Chapter 1

Introduction

Convective heat and mass transfer is one of the major modes of heat and mass transfer in fluids. This phenomenon takes place both by diffusion and by advection, in which matter or heat is transported by the larger scale motion of currents in the fluid. Convection can be qualified in terms of being natural, forced, gravitational, granular, or thermomagnetic. The natural convection has been considered most of our problems since natural convection plays a role in broad range of applications, because in practice, convection occurs for very small temperature gradients. Important in environmental science climate, building ventilation; geophysics e.g. mantle convection; industrial processes e.g. crystal growth. It is known fact that, controlling convection is mainly concerned with space dependent temperature gradients. There are many interesting situations of practical problems in which the temperature gradient is a function of both space and time. This uniform temperature gradient can be determined by solving the energy equation with suitable time dependent thermal boundary conditions and can be used as an effective external mechanism to control the convective flow. However, in practice, the nonuniform temperature gradient finds its origin in transient heating or cooling at the boundaries. Hence the basic temperature profile depends explicitly on position and time, this problem is known thermal modulation problem, involves the solution of the energy equation under
suitable time dependent boundary conditions. Predictions exist for a variety of responses to modulation depending on the relative strength and rate of forcing. Among these, there is the upward or downward shift of convective threshold compared to the unmodulated problems. An excellent review related to this problem is given by Davis (1976). The excellent review and analysis of Rayleigh–Bénard convection under various physical configurations given by Chandrasekhar (1961), Drazin and Reid (2004).

Bénard–Rayleigh convection and its porous medium analog, namely, Bénard–Darcy convection. The study of natural convection in a Newtonian fluid saturated porous medium is now well understood and documented phenomenon. A comprehensive account of the problem and their applications are available presently in the excellent books of Ingham and Pop (1998, 2005), Vafai (2000, 2005), Straughan (2004), Nield and Bejan (2013) and Vadasz (2008). However, in most of the above studies the temperature gradient across the porous medium has been considered to be uniform, which is not so in many practical problems. Thus, keeping in mind the industrial applications of the study, it is more appropriate to assume temperature gradient to be a function of both space and time. In the next section the review of literature has been presented according to the types of modulations either by considering Rayleigh–Bénard or Darcy Bénard convection.

1.1 Review of Literature

1.1.1 Temperature modulation

Temperature modulation in fluid layer

Many researchers, under different physical models have investigated thermal instability in a horizontal fluid layer with temperature modulation. Donnelly (1964) investigated the effect of rotation speed modulation on the onset of instability in fluid flow between two concentric cylinders. However, the rotation speed modulation was the originating idea of the temperature, as well as gravity modulation. Venezian (1969) was motivated by the experiment of Donnelly (1964) and performed a linear stability analysis of Rayleigh–Bénard convection for the case of small amplitude temperature modulation.
Using perturbation method and considering free–free surfaces, he calculated the shift in the critical value of the Rayleigh number and found that, the system can be stabilized or destabilized by suitably tuning the frequency of modulation. A similar problem was studied earlier by Gershuni and Zhukhovitskii (1963), for a temperature profile obeying rectangular law. According to them the convective instability of fluid in a gravity field is usually investigated under the assumption that the equilibrium temperature does not depend on time. However, unsteady equilibrium of a fluid is also possible when the equilibrium temperature varies with time according to the law that is determined by unsteady heating conditions. Rosenblat and Herbert (1970), performed a linear stability problem and found an asymptotic solution by considering low frequency modulation and free-free boundaries. Rosenblat and Tanaka (1971), studied the linear stability for a fluid in a classical geometry of Bénard by considering the temperature modulation of rigid–rigid boundaries. Using Galerkin technique and discussed the stability of the system using Floquet theory. Finucane and Kelly (1976), performed a theoretical and experimental investigation of the thermal modulation in a horizontal fluid layer. They found both experimentally and numerically that, at low frequencies the modulation is destabilizing, where as at high frequencies it is stabilizing. Niemela and Donnelly (1987), Schmitt and Lucke (1991), Or and Kelly (1999) and Or (2001) have also investigated the effect of external modulation on the thermal convection in a horizontal fluid layer. The first nonlinear stability problem in a horizontal fluid layer, under temperature modulation of the boundaries was studied by Roppo et al. (1984). Bhadauria and Bhatia (2002), studied the effect of temperature modulation on thermal instability by considering rigid–rigid boundaries and different types of temperature profiles. Bhadauria (2006), studied the effect of temperature modulation under vertical magnetic field by considering rigid boundaries and using Floquet theory. Further, he also found that it is possible to advance or delay the onset of convection by proper tuning of the frequency of modulation of the wall’s temperature. Malashetty and Swamy (2008), investigated thermal instability of a heated fluid layer subject to both temperature modulation and rotation effects. It is established that, the instability can be enhanced by the rotation at low frequency symmetric modulation.
and with moderate to high frequency lower wall temperature modulation, whereas the stability can be enhanced by the rotation in case of asymmetric modulation. They also found that by proper adjusting the system parameter values it is possible to advance or delay the onset of convection. Bhadauria et al. (2009), studied the nonlinear aspects of thermal instability under temperature modulation, considering various temperature profiles. Raju and Bhattacharyya (2010), investigated onset of thermal instability in a horizontal layer of fluid with modulated boundary temperatures by considering rigid boundaries. Bhadauria et al. (2012) studied thermally or gravity modulated nonlinear stability problem in a rotating viscous fluid layer, using Ginzburg–Landau equation for stationary mode of convection. Recently Bhadauria and Kiran (2014d) investigated an oscillatory mode of double diffusive convection under thermal modulation using complex non-autonomous Ginzburg–Landau equation. They have presented a very good results where oscillatory convection advances the convection and enhances the heat transfer than stationary. They also have presented how phase angle and frequency of modulation affects heat and mass transfer in the system.

