CHAPTER 5

AN EOQ MODEL FOR DETERIORATING ITEMS FOR SHORTAGES WITH PERIODIC TIME DEPENDENT DEMAND, DETERIORATION AND UNIT PRODUCTION COST
5.1 Introduction

In this chapter we developed an inventory model for a deteriorating item with a periodic time-dependent demand. It is also assumed that the finite production rate is proportional to the time-dependent demand rate and the deterioration rate is time-proportional. The unit production cost is inversely proportional to the demand rate. To make the model more physical and realistic, storages are assumed.

In this chapter, the following assumptions are made

1. The demand rate is periodic time-dependent function.
2. The production rate is finite and varies with periodic time dependent demand rate.
3. The unit cost of production is inversely proportional with the time dependent demand rate.
4. The deterioration rate is time proportional.
5. Shortages in inventory are allowed.

5.2 Assumptions and Notations

The proposed model has been developed with the following assumptions

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1. The demand rate $D(t) = a + bt + cs\sin t$, where $t \geq 0$, $a \geq 0$, $b \neq 0$ and $c \neq 0$. Also assumed that $|b| < a$ and $|c| < a$ to ensure that the demand rate is non-negative.

2. The production rate is given by $k(t) = \beta D(t)$, where $\beta > 1$, a constant.

3. A variable fraction $\theta(t) = \alpha \ t$ is the on-hand inventory deterioration per unit time, $0 < \alpha << 1$.

4. The lead time is zero.

5. The unit production cost $\delta$ is inversely proportional to the demand rate as $\theta = \delta D^{\gamma}$, where $\delta > 0$, $\gamma > 0$.

6. $C_1$ is the constant holding cost per item per unit of time.

7. $C_2$ is the shortage cost per unit of time.

8. $C_3$ is the deterioration cost per unit per unit of time.

9. Shortages are fully backlogged.

5.3 Model Development and Analysis

The amount of stock is zero at time $t = 0$. Production begins at $t = 0$ and continues up to $t = t_1$ when the stock level is $S_1$. After meeting the demand in $[0, t_1]$, accumulated inventory is used in $[t_1, t_2]$. During $[t_2, t_3]$, the inventory level gradually decreases mainly to meet the demands and partly because of deterioration. Now, let the stock level reduces at zero at time $t = t_2$. Then the shortage begins to start and reaches the level $S_2$ at $t = t_3$. Again production starts at time $t = t_3$ and demands and backlog quantities for $[t_2, t_3]$ are satisfied in the interval $[t_3, t_4]$. The inventory level gradually
falls to zero at time $t = t_4$. This cycle repeats after a time $t_4$. Consider the inventory level $Q(t)$ in $0 \leq t \leq t_4$. Therefore, the instantaneous state of $Q(t)$ is described by the following differential equations

$$\frac{dQ(t)}{dt} + \alpha t Q(t) = (\beta - 1)(a + bt + csint) \quad 0 \leq t \leq t_1$$  \hspace{1cm} (5.1)

Where $Q(0) = 0$ and $Q(t_1) = S_1$;

$$\frac{dQ(t)}{dt} + \alpha t Q(t) = -(a + bt + csint), \quad t_1 \leq t \leq t_2$$  \hspace{1cm} (5.2)

Where $Q(t_1) = S_1$ and $Q(t_2) = 0$;

$$\frac{dQ(t)}{dt} = -(a + bt + csint), \quad t_2 \leq t \leq t_3$$  \hspace{1cm} (5.3)

Where $Q(t_2) = 0$ and $Q(t_3) = -S_2$;

$$\frac{dQ(t)}{dt} = (\beta - 1)(a + bt + csint), \quad t_3 \leq t \leq t_4$$  \hspace{1cm} (5.4)

Where $Q(t_3) = -S_2$ and $Q(t_4) = 0$.

Solving equation number (5.1), subject to the condition $Q(0) = 0$, we get
\[ Q(t) = (1 - \beta)(c + at + \frac{1}{2}bt^2 + c_t^2 - \frac{1}{3}at^3 - \frac{1}{8}bt^4 - c_t \cos t + ct \sin t) \] ... (5.5)

Solving equation number (5.2), subject to the conditions \( Q(t_1) = S_1 \) and \( Q(t_2) = 0 \), we get:

\[
Q(t) = a(t_2 - t) + \frac{1}{2}b(t_2^2 - t^2) + a_0 \left( \frac{1}{3}t^3 - \frac{1}{2}t^2t_2 + \frac{1}{6}t_2^3 \right) \\
+ b_0 \left( \frac{1}{8}t^4 - \frac{1}{4}t^2t_2^2 + \frac{1}{8}t_2^4 \right) + c((\cos t - \cos t_2)(1 - \alpha) \\
+ \left(\frac{1}{2}\alpha \cos t_2(t_2^2 - t_2^2) - \alpha \sin t_2 \sin t_2 \right) 
\]

