CHAPTER 4

A NOTE ON INVENTORY MODEL
4.1 Introduction

In the classical inventory model (Harris-Wilson Formula 1915) all the cost associated with the formula was taken to be constant and which does not depend on any quantity. There are many practical situations where this is not true. The Harris-Wilson model has attracted many researches to the inventory modeling area. The (Harris-Wilson Formula 1915) for determining the optimum lot-size, the quantity in which an item of inventory should be purchased or produced for stock is

\[
q = \sqrt{\frac{2 \times D \times C_3}{C_1}}
\]

... (4.1)

\(D = \) annual sales or demand.

\(C_3 = \) set-up cost per order.

\(C_1 = \) the carrying cost for a unit for one year.

This formula follows from the propositions, the optimal quantity minimizes the sum of the annual set-up and carrying costs, and the costs are

\[
TC(q) = \frac{q \times C_1}{2} + \frac{D \times C_3}{q}
\]

... (4.2)

The Wilson formula is popular for its simplicity i.e. the costs considered here are
assumed to be constant. But for the wholesalers and manufacturers, an item for which sales are of unit size and at a constant rate is the exception. When sales vary in size, take place irregularly, and the time and the size of the sales are uncertain equation (4.2) is not a correct statement of annual inventory costs.

In this chapter, we consider a situation where lot-size depends on the carrying / holding cost. If the carrying costs are incorrectly stated, then in this case if any inventory model was developed by someone with same assumption the cost of carrying will be very high or very low, which results in the increase of the cost of the goods. There are many real situations where carrying cost depends on the lot-size and various extension of non-constant carrying cost can be seen in (Beranek, 1967). Let us consider an example, that any furnished goods may not be manufactured everywhere. For this it requires raw materials which are to be transported from one place to another and again those furnished goods need to be transported from factory to the retailer i.e. carrying or holding cost may not be same in all cases. Usually the goods are shipped in a truck, train, van etc. As the lot-size increases the carrying cost also increases. Proper carrying cost may reduce the substantial error in both lot-size and cost.

When items are received they are to be checked, inspected etc. These costs would generally increases as the size of the lot received increases and it would costs the organization a certain fixed cost such as wages per hour or per shift both for loading and unloading the items. The labour charges would increase as the run-size increases.

In practical field the carrying cost is dependent on the lot-size in different ways like linearly, non-linearly, exponentially, periodically etc. Considering carrying cost as a linear function of the lot-size we may have,
\[ C_1(q) = a\cdot q + b, \quad \ldots \ (4.3) \]

Where \( a \) and \( b \) are constants, where \( b \) may be assumed as constant cost related with the packing costs, checking costs etc. And \( q \) is given by,

\[ q = \sqrt{\frac{2 \cdot D \cdot C_3}{C_1}} \]

\[ TC(q) = \frac{D \cdot C_3}{q} + \frac{q \cdot C_1}{2} \quad \ldots \ (4.4) \]

Putting the value of \( C_1 \) in equation (4.4), the required result is

\[ TC(q) = \frac{a \cdot q^3 + b \cdot q^2 + 2 \cdot D \cdot C_3}{2 \cdot q} \quad \ldots \ (4.5) \]

Now, \( TC(q) \) will be maximum if,

\[ \frac{d\{TC(q)\}}{dq} = 0 \quad \ldots \ (4.6.1) \]

\[ \frac{d^2\{TC(q)\}}{dq^2} < 0. \quad \ldots \ (4.6.2) \]
By solving equations (4.5), (4.6.1) and (4.6.2), the result obtained is,

\[ a \times q + \frac{b}{2} - \frac{D \times C_3}{q^2} = 0 \]

... (4.7)

And also,

\[ a + \frac{2 \times D \times C_3}{q^3} > 0, \]

For the arbitrary values of \( a \in \mathbb{N} \) and \( b \geq 0 \) problem is formulated and solved.

In this model, we develop an inventory model where the carrying cost depends on lot-size. As the lot-size increases stepwise the carrying cost also increases accordingly. However, in the proposed model all other assumptions of the Harris-Wilson EOQ model remain valid.

### 4.2 Model Development and Analysis

The proposed model has been developed with the following assumptions

- \( D = \text{Total / Annual Demand} \)
- \( C_3 = \text{Setup cost per order} \)
- \( H_j = \text{Carrying cost for the lot-size} Q_j \text{ if } q_{j-1} < Q_j < q_j \)
Where \( j = 1, 2, 3, \ldots, m \), \( q_0 = 0 \) and \( q_m = \infty \).

Also assume \( H_1 < H_2 < \ldots < H_m \)

For carrying cost \( H_j \), Harris-Wilson EOQ is given by

\[
Q_j = \sqrt{\frac{2 \times D \times C_3}{H_j}}
\]

... (4.8)

If \( Q_j \) does not lie between the interval \([q_{j-1}, q_j]\) i.e. \( Q_j \) is not order feasible, then the optimal lot-size will be determined by

\[
q_{j-1} \quad \text{if} \quad Q_j \leq q_{j-1}
\]

... (4.9.1)

\[
q_j \quad \text{if} \quad Q_j > q_j
\]

... (4.9.2)

With the known value of \( Q_j \), thus obtained from the equation (4.8) or (4.9.1) or (4.9.2), \( TC(Q_j) \) can be calculated at the carrying cost \( H_j \) by

\[
TC(Q_j) = \frac{Q_j \times H_j}{2} + \frac{D \times C_3}{Q_j}
\]

... (4.10)

\( Q^* \) can be obtained from the equation (4.8). Since the value of \( H_j \) is strictly increasing, so one would order with a small value of \( H_j \) as possible. The
corresponding economic order quantity $Q_j$ can be obtained from the equation (4.8). However, the value of $Q_j$ may not be feasible, as the carrying cost $H_j$ is applicable only within a specific range of order quantity. Once the optimal lot size is obtained, it can be used to eliminate the higher values of $H_j$.

