Chapter 5

Weak nonlinear thermal instability under magnetic field and rotational speed modulation
5.1 Weak nonlinear analysis of magneto-convection under magnetic field modulation

5.1.1 Introduction

Aniss et al. (2001) studied a magnetic field modulation in an electrically conducting fluid layer with linear stability analysis and show that magnetic field modulation is much effective to handle theoretical and experimental investigation rather than the gravity modulation (Gresho and Sani 1970). In which the amplitudes and frequencies of modulations are more difficult to control. Keeping in mind the linear theory analysis could give only onset of convection but, fails at heat transfer in the system. Due to this reason we have investigated a weakly nonlinear analysis in an electrically conducting fluid layer under magnetic field modulation where we have found external regulations to regulate heat transfer in the system. Using the non-autonomous Ginzburg–Landau equation, we obtain an amplitude equation for convection as a function of system parameters and quantify heat transport in terms of the Nusselt number.

5.1.2 Mathematical Formulation

We consider an electrically conducting liquid of depth $d$, confined between two infinite, parallel, horizontal planes at $z = 0$ and $z = d$. Cartesian co-ordinates have been taken with the origin at the bottom of the liquid layer, and the $z$-axis vertically upwards. The surfaces are maintained at a constant temperature gradient $\Delta T$. Under the Boussinesq approximation, the dimensional governing equations for the study of magnetoconvection in an electrically conducting liquid are (Siddheshwar et al. 2012)

\[
\nabla \cdot \vec{q} = 0, \quad (5.1.1)
\]

\[
\nabla \cdot \vec{H} = 0, \quad (5.1.2)
\]

\[
\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = \frac{1}{\rho_0} \nabla p + \frac{\rho}{\rho_0} \vec{g} - \frac{\mu}{\rho_0} \nabla^2 \vec{q} + \frac{\mu_m}{\rho_0} \vec{H} \cdot \nabla \vec{H}, \quad (5.1.3)
\]

Ph.D. Thesis/Palle Kiran/2014
Chapter-5.1: \textit{Weak nonlinear magneto-convection.....}

\begin{align*}
\gamma \frac{\partial T}{\partial t} + \left( \vec{q} \cdot \nabla \right) T &= \kappa T \nabla^2 T, \tag{5.1.4} \\
\frac{\partial \vec{H}}{\partial t} + \left( \vec{q} \cdot \nabla \right) \vec{H} - (\vec{H} \cdot \nabla) \vec{q} &= \nu_m \nabla^2 \vec{H}, \tag{5.1.5} \\
\rho &= \rho_0 \left[ 1 - \alpha_T (T - T_0) \right]. \tag{5.1.6}
\end{align*}

where the physical variables have their usual meanings given in list of symbols. The externally imposed thermal boundary conditions are given by:

\begin{align*}
T &= T_0 + \Delta T \quad \text{at } z = 0 \\
T &= T_0 \quad \text{at } z = 1 \tag{5.1.7}
\end{align*}

Vertically imposed sinusoidally varying time dependent magnetic field is given by (Aniss et al. 2001)

\[ H_0 \left[ 1 + \delta \chi^2 \cos (\Omega t) \right] \tag{5.1.8} \]

where \( \delta \) is the small amplitude of magnetic modulation, \( \Omega \) is modulation frequency and \( \chi \) indicates the smallness of magnetic modulation. The basic state is assumed to be quiescent and the quantities in this state are given by

\begin{align*}
\vec{q}_b &= 0, \quad \rho = \rho_b(z, t), \quad p = p_b(z, t), \quad T = T_b(z, t), \quad \vec{H} = H_0, \\
\frac{\partial p_b}{\partial z} &= -\rho_b g, \tag{5.1.10} \\
T_b &= T_0 + \Delta T \left( 1 - \frac{z}{d} \right) \tag{5.1.11} \\
\rho_b &= \rho_0 \left[ 1 - \alpha_T (T_b - T_0) \right] \tag{5.1.12}
\end{align*}

Now, we impose finite perturbations to the basic state given in Eq.(5.1.9) as:

\begin{align*}
\vec{q} = q_b + \vec{q}', \quad \rho = \rho_b + \rho', \quad p = p_b + p', \quad T = T_b + T', \quad \vec{H} = H_0 + \vec{H}' \tag{5.1.13}
\end{align*}

where primes denote the quantities at the perturbations. Substituting the Eq.(5.1.13) in Eqs.(5.1.1)-(5.1.6) and using the basic state results, consider only two dimensional disturbances and hence the stream function \( \psi \) and magnetic potential \( \Phi \) are introduced as \((u', w') = \left( \frac{\partial \psi}{\partial z}, -\frac{\partial \psi}{\partial x} \right)\) and \((\vec{H}', \vec{H}) = \left( \frac{\partial \Phi}{\partial z}, -\frac{\partial \Phi}{\partial x} \right)\). Further, eliminating density and
Chapter 5.1: Weak nonlinear magneto-convection

pressure terms from Eqs. (5.1.1)-(5.1.6) then we obtain the following non-dimensionalized governing equations are:

\[- \nabla^4 \psi + Ra \frac{\partial T}{\partial x} - QP_m g_m \frac{\partial \nabla^2 \Phi}{\partial z} = - \frac{1}{Pr} \frac{\partial \nabla^2 \psi}{\partial t} + \frac{1}{Pr} \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} - QP_m \frac{\partial (\Phi, \nabla^2 \Phi)}{\partial (x, z)},\]

(5.1.14)

\[- \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial z} - (\nabla^2)T = - \frac{\partial T}{\partial t} + \frac{\partial (\psi, T)}{\partial (x, z)},\]

(5.1.15)

\[- g_m \frac{\partial \psi}{\partial z} - P_m \nabla^2 \Phi = - \frac{\partial \Phi}{\partial t} + \frac{\partial (\psi, \Phi)}{\partial (x, z)},\]

(5.1.16)

where

\[g_m = [1 + \delta \chi^2 \cos (\Omega t)]\]

and

\[Q = \frac{\mu_m H_0^2 \mu B_m^2}{\rho_0 \nu_m^3}\]

is the Chandrasekhar number. Since, we assume small variations of time, therefore re-scaling it as \(\tau = \chi^2 t\). Now, to study the stationary mode of convection of the system, we write the nonlinear Eqs. (5.1.14)-(5.1.16) in the matrix form as given below

\[
\begin{bmatrix}
- \nabla^4 & Ra \frac{\partial}{\partial x} & - QP_m g_m \frac{\partial \nabla^2}{\partial z} \\
- \frac{\partial}{\partial x} & - \nabla^2 & 0 \\
- g_m \frac{\partial}{\partial z} & 0 & - P_m \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi \\
T \\
\Phi
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{Pr} \left( \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} - \frac{\partial \nabla^2 \psi}{\partial t} \right) - QP_m \frac{\partial (\Phi, \nabla^2 \Phi)}{\partial (x, z)} \\
- \frac{\partial T}{\partial t} + \frac{\partial (\psi, T)}{\partial (x, z)} \\
- \frac{\partial \Phi}{\partial t} + \frac{\partial (\psi, \Phi)}{\partial (x, z)}
\end{bmatrix}
\]

(5.1.17)

The considered stress free and isothermal boundary conditions to solve the Eq. (5.1.17) are

\[\psi = D^2 \psi = 0 \quad \text{and} \quad \Phi = D \Phi = 0 \quad \text{at} \quad z = 0, \quad z = 1,\]

(5.1.18)

where \(D = \frac{\partial}{\partial z}\).