Temperature modulation in porous medium

Caltaoirone (1976), was the first to study the stability of a horizontal porous layer, the temperature of which at the inner side is a periodical function of time, has been theoretically studied by using the Galerkin technique. He showed a correlation between the Rayleigh number, the reduced frequency of the signal on the wall and its amplitude. Different types of evolution of the instabilities within the porous layer have observed. The stability of a horizontal porous layer bounded by two impermeable planes is investigated by Chhuon and Caltagirone (1979). A time dependent periodic temperature profile is imposed on the lower boundary while the upper plane is kept at constant temperature. Using the linear stability theory, a criterion for the onset of convection is defined as a function of the perturbation wavenumber and of the amplitude and frequency of the temperature oscillation. Experimental work with a setup allowing both the amplitude and the frequency of the thermal signal to vary is done. Rudraiah et al. (1990) have investigated the effect of thermal modulation on the onset of convection in a viscoelastic fluid-saturated porous
medium using Oldroyd model. Antohe and Lage (1996), heat and momentum transport is investigated theoretically and numerically considering a rectangular enclosure filled with clear fluid or with fully saturated porous medium, under time-periodic horizontal heating. It is shown that the convection intensity within the enclosure increases linearly with heating amplitude for a wide range of parameters. Moreover, the flow response to pulsating heat is continuously enhanced as the system becomes more permeable. Using Venezian (1969) approach and considering Forchheimer flow model with effective viscosity larger than the fluid viscosity, Malashetty and Wadi (1999), investigated the problem of thermal instability under thermal modulation, and calculated the correction in the critical Rayleigh number as a function of system parameters. It is shown that, the system is most stable when the boundary temperature is modulated out of phase. It is also found that the low frequency thermal modulation can have a significant effect on the stability of the system. Malashetty and Basavaraja (2002, 2003), using Brinkman model with effective viscosity larger than the viscosity of the fluid they have investigated linear theory analysis for anisotropic porous medium. They have found that, it is possible to advance or delay the onset of convection by wall temperature and to advance convection by gravity modulation. They also found, the small anisotropy parameter has a strong influence of the stability of the system. Bhadauria (2007) also studied the convection in a sparsely packed porous medium under temperature modulation. Considering rigid—rigid boundaries and by Galerkin method, he calculated the critical Rayleigh number for the onset of convection. Considering thermal modulation on the boundaries of the porous medium a series of the work has been investigated by Malashetty et al.(2006), Bhadauria (2007), Bhadauria (2008), Suthar and Bhadauria (2009), Bhadauria and Srivastava (2010), Shivakumara et al. (2011), Malashetty and Begum (2011), Siddheshwar et al. (2013), Bhadauria et al. (2013) and Bhadauria and Kiran (2013a). In these papers they have considered the time periodic temperature at the boundaries as proposed by Venezian (1969) and it was discussed onset criteria for linear theory and heat and mass transfer for nonlinear theory for various physical configuration of the problem. Recently Bhadauria and Kiran (2014c,d) investigated an oscillatory mode of convection under thermal modulation, using complex
non-autonomous Ginzburg–Landau equation. They have found that, an oscillatory convection advances the convection and enhances the heat transfer than stationary. They also have presented how phase angle and frequency of modulation affects heat transfer in the system.

1.1.2 Gravity modulation

The problem of convection in a fluid layer in the presence of complex body forces has gained considerable attention in recent decades due to its important applications in engineering and technology. The time dependent gravitational field, one of the complex forces, is of interest in space laboratory experiments, in areas of crystal growth and others. It is also of importance in the large scale convection in atmosphere. The random fluctuations of gravity field, both in magnitude and direction, can be seen in space laboratories, significantly influence natural convection. Existence of adverse density variations with in the fluid and a body force are the necessary conditions to initiate natural convection. The idea of using mechanical vibration as a tool to improve the heat transfer rate has received much attention. However, the regulation of convection is important from the applications point of view and thermo–gravitational vibration is known to be an effective means of controlling instabilities. The gravity modulation of the system leads to the variable coefficients in the governing equations of thermal instability and involves the vertical time periodic vibrations of the system and gravity modulation is known as g–jitter in literature.

**Gravity modulation in fluid layer**

Gershuni and Zhukhovitskii (1963) and Gresho and Sani (1970) were the first to study the effect of gravity modulation in a fluid layer. They studied, the stability of Rayleigh–Bénard convection for the case of a time dependent buoyancy force which is generated by shaking the fluid layer, thus causing a sinusoidal modulation of the gravitational field. A linearized stability analysis is performed to show that gravity modulation can significantly affect the stability limits of the system. Biringen and Peltier (1990), investigated the nonlinear three dimensional Rayleigh–Bénard problem under gravity.
modulation numerically, and confirmed the result obtained by Gresho and Sani (1970). Wadih and Roux (1988) presented a study on instability of the convection in an infinitely long cylinder with gravity modulation oscillating along the vertical axis. The effect of modulation on the stability limits given by linear theory in the standard steady case is analysed. A method based on Floquet theory is proposed in the case of small values of the modulation amplitude for a fixed value of the frequency of modulation. Saunders et al. (1992) have discussed the effect of gravity modulation on thermosolutal convection in an infinite layer of fluid. Clever et al. (1993) performed a detailed non-linear analysis of Rayleigh–Bénard convection under $g$–jitter and presented the stability limits to a much wider region of parameter space. Chen and Chen (1999) have studied the effect of gravity modulation on the stability of convection in a vertical slot. They have examined the stability for fluids of different Prandtl numbers. Rogers et al. (2000) have observed super-lattice patterns in vertically oscillated Rayleigh–Bénard convection. Rogers et al. (2005), Bhadauria et al. (2005) showed that the gravitational modulation, which can be realized by vertically oscillating a horizontal liquid layer, acts on the entire volume of liquid and may have a stabilizing or destabilizing effect depending on the amplitude and frequency of the forcing. Shu et al. (2005) examined the effects of modulation of gravity and thermal gradients on natural convection in a cavity, numerically as well as experimentally. They found that for low Prandtl number fluids, modulations in gravity and temperature produce the same flow field both in structure and in magnitude. Gravity modulation in a fluid layer has been studied by Bhadauria (2006). Boulal et al. (2007), have given attention on the influence of a quasi periodic gravitational modulation on the convective instability. They predicted that, the threshold of convection corresponds precisely to quasi periodic solutions. Bhadauria et al. (2012), studied thermally or gravity modulated nonlinear stability problem in a rotating viscous fluid layer, using Ginzburg–Landau equation for stationary mode of convection. Siddheshwar et al. (2012a), have investigated thermal/gravitation modulation on electrically conducting fluid layer as a magnetoconvection considering nonlinear analysis of stationary mode. Bhadauria et al. (2013) studied internal heating effects on weak nonlinear Rayleigh–Bénard convection under
gravity modulated, using Ginzburg–Landau equation for stationary mode of convection. They found that, the gravity modulation works for both enhancing or diminishing heat transfer in the system. Recently Bhadauria and Kiran (2014a) investigated an oscillatory mode of convection under gravity modulation in a viscoelastic fluid layer using complex non-autonomous Ginzburg–Landau equation. They found that, an oscillatory convection advances the convection and enhances the heat transfer than stationary.

