t) = q_{01}(t) \odot B_{r1}(t) + q_{02}(t) \odot B_{r2}(t) \]
\[ B_{r1}(t) = q_{i3}(t) \odot B_{r3}(t) + q_{i4} \odot B_{r4}(t) \]
\[ B_{r2}(t) = W_{2}(t) + q_{20}(t) \odot B_{r0}(t) \]
\[ B_{r3}(t) = W_{3}(t) + q_{30}(t) \odot B_{r0}(t) + q_{32}(t) \odot B_{r2}(t) \]
\[ B_{r4}(t) = q_{45}(t) \odot B_{r5}(t) + q_{46}(t) \odot B_{r6}(t) \]
\[ B_{r5}(t) = W_{5}(t) + q_{50}(t) \odot B_{r0}(t) \]
\[ B_{r6}(t) = W_{6}(t) + q_{60}(t) \odot B_{r0}(t) \]
where
\[ W_2(t) = \bar{G}(t) ; \quad W_3(t) = \bar{G}_1(t) e^{-\lambda t} ; \]
\[ W_5(t) = \bar{G}_2(t) ; \quad W_6(t) = \bar{G}_3(t) \]

These equations are solved using Laplace transform and \( B_{ro}^*(s) \), is obtained as under:
\[ B_{ro}^*(s) = \frac{N_4(s)}{D_1(s)} , \]

where
\[ N_4(s) = q_{02}(s)W_2^*(s) + q_{01}(s)\left[ q_{13}^*(s)(q_{32}(s)W_2^*(s) + W_3^*(s)) \right. \\
\left. + q_{14}^*(s)(q_{45}(s)W_2^*(s) + W_5^*(s)) \right] + q_{46}^*(s)(q_{62}(s)W_2^*(s) + W_6^*(s)) \]

and
\[ D_1(s) \]

is as defined earlier.

“In steady state busy period of the ordinary repairman for repair of the system is defined by
\[ B_{ro} = \lim_{s \to 0} (sB_{ro}^*(s))'' \].

Thus, we have
\[ B_{ro} = \frac{N_4}{D_1} , \]

where
\[ N_4 = p_{02}(\mu_2 + \mu_3) + p_{01}\left[ p_{13}(p_{33}\mu_2 + \mu_3) + p_{14}\left( p_{45}(p_{53}\mu_2 + \mu_5) + p_{46}(p_{63}\mu_2 + \mu_6) \right) \right] \]

and
\[ D_1 \]

is as defined earlier.

**PROFIT ANALYSIS**

For the system, the expected profit incurred is given by
\[ P = C_0A_0 + C_1A_0 - C_2B_{i0} - C_3B_{r0} - C_4 \]

where
\[ C_0 = \text{revenue per unit uptime of the system working with full capacity} \]
\[ C_1 = \text{revenue per unit uptime of the system working with reduced capacity} \]
\[ C_2 = \text{cost per unit time of inspection by the ordinary repairman} \]
\[ C_3 = \text{cost per unit time of repair by the ordinary repairman} \]
\[ C_4 = \text{cost per unit installation} \]
The graphical analysis of the system has been carried out using the following particular case along with the estimated values of the parameters given in Chapter 2:

\[ i_1(t) = \alpha_1 e^{-\alpha_1 t}; \quad i_2(t) = \alpha_2 e^{-\alpha_2 t}; \]
\[ g(t) = \beta e^{-\beta t}; \quad g_i(t) = \beta_i e^{-\beta_i t}; \]
\[ g_2(t) = \beta_2 e^{-\beta_2 t}; \quad g_3(t) = \beta_3 e^{-\beta_3 t} \]

Then, we get

\[ p_{01} = \frac{\lambda_2}{\lambda_1 + \lambda_2} \quad p_{02} = \frac{\lambda_1}{\lambda_1 + \lambda_2} \]
\[ p_{13} = x \quad p_{14} = y \]
\[ p_{20} = 1 \quad p_{30} = p_1 \]
\[ p_{32} = q_1 \quad p_{45} = a \]
\[ p_{46} = b \quad p_{50} = p_2 \]
\[ p_{52} = q_2 \quad p_{60} = p_3 \]
\[ p_{62} = q_3 \quad \mu_0 = \frac{1}{\lambda_1 + \lambda_2} \]
\[ \mu_1 = \frac{1}{\alpha_1} \quad \mu_2 = \frac{1}{\beta} \]
\[ \mu_3 = \frac{q_1}{\lambda_2} - p_1 \quad \mu_4 = \frac{1}{\alpha_2} \]
\[ \mu_5 = \frac{q_2}{\lambda_2} - p_2 \quad \mu_6 = \frac{q_3}{\lambda_2} - p_3 \]

Various graphs are plotted for “Mean time to system failure”, “Expected uptime of the system with full capacity”, “Expected uptime of the system with reduced capacity”, “Busy periods of the ordinary repairman” and “Profit” of the system by taking rates of different faults (\( \lambda_1 \) and \( \lambda_2 \)), repair rates (\( \beta, \beta_1, \beta_2 \) and \( \beta_3 \)) and inspection rates (\( \alpha_1 \) and \( \alpha_2 \)) and various probabilities (a, b, x, y, \( p_1, p_2, p_3, q_1, q_2 \) and \( q_3 \)).
Following interpretations and conclusions have been made from the graphs:

**Fig.4.2** gives the graph between MTSF ($T_0$) and rate of minor faults ($\lambda_2$) for different values of rate of major faults ($\lambda_1$). It is concluded that that the MTSF decreases with increase in the values of the rate of major faults and MTSF has lower values for higher values of the rate of minor faults.

**Fig.4.3**
The graph in fig.4.3 gives the behavior of MTSF ($T_0$) with respect to rate of major faults ($\lambda_1$) for different values of probability of fault in Type-I redundant subsystem ($a$). The graph reveals that the MTSF decreases with increase in the values of the rate of major faults. However, MTSF has higher values for higher values of the probability of fault in Type-I redundant subsystem.

![MTSF VERSUS PROBABILITY OF PROPER REPAIR OF TYPE-II REDUNDANT SUBSYSTEM ($p_3$) FOR DIFFERENT VALUES OF PROBABILITY OF FAULTS IN NON-REDUNDANT SUBSYSTEM ($x$)](image)

**Fig.4.4**

The curves in fig.4.4 show the behaviour of MTSF ($T_0$) with respect to probability of proper repair of Type-II redundant subsystem ($p_3$) for different values of probability of fault in non-redundant subsystem ($x$). The graph reveals that the MTSF increases with increase in the values of the probability of proper repair of Type-II redundant subsystem and MTSF has lower values for higher values of the probability of fault in non-redundant subsystem.

**Fig.4.5** presents the graph between expected uptime of the system with full capacity ($AF_0$) and rate of minor faults ($\lambda_2$) for different values of rate of major faults ($\lambda_1$). It can be concluded that the expected uptime of the system with full capacity decreases with increase in the values of the rate of minor faults and the expected uptime of the system with full capacity has lower values for higher values of the rate of major faults.
The curves in Fig.4.6 show the behavior of expected uptime of the system with full capacity ($AF_0$) with respect to rate of major faults ($\lambda_1$) for different values of probability of fault in Type-I redundant subsystem (a). The graph reveals that the expected uptime of the system with full capacity decreases with increase in the values of...
the rate of major faults while the expected uptime of the system with full capacity has higher values for higher values of the probability of fault in Type-I redundant subsystem.

**Fig.4.7**

**Fig.4.7** gives the graph between expected uptime of the system with full capacity (AF₀) and probability of proper repair of non-redundant subsystem (p₁) for different values of inspection rate (α₁). The graph reveals that the expected uptime of the system with full capacity decreases with increase in the values of the probability of proper repair of non-redundant subsystem and the expected uptime of the system with full capacity has higher values for higher values of the inspection rate.