5.1.3 Finite amplitude equation and heat transport

Using an asymptotic expansion given in Eq. (2.3.1) for physical quantities \((Ra, \psi, T, \Phi)\) the above system Eq. (5.1.17) is solved for every order of \(\chi\).

At the lowest order, we have

\[
\begin{bmatrix}
- \nabla^4 & Ra \frac{\partial}{\partial x} & - QP_m \frac{\partial (\nabla^2)}{\partial z} \\
- \frac{\partial}{\partial x} & - \nabla^2 & 0 \\
- \frac{\partial}{\partial z} & 0 & - P_m \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_1 \\
T_1 \\
\Phi_1
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(5.1.19)
Chapter-5.1: Weak nonlinear magneto-convection….. 110

The solutions of the lowest order system subject to the boundary conditions Eq.(5.1.18) is

\[
\psi_1 = A(\tau) \sin(a_c x) \sin(\pi z), \\
T_1 = -\frac{a_c}{\delta^2} A(\tau) \cos(a_c x) \sin(\pi z), \\
\Phi_1 = \frac{\pi}{Pm \delta^2} A(\tau) \sin(a_c x) \cos(\pi z),
\]

where \( \delta^2 = a_c^2 + \pi^2 \). The critical value of the Rayleigh number for the onset of magneto-convection in the absence of temperature modulation is:

\[
R_0 = \frac{\delta^2 (\delta^4 + Q \pi^2)}{a_c^2}.
\]

If \( Q = 0 \), we obtained the classical results of Rayleigh–Bénard convection obtained by Chandrasekhar (1961).

At the second order, we have

\[
\begin{bmatrix}
-\nabla^4 & R_0 \frac{\partial}{\partial x} & -QPm \frac{\partial}{\partial z} (\nabla^2) \\
-\frac{\partial}{\partial x} & -\nabla^2 & 0 \\
-\frac{\partial}{\partial z} & 0 & -Pm \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_2 \\
T_2 \\
\Phi_2
\end{bmatrix}
= 
\begin{bmatrix}
R_{21} \\
R_{22} \\
R_{23}
\end{bmatrix}
\]

\[
R_{21} = 0, \\
R_{22} = \frac{\partial \psi_1}{\partial x} \frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial T_1}{\partial x}, \\
R_{23} = \frac{\partial \psi_1}{\partial x} \frac{\partial \Phi_1}{\partial z} - \frac{\partial \psi_1}{\partial z} \frac{\partial \Phi_1}{\partial x}.
\]

The second order solutions subjected to the boundary conditions Eq.(5.1.18) is obtained as follows:

\[
\psi_2 = 0, \\
T_2 = -\frac{a_c^2}{8\pi \delta^2} A^2(\tau) \sin(2\pi z), \\
\Phi_2 = -\frac{\pi^2}{8a_c Pm \delta^2} A^2(\tau) \sin(2a_c x).
\]

The horizontally averaged Nusselt number, \( \text{Nu}(\tau) \), for the stationary mode of convection is given by using Eq.(2.3.12) as

\[
\text{Nu}(\tau) = 1 + \frac{a_c^2}{4\delta^2} A^2(\tau).
\]
Here, we notice that \( \delta \cos(\Omega \tau) \) is effective at second order and affects the above Nusselt number through factor \( \hat{A}(\tau) \) as shown later.

**At the third order**, we have

\[
\begin{bmatrix}
-\nabla^4 & R_0 \frac{\partial}{\partial x} & -QPm \frac{\partial}{\partial z}(\nabla^2) \\
-\frac{\partial}{\partial x} & -\nabla^2 & 0 \\
-\frac{\partial}{\partial z} & 0 & -Pm \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_3 \\
T_3 \\
\Phi_3
\end{bmatrix}
= \begin{bmatrix}
R_{31} \\
R_{32} \\
R_{33}
\end{bmatrix}
\tag{5.1.31}
\]

where

\[
R_{31} = \frac{-1}{Pr} \frac{\partial \nabla^2 \psi_1}{\partial \tau} + QPm \delta \cos(\Omega \tau) \frac{\partial \nabla^2 \Phi_1}{\partial z} - R_2 \frac{\partial T_1}{\partial x} - QPm \left( \frac{\partial \Phi_1}{\partial z} \frac{\partial \nabla^2 \Phi_2}{\partial x} - \frac{\partial \Phi_2}{\partial x} \frac{\partial \nabla^2 \Phi_1}{\partial z} \right),
\]

\[
R_{32} = - \frac{\partial T_1}{\partial \tau} + \frac{\partial \psi_1}{\partial x} \frac{\partial T_2}{\partial z},
\]

\[
R_{33} = - \frac{\partial \Phi_1}{\partial \tau} - \frac{\partial \psi_1}{\partial z} \frac{\partial \Phi_2}{\partial x} + \delta \cos(\Omega \tau) \frac{\partial \psi_1}{\partial z}.
\]

Substituting \( \psi_1, T_1 \) and \( T_2 \) into Eqs.(5.1.32)-(5.1.34), we can obtain expressions for \( R_{31}, R_{32} \) and \( R_{33} \) easily. Now by applying the solvability condition for the existence of third order solution, we get the Ginzburg–Landau equation for stationary convection with time-periodic coefficients in the form:

\[
A_1 \hat{A}'(\tau) = A_2 \hat{A}(\tau) - A_3 \hat{A}(\tau)^3,
\]

where

\[
A_1 = \frac{\delta^2}{Pr} + \frac{R_0 a^2}{Pr^4} - \frac{Q \pi^2}{Pr^2}, \quad A_2 = \left[ \frac{R_0 a^2}{Pr^4} - Q \pi^2 \delta \cos(\Omega t) \right], \quad A_3 = \frac{Q \pi^4 a^2}{4 Pr^2 m^2} + \frac{R_0 a^4}{8 m^4} - \frac{Q \pi^4}{4 Pr^2 m^2}.
\]

The Ginzburg-Landau equations given in Eq.(5.1.35) is Bernoulli equation and obtaining its analytical solution is not an easy task, due to its non-autonomous nature. So it has been solved numerically using the in-built function NDSolve of Mathematica, subjected to the initial condition \( A(0) = b_0 \), where \( b_0 \) is the chosen initial amplitude of convection.