**Gravity modulation in porous layer**

Malashetty and Padmavathi (1997), studied the effect of small amplitude gravity modulation on the onset of convection in fluid and fluid saturated porous layer. They found that low frequency oscillations have significant effect on the stability of the system. Bardan and Mojtabi (2000), made an analytical and numerical study of convection in a porous cavity in the presence of vertical vibrations. They found that the vibrations stabilize the quiescent state. Rees (2000), considered the boundary layer flow induced by a constant temperature vertical surface embedded in a porous medium is modified by time periodic variations in the gravitational acceleration. Using an amplitude expansion to determine the detailed effect of g–jitter, and the expansion is carried through to fourth order. The numerical and asymptotic solutions show that the g–jitter effect is eventually confined to a thin layer embedded within the main boundary layer, but it becomes weak at increasing distances from the leading edge. Later Rees and Pop (2003), the nonsimilar boundary layer equations are solved using the Keller box method after using a Fourier decomposition in time to reduce the system to parabolic form with only two independent variables. The main effect of such g–jitter is confined mainly to the region near the leading edge and becomes weak at larger distances from the leading edge. Rees and Pop (2001), the effect of periodical gravity modulation on the free convection near the forward stagnation point of a cylindrical surface which is embedded in a porous medium has been investigated while considering nonlinear theory. Govender (2004), made stability analysis to investigate the effect of low amplitude gravity modulation on convection in a porous layer heated from below. It was shown that increasing the frequency of vibration stabilizes the convection. Govender (2005a), using linear theory he demonstrated that
increasing the excitation frequency rapidly stabilizes the convection up to the transition point from synchronous to subharmonic convection. Beyond the transition point, the effect of increasing the frequency is to slowly destabilize the convection. Govender (2005b) using weak nonlinear analysis he found that, increasing the vibration frequency causes the convection amplitude to approach zero, i.e., increasing the vibration frequency stabilizes the convection. The effect of vertical vibration on the stability of a dilute suspension of negatively geotactic microorganisms in a fluid layer of finite depth is investigated by Kuznetsov (2005). Kuznetsov (2006) investigated analytically the potential of utilizing the vertical vibration for controlling bioconvection. A shallow horizontal fluid saturated porous layer that contains a suspension of oxytactic bacteria, such as Bacillus subtilis, is considered. He said that, the linear stability analysis indicates that vertical vibration has a stabilizing effect on the suspension. Siddhavaram and Homsy (2006), they studied the effects of gravity modulation on the mixing characteristics of two inter diffusing miscible fluids initially in two vertical regions separated by a thin diffusion layer. Strong (2008), the effect of vertical harmonic vibration on the onset of convection in an infinite horizontal layer of fluid saturating a porous medium is studied. His investigation is carried out that the vertical vibration can significantly affect the stability of the system by increasing or decreasing its susceptibility to convection. The same problem i.e gravity modulation extended for double diffusive convection in porous medium by Strong (2009). Saravanan and Arunkumar (2010), the effect of gravity modulation on the onset of convection in a horizontal fluid saturated porous layer in which the applied temperature gradient is opposite to that of gravity is investigated. The flow through the porous layer is governed by an extended form of Darcys law incorporating Brinkmans boundary layer correction. Their study is focussed on low amplitude gravity modulation and the thresholds are found using Mathieus functions. The emergence of instability via the synchronous and subharmonic modes and the transition between them are discussed as a function of the physical parameters. Malashetty and Swamy (2011), investigated the effect of gravity modulation on the onset of thermal convection in a fluid or porous layer using linear stability. They show that, in general the gravity modulation produces a stabilizing effect in case
of viscous fluid layer and both destabilizing and stabilizing effects in case of Brinkman porous medium while it produces a destabilizing effect in case of Darcy porous medium. The low frequency gravity modulation is found to have a significant effect on the stability of the problem. It is also shown that, the onset can be advanced or delayed by proper tuning of various governing parameters. Saravanan and Sivakumara (2010, 2011) studied the effect vibrations on the onset of convection in a horizontal fluid saturated porous layer considering an arbitrary amplitude and frequency. It is demonstrated that vibrations can produce a stabilizing or a destabilizing effect depending on their amplitude and frequency for a porous layer heated from below. The temperature sensitivity of viscous fluid saturated porous medium under vertical vibrations of Rayleigh–Bénard convection is investigated by Siddheshwar et al. (2012b). They found that the variable viscous parameter has a tendency of enhancing heat transfer in the system. Srivastava et al. (2013), have investigated the temperature sensitivity of viscous fluid saturated porous medium under gravity modulation along with internal heating effects. They found that, the variable viscosity and internal Rayleigh number is to enhance the heat transfer in the system. Recently Bhadauria and Kiran (2014b) investigated an oscillatory mode of convection under gravity modulation in a viscoelastic fluid saturated porous medium using complex non-autonomous Ginzburg–Landau equation. They found that, an oscillatory mode of convection enhances the heat transfer more than stationary mode of convection.

1.1.3 Rotation speed modulation

The combined effect of both thermal modulation and rotation on the onset of convection in a rotating fluid layer was made by Rauscher and Kelly (1975). Liu and Ecke (1997), analyzed heat transport of turbulent Rayleigh–Bénard convection under rotational effect. Malashetty and Swamy (2008), investigated thermal instability of a heated rotating fluid layer subjected to temperature modulation. They found that, by proper tuning of modulation frequency, Taylor number and Prandtl number it is possible to advance or delay the onset of convection. Kloosterziel and Carnevale (2003), investigated the effect of rotation on the stability of thermally modulated system. They determined
analytically critical points on the marginal stability boundary above which an increase of either viscosity or diffusivity is destabilizing. Finally, they show that, when the fluid has zero viscosity the system is always unstable, in contradiction to Chandrasekhar (1961) conclusion. Malashetty and Swamy (2007), studied the effect of rotation on the stability of thermally modulated system. They found that, the symmetric modulation destabilizes the system at low frequencies while it stabilizes the system at moderate and high frequencies. They also found, an asymmetric modulation is the most stable situations, for all frequencies. Finally the lower wall temperature has stabilizing effect for low and higher values of frequency and destabilizing effect for moderate values of frequency. Bhadauria (2008), investigated rotational influence on Darcy convection and found that both rotation and permeability suppress the onset of thermal instability. Bhadauria et al. (2012) investigated the nonlinear thermal instability in a rotating viscous fluid layer under temperature/gravity Modulation. They found that the effect of rotation is to diminish the heat transfer in the system and modulation is to enhance the heat transfer or diminish the heat transfer depending on the amplitude and frequency.

The rotation speed modulation was the originating idea of the temperature as well as gravity modulation, but not much research work has implemented in this field. The effect of temperature modulation on the Rayleigh–Bénard instability and the effect of modulation of the rotation speed in the Taylor–Couette instability has been investigated in detail both theoretically and experimentally by Ahlers et al. (1985), Niemela and Donnelly (1986), Kumar et al. (1986), Meyer et al. (1988), Walsh and Donnelly (1988). For Rayleigh–Bénard convection the temperature modulation is supposed to stabilize the conduction state. However, since the temperature modulation breaks the reflection symmetry about the midplane and hexagons, rather than cylinders, takes the pattern in which convection occurs immediately above the threshold. The Rayleigh–Bénard problem with rotation, the above problem can be avoided if the rotation speed is modulated in time periodic manner. This leads to a simple problem for the study of the effect of modulation on the threshold. When we study the rotational effect, then one more parameter in the form of rotation speed exists, which can affect the stability of convective flow. From

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the literature, the study due to Bhattacharjee (1989) is of great importance, in which he studied the effect of rotation speed modulation on Rayleigh–Bénard convection in an ordinary fluid layer. He found that the effect of modulation is stabilizing for most of the configurations. Bhadauria and Suthar (2009) investigated the effect of the rotation speed modulation on the onset of free convection in a rotating porous layer placed farther away from the axis of rotation. Suthar et al. (2011), investigated the time periodic rotational speed on the onset of free convection in a rotating porous layer about \(z\)-axis. They conclude that, the effect of modulated rotation speed is found to have a stabilizing effect on the onset of convection for different values of modulation frequency.