... (5.6)

Solving equation number (5.3), subject to the condition \( Q(t_2) = 0 \), we get:

\[
Q(t) = a(t_2 - t) + \frac{1}{2}b(t_2^2 - t^2) - c(\cos t_2 - \cos t) 
\]

... (5.7)

Solving equation number (5.4), subject to the condition \( Q(t_4) = 0 \), we get:

\[
Q(t) = (\beta - 1)(a(t - t_4) + \frac{1}{2}b(t^2 - t_4^2) - c(\cos t - \cos t_4)) 
\]

... (5.8)

\[ D_1 = \text{Production in } [0, t_1] - \text{Demand in } [0, t_2]. \]
\[ = \int_{0}^{t_1} \beta D(t) \, dt - \int_{0}^{t_2} D(t) \, dt \]

\[ = c(\beta - 1) + at_1\beta - at_2 + \frac{1}{2} bt_1^2 \beta - \frac{1}{2} bt_2^2 - c\beta \cos t_1 + c \cos t_2 \]

\[ \text{... (5.9)} \]

The holding cost in carrying the inventory for the entire circle is

\[ C_{h(o)} = c_1 \left[ \int_{0}^{t_1} Q(t) \, dt + \int_{t_1}^{t_2} Q(t) \, dt \right] \]

\[ C_{h(o)} = c_1 \left\{ (1 - a)(1 - \beta) - at_1 t_2 + at_2^2 + \frac{1}{6} bt_1 t_2^2 + \frac{1}{3} bt_2^3 - \frac{1}{6} ct_1^3 \alpha \right. \]

\[ + \frac{1}{6} at_1^3 t_2 \alpha + \frac{1}{12} bt_1^3 t_2^2 \alpha - \frac{1}{6} at_1 t_2^2 \alpha + \frac{1}{12} at_2^4 \alpha - \frac{1}{8} bt_1 t_2^4 \alpha \]

\[ + \frac{1}{15} bt_2^5 \alpha + \frac{1}{2} at_1^2 \beta + \frac{1}{6} bt_1^3 \beta \]

\[ + a\beta \left\{ \frac{1}{6} ct_1^3 - \frac{1}{12} at_1^4 - \frac{1}{40} bt_1^5 - ct_1 \cos t_1 \right\} + c \cos t_2 (t_1 - t_2) \}

\[ \text{... (5.10)} \]

The unit production cost is \([t, t + \delta t] \text{ is } k \delta t\)

\[ k \delta t = \frac{\delta \beta D(t)}{D(t)^2} \, dt \]

\[ = \delta \beta \frac{1}{D(t)^{n-1}} \, dt \]

\[ = \frac{\delta \beta}{a^{n-1}} \left[ 1 - \frac{t_{n-1}}{a} (bt + csint) \right] dt \]
The production cost in \([0, t_4]\) is

\[
P = \int_0^{t_1} k \varrho \ dt + \int_{t_3}^{t_4} k \varrho \ dt
\]

\[
= (\delta \beta a^{-1})(t_1 - t_3 + t_4) - (\gamma - 1)(\delta \beta a^{-2})\left(\frac{b}{2} (t_1^2 - t_3^2 + t_4^2) - c\text{cost}_3 - \text{cost}_4 + 1\right)
\]

... (5.11)

The total shortage cost in \([0, t_4]\) is

\[
C_{sho} = -\left(\int_{t_2}^{t_3} Q(t) \ dt + \int_{t_3}^{t_4} Q(t) \ dt\right)
\]

Using equations (5.7) and (5.8) respectively, we get

\[
= (1 - \beta) \left\{ a_3 t_3 - \frac{a}{2} t_4^2 + \frac{b}{2} t_4 t_3 - \frac{b}{3} t_4^3 - c \text{cost}_4 (t_4 - t_3) - c \text{sint}_4 \right\}
\]

\[
+ \frac{b}{3} t_4^2 - \frac{b}{2} t_4 t_3 + \frac{b \beta}{6} t_3^3 + \frac{a}{2} t_2^2 + \frac{2 \beta}{2} t_4^2 - a t_2 t_3
\]

\[- c (\text{cost}_2 (t_2 - t_3) + \beta \text{sint}_3 - c \text{sint}_2)
\]