In our calculations we may use \( R_2 = R_0 \), to keep the parameters to the minimum.

### 5.1.4 Analytical solution for Unmodulated case

In the case of unmodulated fluid layer, the above Ginzburg–Landau equation can be written as

\[
A_1 \hat{A}'(\tau) = A_2 \hat{A}(\tau) - A_3 \hat{A}(\tau)^3,
\]

\[
A_1 = \frac{\delta^2}{Pr} + \frac{R_0 a^2}{Pr^4} - \frac{Q \pi^2 \delta}{Pr^2}, \quad A_2 = \left[ \frac{R_0 a^2}{Pr^4} - Q \pi^2 \delta \cos(\Omega t) \right], \quad A_3 = \frac{Q \pi^4 a^2}{4 Pr^2 m^2} + \frac{R_0 a^4}{8 m^4} - \frac{Q \pi^4}{4 Pr^2 m^2}.
\]
Chapter 5.1: Weak nonlinear magneto-convection

where $A_u(\tau)$ is an amplitude of convection for unmodulated case and $A_1, A_3$ have the same expression as given in the Eq.(5.1.35) and $A_2 = \frac{R_0 c^2}{\delta}$. The solution of Eq.(5.1.36) is given by

$$A_u(\tau) = \sqrt{\frac{1}{\left(\frac{A_3}{2A_2} + C_1 \exp \left[-\frac{2A_2}{A_1}\right]\right)}}$$

where $C_1$ is a parameter, it can be calculated for given suitable initial condition. The horizontal averaged Nusselt number in this case is obtained from Eq.(5.1.30) by using the value of $A_u(\tau)$ in the place of $A(\tau)$.

5.1.5 Results and discussion

In this section, we study the Rayleigh–Bénard magneto-convection under time-periodic magnetic field. A weakly nonlinear stability analysis has been performed to investigate the effect of magnetic modulation on heat transport. The effect of magnetic modulation on the Rayleigh-Bénard system has been assumed to be of order $O(\chi^2)$. This means we consider only small amplitude of magnetic field modulation. Such an assumption will help us in obtaining the amplitude equation of magneto-convection in simple and elegant manner and is much easier to obtain than in the case of the Lorenz model. The physical variables which appear in our analysis are $Pr, Pm, Q, \delta$ and $\Omega$. The effect of various parameters has been observed keeping while fixing the others parameters. We fix the parameter values as $Pr = 1.0, Pm = 1.6, Q = 20, \delta = 0.3$ and $\Omega = 2.0$.

From the figure 5.1a, we observe that the effect of Prandtl number, which is ratio of kinematic viscosity and thermal diffusivity, is to enhance the heat transport for lower values of time $\tau$. Similar effect is also observed for higher values of time $\tau$. It is clear that when $Pr$ increases, then either kinematic viscosity increases or thermal diffusivity decreases, which means in both the cases heat transfer increases. We take small values of $Pr$ to include the time-derivative term as a coefficient in momentum Eq.(5.1.14). Though the critical value of Rayleigh number is independent of Prandtl number, the heat transport is affected due to the time derivative $\frac{1}{Pr} \frac{\partial}{\partial \tau}$. The reason for considering $Pr$ around 1 to retain the time derivative in momentum equation. Aniss et al. (2001) considered
Figure 5.1: Nu versus \( \tau \) for different values of system parameters
Pr=7 in the case of linear theory of same problem in which one can not see the effect of time-derivative of stream functions.

Further, from the figure 5.1b, we find that the effect of magnetic Prandtl number $P_m$ which is the ratio of viscous diffusion rate to the magnetic diffusion rate is to increase the heat transfer. When $P_m$ increases either viscous diffusion rate may increase or magnetic diffusion rate may decrease in both cases heat transfer increases. The Chandrasekhar number is to represent ratio of the Lorentz force to the viscous force. The Lorentz force is the combination of electric and magnetic force on a point charge due to electromagnetic fields. In figure 5.1c, we depict the effect of Chandrasekher number $Q$ on Nu for fixed values of other parameters. We know that on increasing $Q$ the value of critical Rayleigh number $R_0$ also increases, so it delays the onset of convection that means stabilize the system, and hence decreases the heat transport. It is clear from the figure 5.1c, that as Chandrasekher number $Q$ increases, the amplitude of modulation is also increasing, so the effect of $Q$ is also reflecting on the amplitude of modulation. From the Eq.(5.1.35), it is clear that the Chandrasekher number is multiple of amplitude of modulation, which means that an amplitude of magnetic modulation is affected by Chandrasekher number.

From figure 5.1d, we find that an increment in the amplitude of magnetic modulation increases the value of Nu, hence advances the convection, and so the heat transport. We also obtained analytically the amplitude of convection for unmodulated case, given in Eq.(5.1.37) and depicted the result in figure 5.1d. The nature of graph is found to be non oscillatory.

In figure 5.1e, we have shown the effect of frequency of magnetic modulation on heat transport.
transport. We find that for small values of $\Omega$ the heat transport is more. As $\Omega$ increases, we observe that the amplitude of modulation decreases, and so the magnitude of $\text{Nu}(\tau)$. As the frequency increases from 10 to 100, the magnitude of $\text{Nu}(\tau)$ decreases considerable, and the effect of modulation on heat transport diminishes. On further increasing the value of $\Omega$, the effect of modulation on magneto-convection disappears altogether. Hence the effect of $\Omega$ is to stabilize the system. From the figure 5.1f, it is observed that for small values of $Q$, the system is having destabilizing effect, while at large values of $Q$ it has stabilizing effect. In order to verify the accuracy of our results, we have compared the results in figure 5.2, by solving the amplitude Eq.(5.1.35) using both RKF45 method and NDSolve Mathematica 8, which conforms our results with good approximation to RKF45 method.

In figures 5.3 and 5.4, the stream lines and the corresponding isotherms are depicted for magnetic field modulation, respectively at $\tau = 0.0, 0.1, 0.3, 0.5, 1.0$ and $2.0$ for $Pr = 1.0, Q = 20.0, \delta = 0.3$ and $\Omega = 2.0$. From the figures, we found that initially when time is small, the magnitude of streamlines is also small figures 5.3a-b, and isotherms are straight that is the system is in conduction state figures 5.4a-b. However, as time increases, the magnitude of streamlines increases and the isotherms loses their evenness. This shows that the convection is taking place in the system. Convection becomes faster on further increasing the value of time $\tau$. However, the system achieves the study state beyond $\tau = 1.0$ as there is no change in the streamlines and isotherms figures 5.3-5.4d-f.