1.1.4 Magnetocovective/Magnetic modulation

In general the magnetic fluids are differ from the ordinary fluids by showing magnetic as well as flow properties. The magnetococonvection arises due to the interaction of electrically conducting fluid flow and the applied magnetic field. Convection can also take place in these fluids due to temperature dependence of their magnetization. This property is useful in space research, where the role of gravity can be replaced by a magnetic body force. The magnetic force can be used to create circulation in small passages where natural convection is either absent or ineffective. Generally, the magnetization depends on the magnetic field, temperature and density. Hence, the magnetic force depend on the thermal state of the fluid and may lead to convection.

Thompson (1951) was the first to study magnetoconvection in horizontal fluid layer. Using Galerkin method, Nakagawav (1955, 1957) and Jirlow (1956), investigated that, the vertical magnetic field suppresses the onset of convection. Finlayson (1970) considered a magnetic horizontal fluid layer heated from below in the presence of an uniform vertical magnetic field. He studied the linear stability analysis by considering free–free and rigid–rigid boundaries, and predicted the critical gradient of temperature corresponding to the onset of convection, considering both buoyancy and magnetic forces. Gotoh and Yamada (1982) the linear instability is investigated for a horizontal magnetic fluid layer confined between two ferromagnetic boundaries and heated from below in the presence of
a vertical magnetic field. Galerkin method is used for solving the disturbance equations. It is concluded that the magnetization of the boundaries and the nonlinearity of fluid magnetization both reduce critical Rayleigh number, and that the effects of magnetic force and buoyancy compensate each other. Schwab et al. (1983) analyzed the Finlayson’s problem experimentally in the presence of strong magnetic field and discussed the onset of convection. Later, Stiles and Kagan (1990) examined the problem reported by Schwab et al. (1983) and generalized the Finlayson’s model assuming that under a strong magnetic field, the rotational viscosity augments the shear viscosity. Rudraiah and Sekhar (1991) treated the Finlayson’s problem with internal heat source and showed that the variation of temperature, due to heat source, induces a variation in the magnetic field. These variations can be used to control magnetic convection.

Another interesting case consists of applying an external magnetic field of a constant or spatially varying gradient. Aniss et al. (1993) and Souhar et al. (1999) proposed theoretical and experimental investigations of the Rayleigh–Bénard convection in a magnetic fluid layer confined in a horizontal annular Hele-Shaw cell and submitted simultaneously to radial temperature and magnetic field gradients. With their geometrical configuration, they showed the possibility to simulate theoretically and experimentally the Rayleigh–Bénard convection and its control by an external magnetic field gradient in the absence of gravity. Siddheshwar and Pranesh (1999, 2000) examined the effects of time periodic temperature/gravity modulation on the onset of magnetoconvection in electrically conducting fluids with internal angular momentum by making a linear stability analysis. Bhadauria (2006) studied the effect of temperature modulation on magnetoconvection using rigid boundaries and Fluquet theory. Further, Bhadauria (2007, 2008) studied the combined effect of temperature modulation and magnetic field on the onset of convection in an electrically conducting fluid saturated porous medium using rigid–rigid and free–free boundaries. Bhadauria and Sherani (2008, 2010) investigated the onset of Darcy convection in a magnetic fluid saturated porous medium subject to temperature modulation of the boundaries and magnetoconvection. Siddheshwar et al. (2012) investigated heat transport for a stationary magnetoconvection in a Newtonian liquid under
temperature or gravity modulation by performing a weak nonlinear stability analysis and using Ginzburg–Landau model.

Aniss et al. (2000) showed that, the vertical oscillations of the cell generate parametric convective instability only for small Prandtl numbers. Consequently, using the similarity between gravitational and magnetic modulations, the parametric convective instability cannot occur in a Hele–Shaw magnetic liquid layer. Most of the above studies are concern with linear stability analysis. It is observed that an imposed vertically time periodic magnetic field is just like gravity modulation. According to Aniss et al. (2001) magnetic modulation is much effective to handle theoretical and experimental investigation rather than the gravity modulation, in which the amplitudes and frequencies are more difficult to control. When we consider the system in the space (where magnetoconvection occurs in stars: near the surface of cool, surrounding nuclear burning shells in the late stages of stellar evolution, in supernova explosions, in accretion disks during the formation of stars and planets and in accretion onto black holes and neutron stars, in the hot plasma in clusters of galaxies etc.) handling temperature modulation is some what difficult.

1.2 Equations of hydrodynamics

In fluid mechanics the following are the basic properties that every fluid obeys where the hydrodynamical flow of a viscous fluid of varying density and temperature:

1. Equation of continuity

A continuity equation is an equation that describes the transport of a conserved quantity. Since mass, momentum, energy, electric charge and other natural quantities are conserved under their respective appropriate conditions, a variety of physical phenomena may be described using continuity equations. The continuity equation states that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. The differential form of the continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{q} = 0,$$

(1.2.1)
where $\rho$ is fluid density, $t$ is time, $\vec{q}$ is the flow velocity vector field. This equation is one of Euler equations. For incompressible fluid flow, the mass continuity equation simplifies to a volume continuity equation:

$$\nabla \cdot \vec{q} = 0,$$

(1.2.2)

which means that the divergence of velocity field is zero everywhere.

2. **Equation of momentum**

The general form of the equations of fluid motion is:

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{q} + F,$$

(1.2.3)

where $\nu$ is kinetic viscosity, $F$ is the body force term, represents an external forces that act on the fluid; for example: gravity, wind, etc. The time derivative term $\frac{\partial}{\partial t}$ is called the local derivative, which is physically the time rate of change at a fixed point, $(\vec{q} \cdot \nabla)$ is called the convective derivative, which is physically the time rate of change due to the movement of the fluid element from one. This is a statement of the conservation of momentum in a fluid and it is an application of Newton’s second law to a continuum.

3. **Equation of energy**

The energy equation is defined as

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T = \kappa T \nabla^2 T.$$

(1.2.4)

It is based on the law of conservation of energy, $\kappa_T$ is the thermal conductivity, which is proportional constant in Fourier’s law of heat conduction.