The curves in **Fig.4.8** depict the behaviour of expected uptime of the system with full capacity (AF₀) with respect to probability of proper repair of Type-I redundant subsystem (p₂) for different values of inspection rate (α₂). It can be concluded that the expected uptime of the system with full capacity increases with increase in the values of the probability of proper repair of Type-I redundant subsystem and the expected uptime of the system with full capacity has higher values for higher values of inspection rate.
The graph in **Fig.4.9** shows the pattern of expected uptime of the system with full capacity ($AF_0$) with respect to probability of proper repair of Type-II redundant subsystem ($p_3$) for different values of probability of fault in non-redundant subsystem ($x$). From the graph, it has been concluded that the expected uptime of the system with full
capacity increases with increase in the values of the probability of proper repair of Type-II redundant subsystem and the expected uptime of the system with full capacity has lower values for higher values of probability of fault in non-redundant subsystem.

![Graph](image)

**Fig.4.10**

**Fig.4.10** presents the graph between expected uptime of the system with reduced capacity (\(AR_0\)) and rate of minor faults (\(\lambda_2\)) for different values of the rate of major faults (\(\lambda_1\)). The graph reveals that the expected uptime of the system with reduced capacity increases with increase in the values of the rate of minor faults while the expected uptime of the system with reduced capacity has lower values for higher values of the rate of major faults.

The graph in **Fig.4.11** exhibits the behaviour of expected uptime of the system with reduced capacity (\(AR_0\)) with respect to rate of major faults (\(\lambda_1\)) for different values of probability of fault in Type-I redundant subsystem (a). The graph reveals that the expected uptime of the system with reduced capacity decreases with increase in the values of the rate of major faults and the expected uptime of the system with reduced capacity has lower values for higher values of probability of faults in Type-I redundant subsystem.
The curves in Fig.4.12 reveal the behaviour of expected uptime of the system with reduced capacity (AR₀) with respect to probability of proper repair of non-redundant subsystem (p₁) for different values of inspection rate (α₁). It can be concluded from the
graph that the expected uptime of the system with reduced capacity increases with increase in the values of the probability of proper repair of non-redundant subsystem and the expected uptime of the system with reduced capacity has higher values for higher values of the inspection rate.

**Fig. 4.13**

**Fig. 4.13** gives the graph between expected uptime of the system with reduced capacity ($AR_0$) and probability of proper repair of Type-I redundant subsystem ($p_2$) for different values of inspection rate ($\alpha_2$). The graph reveals that the expected uptime of the system with reduced capacity increases with increase in the values of the probability of proper repair of Type-I redundant subsystem and the expected uptime of the system with reduced capacity has higher values for higher values of the inspection rate.

The patterns in **fig. 4.14** give the behaviour of the profit incurred from the system (P) with respect to the rate of minor faults ($\lambda_2$) for the different values of rate of major faults ($\lambda_1$).

Following has been concluded from the graph:

(i) The profit decreases with increase in the values of the rate of minor faults. Further the profit has lower values for higher values of the rate of major faults when other parameters remain fixed.
For $\lambda_1 = 0.001$, the profit is positive or zero or negative according as $\lambda_2$ is less than or equal or greater than 0.0016. Thus, the system will give profit for this when $\lambda_2$ is less than 0.0016.

For $\lambda_1 = 0.0012$, the profit is positive or zero or negative according as $\lambda_2$ is less than or equal or greater than 0.0018. Thus, the system will give profit for this when $\lambda_2$ is less than 0.0018.

For $\lambda_1 = 0.0014$, the profit is positive or zero or negative according as $\lambda_2$ is less than or equal or greater than 0.002. Thus, the system will give profit for this when $\lambda_2$ is less than 0.002.

The curves in the Fig.4.15 show the behaviour of profit (P) with respect to rate of major faults ($\lambda_1$) for different values of probability of faults in Type-I redundant subsystem (a).

Following conclusions have been drawn from the graph:

(i) The profit decreases with increase in the values of the rate of major faults. Further the profit has higher values with the higher values of the probability of faults in Type-I redundant subsystem when other parameters remain fixed.
(ii) For \( a = 0.1 \), the profit is positive or zero or negative according as \( \lambda_1 \) is less than or equal or greater than 0.00064. Thus, the system will give profit for this when \( \lambda_1 \) is less than 0.00064.

(iii) For \( a = 0.3 \), the profit is positive or zero or negative according as \( \lambda_1 \) is less than or equal or greater than 0.00067. Thus, the system will give profit for this when \( \lambda_1 \) is less than 0.00067.

(iv) For \( a = 0.5 \), the profit is positive or zero or negative according as \( \lambda_1 \) is less than or equal or greater than 0.00078. Thus, the system will give profit for this when \( \lambda_1 \) is less than 0.00078.

**Fig.4.15**

The **fig.4.16** presents graph depicting the behaviour of profit of the system (P) with respect to the probability of proper repair of non-redundant subsystem \((p_1)\) for different values of inspection rate \((\alpha_1)\).

Following has been concluded from the graph:

(i) The profit increases with increase in the values of the probability of proper repair of non-redundant subsystem. Moreover, the profit has lower values with the higher values of the inspection rate when other parameters remain fixed.
(ii) For $\alpha_1 = 1.5$, the profit is negative or zero or positive according as $p_1$ is less than or equal or greater than 0.694030. Thus, the system will give profit for this when $p_1$ is greater than 0.694030.

(iii) For $\alpha_1 = 1.7$, the profit is negative or zero or positive according as $p_1$ is less than or equal or greater than 0.532147. Thus, the system will give profit for this when $p_1$ is greater than 0.532147.

(iv) For $\alpha_1 = 1.9$, the profit is negative or zero or positive according as $p_1$ is less than or equal or greater than 0.385979. Thus, the system will give profit for this when $p_1$ is greater than 0.385979.

![Graph](image-url)

**Fig.4.16**

The curves in **Fig.4.17** highlight the behaviour of profit incurred from the system (P) with respect to probability of proper repair of Type-I redundant subsystem ($p_2$) for different values of inspection rate ($\alpha_2$).

From the graph, following conclusions have been made:

(i) The profit increases with increase in the values of the probability of proper repair of non-redundant subsystem. Also, the profit has lower values with the higher values of the inspection rate when other parameters remain fixed.
(ii) For $\alpha_2 = 2.2$, the profit is negative or zero or positive according as $p_2$ is less than or equal or greater than 0.717649. Thus, the system will give profit for this when $p_2$ is greater than 0.717649.

(iii) For $\alpha_2 = 2.6$, the profit is negative or zero or positive according as $p_2$ is less than or equal or greater than 0.517551. Thus, the system will give profit for this when $p_2$ is greater than 0.517551.

(iv) For $\alpha_2 = 3.0$, the profit is negative or zero or positive according as $p_2$ is less than or equal or greater than 0.328838. Thus, the system will give profit for this when $p_2$ is greater than 0.328838.

Fig. 4.17

The fig. 4.18 gives graph that show the behaviour of profit with respect to revenue per unit uptime of the system with full capacity ($C_0$) for different values of probability of proper repair of non-redundant subsystem ($p_1$).

Following has been concluded from the graph:

(i) The profit increases with increase in the values of the revenue per unit uptime with full capacity of the system. Also the profit has higher values with the higher values of the probability of proper repair of non-redundant subsystem when other parameters remain fixed.
(ii) For \( p_1 = 0.1 \), the profit is negative or zero or positive according as \( C_0 \) is less than or equal or greater than Rs.11716.28. Thus, the system will give profit for this when \( C_0 \) is greater than Rs.11716.28.

(iii) For \( p_1 = 0.5 \), the profit is negative or zero or positive according as \( C_0 \) is less than or equal or greater than Rs.9581.08. Thus, the system will give profit for this when \( C_0 \) is greater than Rs.9581.08.

(iv) For \( p_1 = 0.9 \), the profit is negative or zero or positive according as \( C_0 \) is less than or equal or greater than Rs.7445.88. Thus, the system will give profit for this when \( C_0 \) is greater than Rs.7445.88.