Therefore for all the values of $H_j$, which are greater than the $\frac{TC(Q_j)^2}{2x D \times C_3}$ can be eliminated from further calculations. Moreover $H_j$ can be further tightened as we improve it in the successive steps.

### 4.3 Algorithm

1. Set $j = 1$.
2. Compute $Q_j$, using equation (4.8)
3. If $Q_j$ is order feasible, Set $Q^* = Q_j$ and stop.
4. If $Q_j$ is not order feasible, then make $Q_j$ order feasible, by equation (4.9.1) and (4.9.2).
5. Set $Q^* = Q_j$, and calculate $TC(Q_j^*) = \frac{Q_j^* H_j}{2} + \frac{D \times C_3}{Q_j}$ by equation (4.10)
6. Compute $r_j = \frac{TC(Q_j^*)}{2x D \times C_3}$, and ignore all the values of $H_j$, which are greater then $r_j$.
8. If no more $H_j$'s are left, then the $Q_j$ obtained from the last step will give the optimal $Q_j$ i.e. $Q^* = Q_j^*$, and thus $TC(Q_j^*) = \frac{Q_j^* H_j}{2} + \frac{D \times C_3}{Q_j}$, can be obtained by equation (4.10).
4.4 Numerical Example

Consider the following example with $C_3 = \text{Rs } 350.00$ and $D = 1800$.

Table 4.A

| J | Range | $H_j$ | $Q_j = \sqrt{\frac{2DC_j}{H_j}}$ | Feasible $Q_j$ | TC($Q_j$)  
|---|-------|-------|---------------------------------|----------------|-------------
| 1. | 1-30  | 110   | 107.02                          | 30             | 22650.00
| 2. | 31-60 | 130   | 98.44                           | 60             | 14400.00
| 3. | 61-90 | 150   | 91.65                           | 90             | 13750.00
| 4. | 91-20 | 180   | -                               | -              | -           
| 5. | 121-150 | 200  | -                               | -              | -           
| 6. | 151 or more | 220 | -                               | -              | -           

In the classical Harris-Wilson formula all the costs are considered to be constant. Gupta (1994) considered an inventory model with lot-size dependent ordering cost following the similar procedure as in (Gupta, 1994).

We get for, $D = 1800$ units and $C_3 = \text{Rs } 350.00$

$\Rightarrow H_1 = 110$, $Q_1 = 107.02$, which is not feasible. Therefore $Q_1^* = 30$ and $\text{TC}(Q_1^*) = 22650.00$

Now computing $r_1 = \frac{22650^2}{2 \times 1800 \times 350} = 407.61$

Further calculations are required for all $H_j$'s, $j \neq 1$.

$\Rightarrow H_2 = 130$, $Q_2 = 98.44$, which is not feasible. Therefore $Q_2^* = 60$ and $\text{TC}(Q_2^*) = 14400.00$
Now computing \( r_2 = \frac{14400^2}{2 \times 1800 \times 350} = 164.57 \)

Since \( H_4, H_5 \) and \( H_6 \) are greater than \( r_2 \), so they require no further consideration.

Further calculation is required for \( H_3 \)

\( \Rightarrow \) \ For \( H_3 = 150, \) \( Q_3 = 91.65 \), which is not feasible. Therefore \( Q_3^* = 90 \) and \( TC(Q_3^*) = 13750.00 \)

Now computing \( r_3 = \frac{13750^2}{2 \times 1800 \times 350} = 150.01 \)

Since no more \( H_j \)'s are left for further calculation, therefore the optimal lot-size is \( Q^* = Q_3^* = 90 \) units and \( TC(Q^*) = Rs 13750.00 \)

4.5 Discussion

From the Table - 4.A, we observed that the carrying cost will be optimal when the difference between the respective lot-size is more with same carrying cost. Also the total cost will be optimal if the demand size and the set-up cost is less. And when the carrying costs are not in a increasing order, then for the lot-size having least carrying cost will give the optimal carrying cost. So, in this chapter and in the chapter 3, the carrying cost is depends on lot-size and increases in steps as the lot-size increases. But the numerical result of this chapter is same as chapter 3, numerical example, which is solved by using the calculus method (Maxima & Minima).
4.6 Conclusion

In this chapter, we have extended the classical Harris-Wilson model with carrying costs depending on lot-size. It is not necessary that, if the calculated value of $Q_j$ lying within the interval, may give the optimal lot-size. From the above example, it can be shown that, for $C_3=350$ and $D=1000$, $Q_3 = 68.31$ which lies in the interval 61-90, but $TC(Q_3) = 10246.95 > 9733.33 = TC(Q_2)$, so $Q_2 = 60$ would be the optimal lot-size. So the value of $a$ and $b$ are 1 and 30 respectively. From the table it can be observed that, when the demand and the set-up costs vary, the optimal lot-size and the carrying costs also vary.