4. **Boussinesq approximation**

The Boussinesq approximation states that, the density differences are sufficiently small to be neglected, except where they appear in terms multiplied by $\vec{g}$, the acceleration due to gravity. The essence of the Boussinesq approximation is that the difference in inertia is negligible but gravity is sufficiently strong to make the
specific weight appreciably different between the two fluids. The accounted density according to Boussinesq is given by
\[ \rho = \rho_0 [1 - \alpha_T (T - T_0)], \] (1.2.5)
where \( \alpha_T \) denotes the thermal expansion coefficient, and subscript 0 refers to some reference value of the quantity.

1.3 Basics & different models for porous medium

1. **Porosity**

Porosity of the porous medium is a basic quantity for characterizing a porous medium. Porosity is defined as the ratio between the volume occupied by the fluid (voids) and the total volume of the material (including voids and solid). It is also defined the ratio of pore volume to its total volume. Let, \( V_f \) denotes the volume of the fluid (or voids), and \( V_m \) denotes the volume of the material, then the porosity \( \delta_\phi \) is given by,
\[ \delta_\phi = \frac{V_f}{V_m}. \] (1.3.1)
Clearly, the porosity of any porous domain lies in the interval (0,1). The porosities of most commonly occurring porous media are less than 0.6.

2. **Darcys Law & Permeability**

Darcy’s law describes the flow of a fluid through a porous medium. The law was derived by Henry Darcy (1856) based on the results of experiments on the flow of water through beds of sand. It also forms the scientific basis of fluid permeability used in the earth sciences, particularly in hydrogeology. Henry Darcy found a proportionality between the seepage velocity and the applied pressure gradient, which is given by
\[ \bar{q} = -\frac{K}{\mu} \nabla p + F, \] (1.3.2)
where \( K \) specific permeability or intrinsic permeability of medium, is independent of nature of the fluid but depends on the geometry of the medium. It measures
the flow conductance of a porous domain. For an anisotropic porous domain, it is generally a tensor of second order and this permeability $K$ varies, and this variable permeability, enhanced within a region of constant thickness. According to Rees and Pop (2000), near the leading edge the flow enhanced and the rate of heat transfer is more than in non-uniform permeability case. Here $\mu$ is the dynamic viscosity of the fluid, $p$ is the pressure and $F$ is the external body force.

3. Modifications in Darcys Law

The Darcys law is applicable only when seepage velocity $\vec{q}$ is sufficiently small and is linear or gradually looses it’s validity for high velocities, i.e. for high Reynold numbers. Thus, extensions of the Darcys law was modified in the form of Darcy-Forchheimer model and Brinkmans model.

(a) Brinkman—extended Darcy model

The Brinkman equation is defined from Darcy equation as

$$\nabla p = -\frac{\mu}{K}\vec{q} + \mu\nabla^2 \vec{q}$$

(1.3.3)

where $K$ is the permeability and it is the reciprocal of the shear factor. The above equation is the working version of the Brinkman equation where the viscosity associated with the viscous diffusion term is the same as the viscosity of the fluid. The added diffusion term simply to meet the boundary specifications and hence the viscosity was not defined. Brinkmans first version of the flow equation is given by

$$\nabla p = -\frac{\mu}{K}\vec{q} + \bar{\mu}\nabla^2 \vec{q},$$

(1.3.4)

where $\bar{\mu}$ is a quantity having the dimension of viscosity and it was named the effective viscosity. One should acknowledge that Eq. (1.3.4) is a general form of volume averaged Stokes equation. In general, the effective viscosity is not expected to be the same as the viscosity of the fluid owing to the effect of tortuosity and the dispersion of viscous diffusion flux.

(b) Brinkman—Forchheimer—extended Darcy model

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The Darcy–Brinkman–Forchheimer equation is defined as

\[-\nabla p = \frac{\mu K}{q} + \frac{c_F \rho |q|}{\sqrt{K}} q - \mu \nabla^2 q, \tag{1.3.5}\]

where $c_F$ is the form drag coefficient. Equation (1.3.5) is thought as an extension from the Brinkman’s equation by accounting for the inertial effects on the internal shear loss term, however, the dispersion of momentum is not accounted for. Owing to the omission of the momentum dispersion, one should note that Eq. (1.3.5) is useful only for systems where the flow domain is large, that is, when Darcy’s law is valid at creeping flow. Hence, strict restrictions apply to the use of the Brinkman–Forchheimer equation. When the diffusion term is dropped out, Eq. (1.3.5) becomes the Darcy–Forchheimer equation. That is,

\[-\nabla p = \frac{\mu}{K} q + \frac{c_F \rho |q|}{\sqrt{K}} q. \tag{1.3.6}\]

When an interface is encountered, an additional empirical model on the velocity jump condition needs to be provided in connection with the Darcy’s law or Darcy–Forchheimer equation to account for the inconsistency of the governing equation and the physical description of the flow.

1.3.1 Fundamental equations for porous medium

1. **Continuity equation**

\[\nabla \cdot \vec{q} = 0. \tag{1.3.7}\]

2. **Equation of momentum**

\[\frac{\rho}{\delta_t} \frac{\partial \vec{q}}{\partial t} = -\nabla p - \frac{\mu}{K} \vec{q} + \rho_f \vec{g}. \tag{1.3.8}\]

3. **Equation of energy**

\[(\rho c)_m \frac{\partial T}{\partial t} + (\rho c_p)_f (\vec{q} \cdot \nabla) T = \kappa_m \nabla^2 T. \tag{1.3.9}\]
4. **Oberbeck-Boussinesq approximation**

\[ \rho_f = \rho_0 [1 - \alpha_T (T - T_0)], \]

(1.3.10)

where \( \vec{q} \) is seepage velocity, \( \delta_1 \) is porosity of the porous medium, \( p \) is pressure, \( \mu \) is the dynamic viscosity, \( K \) is permeability, \( c \) is specific heat, \( \kappa_m \) is overall thermal conductivity, and \( \alpha_T \) is the thermal volume expansion coefficient.

## 1.4 Basics of heat transfer

Heat transfer is nothing but, heat moving from one object to another. Heat generally transfers from a high temperature object to a lower temperature object. Thermal instability happens mainly due to heat transfer. It happens whenever there exists a temperature difference in a medium or between media. Heat transfer can occur in the following three ways.

1. **Conduction**

   The heat transfer by means of conduction, occurs when the objects are in physically contacts, the heat, in the form of kinetic energy, is transferred at microscopic level, between the molecules by their collisions with each other without any motion of the object as a whole. Thermal conductivity is the property of a material to conduct heat and evaluated primarily in terms of Fourier’s Law for heat conduction. The rate of heat transfer via conduction is different for different materials, and is measured by the thermal conductivity of the material.