**MODEII**

The state transition diagram depicting the various states of transition is shown in [fig.4.19](#). The epochs of entry in to state 0, 1, 3, 4, 5, 6, 7, 8, 9, 14 and 15 are regenerative points, i.e. the states are regenerative states while the states 2, 10, 11, 12, 13, 17 and 19 are the failed states.
TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The state transition probabilities are obtained as under:

\[ dQ_{10}(t) = \lambda_2 e^{-(\lambda_1 + \lambda_2) t} dt \]
\[ dQ_{07}(t) = x_1 \lambda_1 e^{-(\lambda_1 + \lambda_2) t} dt \]
\[ dQ_{14}(t) = y_1(t) dt \]
\[ dQ_{13}(t) = x_1(t) dt \]
\[ dQ_{213}(t) = q_1 g_1(t) dt \]
\[ dQ_{310}(t) = q_1 g_1(t) dt \]
\[ dQ_{30}(t) = p_1 g_1(t) dt \]
\[ dQ_{345}(t) = a_2(t) dt \]
\[ dQ_{46}(t) = b_2(t) dt \]
\[ dQ_{50}(t) = p_2 g_2(t) dt \]
\[
dQ_{5,11}(t) = q_2 g_2(t) dt \\
\]
\[
dQ_{6,12}(t) = q_3 g_3(t) dt \\
\]
\[
dQ_{79}(t) = b_2(t) dt \\
\]
\[
dQ_{8,14}(t) = q_2 e^{-(\lambda_1 + \lambda_2)} g_2(t) dt \\
\]
\[
dQ_{8,17}(t) = \lambda_1 e^{-(\lambda_1 + \lambda_2)} G_2(t) dt \\
\]
\[
dQ_{9,15}(t) = q_3 e^{-(\lambda_1 + \lambda_2)} g_3(t) dt \\
\]
\[
dQ_{9,19}(t) = \lambda_1 e^{-(\lambda_1 + \lambda_2)} G_3(t) dt \\
\]
\[
dQ_{11,0}(t) = g_e(t) dt \\
\]
\[
dQ_{13,0}(t) = g_e(t) dt \\
\]
\[
dQ_{15,0}(t) = g_e(t) dt \\
\]
\[
dQ_{17,0}(t) = g_e(t) dt \\
\]
\[
dQ_{19,0}(t) = g_e(t) dt \\
\]

Then the non-zero elements \( p_{ij} \) are given by

\[
p_{ij} = \lim_{s \to 0} Q_{ij}^{**}(s),
\]

where \( Q_{ij}^{**}(s) \) is the Laplace Stieltjes transformation of \( Q_{ij}(t) \).

Thus, we have

\[
p_{01} = \frac{\lambda_2}{\lambda_1 + \lambda_2} \\
p_{07} = \frac{x_1 \lambda_1}{\lambda_1 + \lambda_2} \\
p_{14} = y_1^{*}(0) \\
p_{2,13} = q_1 g_1^{*}(0) \\
p_{3,10} = q_1 g_1^{*}(0) \\
p_{46} = b_2^{*}(0) \\
p_{5,11} = q_2 g_2^{*}(0) \\
p_{6,12} = q_3 g_3^{*}(0) \\
p_{78} = a_2^{*}(0) \\
p_{80} = p_2 e^{-(\lambda_1 + \lambda_2)} g_2(t) dt \\
p_{8,16} = \lambda_2 e^{-(\lambda_1 + \lambda_2)} G_2(t) dt \\
p_{9,18} = \lambda_2 e^{-(\lambda_1 + \lambda_2)} G_3(t) dt \\
p_{9,19} = \lambda_1 e^{-(\lambda_1 + \lambda_2)} G_3(t) dt \\
p_{10,0} = g_e(t) dt \\
p_{12,0} = g_e(t) dt \\
p_{14,0} = g_e(t) dt \\
p_{16,0} = g_e(t) dt \\
p_{18,0} = g_e(t) dt \\
\]
\[ p_{79} = b_i^* (0) \]
\[ p_{814} = q_2 g_2^* (\lambda_1 + \lambda_2) \]
\[ p_{817} = \frac{\lambda_1 \left[ 1 - g_2^* (\lambda_1 + \lambda_2) \right]}{\lambda_1 + \lambda_2} \]
\[ p_{915} = q_3 g_3^* (\lambda_1 + \lambda_2) \]
\[ p_{919} = \frac{\lambda_1 \left[ 1 - g_3^* (\lambda_1 + \lambda_2) \right]}{\lambda_1 + \lambda_2} \]
\[ p_{10,0} = g_e^* (0) \]
\[ p_{11,0} = g_e^* (0) \]
\[ p_{12,0} = g_e^* (0) \]
\[ p_{13,0} = g_e^* (0) \]
\[ p_{14,0} = g_e^* (0) \]
\[ p_{15,0} = g_e^* (0) \]
\[ p_{16,0} = g_e^* (0) \]
\[ p_{17,0} = g_e^* (0) \]
\[ p_{18,0} = g_e^* (0) \]
\[ p_{19,0} = g_e^* (0) \]

By these transition probabilities, it can be verified that
\[ p_{01} + p_{02} + p_{07} = 1, \quad p_{13} + p_{14} = 1, \]
\[ p_{20} + p_{213} = 1, \quad p_{30} + p_{310} = 1, \]
\[ p_{45} + p_{46} = 1, \quad p_{50} + p_{511} = 1, \]
\[ p_{60} + p_{612} = 1, \quad p_{78} + p_{79} = 1, \]
\[ p_{80} + p_{814} + p_{816} + p_{817} = 1, \quad p_{90} + p_{915} + p_{918} + p_{919} = 1, \]
\[ p_{10,0} = p_{11,0} = p_{12,0} = p_{13,0} = p_{14,0} = p_{15,0} = p_{16,0} = p_{17,0} = p_{18,0} = p_{19,0} = 1 \]

“The unconditional mean time taken by the system to transit for any regenerative state ‘j’, when it is counted from epoch of entrance into that state ‘i’, is mathematically stated by
\[ m_j = \int_0^\infty t dQ_j(t) = -q_j^* (0). \]
Thus, we have

\[ m_{01} + m_{02} + m_{07} = \mu_0 \]
\[ m_{20} + m_{2,13} = \mu_2 \]
\[ m_{45} + m_{46} = \mu_4 \]
\[ m_{60} + m_{6,12} = \mu_6 \]
\[ m_{80} + m_{8,14} + m_{8,16} + m_{8,17} = \mu_8 \]
\[ m_{10,0} = \mu_{10} \]
\[ m_{12,0} = \mu_{12} \]
\[ m_{14,0} = \mu_{14} \]
\[ m_{16,0} = \mu_{16} \]
\[ m_{18,0} = \mu_{18} \]
\[ m_{13} + m_{14} = \mu_1 \]
\[ m_{30} + m_{3,12} = \mu_3 \]
\[ m_{50} + m_{5,11} = \mu_5 \]
\[ m_{78} + m_{79} = \mu_7 \]
\[ m_{90} + m_{9,15} + m_{9,18} + m_{9,19} = \mu_9 \]

“The mean sojourn time (\(\mu_i\)) in the regenerative state ‘i’ is defined as the time of stay in that state before transition to any other state”. If ‘T’ denotes the sojourn time in regenerative state ‘i’, then

\[ \mu_0 = \frac{1}{\lambda_1 + \lambda_2} \]
\[ \mu_1 = -i_1'(0) \]
\[ \mu_2 = -g_1''(0) \]
\[ \mu_3 = -g_1''(0) \]
\[ \mu_4 = -i_2''(0) \]
\[ \mu_5 = -g_2''(0) \]
\[ \mu_6 = -g_3''(0) \]
\[ \mu_7 = -i_2''(0) \]
\[ \mu_8 = \frac{1-g_3''(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2} \]
\[ \mu_9 = \frac{1-g_3''(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2} \]
\[ \mu_{10} = -g_e''(0) \]
\[ \mu_{11} = -g_e''(0) \]
\[ \mu_{12} = -g_e''(0) \]
\[ \mu_{13} = -g_e''(0) \]
\[ \mu_{14} = -g_e''(0) \]
\[ \mu_{15} = -g_e''(0) \]
\[ \mu_{16} = -g_e''(0) \]
\[ \mu_{17} = -g_e''(0) \]
\[ \mu_{18} = -g_e''(0) \]
\[ \mu_{19} = -g_e''(0) \]
MEAN TIME TO SYSTEM FAILURE