5.1.6 Conclusions

1. The effect of time-periodic magnetic modulation on Rayleigh–Bénard convection has been studied by employing the non-linear stability analysis, and using the Ginzburg–Landau model.

2. The effect of increasing Prandtl number $Pr$ is to advance the convection, hence increase the heat transfer.

3. The effect of increasing magnetic Prandtl number $Pm$ is to advance the convection,
Figure 5.3: Streamlines at (a) $\tau = 0.0$ (b) $\tau = 0.1$ (c) $\tau = 0.3$ (d) $\tau = 0.5$ (e) $\tau = 1.0$ (f) $\tau = 2.0$
Figure 5.4: Isotherms at (a) $\tau = 0.0$ (b) $\tau = 0.1$ (c) $\tau = 0.3$ (d) $\tau = 0.5$ (e) $\tau = 1.0$ (f) $\tau = 2.0$
4. The effect of Chandrasekhar number is to delay the onset of convection and hence decrease the heat transfer.

5. The effect of increasing amplitude of modulation $\delta$ is to advance the convection hence heat transfer.

6. The effect of modulation starts vanishing at sufficiently large values of modulation frequency $\Omega$.

7. It can be concluded that the effect of magnetic modulation is highly significant and can be used to delay the onset of convection, and hence to decrease the heat transfer.

8. It was observed that, amplitude and frequency of modulation has no effect on un-modulated system but in the case of modulated system shows sinusoidal behavior.
5.2 Effect of rotational speed modulation on the heat transport in a fluid layer with a temperature dependent viscosity and an internal heat source

5.2.1 Introduction

When we study the rotation effect then one more parameter, in form of rotation speed, exists which can affect the stability of the convective flow. Donnelly (1964) was the first who investigated the effect of rotation speed modulation on the onset of instability in fluid flow between two concentric cylinders as the speed is slowly increased beyond the point at which instability sets in. The purpose of their study is to demonstrate that under certain conditions, Couette flow can be stabilized by modulating sinusoidally the rate of rotation of the inner cylinder. It was shown that the enhancement of stability is connected with the viscous wave set up in the annulus by the modulation, and this connection is further explore by experimenting with various widths of the gap between the cylinders, as well as different frequencies and amplitudes of modulation. However, the rotation speed modulation was the originating idea of the temperature modulation (Venezian 1969), as well as, gravity modulation (Gresho and Sani 1970). But, research work in this field is scarce. Amongst the available studies, the study due to Bhattacharjee (1989) is of great importance, in which he studied the effect of rotation speed modulation on Rayleigh–Bénard convection in ordinary fluid layer. He found that the effect of modulation is stabilizing for most of the configurations. In particular, when the convection is induced by the effect of rotation only then the rotation speed modulation serves as analogues to gravity modulation applied to the natural convection. In the porous media analogue is due to Suthar et al. (2009), who investigated the effect of rotation speed modulation on the onset of centrifugal convection in a rotating vertical porous layer distant from the axis of rotation. No nonlinear study available in the literature in which the effect of rotation speed modulation has been considered where one can analyze the heat transfer in the system. Hence a weakly nonlinear study under rotational speed modulation along
with internal heating and in a temperature dependent viscosity effects has been discussed in this section.

5.2.2 Problem Formulation

We consider an infinitely extended horizontal viscous-incompressible fluid layer, confined between two parallel planes which are at \( z = 0 \), lower plane and \( z = d \), upper plane. The lower surface is heated and upper surface is cooled to maintain an adverse temperature gradient across the fluid layer. We consider the fluid layer is rotating with variable rotational speed \( \vec{\Omega}_r = (0, 0, \Omega_r(t)) \), about the \( z \)-axis. The effect of rotation is restricted to Coriolis term, thus we neglect the centrifugal force term. The effect of density variation is given by Boussinesq approximation. With these assumptions the basic governing equations are:

\[
\nabla \cdot q = 0, \quad (5.2.1)
\]
\[
\frac{\partial q}{\partial t} + (q \cdot \nabla)q + 2(\vec{\Omega}_r \times \hat{q}) = - \frac{1}{\rho_0} \nabla p - \frac{\rho}{\rho_0} \hat{g} + \frac{\mu(T)}{\rho_0} \nabla^2 \hat{q}, \quad (5.2.2)
\]
\[
\frac{\partial T}{\partial t} + (q \cdot \nabla)T = \kappa \nabla^2 T + Q(T - T_0), \quad (5.2.3)
\]
\[
\rho = \rho_0 [1 - \alpha_T (T - T_0)], \quad (5.2.4)
\]
\[
\mu(T) = \frac{\mu_0}{1 + \chi^2 \delta_0 (T - T_0)}. \quad (5.2.5)
\]

The considered rotational speed which is varying sinusoidally with respect to time is defined as:

\[
\vec{\Omega}_r(t) = \Omega_0 \left( 1 + \chi^2 \delta \cos(\Omega t) \right) \hat{k}. \quad (5.2.6)
\]

The constants and variables used in the above Eqs.\((5.2.1)\)–\((5.2.6)\) have their usual meanings and are given in the nomenclature. The thermo-rheological relationship given in Eq.\((5.2.5)\) is guided by (Nield,1996). The considered thermal boundary conditions at the plates are:

\[
T = T_0 + \Delta T \quad \text{at} \quad z = 0 \quad T = T_0 \quad \text{at} \quad z = d. \quad (5.2.7)
\]
Chapter-5.2: Weak nonlinear theory under rotation speed modulation......

At the steady state the fluid is at rest \( \vec{q}_b = (0, 0, 0) \) and the heat transfer will be in the form of conduction. The other quantities of the conduction state are:

\[
\rho = \rho_b(z), \quad p = p_b(z) \quad \text{and} \quad T = T_b(z). \quad (5.2.8)
\]

Substituting the Eq.(5.2.8) into Eqs.(5.2.1)−(5.2.4), we get the following relations which help us to define basic state pressure and temperature

\[
\frac{dp_b}{dz} = -\rho_b g, \quad (5.2.9)
\]

\[
\kappa_T \frac{d^2(T_b - T_0)}{dz^2} + Q(T_b - T_0) = 0, \quad (5.2.10)
\]

\[
\rho_b = \rho_0 [1 - \alpha_T (T_b - T_0)], \quad (5.2.11)
\]

where \( b \) refers the basic state. The Eq.(5.2.10) is solved for \( T_b(z) \) subject to the boundary condition given in Eq.(5.2.7), we get:

\[
T_b = T_0 + \Delta T \frac{\sin \sqrt{\frac{Q}{\kappa_T}}(1 - \frac{z}{d})}{\sin \sqrt{\frac{Q}{\kappa_T}}}. \quad (5.2.12)
\]