2. **Radiation**

   Radiation is heat transfer in the form of emission of electromagnetic waves which carry energy away from the emitting object. This emission may be attributed to changes in the electron configurations of the constituent atoms or molecules. The heat of the Sun is a good example of the radiative heat transfer. The major part of our study is the naturally convective heat transfer and thus we stick to it.
3. **Convection**

In the case of convection, heat transfer occurs via macroscopic motion of the fluid from a hot to a cool region, i.e., when the heated fluid is caused to move away from the source of heat, carrying energy with it. In convection, there are two types of mechanisms. Energy transfer due to random molecular motion (diffusion); in this case energy is transferred by the bulk, or macroscopic, motion of the fluid. Energy is also transformed due to temperature gradient. Boiling of water is a very common example of convective heat transfer. This convective phenomenon divided into; Natural (or free) convection, Forced convection and Mixed convection.

(a) **Natural convection**

If the motion of the fluid is caused by the buoyant force differences, resulting from a temperature or concentration gradient, in a body force field, like a gravitational field, then this process of heat transfer is known as Natural or Free convection. Through the density differences, the buoyancy force come into existence which give rise to the fluid motion.

(b) **Forced convection**

Forced convection is a mechanism, or type of heat transport, in which the fluid motion is generated by any external source (like a pump, fan, suction device, etc.). For example, the use of a fan to provide forced convection air cooling of hot electrical components on a stack of printed circuit boards.

(c) **Mixed convection**

Mixed convection is a combination of both forced and free convection’s which is the general case of convection when a flow is determined simultaneously by both an outer forcing system (i.e., outer energy supply to the fluid streamlined body system) and inner volumetric (mass) forces, viz., by the nonuniform density distribution of a fluid medium in a gravity field.
1.4.1 Rayleigh-Bénard convection

Rayleigh-Bénard convection is a type of natural convection (gravity induced free convection), occurring in a plane horizontal fluid layer heated from below, in which the fluid develops a regular pattern of convection cells known as Bénard cells. Buoyancy, and hence gravity, is responsible for the appearance of convection cells. There are three assumptions one can make for the Rayleigh-Bénard convection, viz., (a) the fluid is incompressible, (b) the density of the fluid is the only property that gets affected by the change in the temperature across it (c) an uniform gravitational force it experiences over its entire volume. The second assumptions is further limited to the degree that the density variations are given by the Boussinesq approximation. The Rayleigh-Bénard convection mainly depends on the buoyancy force and the viscous force. The ratio of these two forces is called the Rayleigh number which is defined as

\[ Ra = \frac{\alpha T g \Delta T d^3}{\nu \kappa_m} \]  

where \( d \) is the distance between the plates, \( \Delta T \) is the temperature difference, \( \nu \) is the kinematic viscosity and \( \kappa_m \) is the thermal diffusivity of the fluid. When the Rayleigh number exceeds a certain value the convection takes place in the system. This value is called critical Rayleigh number.

1.4.2 Hoton-Rogers-Lapwood convection

Rayleigh-Bénard convection for porous media analogue of is known as Horton-Rogers-Lapwood convection it was named by Horton and Rogers (1945) and Lapwood (1948). The assumption other than the Rayleigh-Bénard convection is that the porous layer is assumed to be isotropic and the fluid and solid phases are in local thermal equilibrium. The onset of convection in this case is governed by Rayleigh-Darcy number given by

\[ Ra_D = \frac{\alpha T g \Delta T K d}{\nu \kappa} \]  

In general \( Ra_D \) is the product of the Darcy number \( Da = \frac{K}{\varphi^2} \) and usual Rayleigh number for clear viscous fluid. Here \( K \) is permeability, \( \alpha \) is the thermal volume expansion.
coefficient, $\kappa_T$ is thermal diffusivity and $\nu$ is kinematic viscosity.

1.5 Important definitions

1. Double-diffusive convection

The density stratification in thermal convection is due to the variation of only one component (temperature), then the system is called single diffusive system. If it is due to two components such as temperature and salinity, then it is known as Double diffusive system. These two components will have different diffusivities, and opposite contribution to the density; i.e. one component is to stabilize and other is to destabilize the system. In this case both thermal and concentration (solute) gradients are present, then the Boussinesq equation takes the form

$$\rho = \rho_0[1 - \alpha_T(T - T_0) + \beta_T(S - S_0)],$$

(1.5.1)

where $\alpha_T$ and $\beta_T$ are the thermal and solute expansion coefficients, $T$ and $S$ are the temperature and the solute content, respectively. It is to be noted that when there is direct coupling between the two diffusing components, then it is called cross-diffusion. In this the flux of one component is caused by the another component spatial gradient.

2. Magneto-convection

Thermal convection in the presence of an imposed vertical magnetic field and the fluid is of electrically conducted is known as magneto-convection. The effect of magnetic field on thermal instability is characterized by Chandrasekhar number $Q$, which is defined as a ratio of Lorentz force exerted by the magnetic field and pressure forces

$$Q = \frac{\nu_m B_0^2 d^2}{\rho_0 \nu \nu_m},$$

(1.5.2)

where $\nu_m$ is the magnetic viscosity, $\nu$ kinematic viscosity and $H_0$ characteristic magnetic field. If the Lorentz force exerted by the magnetic field is weaker than the force exerted by the moving plasma (turbulent pressure), then the convective
motions twist and stretch the magnetic field, which in a turbulent flow increases its strength (dynamo action). If the Lorentz forces are stronger than the turbulent pressure forces, then the magnetic field channels the plasma motions along the field direction and inhibits the convection.

3. Rotation

When we consider the effect of rotation on the thermal instability introduces new elements into the problem as Taylor number $Ta$. Where Taylor number is defined as

$$Ta = \frac{4\Omega_r^2d^4}{\nu^2}$$

(1.5.3)

where $\Omega_r$ is a characteristic angular velocity, $d$ is a characteristic linear dimension perpendicular to the rotation axis, and $\nu$ is the kinematic viscosity. And $Ta$ characterizes the importance of centrifugal “forces” or so-called inertial forces due to rotation of a fluid about an axis, relative to viscous forces. Most generally $Ta$ has stabilizing effect for sufficient large values, when it exceeds it’s critical value there is an opposite effect and this instability known as K"upper–Lortz instability.

4. Non-Newtonian fluids

Non-Newtonian fluids are differ from Newtonian fluids most commonly the viscosity of non-Newtonian fluids are dependent of shear rate or shear rate history. The relation between the shear stress and the shear rate is linear, passing through the origin, in the case of a Newtonian fluid. But, in the case of a non-Newtonian fluids the relation is different and can even be time dependent. Viscoelastic fluids are the fluids that behaves as solid and as well as liquid too. They are elastic in nature and they will regain back when applied stress is removed. Some of the examples are as ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo. In 19th century, some physicists like Maxwell, Boltzmann, and Kelvin researched and experimented with creep and recovery of glasses, metals, and rubbers. Viscoelasticity was further examined in the late twentieth century when synthetic polymers were engineered and used in a variety of applications. These fluids exhibits the
oscillatory nature on thermal instability.