The mean time to system failure (MTSF) has been obtained by considering the failed states of the system as absorbing states. The following recursive relations for \( \phi_i(t) \), c.d.f of the first passage time from regenerative state ‘i’ to failed state are obtained using concepts of probability:

\[
\begin{align*}
\phi_0(t) &= Q_{01}(t) + Q_{07}(t) + Q_{02}(t) \\
\phi_1(t) &= Q_{13}(t) + Q_{14}(t) + Q_{1}(t) \\
\phi_3(t) &= Q_{30}(t) + Q_{310}(t) \\
\phi_4(t) &= Q_{45}(t) + Q_{46}(t) + Q_6(t) \\
\phi_5(t) &= Q_{50}(t) + Q_{511}(t) \\
\phi_6(t) &= Q_{60}(t) + Q_{612}(t) \\
\phi_7(t) &= Q_{78}(t) + Q_{79}(t) + Q_9(t) \\
\phi_8(t) &= Q_{80}(t) + Q_{814}(t) + Q_{14}(t) + Q_{816}(t) + Q_{817}(t) + Q_9(t) \\
\phi_9(t) &= Q_{90}(t) + Q_{915}(t) + Q_{918}(t) + Q_{919}(t) \\
\phi_{14}(t) &= Q_{140}(t) + Q_0(t) \\
\phi_{15}(t) &= Q_{150}(t) + Q_0(t) \\
\phi_{16}(t) &= Q_{160}(t) + Q_0(t) \\
\phi_{18}(t) &= Q_{180}(t) + Q_0(t)
\end{align*}
\]

These equations are solved using Laplace Stieltjes transformation and \( \phi_0^{**}(s) \), is obtained as under:

\[
\phi_0^{**}(s) = \frac{N(s)}{D(s)},
\]

where

\[
N(s) = Q_{02}^{**}(s) + Q_{01}^{**}(s)\left[Q_{13}^{**}(s)Q_{310}^{**}(s) + Q_{14}^{**}(s)\left(Q_{45}^{**}(s)Q_{511}^{**}(s) + Q_{46}^{**}(s)Q_{612}^{**}(s)\right)\right]
\]

\[
+ Q_{07}^{**}(s)\left(Q_{78}^{**}(s)Q_{817}^{**}(s) + Q_{79}^{**}(s)Q_{919}^{**}(s)\right)
\]

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and

\[ D(s) = 1 - Q_{01}^{**}(s) \left[ Q_{13}^{**}(s)Q_{30}^{**}(s) + Q_{14}^{**}(s) \left( Q_{45}^{**}(s)Q_{50}^{**}(s) + Q_{46}^{**}(s)Q_{60}^{**}(s) \right) \right] \]

\[ - Q_{07}^{**}(s) \left[ Q_{78}^{**}(s)Q_{80}^{**}(s) + Q_{8,16}^{**}(s) + Q_{8,14}^{**}(s) \right] \]

\[ + Q_{79}^{**}(s) \left[ Q_{90}^{**}(s) + Q_{9,15}^{**}(s) + Q_{9,18}^{**}(s) \right] \]

"The mean time to system failure when the system starts from the state 0, is defined by

\[ T_0 = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s}, \]

Substituting the value of \( \phi_0^{**}(s) \) and using L’ Hospital Rule, we get

\[ T_0 = \frac{N}{D}, \]

where

\[ N = \mu_0 + p_{01} \left[ \mu_1 + p_{13}\mu_3 + p_{14} \left( \mu_4 + p_{45}\mu_5 + p_{46}\mu_6 \right) \right] \]

\[ + p_{07} \left[ \mu_7 + p_{78} \left( \mu_8 + p_{8,14}\mu_{14} + p_{8,16}\mu_{16} \right) + p_{79} \left( \mu_9 + p_{9,15}\mu_{15} + p_{9,18}\mu_{18} \right) \right] \]

and

\[ D = 1 - p_{01} \left[ p_{13}p_{30} + p_{14} \left( p_{45}p_{50} + p_{46}p_{60} \right) \right] \]

\[ - p_{07} \left[ p_{78} \left( p_{80} + p_{8,14} + p_{8,16} \right) + p_{79} \left( p_{90} + p_{9,15} + p_{9,18} \right) \right] \]

**EXPECTED UPTIME OF THE SYSTEM WITH FULL CAPACITY**

The expected uptime of the system with full capacity \( AF_i(t) \), the probability that the system is up at instant ‘t’ with full capacity given that it entered regenerative state ‘i’ at \( t = 0 \), by making use of the concepts of probability in theory of regenerative processes, gives the following recursive relations:

\[ AF_0(t) = M_0(t) + q_{01}(t) \odot AF_1(t) + q_{02}(t) \odot AF_2(t) + q_{07}(t) \odot AF_7(t) \]

\[ AF_1(t) = q_{13}(t) \odot AF_3(t) + q_{14}(t) \odot AF_4(t) \]

\[ AF_2(t) = q_{20}(t) \odot AF_0(t) + q_{2,13}(t) \odot AF_{13}(t) \]

\[ AF_3(t) = q_{30}(t) \odot AF_0(t) + q_{3,10}(t) \odot AF_{10}(t) \]

\[ AF_4(t) = q_{45}(t) \odot AF_5(t) + q_{46}(t) \odot AF_6(t) \]
\[ \text{AF}_2(t) = q_{50}(t) \odot \text{AF}_0(t) + q_{5,11}(t) \odot \text{AF}_1(t) \]
\[ \text{AF}_5(t) = q_{60}(t) \odot \text{AF}_0(t) + q_{6,12}(t) \odot \text{AF}_2(t) \]
\[ \text{AF}_7(t) = M_7(t) + q_{78}(t) \odot \text{AF}_8(t) + q_{79}(t) \odot \text{AF}_9(t) \]
\[ \text{AF}_8(t) = M_8(t) + q_{80}(t) \odot \text{AF}_0(t) + q_{8,14}(t) \odot \text{AF}_14(t) + q_{8,16}(t) \odot \text{AF}_16(t) \]
\[ + q_{8,17}(t) \odot \text{AF}_7(t) \]
\[ \text{AF}_9(t) = M_9(t) + q_{90}(t) \odot \text{AF}_0(t) + q_{9,15}(t) \odot \text{AF}_15(t) + q_{9,18}(t) \odot \text{AF}_18(t) \]
\[ + q_{9,19}(t) \odot \text{AF}_9(t) \]
\[ \text{AF}_{10}(t) = q_{10,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{11}(t) = q_{11,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{12}(t) = q_{12,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{13}(t) = q_{13,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{14}(t) = M_{14}(t) + q_{14,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{15}(t) = M_{15}(t) + q_{15,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{16}(t) = q_{16,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{17}(t) = q_{17,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{18}(t) = q_{18,0}(t) \odot \text{AF}_0(t) \]
\[ \text{AF}_{19}(t) = q_{19,0}(t) \odot \text{AF}_0(t) , \]

where
\[ M_0(t) = e^{-(\lambda_1 + \lambda_2) t} ; \]
\[ M_7(t) = \overline{I}_2(t) ; \]
\[ M_8(t) = \overline{G}_2(t) ; \]
\[ M_9(t) = \overline{G}_3(t) ; \]
\[ M_{14}(t) = \overline{M}_{15}(t) = \overline{G}_e(t) \]

These equations are solved using Laplace transform and \( \text{AF}_0^*(s) \), is obtained as under:
\[ \text{AF}_0^*(s) = \frac{N_i(s)}{D_1(s)} , \]
where

\[ N_1(s) = M_0^*(s) + q_{07}^*(s) \left[ M_7^*(s) + q_{78}^*(s) \left( M_8^*(s) + q_{814}^*(s) M_{14}^*(s) \right) \right] + q_{79}^*(s) \left[ M_9^*(s) + q_{515}^*(s) M_{15}^*(s) \right] \]

and

\[ D_1(s) = 1 - q_{02}^*(s) \left[ q_{20}^*(s) + q_{213}^*(s) q_{130}^*(s) \right] - q_{01}^*(s) \left[ q_{113}^*(s) \left( q_{310}^*(s) + q_{310}^*(s) q_{110}^*(s) \right) \right] + q_{14}^*(s) \left( q_{45}^*(s) \left( q_{511}^*(s) + q_{511}^*(s) q_{110}^*(s) \right) \right) + q_{46}^*(s) \left( q_{612}^*(s) + q_{612}^*(s) q_{120}^*(s) \right) \]

\[ - q_{07}^*(s) \left[ q_{78}^*(s) \left( q_{814}^*(s) q_{140}^*(s) + q_{516}^*(s) q_{610}^*(s) + q_{817}^*(s) q_{170}^*(s) \right) \right] + q_{79}^*(s) \left[ q_{950}^*(s) + q_{915}^*(s) q_{150}^*(s) + q_{918}^*(s) q_{180}^*(s) + q_{919}^*(s) q_{190}^*(s) \right] \]

“In steady state the expected uptime of the system with full capacity is defined by

\[ AF_0 = \lim_{s \to 0} \left( s AF^*_0(s) \right) \].