The finite amplitude perturbations on the basic state are superposed in the following form:

\[
\vec{q} = \vec{q}_b + \vec{q}^\prime, \quad \rho = \rho_b + \rho^\prime, \quad p = p_b + p^\prime, \quad T = T_b + T^\prime. \quad (5.2.13)
\]

Substituting the Eq.(5.2.13) in Eqs.(5.2.1)-(5.2.4), and using the basic state solutions, we get:

\[
\nabla . \vec{q}^\prime = 0, \quad (5.2.14)
\]

\[
\frac{\partial \vec{q}^\prime}{\partial t} + (\vec{q}^\prime . \nabla) \vec{q} + 2 \left( \vec{\Omega}_r \times \vec{q}^\prime \right) \vec{k} = -\frac{1}{\rho_0} \nabla p + \alpha_T g T^\prime + \frac{\mu(T)}{\rho_0} \nabla^2 \vec{q}^\prime, \quad (5.2.15)
\]

\[
\frac{\partial T^\prime}{\partial t} + W' \frac{dT_b}{dz} + (\vec{q}^\prime . \nabla) T^\prime = \kappa_T \nabla^2 T^\prime + R_t T^\prime. \quad (5.2.16)
\]

For two dimensional convection, one can introduce stream function \( \psi \) as \( U' = \frac{\partial \psi}{\partial z} \), \( W' = -\frac{\partial \psi}{\partial x} \). Non dimensionalizing the physical variables as;

\[
(x, y, z) = d(x^*, y^*, z^*), \quad t = \frac{d^2}{\kappa_T} t^*, \quad \vec{q} = \frac{\kappa_T}{d} \vec{q}^*, \quad \psi = \kappa_T \psi^*, \quad T' = \Delta T T^* \quad \text{and} \quad \vec{\Omega}_r = \frac{\kappa_T}{d^2} \vec{\Omega}_r^*.
\]

Ph.D. Thesis/Palle Kiran/2014
then eliminating the pressure term and finally dropping the asterisk, we obtain the nondimensional governing system as

\[
\frac{1}{Pr} \frac{\partial}{\partial t} (\nabla^2 \psi) - \frac{1}{Pr} \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, z)} = -Ra \frac{\partial T}{\partial x} \bar{p}(T) \nabla^4 \psi + \sqrt{T}\alpha (1 + \chi^2 \delta \cos(\Omega t)) \frac{\partial V}{\partial z} + \frac{\partial \pi \partial \nabla^2 \psi}{\partial z},
\]

\[ (5.2.17) \]

\[- \frac{dT_b}{dz} \frac{\partial \psi}{\partial x} - (\nabla^2 + R_i) T = - \frac{\partial T}{\partial t} + \frac{\partial (\psi, T)}{\partial (x, z)}. \]  

\[ (5.2.18) \]

Also from the Eq. (5.2.15), we may write the following equation for \( V \):

\[
\frac{1}{Pr} \frac{\partial V}{\partial \tau} - \frac{1}{Pr} \frac{\partial (\psi, \nabla^2 \psi)}{\partial (x, z)} = - \sqrt{\alpha} (1 + \chi^2 \delta \cos(\Omega t)) \frac{\partial \psi}{\partial z} + \frac{1}{Pr} \frac{\partial (\psi, V)}{\partial (x, z)}. \]  

\[ (5.2.19) \]

The non-dimensionalized parameters are given in list of symbols. The non-dimensional basic temperature \( T_b(z) \) which appears in the Eq. (5.2.18), can be obtained from the Eq. (5.2.12) as

\[
\frac{dT_b}{dz} = - \sqrt{R_i} \cos \sqrt{R_i}(1 - z) \sin \sqrt{R_i}.
\]

\[ (5.2.20) \]

We assume small variations in time, and re-scaling it as \( \tau = \chi^2 t \). To study the stationary convection of the system, we write the non-linear Eqs. (5.2.17)-(5.2.19) in the matrix form as given bellow

\[
\begin{bmatrix}
\frac{\chi^2}{Pr} \frac{\partial}{\partial \tau} \nabla^2 - \bar{p} \nabla^4 & Ra \frac{\partial}{\partial x} & - \sqrt{\alpha} \frac{\partial}{\partial z} \\
\frac{\partial}{\partial x} & \chi^2 \frac{\partial}{\partial \tau} - (\nabla^2 + R_i) & 0 \\
\sqrt{\alpha} \frac{\partial}{\partial z} & 0 & \frac{\chi^2}{Pr} \frac{\partial}{\partial \tau} - \bar{p} \nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi \\
T \\
V
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial(\psi, \nabla^2 \psi)}{Pr \partial (x, z)} + \sqrt{\alpha} \chi^2 \delta \cos(\Omega t) \frac{\partial V}{\partial z} + \frac{\partial \pi \partial \nabla^2 \psi}{\partial z} \\
\frac{\partial(\psi, T)}{\partial (x, z)} - \sqrt{T}\alpha \chi^2 \delta \cos(\Omega t) \frac{\partial \psi}{\partial z} \\
\frac{1}{Pr} \frac{\partial (\psi, V)}{\partial (x, z)} - \sqrt{T}\alpha \chi^2 \delta \cos(\Omega t) \frac{\partial \psi}{\partial z}
\end{bmatrix}
\]

\[ (5.2.21) \]

To solve the system of Eqs. (5.2.21), we consider stress free and isothermal boundary conditions as given bellow

\[
\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial V}{\partial z} = T = 0 \text{ at } z = 0 \text{ and } z = 1.
\]

\[ (5.2.22) \]
5.2.3 Finite amplitude equation and heat transport

We introduce the following asymptotic expansions in Eqs. (5.2.21) as we have used in Eqs. (2.3.1) and solve the above system Eq. (5.2.21) for different orders of $\chi$.