## 1.6 Boundary conditions

In general for mathematical modeling of any dynamical system, the boundary conditions of the dependent variable is very important. Depending upon the configuration of the problem (flow between two parallel horizontal plates) there are two types of physical boundaries: rigid (impermeable) and free (permeable). We can study different types of boundary conditions such as the upper and lower boundaries are rigid—rigid, free—free, rigid—free and free—rigid.

1. **Zero normal velocity**: \( w = 0 \), for both rigid and free boundaries.

2. (a) **zero tangential velocity (no slip)**: \( \frac{\partial w}{\partial z} = 0 \), for rigid boundaries.
   (b) **zero tangential stress**: \( \frac{\partial^2 w}{\partial z^2} = 0 \), for free boundaries.

For thermal boundary conditions such as isothermal and adiabatic.

1. **Isothermal**: For isothermal wall, the temperature disturbances must be zero at the boundary.

2. **Adiabatic**: For adiabatic wall, the temperature of the wall change, but there should be no through-flow of temperature.

While dealing the convective flow in porous domains, and in the simplest case, we need to have information about the velocity, as well as, temperature at the boundaries. Suppose the boundary of a Darcy-porous domain at \( x = 0 \) is rigid, then the normal component of the velocity at the boundary must vanish, i.e. \( q\hat{e}_x = 0 \). Since we consider Darcy flow only one condition applied at a given boundary, the other velocity components may have arbitrary values at the boundary, and thus we have slip at the boundary. In the free boundary case the pressure is constant at the boundary along \( y, z \) i.e. \( \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \). Which gives \( v = w = 0 \) for all \( y \) and \( z \), which further implies that \( \frac{\partial v}{\partial y} = \frac{\partial w}{\partial z} = 0 \). And by continuity equation we have \( \frac{\partial u}{\partial x} = 0 \) at \( x = 0 \), for the \( x \)-component of the fluid velocity.

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1.7 Methods

1.7.1 Analytical methods

1. Normal mode technique

Normal mode analysis is a harmonic analysis. It is one of the major simulation techniques used to probe the large-scale, shape-changing motions in dynamical system. Mainly this method is used to study the oscillations and instability of a dynamical system, under the assumption that all particles move with the same frequency and phase. In our thesis we have used the normal mode expansion to express the perturbations in physical quantities, such that the frequency of perturbation is same in all. The normal mode expansion is expressed in the form 

\[ \exp[i(a_x x + a_y y) + \sigma t], \]

where \( \sigma \) is the frequency of perturbation, \( a = \sqrt{a_x^2 + a_y^2} \) is the wave number. The frequency of perturbation \( \sigma \), decides whether the system is stable or unstable. The growth rate \( \sigma \) is in general complex such that \( \sigma = \sigma_r + i\sigma_i \). The system with \( \sigma_r < 0 \) is always stable, while for \( \sigma_r > 0 \) it will become unstable. When \( \sigma = 0 \), that is \( \sigma_r = 0 \) and \( \sigma_i = 0 \), the system is marginally stable. With \( \sigma_r = 0 \) and \( \sigma_i \neq 0 \), the overstable motion may occur.

2. Perturbation method

In many practical dynamical problems, any mathematical model cannot be solved exactly or, if the exact solution is available, it exhibits such a very complicated dependency on the parameters that it is very hard to use as such. However, to simplify the problem a parameter can be introduced and identified, say \( \chi \), such that the solution is available and reasonably simple for \( \chi = 0 \). This small quantity \( \chi \) is called perturbation parameter and this method is known as regular perturbation method. There is also a singular perturbation problem, in which a small parameter that cannot be approximated by setting the parameter value to zero. This is in contrast to regular perturbation problems, for which an approximation can be obtained by simply setting the small parameter to zero. More precisely, the solution of the

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problem cannot be uniformly approximated by an asymptotic expansion. Generally we have for the approximation to the full solution $A$, a series in the small parameter $\chi$, in the following form

$$A = A_0 + \chi A_1 + \chi^2 A_2 + \ldots$$

(1.7.1)

In the above expression, $A_0$ would be the known solution to the exactly solvable initial problem and $A_1, A_2$ ... represent the higher-order terms which may be found iteratively by some systematic procedure. For small values of $\chi$ the higher order terms in the above series become successively smaller. In general an approximate “perturbation solution” is obtained by truncating the series, usually by keeping only the first two terms, the initial solution and the “first-order” perturbation correction.

3. **A truncated representation of Fourier series method**

   The linear stability analysis is sufficient only for obtaining the stability condition of the motionless solution and the corresponding eigenfunctions describing qualitatively the convective flow. But, it cannot provide an information about the values of the convective amplitudes, nor even regarding the rate of heat and mass transfer. In order to obtain this additional information about heat and mass transfer in the system, we need to perform the nonlinear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward toward understanding full nonlinear problem.

4. **Nonlinear stability analysis**

   The stability of a system is tested by applying infinitesimal disturbances. If the governing equations are approximated just to include the linear terms in the applied disturbances then the theory used to predict the stability of the system is called linear stability analysis. When the non-linear terms are remained in the governing equations, then we call it nonlinear stability analysis. The linear stability theory basically provide us the information that when the flow will be unstable to infinitesimal disturbances. The Nonlinear stability theory provides an information to measure the amount of heat or mass transfer is taking place in the system in
1.7.2 Numerical methods

1. **Galerkin method**

Galerkin methods are a class of methods for converting a continuous operator problem (such as a differential equation) to a discrete problem. In principle, it is the equivalent of applying the method of variation of parameters to a function space, by converting the equation to a weak formulation. Galerkin method is basically a weighted residual method used to solve boundary value problems. We apply the following steps:

(a) Expand the unknown solution in a set of basis functions, with unknown coefficients or parameters; this is called the trial solution.

(b) Make the trial solution satisfy the boundary conditions (usually) and initial conditions.

(c) Define the residual.

(d) Set the weighted residual to zero and solve the equations.

(e) Examine the error by constructing successive approximations, and show convergence as the number of basis functions increases.

2. **Runge–Kutta method**

In numerical analysis, the Runge–Kutta methods are an important family of implicit and explicit iterative methods, which are used in temporal discretization for the approximation of solutions of ordinary differential equations. These techniques were developed by the German mathematicians C. Runge and M. W. Kutta during 19th century. One member of the family of Runge–Kutta methods is so commonly used and often referred as RK4, classical Runge–Kutta method or simply as the Runge–Kutta method. One of our problem in the thesis the Runge–Kutta–Fehlberg method has been used to find accuracy of our results.
1.8 Scope of the thesis

The thesis deals with the thermal instability under different hydrodynamic configurations while considering Rayleigh–Bénard and Darcy–Bénard convection. The problems have been studied analytically/numerically under various physical conditions, for different fluids and boundary conditions. The following assumptions are considered in the present thesis:

1. The systems considered are supposed to have the characteristic length much larger than the mean free path of the fluid molecules, so that the continuum hypothesis can be applied.