Thus, we have \( AF_0 = \frac{N_1}{D_1} \),

where

\[ N_1 = \mu_0 + p_{07} \left[ \mu_7 + p_{78} (\mu_8 + p_{814} \mu_{14}) + p_{79} (\mu_9 + p_{915} \mu_{15}) \right] \]

and

\[ D_1 = \mu_0 + p_{02} \left( \mu_2 + p_{213} \mu_{13} \right) + p_{01} \left[ \mu_1 + p_{13} (\mu_5 + p_{515} \mu_{15}) \right] + p_{14} \left[ \mu_4 + p_{45} (\mu_5 + p_{511} \mu_{11}) + p_{46} (\mu_6 + p_{612} \mu_{12}) \right] + p_{07} \left[ \mu_7 + p_{78} (\mu_8 + p_{814} \mu_{14} + p_{816} \mu_{16} + p_{817} \mu_{17}) \right] + p_{79} (\mu_9 + p_{915} \mu_{15} + p_{918} \mu_{18} + p_{919} \mu_{19}) \]

**EXPECTED UPTIME OF THE SYSTEM WITH REDUCED CAPACITY**

In the similar way, the expected uptime of the system with reduced capacity \( AR_i(t) \), the probability that the system is up at instant ‘t’ with reduced capacity given that it entered regenerative state ‘i’ at \( t = 0 \), can be obtained by using the following recursive relations:
\[ \begin{align*}
AR_0(t) &= q_{01}(t) \odot AR_1(t) + q_{02}(t) \odot AR_2(t) + q_{07}(t) \odot AR_7(t) \\
AR_1(t) &= M_1(t) + q_{13}(t) \odot AR_3(t) + q_{14}(t) \odot AR_4(t) \\
AR_2(t) &= q_{20}(t) \odot AR_0(t) + q_{2,13}(t) \odot AR_{13}(t) \\
AR_3(t) &= M_1(t) + q_{30}(t) \odot AR_0(t) + q_{3,10}(t) \odot AR_{10}(t) \\
AR_4(t) &= M_4(t) + q_{45}(t) \odot AR_5(t) + q_{46}(t) \odot AR_6(t) \\
AR_5(t) &= M_5(t) + q_{50}(t) \odot AR_0(t) + q_{5,11}(t) \odot AR_{11}(t) \\
AR_6(t) &= M_6(t) + q_{60}(t) \odot AR_0(t) + q_{6,12}(t) \odot AR_{12}(t) \\
AR_7(t) &= q_{78}(t) \odot AR_8(t) + q_{79}(t) \odot AR_9(t) \\
AR_8(t) &= q_{80}(t) \odot AR_0(t) + q_{8,14}(t) \odot AR_{14}(t) + q_{8,16}(t) \odot AR_{16}(t) + q_{8,17}(t) \odot AR_{17}(t) \\
AR_9(t) &= q_{90}(t) \odot AR_0(t) + q_{9,15}(t) \odot AR_{15}(t) + q_{9,18}(t) \odot AR_{18}(t) + q_{9,19}(t) \odot AR_{19}(t) \\
AR_{10}(t) &= q_{10,0}(t) \odot AR_0(t) \\
AR_{11}(t) &= q_{11,0}(t) \odot AR_0(t) \\
AR_{12}(t) &= q_{12,0}(t) \odot AR_0(t) \\
AR_{13}(t) &= q_{13,0}(t) \odot AR_0(t) \\
AR_{14}(t) &= q_{14,0}(t) \odot AR_0(t) \\
AR_{15}(t) &= q_{15,0}(t) \odot AR_0(t) \\
AR_{16}(t) &= M_{16}(t) + q_{16,0}(t) \odot AR_0(t) \\
AR_{17}(t) &= q_{17,0}(t) \odot AR_0(t) \\
AR_{18}(t) &= M_{18}(t) + q_{18,0}(t) \odot AR_0(t) \\
AR_{19}(t) &= q_{19,0}(t) \odot AR_0(t),
\end{align*} \]

where
\[ \begin{align*}
M_1(t) &= \bar{I}_1(t); & M_3(t) &= \bar{G}_1(t); \\
M_4(t) &= \bar{I}_2(t); & M_5(t) &= \bar{G}_2(t); \\
M_6(t) &= \bar{G}_3(t); & M_{16}(t) &= M_{18}(t) = \bar{G}_e(t).
\end{align*} \]
These equations are solved using Laplace transform and $AF_0^*(s)$, is obtained as under:

$$AR_0^*(s) = \frac{N_2(s)}{D_1(s)},$$

where

$$N_2(s) = q_{01}^*(s)\left[M_1^*(s) + q_{13}^*(s)M_3^*(s) + q_{14}^*(s)\right] + q_{14}^*(s)\left[M_4^*(s) + q_{45}^*(s)M_5^*(s) + q_{46}^*(s)M_6^*(s)\right] + q_{07}^*(s)\left[q_{78}^*(s)q_{8,16}^*(s)M_{16}^*(s) + q_{79}^*(s)q_{9,18}^*(s)M_{18}^*(s)\right]$$

and

$$D_1(s)$$ is as defined earlier.

“In steady state the expected uptime of the system with reduced capacity is defined by

$$AR_0 = \lim_{s \to 0} (sAR_0^*(s)).$$

Thus, we have $AR_0 = \frac{N_2}{D_1}$,

where

$$N_2 = p_{01}\left[\mu_1 + p_{13}\mu_3 + p_{14}\left(\mu_4 + p_{45}\mu_5 + p_{46}\mu_6\right)\right] + p_{07}\left(p_{78}p_{8,16}\mu_{16} + p_{79}p_{9,18}\mu_{18}\right)$$

and

$D_1$ is as defined earlier.

**BUSY PERIOD OF ORDINARY REPAIR MAN (INSPECTION TIME ONLY)**

The following recursive relations for $B_{i0}(t)$ are obtained using the concepts of probability in regenerative process:

$$B_{i0}(t) = q_{01}(t)\odot B_{i1}(t) + q_{02}(t)\odot B_{i2}(t) + q_{07}(t)\odot B_{i7}(t)$$

$$B_{i1}(t) = W_i(t) + q_{13}(t)\odot B_{i3}(t) + q_{14}(t)\odot B_{i4}(t)$$

$$B_{i2}(t) = q_{20}(t)\odot B_{i0}(t) + q_{2,13}(t)\odot B_{i13}(t)$$

$$B_{i3}(t) = q_{30}(t)\odot B_{i0}(t) + q_{3,10}(t)\odot B_{i10}(t)$$

$$B_{i4}(t) = W_4(t) + q_{45}(t)\odot B_{i5}(t) + q_{46}(t)\odot B_{i6}(t)$$

$$B_{i5}(t) = q_{50}(t)\odot B_{i0}(t) + q_{5,11}(t)\odot B_{i11}(t)$$

$$B_{i6}(t) = q_{60}(t)\odot B_{i0}(t) + q_{6,12}(t)\odot B_{i12}(t)$$

$$B_{i7}(t) = W_7(t) + q_{78}(t)\odot B_{i8}(t) + q_{79}(t)\odot B_{i9}(t)$$
\[ B_{i8}(t) = q_{80}(t)B_{i0}(t) + q_{8,14}(t)B_{i14}(t) + q_{8,16}(t)B_{i16}(t) + q_{8,17}(t)B_{i17}(t) \]
\[ B_{i9}(t) = q_{90}(t)B_{i0}(t) + q_{9,15}(t)B_{i15}(t) + q_{9,18}(t)B_{i18}(t) + q_{9,19}(t)B_{i19}(t) \]
\[ B_{i10}(t) = q_{i10,0}(t)B_{i0}(t) \]
\[ B_{i11}(t) = q_{i1,0}(t)B_{i0}(t) \]
\[ B_{i12}(t) = q_{i2,0}(t)B_{i0}(t) \]
\[ B_{i13}(t) = q_{i3,0}(t)B_{i0}(t) \]
\[ B_{i14}(t) = q_{i4,0}(t)B_{i0}(t) \]
\[ B_{i15}(t) = q_{i5,0}(t)B_{i0}(t) \]
\[ B_{i16}(t) = q_{i6,0}(t)B_{i0}(t) \]
\[ B_{i17}(t) = q_{i7,0}(t)B_{i0}(t) \]
\[ B_{i18}(t) = q_{i8,0}(t)B_{i0}(t) \]
\[ B_{i19}(t) = q_{i9,0}(t)B_{i0}(t) \]

where

\[ W_1(t) = \tilde{I}_1(t) \]
\[ W_4(t) = W_7(t) = \tilde{I}_2(t) \]