At the lowest order, we have

\[
\begin{bmatrix}
-\nabla^4 & R_0 \frac{\partial}{\partial x} & -\sqrt{T_a} a \frac{\partial}{\partial z} \\
-\frac{\partial}{\partial x} \frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 \\
\sqrt{T_a} a \frac{\partial}{\partial z} & 0 & -\nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_1 \\
T_1 \\
V_1
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}.
\] (5.2.23)

The solution of the lowest order system subject to the boundary conditions given in Eq. (5.2.22), is

\[
\psi_1 = A(\tau) \sin(a_c x) \sin(\pi z),
\] (5.2.24)

\[
T_1 = -\frac{4\pi^2 a_c}{\beta_2^2 (4\pi^2 - R_i)} A(\tau) \cos(a_c x) \sin(\pi z),
\] (5.2.25)

\[
V_1 = -\frac{\pi \sqrt{T_a}}{\beta_1^2} A(\tau) \sin(a_c x) \cos(\pi z),
\] (5.2.26)

where $\beta^2 = a_c^2 + \pi^2$ and $\beta_1^2 = \beta^2 - R_i$. The critical value of the Rayleigh number for the onset of stationary convection is calculated numerically, and the expression is given by

\[
R_0 = \frac{\beta_1^2 (4\pi^2 - R_i)(\beta^6 + \pi^2 T_a)}{4\beta^2 \pi^2 a_c^2}.
\] (5.2.27)

For the system without rotation ($T_a = 0$) and internal heating ($R_i = 0$), we get:

\[
R_0 = \frac{\beta^6}{a_c^2},
\]

\[
a_c = \frac{\pi}{\sqrt{2}},
\]

which are classical results of Chandrasekhar (1961).

At the second order, we have

\[
\begin{bmatrix}
-\nabla^4 & R_0 \frac{\partial}{\partial x} & -\sqrt{T_a} a \frac{\partial}{\partial z} \\
-\frac{\partial}{\partial x} \frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 \\
\sqrt{T_a} a \frac{\partial}{\partial z} & 0 & -\nabla^2
\end{bmatrix}
\begin{bmatrix}
\psi_2 \\
T_2 \\
V_2
\end{bmatrix}
= \begin{bmatrix}
R_{21} \\
R_{22} \\
R_{23}
\end{bmatrix}.
\] (5.2.28)
where

\[
R_{21} = 0
\]  
\[
R_{22} = \frac{\partial \psi_1 \partial T_1}{\partial x \partial z} - \frac{\partial \psi_1 \partial T_1}{\partial z \partial x},
\]  
\[
R_{23} = \frac{\partial \psi_1 \partial V_1}{\partial x \partial z} - \frac{\partial \psi_1 \partial V_1}{\partial z \partial x}.
\]

(5.2.29)  
(5.2.30)  
(5.2.31)

The second order solutions subjected to the boundary conditions given in Eq. (5.2.22), is obtained as:

\[
\psi_2 = 0
\]  
\[
T_2 = -\frac{2\pi^2 a^2}{\beta_i^2 (4\pi^2 - R_i)^2} A^2(\tau) \sin(2\pi z),
\]  
\[
V_2 = \frac{\pi^2 \sqrt{T_a}}{8a_c Pr \beta^2} A^2(\tau) \sin(2a_c x).
\]

(5.2.32)  
(5.2.33)  
(5.2.34)

The horizontally averaged Nusselt number \(\text{Nu}(\tau)\), for the stationary mode of convection is determined by substituting the expression of \(T_2\) and \(\frac{dT_b}{dz}\) in Eq. (2.3.12) and simplifying, we get the Nusselt number as:

\[
\text{Nu}(\tau) = 1 + \frac{4\pi^2 a^2 \sin \sqrt{R_i}}{\beta_i^2 (4\pi^2 - R_i)^2 \sqrt{R_i} \cos \sqrt{R_i}} A^2(\tau).
\]

(5.2.35)

At the third order, we have

\[
\begin{bmatrix}
-\nabla^4 & R_0 \frac{\partial}{\partial x} & -\sqrt{T_a} \frac{\partial}{\partial z} & 0 \\
-\frac{\partial}{\partial z} \frac{\partial}{\partial x} & -(\nabla^2 + R_i) & 0 & -\nabla^2 \\
\sqrt{T_a} \frac{\partial}{\partial z} & 0 & -\nabla^2 & 0
\end{bmatrix}
\begin{bmatrix}
\psi_3 \\
T_3 \\
V_3
\end{bmatrix}
= \begin{bmatrix}
R_{31} \\
R_{32} \\
R_{33}
\end{bmatrix}
\]

(5.2.36)

where

\[
R_{31} = -\frac{1}{Pr} \frac{\partial}{\partial \tau} (\nabla^2 \psi_1) + \sqrt{T_a} \delta \cos(\Omega t) \frac{\partial V_1}{\partial z} - R_2 \frac{\partial T_1}{\partial x} + V_T T_b \nabla^4(\psi_1)
\]  
\[
- 2R_0 V_T T_b \frac{\partial T_1}{\partial x} + 2V_T \sqrt{T_a T_b} \frac{\partial V_1}{\partial x} - V_T \frac{\partial T_1}{\partial z} \frac{\partial \nabla^2 \psi}{\partial z}
\]

(5.2.37)

\[
R_{32} = -\frac{\partial T_1}{\partial \tau} + \frac{\partial \psi_1}{\partial x} \frac{d T_2}{d z},
\]

(5.2.38)

\[
R_{33} = -\frac{1}{Pr} \frac{\partial V_1}{\partial \tau} - \frac{1}{Pr} \frac{\partial \psi_1}{\partial z} \frac{\partial V_2}{\partial x} - \sqrt{T_a (V_T T_b + \delta \cos(\Omega t))} \frac{\partial \psi_1}{\partial z}.
\]

(5.2.39)

Substituting the first and second order solutions into Eqs. (5.2.37) – (5.2.39), we can easily simplify the expressions \(R_{31}, R_{32}\) and \(R_{33}\). Now, by applying the solvability condition
Chapter 5.2: Weak nonlinear theory under rotation speed modulation

for the existence of third order solutions, we get the non autonomous Ginzburg–Landau equation for stationary mode of convection, with time-periodic coefficients in the form:

\[ A_1 \dot{A}(\tau) - A_2 A(\tau) + A_3 A(\tau)^3 = 0, \quad (5.2.40) \]

where

\[ A_1 = \left[ \frac{\beta^2}{Pr} + \frac{4 R_0 \pi^2 a_c^2}{\beta^2 (4 \pi^2 - R_i)} - \frac{T a \pi^2}{\beta^2} \right], \quad A_2 = \left[ \frac{4 R_0 \pi^2 a_c^2}{\beta^2 (4 \pi^2 - R_i)} - \frac{2 Ta \pi^2}{\beta^2} \delta \cos(\Omega t) - H_1 \right], \]

\[ H_1 = \frac{4 \pi^2 V_T (\cos \sqrt{R_i} - 1)}{(4 \pi^2 - R_i) \sin \sqrt{R_i}} \left[ \frac{\beta^2 R_0 a_c}{\beta^2 (4 \pi^2 - R_i) \sqrt{R_i}} - \frac{3 \pi^2 Ta (\cos \sqrt{R_i} - 1)}{\beta^2 \sqrt{R_i}} - \frac{\delta}{2} \right], \]

\[ A_3 = \left[ \frac{2 R_0 \pi^4 a_c^4}{\beta^4 (4 \pi^2 - R_i)^2} + \frac{T a \pi^4}{8 Pr^2 \beta^4} \right]. \]

The Ginzburg–Landau equation given in Eq.(5.2.40) is Bernoulli equation, and obtaining its analytical solution is difficult, due to its non-autonomous nature. Therefore, it has been solved numerically using the in-built function NDSolve of Mathematica 8, subjected to the initial condition \( A(0) = b_0 \), where \( b_0 \) is the chosen initial amplitude of convection.