2. The density variations are assumed to govern by the Boussinesq approximations for both Newtonian/non-Newtonian fluids.

3. The porous medium considered is assumed to be isotropic unless it is specified.

4. The fluid considered is Newtonian for stationary convection, for oscillatory mode of convection is non-Newtonian fluid.

5. The central part of the thesis is ‘thermal instability under nonlinear oscillatory mode of convection’, we are the initiators of this model. A lot of scope can be seen in this aspect of the problem.

6. Temperature, Gravity, Rotational speed and Magnetic field modulations have been presented in the current thesis.

7. Nonlinear vertical throughflow effects under oscillatory mode and chaotic convection have been presented.

Since oscillatory mode of convection is strict to additional external constraint to the system (like rotation, magnetic field and non-Newtonian fluids etc..) one can see future scope in this direction (present thesis).
1.9 Preface

The thesis entitled “Nonlinear Thermal Instability Under Various Physical Configurations” comprising of analytical/numerical solutions of some problems related with the topic, is an outcome of the research work carried out by me during the course of investigations under the guidance of Dr. B.S. Bhadauria, Professor, Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow.

Rayleigh–Bénard convection is a paradigmatic example of convective thermal instability in ordinary fluid layers. The porous media analogue of this problem is known as Horton–Rogers–Lapwood convection, and it is of paramount interest due to its applications in various fields of engineering, thermal sciences and geophysics. Regulating the convective phenomenon in thermal sciences is of considerable importance due to its numerous application in many engineering problems. Keeping in mind the regulations of heat and mass transfer we have presented our results in the following chapters in which some of the work has been published.

The first chapter is of introductory part. The key features of the discipline are stated in this chapter. We describe the governing equation of dynamical systems. This chapter also consists of the basic definitions, relevant to the thesis topic. The literature survey of thermal convection in different hydrodynamic configurations and different kinds of modulations has been explained.

In chapter 2, we have presented thermal instability in anisotropic horizontal porous medium saturated with temperature dependant viscous fluid with time periodic temperature modulation. A weak non-linear stability analysis has been performed for the stationary mode of convection, and heat transport in terms of the Nusselt number is calculated. The effects of thermo rheological parameter, amplitude and frequency of modulation, thermo–mechanical anisotropies and Vadasz number on heat transport have been analyzed and depicted graphically. It is also found that, the heat transport can be controlled effectively by a mechanism that is external to the system.
In chapter 3, using complex non-autonomous Ginzburg–Landau equation, we have investigated nonlinear oscillatory convection in fluid layer (section 3.1) and porous medium (section 3.2) under Gravity modulation, considering viscoelastic fluids in the layer. The influence of (stress) relaxation and (strain) retardation times of viscoelastic fluid on heat transfer has been discussed. The study establishes that the heat transport can be controlled effectively by a mechanism that is external to the system. Modulation has a destabilizing effect at low frequencies and a stabilizing effect at high frequencies, which increases with increasing the amplitude of modulation. We also found that overstability advances the onset of convection, hence increases heat transfer.

In chapter 4, a nonlinear oscillatory convection in viscoelastic fluid saturated porous medium (section 4.1) and double diffusive convection in viscoelastic fluid layer (section 4.2) under temperature modulation has been investigated. The time periodic temperature profile on the boundaries has been considered and its effect on the system has been investigated. The effect of relaxation and retardation times of viscoelastic fluid on heat transfer and mass transfer has been discussed. The average value of Nusselt number is obtained numerically while using the value of Nusselt number and found the good approximation (or combination) of frequency and phase angle where heat and mass transfer is enhances or diminishes.

In chapter 5, the influence of sinusoidally varying magnetic field and rotational speed effects on Rayleigh–Bénard convection is carried out. In section 5.1, we have developed an analytic study of heat transport in an electrically conducting fluid layer under nonuniform time dependent magnetic field. The applied vertical magnetic field consists of two parts; constant part, and a time dependent periodic part, which varies sinusoidally with time. Using weakly nonlinear theory, the Ginzburg–Landau equation is solved through NDSolve Mathematica 8, and the results are verified using Runge–Kutta–Fehlberg method. The Nusselt number is obtained in terms of various system parameters and the effect of each parameter on heat transport is reported in detail. The effect of magnetic Prandtl number $Pm$, amplitude of modulation $\delta$ is to enhance the heat transfer. The Chandrasekhar number $Q$, modulation frequency $\Omega$ is to stabilize the system. Further, it is found that
magnetic modulation can effectively be used in either enhancing the heat transfer or diminishing it. In section 5.2, a theoretical investigation has been carried out to study the combined effect of rotation speed modulation and internal heating on thermal instability in a temperature dependent viscous horizontal fluid layer. Using Ginzburg–Landau analysis it is found that, the modulated rotation speed has a stabilizing effect for different values of modulation frequency. Further, internal heating and thermo–rheological parameter is found to destabilize the system.

In chapter 6, in the light of earlier work proposed by Johnathan et al. (2014) motivated us to make a chaotic mode of convection under temperature modulation. The analysis of buoyancy driven convection for moderate Prandtl number in a fluid saturated porous layer heated from below and subject to thermal modulation is presented. It’s been investigated a better combination of values of $\Omega, \delta$ and scaled Rayleigh number $R$ provides a way to chaos. It is also found that temperature modulation of the boundaries is to enhance the behaviour of the chaotic motions.

In chapter 7, thermal instability has been investigated in non-Newtonian fluids. In section 7.1, we study nonlinear convection in a porous medium saturated with nanofluid under gravity modulation, and calculate heat and mass transport across the porous medium. The nonuniform vertical vibrations of the system, which can be realized by oscillating the system vertically, are considered to vary sinusoidally with time. A nonlinear stability analysis has been performed to obtained the Nusselt number, which is found to be the function of thermal Rayleigh number, concentration Rayleigh number, Lewis number, modified diffusivity ratio, amplitude and frequency of modulation. The effects of various physical parameters have been investigated on heat and mass transfer. It is found that gravity modulation can be used effectively to regulate the stability of the system. In section 7.2, the effect of vertical throughflow on oscillatory convection in a viscoelastic fluid saturated porous medium has been investigated. The heat transport is investigated in terms of both the Nusselt and average Nusselt numbers, governed by the non–autonomous complex Ginzburg–Landau equation using weak nonlinear stability analysis. The effect of vertical throughflow is found to stabilize the system irrespective of
the direction of throughflow. The time relaxation parameter $\lambda_1$ has destabilizing effect, while the time retardation parameter $\lambda_2$ has stabilizing effect on the system. Further, it is also found that heat transfer is more in the oscillatory mode of convection rather than stationary.