These equations are solved using Laplace transform and \( B_{i0}^*(s) \), is obtained as under:

\[ B_{i0}^*(s) = \frac{N_3(s)}{D_1(s)} \]

where

\[ N_3(s) = q_{i01}^*(s)\left[ W_1^*(s) + q_{i4}^*(s)W_4^*(s) \right] + q_{i07}^*(s)W_7^*(s) \]

and \( D_1(s) \) is as defined earlier.

“In steady state busy period of the ordinary repairman for inspection of the system is defined by

\[ B_{i0} = \lim_{s \to 0} \left(sB_{i0}^*(s)\right) \].

Thus, we have \( B_{i0} = \frac{N_3}{D_1} \),

where

\[ N_3 = \mu_1(p_{01} + \mu_4p_{14}) + \mu_7p_{07} \]

and \( D_1 \) is as defined earlier.
BUSY PERIOD OF ORDINARY REPAIRMAN (REPAIR TIME ONLY)

The following recursive relations for $B_{r0}(t)$ are obtained using the concepts of probability in regenerative process:

$$B_{r0}(t) = q_{01}(t)B_{r1}(t) + q_{02}(t)B_{r2}(t) + q_{07}(t)B_{r7}(t)$$

$$B_{r1}(t) = q_{13}(t)B_{r3}(t) + q_{14}(t)B_{r4}(t)$$

$$B_{r2}(t) = W_2(t) + q_{20}(t)B_{r0}(t) + q_{2,13}(t)B_{r13}(t)$$

$$B_{r3}(t) = W_3(t) + q_{30}(t)B_{r0}(t) + q_{3,10}(t)B_{r10}(t)$$

$$B_{r4}(t) = q_{45}(t)B_{r5}(t) + q_{46}(t)B_{r6}(t)$$

$$B_{r5}(t) = W_5(t) + q_{50}(t)B_{r0}(t) + q_{5,11}(t)B_{r11}(t)$$

$$B_{r6}(t) = W_6(t) + q_{60}(t)B_{r0}(t) + q_{6,12}(t)B_{r12}(t)$$

$$B_{r7}(t) = q_{76}(t)B_{r8}(t) + q_{79}(t)B_{r9}(t)$$

$$B_{r8}(t) = W_8(t) + q_{80}(t)B_{r0}(t) + q_{8,14}(t)B_{r14}(t) + q_{8,16}(t)B_{r16}(t) + q_{8,17}(t)B_{r17}(t)$$

$$B_{r9}(t) = W_9(t) + q_{90}(t)B_{r0}(t) + q_{9,15}(t)B_{r15}(t) + q_{9,18}(t)B_{r18}(t) + q_{9,19}(t)B_{r19}(t)$$

$$B_{r10}(t) = q_{10,0}(t)B_{r0}(t)$$

$$B_{r11}(t) = q_{11,0}(t)B_{r0}(t)$$

$$B_{r12}(t) = q_{12,0}(t)B_{r0}(t)$$

$$B_{r13}(t) = q_{13,0}(t)B_{r0}(t)$$

$$B_{r14}(t) = q_{14,0}(t)B_{r0}(t)$$

$$B_{r15}(t) = q_{15,0}(t)B_{r0}(t)$$

$$B_{r16}(t) = q_{16,0}(t)B_{r0}(t)$$

$$B_{r17}(t) = q_{17,0}(t)B_{r0}(t)$$

$$B_{r18}(t) = q_{18,0}(t)B_{r0}(t)$$

$$B_{r19}(t) = q_{19,0}(t)B_{r0}(t)$$
where
\[ W_2(t) = \tilde{G}(t); \quad W_3(t) = \tilde{G}_1(t); \]
\[ W_5(t) = W_8(t) = \tilde{G}_2(t); \quad W_6(t) = W_9(t) = \tilde{G}_3(t); \]

These equations are solved using Laplace transform and \( B_{r0}^*(s) \), is obtained as under:
\[ B_{r0}^*(s) = \frac{N_4(s)}{D_1(s)}, \]

where
\[ N_4(s) = q_{01}^*(s) \left[ q_{13}^*(s) W_3^*(s) + q_{14}^*(s) \left( q_{45}^*(s) W_5^*(s) + q_{46}^*(s) W_6^*(s) \right) \right] \]
\[ + q_{02}^*(s) W_2^*(s) + q_{07}^*(s) \left( q_{78}^*(s) W_8^*(s) + q_{79}^*(s) W_9^*(s) \right) \]
and
\[ D_1(s) \] is as defined earlier.

“In steady state busy period of the ordinary repairman for repair of the system is defined by
\[ B_{r0} = \lim_{s \to 0} (sB_{r0}^*(s))'. \quad \text{Thus, we have} \quad B_{r0} = \frac{N_4}{D_1}, \]

where
\[ N_4 = p_{02} \mu_2 + p_{01} \left[ p_{13} \mu_3 + p_{14} \left( p_{45} \mu_5 + p_{46} \mu_6 \right) \right] + p_{07} \left( p_{78} \mu_8 + p_{79} \mu_9 \right) \]
and \( D_1 \) is as already defined.

**BUSY PERIOD OF EXPERT REPAIR MAN (REPAIR TIME ONLY)**

The following recursive relations for \( B_{re0}(t) \) are obtained using the concepts of probability in regenerative process:
\[ B_{re0}(t) = q_{01}(t) \otimes B_{re1}(t) + q_{02}(t) \otimes B_{re2}(t) + q_{07}(t) \otimes B_{re7}(t) \]
\[ B_{re1}(t) = q_{13}(t) \otimes B_{re3}(t) + q_{14}(t) \otimes B_{re4}(t) \]
\[ B_{re2}(t) = q_{20}(t) \otimes B_{re5}(t) + q_{213}(t) \otimes B_{re13}(t) \]
\[ B_{re3}(t) = q_{30}(t) \otimes B_{re6}(t) + q_{310}(t) \otimes B_{re10}(t) \]
\[ B_{re4}(t) = q_{45}(t) \otimes B_{re5}(t) + q_{46}(t) \otimes B_{re6}(t) \]
\[ B_{re5}(t) = q_{50}(t)B_{re0}(t) + q_{5,11}(t)B_{re11}(t) \]
\[ B_{re6}(t) = q_{60}(t)B_{re0}(t) + q_{6,12}(t)B_{re12}(t) \]
\[ B_{re7}(t) = q_{78}(t)B_{re8}(t) + q_{79}(t)B_{re9}(t) \]
\[ B_{re8}(t) = q_{80}(t)B_{re0}(t) + q_{8,14}(t)B_{re14}(t) + q_{8,16}(t)B_{re16}(t) + q_{8,17}(t)B_{re17}(t) \]
\[ B_{re9}(t) = q_{90}(t)B_{re0}(t) + q_{9,15}(t)B_{re15}(t) + q_{9,18}(t)B_{re18}(t) + q_{9,19}(t)B_{re19}(t) \]
\[ B_{re10}(t) = W_{10}(t) + q_{10,0}(t)B_{re0}(t) \]
\[ B_{re11}(t) = W_{11}(t) + q_{11,0}(t)B_{re0}(t) \]
\[ B_{re12}(t) = W_{12}(t) + q_{12,0}(t)B_{re0}(t) \]
\[ B_{re13}(t) = W_{13}(t) + q_{13,0}(t)B_{re0}(t) \]
\[ B_{re14}(t) = W_{14}(t) + q_{14,0}(t)B_{re0}(t) \]
\[ B_{re15}(t) = W_{15}(t) + q_{15,0}(t)B_{re0}(t) \]
\[ B_{re16}(t) = W_{16}(t) + q_{16,0}(t)B_{re0}(t) \]
\[ B_{re17}(t) = W_{17}(t) + q_{17,0}(t)B_{re0}(t) \]
\[ B_{re18}(t) = W_{18}(t) + q_{18,0}(t)B_{re0}(t) \]
\[ B_{re19}(t) = W_{19}(t) + q_{19,0}(t)B_{re0}(t) , \]

where
\[
W_{10}(t) = W_{11}(t) = W_{12}(t) = W_{13}(t) = W_{14}(t) = \overline{G_e}(t) ; \\
W_{15}(t) = W_{16}(t) = W_{17}(t) = W_{18}(t) = W_{19}(t) = \overline{G_e}(t) 
\]