In our calculations we may use \( R_2 = R_0 \), to keep the parameters to the minimum.

### 5.2.4 Analytical solution for unmodulated case

In the case of unmodulated fluid layer, the above Ginzburg–Landau equation Eq.(5.2.40) can be written as

\[ A_1 A_u'(\tau) - A_2 A_u(\tau) + A_3 A_u(\tau)^3 = 0, \quad (5.2.41) \]

where \( A_u(\tau) \) is an amplitude of convection for unmodulated case. Coefficients \( A_1 \) and \( A_3 \) have the same expressions as given in the Eq.(5.2.40), while \( A_2 = \left[ \frac{4 R_0 \pi^2 a_c^2}{\beta^2 (4 \pi^2 - R_i)} - H_1 \right] \).

The solution of the Eq.(5.2.41) is given by

\[ A_u(\tau) = \frac{1}{\sqrt{\left( \frac{A_3}{2A_2} + C_1 e^{\frac{-2A_3}{4A_2}} \right)}}, \quad (5.2.42) \]

where \( C_1 \) is a parameter, it can be calculated for given suitable initial condition. The horizontal averaged Nusselt number in this case is obtained from the Eq.(5.2.35) by using the value of \( A_u(\tau) \) in the place of \( A(\tau) \).
Chapter 5.2: Weak nonlinear theory under rotation speed modulation

5.2.5 Results and discussion

The problem addresses a nonlinear realm of Rayleigh–Bénard convection with a variable viscous liquid and internal heating effects under rotational speed modulation. Here, we have presented a weakly nonlinear stability analysis to investigate the effect of rotational speed modulation, internal heating and thermo-rheological behaviour on heat transport. The modulation of Rayleigh–Bénard system has been assumed to be of order $O(\chi^2)$, which means we consider only small amplitude of rotation speed modulation. At third order only Solvability condition exist to define an amplitude equation Eq.(5.2.40). The modulated term ($\delta \cos(\Omega \tau)$) is effective at $O(\chi^2)$ and affects the system. This assumption will help us in obtaining the amplitude equation in a simple manner, and much easier than the Lorenz model. Before writing the discussion of the results, we mention some features of the following aspects of the problem:

1. The importance and need for nonlinear stability analysis.
2. The relation of the problem to real life application.
3. The selection of all dimensionless parameters utilized in computations.
4. Consideration of numerical values for different parameters.

It is imperative to make a nonlinear study of the problem if one wants to obtain heat transport, which can not be obtained using the linear stability theory. External regulation of convection is important in the study of thermal instability in a fluid layer, therefore, in this section, we have considered rotational speed modulation and internal heating for either enhancing or inhibiting convective heat transport as is required by a real life applications in science and engineering, such as, food process industry, chemical process industry, rotating turbo machinery etc. The effect of rotational speed modulation on heat transport has been depicted in figures (5.5-5.7). The parameters that arise in the problem are $R_i, Pr, V_T, Ta, \delta$ and $\Omega$, and these parameters influence the convective heat transport. The first two parameters are related to the fluid layer and the next three parameters concern the external mechanism of controlling convection. The fluid layer
Figure 5.5: Nu versus $\tau$ for different values of system parameters

Ph.D. Thesis/Palle Kiran/2014
Chapter-5.2: Weak nonlinear theory under rotation speed modulation

Figure 5.6: Streamlines at (a) $\tau = 0.0$ (b) $\tau = 0.2$ (c) $\tau = 0.6$ (d) $\tau = 0.9$ (e) $\tau = 1.5$ (f) $\tau = 2.0$

Ph.D. Thesis/Palle Kiran/2014
Chapter-5.2: Weak nonlinear theory under rotation speed modulation

is not considered to be highly viscous, therefore only moderate values of $Pr$ are taken for calculations. Because of small amplitude modulation, the values of $\delta$ are considered around 0.5. Further, the rotational speed modulation assumed to be of low frequency, as at low range of frequencies, the effect of frequencies on onset of convection as well as on heat transport is maximum. The values of $R_i$ are considered to be moderate so that it will not affect the effect of modulation on the system by dominating it otherwise. The thermo-rheological parameter $V_T$ has taken to be small values.

In figure 5.5, we have plotted the Nusselt number $Nu(\tau)$ with respect to time $\tau$ for the case of rotational speed modulation. From the figures, we find that for lower values time $\tau$, the value of $Nu(\tau)$ does not alter and remains almost constant, then it increases on increasing $\tau$, and finally becomes oscillatory on further increasing $\tau$. It is clear from the figures that $Nu(\tau)$ starts with one showing the conduction state. From figure 5.5a, we observe that the effect of internal heating on thermal instability is destabilizing, as heat transport increases on increasing $R_i$. The heat transport is more at higher values of $R_i$. This confirms the results obtained most recently by Bhadauria et al. (2013a,b,c).

Further, we have

\[
Nu_{R_i=1.0} < Nu_{R_i=1.1} < Nu_{R_i=1.3}
\]

In figure 5.5b, we find that the Nusselt number $Nu(\tau)$ increases upon increasing Prandtl number $Pr$ for fixed values of other parameters. This may happen due to the dominating role of thermal diffusivity $\kappa_T$ over kinematic viscosity $\nu$. As Prandtl number $Pr$ increases, then for no change in kinematic viscosity, probably there is a large decrement in thermal diffusivity, and this makes sudden increase in the temperature gradient. So convection takes place early, and there is an enhancement in heat transfer. Thus, the effect of an increment in Prandtl number $Pr$ is to advance the convection. Similar effect can be seen in the case of thermo-rheological parameter $V_T$ in figure 5.5c. We have

\[
Nu_{Pr=0.5} < Nu_{Pr=0.6} < Nu_{Pr=0.7}
\]
\[
Nu_{V_T=0.1} < Nu_{V_T=0.2} < Nu_{V_T=0.4}
\]

From the figure 5.5d, we depict the effect of Taylor number $Ta$ on $Nu(\tau)$ for fixed
Figure 5.7: Isotherms at (a) $\tau = 0.0$ (b) $\tau = 0.2$ (c) $\tau = 0.6$ (d) $\tau = 0.9$ (e) $\tau = 1.5$ (f) $\tau = 2.0$
values of other parameters. Upon increasing $Ta$ increases the value of critical Rayleigh number $R_0$, and it delays the onset of convection, hence heat transport decreases. It is clear from the figure 5.5d that, on increasing Taylor number $Ta$, the amplitude of modulation also increases, so the effect of $Ta$ is reflecting on amplitude of modulation as well. From the Eq.(5.2.40), it is clear that the rotation is multiple of amplitude of rotation speed modulation. Which means that the amplitude rotation speed modulation is dependant of rotation. Generally, if there is no rotation ($Ta = 0$), it is meaningless to talk about rotation speed modulation. Further, for no rotation $Ta = 0$, the effect of frequency of modulation diminishes, so the effect of frequency of modulation can be seen when rotation is not there. Since we are studying rotation speed modulation, it is necessary to consider $Ta$ as non-zero values, otherwise modulation effect disappears.

\[ \text{Nu}_{Ta=40} < \text{Nu}_{Ta=30} < \text{Nu}_{Ta=25} < \text{Nu}_{Ta=20} \]

In figure 5.5e, we depict the effect of amplitude of modulation for moderate values of $Ta$ and for the fixed values of other parameters. The Nusselt number $Nu$ increases upon increasing the value of $\delta$, hence advancing the convection. Which means that increasing upon $\delta$ increases the heat transfer. In case of unmodulated $\delta = 0$ system shows no influence on heat transport for larger values of time $\tau$. Similar results can be obtained analytically for an unmodulated system the amplitude of convection given by Eq.(5.2.42).

\[ \text{Nu}_{\delta=0.0} < \text{Nu}_{\delta=0.5} < \text{Nu}_{\delta=0.9} < \text{Nu}_{\delta=1.3} \]

From the figure 5.5f, we see the effect of frequency of modulation, for small values of $\Omega$ heat transport is more. Upon increasing the value of $\omega$ decreases the magnitude of $\text{Nu}(\tau)$, and shortens the wavelength of oscillations. As the frequency increases from 2 to 100, the magnitude of $\text{Nu}(\tau)$ decreases, and the effect of modulation on heat transport diminishes. On further increasing the value of $\Omega$, the effect of modulation on thermal instability disappears altogether. Hence the effect of $\Omega$ is to stabilize the system. We have

\[ \text{Nu}_{\Omega=70} < \text{Nu}_{\Omega=30} < \text{Nu}_{\Omega=10} < \text{Nu}_{\Omega=2} \]
Chapter-5.2: *Weak nonlinear theory under rotation speed modulation*....132

The present result of internal heating has been compared with the results of non-internal heating in figure 5.5g. We observe that, in case of internal heating of the system, the heat transport in the system is more than that in the absence of internal heating, thus internal heating advances the onset of convection as well as heat transport.

\[ \text{Nu}_{R_i=0.0} < \text{Nu}_{R_i=1.0} \]

The figure 5.5h show that, the heat transport is more when there is no rotation \( Ta = 0 \) (which means no modulation) than in the presence of rotation and modulation. Hence rotation strongly stabilize the system.

\[ \text{Nu}_{Ta\neq0} < \text{Nu}_{Ta=0,\Omega\neq0} \]

In figures 5.6-5.7, the stream lines and the corresponding isotherms are depicted for rotation speed modulation, respectively at \( \tau = 0.0, 0.2, 0.6, 0.9, 1.5 \) and \( 2.0 \) for \( Pr = 0.5, Ta = 20.0, \delta = 0.5 \) and \( \Omega = 2.0 \). From the figures, we found that initially when time is small, the magnitude of stream function is also small figures 5.6a-b, and isotherms are straight that is the system is in conduction state figures 5.7a-b. However, as time increases, the magnitude of stream function increases and the isotherms loses their evenness. This shows that the convection is taking place in the system. Convection becomes faster on further increasing the value of time \( \tau \). However, the system achieves the study state beyond \( \tau = 1.0 \) as there is no change in the stream function and isotherms figures 5.6-5.7d-f.

5.2.6 Conclusions

The combined effect of internal heating and rotation speed modulation on Rayleigh–Bénard convection in a rotating horizontal temperature dependent viscous fluid layer has been studied by employing nonlinear stability analysis, and using Ginzburg–Landau model. The results have been obtained in terms of the Nusselt number, and the effect of various parameters have been obtained and depicted graphically. We have the following observations

*Ph.D. Thesis/Palle Kiran/2014*
1. The effect of rotation speed modulation on Rayleigh–Bénard convection in a rotating horizontal fluid layer has been studied by employing non-linear stability analysis using Ginzburg–Landau model.

2. The effect of increasing internal Rayleigh number $R_i$ is to increase the value of Nu, thus advancing the convection, hence heat transfer.

3. The effect of increasing Prandtl number $Pr$ is to advance the onset of convection, hence heat transfer.

4. The effect of increasing $V_T$ is to advance the onset of convection, hence heat transfer.

5. Overall, it can be concluded that the effect of rotation speed modulation is highly significant and can be used to delay the onset of convection.

6. In modulated case, the effect of Taylor number is to delay the onset of convection, and hence heat transfer.

7. It was also observed that the effect of modulation starts vanishing at sufficiently large values of modulation frequency.

8. As time $\tau$ increases, the magnitude of streamlines increases, and isotherms loses their evenness, showing that convection is taking place. At $\tau = 1.0$ the system achieves steady state.

9. The thermo-rheological model of Nield (1996), gives physically acceptable results, namely, the destabilizing effect of variable viscosity on Bénard-Darcy convection, and thereby an enhanced heat transport.

The results of this work can be summarized as follows:

1. $\text{Nu}_{R_i=1.0} < \text{Nu}_{R_i=1.1} < \text{Nu}_{R_i=1.3}$

2. $\text{Nu}_{Pr=0.5} < \text{Nu}_{Pr=0.6} < \text{Nu}_{Pr=0.7}$

3. $\text{Nu}_{V_T=0.1} < \text{Nu}_{V_T=0.2} < \text{Nu}_{V_T=0.4}$

*Ph.D. Thesis/Palle Kiran/2014*
4. $\text{Nu}_{T_a=30} < \text{Nu}_{T_a=25} < \text{Nu}_{T_a=20}$

5. $\text{Nu}_{\delta=0.0} < \text{Nu}_{\delta=0.5} < \text{Nu}_{\delta=0.9} < \text{Nu}_{\delta=1.3}$

6. $\text{Nu}_{\Omega=70} < \text{Nu}_{\Omega=30} < \text{Nu}_{\Omega=10} < \text{Nu}_{\Omega=2}$

7. $\text{Nu}_{T_a 
eq 0} < \text{Nu}_{T_a=0}$

8. $\text{Nu}_{R_i=0.0} < \text{Nu}_{R_i=1.0}$. 