These equations are solved using Laplace transform and \( B_{re0}^*(s) \), is obtained as under:

\[ B_{re0}^*(s) = \frac{N_5(s)}{D_1(s)} \]

where
\[
N_5(s) = q_{02}^*(s)q_{13}^*(s)W_{13}^*(s) + q_{01}^*(s)\left[q_{13}^*(s)q_{3,10}^*(s)W_{10}^*(s) + q_{14}^*(s)\left[q_{3,11}^*(s)W_{11}^*(s) + q_{46}^*(s)q_{6,12}^*(s)W_{12}^*(s) \right] \right] + q_{07}^*(s)\left[q_{78}^*(s)\left[q_{8,14}^*(s)W_{14}^*(s) + q_{8,16}^*(s)W_{16}^*(s) + q_{8,17}^*(s)W_{17}^*(s) \right] \right] + q_{78}^*(s)\left[q_{9,15}^*(s)W_{15}^*(s) + q_{9,18}^*(s)W_{18}^*(s) + q_{9,19}^*(s)W_{19}^*(s) \right] 
\]

and \( D_1(s) \) is as defined earlier.
“In steady state busy period of the expert repairman for repair of the system is defined by

\[ B_{re0} = \lim_{s \to 0} \left( sB'_{re0}(s) \right) \].

Thus, we have \[ B_{re0} = \frac{N_5}{D_1} \],

where

\[ N_5 = P_{02}P_{2,13}H_{13} + P_{01}\left[ P_{13}P_{3,10}H_{10} + P_{14}\left( P_{45}P_{5,11}H_{11} + P_{46}P_{6,12}H_{12} \right) \right] \]

\[ + P_{07}\left[ P_{78}\left( P_{8,14}H_{14} + P_{8,16}H_{16} + P_{8,17}H_{17} \right) + P_{79}\left( P_{9,15}H_{15} + P_{9,18}H_{18} + P_{9,19}H_{19} \right) \right] \]

and

\[ D_1 \] is as defined earlier.

**PROFIT ANALYSIS**

For the system, the expected profit incurred is given by

\[ P = C_0AF_0 + C_1AR_0 - C_2B_{i0} - C_3B_{r0} - C_4B_{re0} - C_5 \]

where

\[ C_0 = \text{revenue per unit uptime of the system working with full capacity} \]
\[ C_1 = \text{revenue per unit uptime of the system working with reduced capacity} \]
\[ C_2 = \text{cost per unit time of inspection by the ordinary repairman} \]
\[ C_3 = \text{cost per unit time of repair by the ordinary repairman} \]
\[ C_4 = \text{cost per unit time of repair by the expert repairman} \]
\[ C_5 = \text{cost per unit installation} \]

**GRAPHICAL INTERPRETATIONS AND CONCLUSION**

The graphical analysis of the system has been carried out using the following particular case along with the estimated values of the parameters given in Chapter 2:

\[ i_1(t) = \alpha_1e^{-\alpha_1(t)}; \quad i_2(t) = \alpha_2e^{-\alpha_2(t)}; \]
\[ g(t) = \beta_1e^{-\beta_1(t)}; \quad g_2(t) = \beta_2e^{-\beta_2(t)}; \]
\[ g_3(t) = \beta_3e^{-\beta_3(t)}; \]
Then, we get

\[ p_{01} = \frac{\lambda_2}{\lambda_1 + \lambda_2} \]

\[ p_{07} = \frac{x_1 \lambda_1}{\lambda_1 + \lambda_2} \]

\[ p_{14} = y \]

\[ p_{2,13} = q_i = p_{3,10} \]

\[ p_{46} = b = p_{79} \]

\[ p_{5,11} = q_2 \]

\[ p_{6,12} = q_3 \]

\[ p_{8,14} = \frac{q_1 \beta_2}{(\beta_2 + \lambda_1 + \lambda_2)} \]

\[ p_{8,17} = \frac{\lambda_1 \beta_2}{(\beta_2 + \lambda_1 + \lambda_2)} \]

\[ p_{9,15} = \frac{q_1 \beta_3}{(\beta_3 + \lambda_1 + \lambda_2)} \]

\[ p_{9,19} = \frac{\lambda_1 \beta_3}{(\beta_3 + \lambda_1 + \lambda_2)} \]

\[ \mu_0 = \frac{1}{\lambda_1 + \lambda_2} \]

\[ \mu_2 = \frac{1}{\beta} \]

\[ \mu_4 = \mu_7 = \frac{1}{\alpha_2} \]

\[ \mu_6 = \mu_9 = \frac{1}{\beta_3} \]
Various graphs are plotted for “Mean time to system failure”, “Expected uptime of the system with full capacity”, “Expected uptime of the system with reduced capacity”, “Busy periods of the ordinary/expert repairman” and “Profit” of the system by taking rates of different faults ($\lambda_1$ and $\lambda_2$), repair rates ($\beta, \beta_1, \beta_2, \beta_3$ and $\beta_e$) and inspection rates ($\alpha_1$ and $\alpha_2$) and various probabilities ($a, b, x, y, x_1, y_1, p_1, p_2, p_3, q_1, q_2$ and $q_3$).

Following interpretations and conclusions have been made from the graphs:

**Fig.4.20** gives the graph between MTSF ($T_0$) and rate of minor faults ($\lambda_2$) for different values of rate of major faults ($\lambda_1$). It is concluded that that the MTSF decreases with increase in the values of the rates of major faults and MTSF has higher values for higher values of the rates of minor faults.

The graph in **Fig.4.21** gives the pattern of MTSF ($T_0$) with respect to rate of minor faults ($\lambda_2$) for different values of probability of minor faults in non-redundant subsystem (a). The graph reveals that the MTSF decreases with increase in the values of the rate of minor faults. However, MTSF has higher values for higher values of the values of the probability of minor faults in non-redundant subsystem.
Fig. 4.21

Fig. 4.22 gives the graph between MTSF and rate of major faults ($\lambda_1$) for different values of probability of occurrence of major fault in redundant subsystem ($x_1$). The graph reveals that the MTSF decreases with increase in the values of the rate of major faults.
However, MTSF has higher values for higher values of the probability of major faults in redundant subsystem.

**Fig. 4.23**

The curves in **Fig. 4.23** show the behaviour of MTSF with respect to probability of proper repair of non-redundant subsystem ($p_1$) for different values of probability of minor fault in redundant subsystem ($x_1$). The graph reveals that the MTSF increases with increase in the values of the probability of proper repair of faults in non-redundant subsystem and MTSF has lower values for higher values of the probability of major fault in redundant subsystem.

**Fig. 4.24** presents the graph between expected uptime of the system with full capacity ($AF_0$) and rate of minor faults ($\lambda_2$) for different values of rate of major faults ($\lambda_1$). It can be concluded that the expected uptime of the system with full capacity decreases with increase in the values of the rates of minor faults and expected uptime of the system with full capacity has lower values for higher values of the rate of major faults.
The graph in Fig. 4.25 gives the behavior of expected uptime of the system with full capacity ($\text{AF}_0$) with respect to rate of minor faults ($\lambda_2$) for different values of probability of minor faults in non-redundant subsystem ($x$). The graph reveals that the expected uptime of the system with full capacity decreases with increase in the values of
the rate of minor faults. However expected uptime of the system with full capacity has higher values for higher values of the probability of minor faults in non-redundant subsystem.

**Fig.4.26**

**Fig.4.26** gives the graph between expected uptime of the system with full capacity ($AF_0$) and repair rate of non-redundant subsystem ($\beta_1$) for different values of inspection rate ($\alpha_1$). The graph reveals that the expected uptime of the system with full capacity increases with increase in the values of the repair rate of non-redundant subsystem and expected uptime of the system with full capacity has higher values for higher values of the inspection rate.

The curves in **Fig.4.27** present the behaviour of expected uptime of the system with full capacity ($AF_0$) with respect to repair rate of Type-I redundant subsystem ($\beta_2$) for different values of inspection rate ($\alpha_2$). It is concluded that the expected uptime of the system with full capacity increases with increase in the values of the repair rate of Type-I redundant subsystem and expected uptime of the system with full capacity has higher values for higher values of inspection rate.
The graph in Fig.4.28 gives patterns of expected uptime of the system with full capacity ($AF_0$) with respect to the rate of major faults ($\lambda_1$) for the different values of probability of occurrence of major fault in redundant subsystem ($x_1$). From the graph it
can be concluded that the expected uptime of the system with full capacity decreases with increase in the values of the rate of major faults and expected uptime of the system with full capacity has higher values for higher values of the probability of occurrence of major fault in redundant subsystem.

**Fig.4.29**

Fig.4.29 gives the graph between expected uptime of the system with reduced capacity ($AR_0$) and rate of minor faults ($\lambda_2$) for different values of the rate of major faults ($\lambda_1$). The graph reveals that the expected uptime of the system with reduced capacity increases with increase in the values of the rate of minor faults while expected uptime of the system with reduced capacity has lower values for higher values of the rate of major faults.

The graph in fig.4.30 presents the behavior of expected uptime of the system with reduced capacity ($AR_0$) with respect of rate of minor faults ($\lambda_2$) for different values of probability of faults in Type-I redundant subsystem (a). The graph reveals that the expected uptime of the system with reduced capacity increases with increase in the values of the rate of minor faults and the expected uptime of the system with reduced capacity has lower values for higher values of the probability of faults in Type-I redundant subsystem.
The curves in Fig. 4.31 reveal the behaviour of expected uptime of the system with reduced capacity (ARₐ) with respect to the rate of major faults (λₐ₁) for the different values of probability of occurrence of major fault in redundant subsystem (x₁). It can be concluded from the graph that the expected uptime of the system with reduced capacity
decreases with increase in the values of the rate of major faults and expected uptime of the system with reduced capacity has lower values for higher values of the probability of occurrence of major fault in redundant subsystem.

**Fig.4.32**

*Fig.4.32* gives the patterns of the profit incurred from the system (P) with respect to the rate of major faults ($\lambda_1$) for the different values of rate of minor faults ($\lambda_2$).

Following has been concluded from the graph:

(i) The profit decreases with increase in the values of the rate of major faults. Also the profit has lower values for higher values of the rate of minor faults when other parameters remain fixed.

(ii) For $\lambda_1 = 0.001$, the profit is positive or zero or negative according as $\lambda_1$ is less than or equal or greater than 0.003901. Thus, the system will give profit for this when $\lambda_1$ is less than 0.003901.

(iii) For $\lambda_1 = 0.002$, the profit is positive or zero or negative according as $\lambda_1$ is less than or equal or greater than 0.003085. Thus, the system will give profit for this when $\lambda_1$ is less than 0.003085.

(iv) For $\lambda_2 = 0.003$, the profit is positive or zero or negative according as $\lambda_1$ is less than or equal or greater than 0.002276. Thus, the system will give profit for this when $\lambda_1$ is less than 0.002276.
The curves in the fig.4.33 show the behaviour of profit of the system (P) with respect to rate of major faults ($\lambda_1$) for different values of probability of occurrence of major fault in redundant subsystem ($x_1$).

From the graph, following conclusions have been made:

(i) The profit decreases with increase in the values of the rate of major faults. However, the profit has higher values for higher values of the rate of minor faults when other parameters remain fixed.

(ii) For $x_1 = 0.1$, the profit is positive or zero or negative according as $\lambda_1$ is less than or equal or greater than 0.003089. Thus, the system will give profit for this when $\lambda_1$ is less than 0.003089.

(iii) For $x_1 = 0.3$, the profit is positive or zero or negative according as $\lambda_1$ is less than or equal or greater than 0.003134. Thus, the system will give profit for this when $\lambda_1$ is less than 0.003134.

(iv) For $x_1 = 0.5$, the profit is positive or zero or negative according as $\lambda_1$ is less than or equal or greater than 0.00318. Thus, the system will give profit for this when $\lambda_1$ is less than 0.00318.
The curves in the fig. 4.34 show the behaviour of profit with respect to revenue per unit uptime of the system with full capacity \((C_0)\) for different values of repair rate of expert repairman \((\beta_e)\).

Following has been concluded from the graph:

(i) The profit increases with increase in the values of the revenue per unit uptime of the system with full capacity and the profit has higher values with the higher values of the repair rate of expert repairman when other parameters remain fixed.

(ii) For \(\beta_e = 0.1\), the profit is negative or zero or positive according as \(C_0\) is less than or equal or greater than Rs.3824.37. Thus, the system will give profit for this when \(C_0\) is greater than Rs.3824.37.

(iii) For \(\beta_e = 0.3\), the profit is negative or zero or positive according as \(C_0\) is less than or equal or greater than Rs.3147.55. Thus, the system will give profit for this when \(C_0\) is greater than Rs.3147.55.

(iv) For \(\beta_e = 0.5\), the profit is negative or zero or positive according as \(C_0\) is less than or equal or greater than Rs.2508.48. Thus, the system will give profit for this when \(C_0\) is greater than Rs. 2507.48.
Fig. 4.35 presents a graph depicting the behaviour of profit with respect to revenue per unit uptime with full capacity of the system \((C_0)\) for different values of repair rate of Type-I redundant subsystem \((\beta_2)\).

Following conclusions have been made from the graph:

(i) The profit increases with increase in the values of the revenue per unit uptime of the system with full capacity and the profit has higher values with the higher values of the repair rate of Type-I redundant subsystem.

(ii) For \(\beta_2 = 0.02\), the profit is negative or zero or positive according as \(C_0\) is less than or equal or greater than Rs.7597.30. Thus, the system will give profit for this when \(C_0\) is greater than Rs.7597.30.

(iii) For \(\beta_2 = 0.05\), the profit is negative or zero or positive according as \(C_0\) is less than or equal or greater than Rs.7537.23. Thus, the system will give profit for this when \(C_0\) is greater than Rs.7537.23.

(iv) For \(\beta_2 = 0.08\), the profit is negative or zero or positive according as \(C_0\) is less than or equal or greater than Rs.7380.70. Thus, the system will give profit for this when \(C_0\) is greater than Rs.7380.70.
The curves in the fig.4.36 show the behaviour of profit with respect to revenue per unit uptime of the system with reduced capacity \( (C_1) \) for different values of probability of minor faults in non-redundant subsystem \( (x) \).

From the graph, following conclusions have been made:

(i) The profit increases with increase in the values of the revenue per unit uptime of the system with reduced capacity and the profit has higher values with the higher values of the probability of minor faults in non-redundant subsystem when other parameters remain fixed.

(ii) For \( x = 0.3 \), the profit is negative or zero or positive according as \( C_1 \) is less than or equal or greater than Rs.10909.78. Thus, the system will give profit for this when \( C_0 \) is greater than Rs.10909.78.

(iii) For \( x = 0.5 \), the profit is negative or zero or positive according as \( C_1 \) is less than or equal or greater than Rs.8919.21. Thus, the system will give profit for this when \( C_0 \) is greater than Rs.8919.21.

(iv) For \( x = 0.7 \), the profit is negative or zero or positive according as \( C_1 \) is less than or equal or greater than Rs.7870.42. Thus, the system will give profit for this when \( C_0 \) is greater than Rs.7870.42.