... (5.12)

The total average cost in \([0, t_4]\) is

\[
tcq = \frac{1}{t_4} [C_3 D_1 + C_{ho} + P + C_{sho}]
\]

Using the equations (5.9), (5.10), (5.11) and (5.12) respectively and putting expanding sint and cost with neglecting higher power of ‘t’ more than two, we get
The optimal solutions \( t_1 = t_1^*, \ t_2 = t_2^*, \ t_3 = t_3^* \) and \( t_4 = t_4^* \) can be obtained from the equation (13), provided the sufficient conditions \( H_i > 0 \ (i = 1, 2, 3, 4) \) holds for \( t_1 = t_1^*, \ t_2 = t_2^*, \ t_3 = t_3^* \) and \( t_4 = t_4^* \), where \( H_i \) is the Hessian determinant of order \( i \) given by

\[
H_i = \begin{bmatrix}
    c_{11} & c_{12} & \cdots & c_{1i} \\
    c_{21} & c_{22} & \cdots & c_{2i} \\
    \vdots & \vdots & \ddots & \vdots \\
    c_{i1} & c_{i2} & \cdots & c_{ii} \\
\end{bmatrix}
\]

... (5.13)
Where, \[ c_{ij} = \frac{\partial^2 t_{cq}}{\partial t_i \partial t_j} \quad (i, j = 1, 2, 3, 4) \]

Corresponding minimum average cost during \([0, t_4]\) is \( t_{cq}^* = t_{cq}(t_1^*, t_2^*, t_3^*, t_4^*) \).

From the equation (5.14), we have the following results:

\[ \frac{\partial t_{cq}}{\partial t_1} = 0, \] gives,

\[ => (a^{-}\gamma \beta \delta + a C_3 \beta) t_1^{\frac{1}{4}} + (a^{-}\gamma (b + c)(1-\gamma) \beta \delta + a C_1 \beta + \\
(b + c) C_3 \beta) t_1^{\frac{1}{4}} + \left(\frac{b+c}{2} C_1 \beta\right) t_1^{\frac{1}{4}} + \left(\frac{a C_1 \alpha}{6} \right) t_1^{\frac{3}{4}} - \left(\frac{b C_1 \alpha}{8} \right) t_1^{\frac{3}{4}} - \left(\frac{a C_1 \alpha}{8} \right) t_1^{\frac{3}{4}} - \left(\frac{b C_1 \alpha}{8} \right) t_1^{\frac{3}{4}} + \\
\frac{C_1 \alpha}{t_2} t_1^{\frac{1}{4}} = 0 \]

... (5.15.1)

\[ \frac{\partial t_{cq}}{\partial t_2} = 0, \] gives,

\[ => (a C_3) t_1^{\frac{1}{4}} + (a C_1) t_1^{\frac{1}{4}} + \left(\frac{a C_1 \alpha}{6} \right) t_1^{\frac{3}{4}} + (a C_1 + a C_2 - (b + c) C_3) t_1^{\frac{1}{4}} - \\
(b + c) C_1 t_1^{\frac{1}{4}} + \left(\frac{b+c}{6} C_1 \alpha\right) t_1^{\frac{1}{4}} + (b + c)(C_1 + C_2) t_1^{\frac{1}{4}} - \left(\frac{a C_1 \alpha}{2} \right) t_1^{\frac{1}{4}} + \\
\frac{t_1 t_2^2}{t_4} + \left(\frac{a C_1 \alpha}{3} \right) t_1^{\frac{3}{4}} + \left(\frac{b+c}{3} C_1 \alpha \right) t_1^{\frac{3}{4}} + \left(\frac{b C_1 \alpha}{3} \right) t_1^{\frac{3}{4}} - (a C_2) t_1^{\frac{3}{4}} - (b + \\
c) C_1 t_1^{\frac{1}{4}} = 0 \]

... (5.15.2)
\[ \frac{\partial \tau_{eq}}{\partial t_3} = 0, \text{ gives,} \]

\[
=> -a(b + c) C_2 - (\frac{\partial^2}{\partial t_3}) \frac{1}{t_4} - a C_2 \frac{t_2}{t_4} - \left( \frac{(b + c) C_2}{2} \right) \frac{t_2^2}{t_4} + (a C_2 \beta - a^{-\gamma}(b + c)(1 - \gamma) \beta \delta) \frac{t_3}{t_4} + \left( \frac{b + c C_2 \beta t_3^2}{2} \right) \frac{t_2}{t_4} - \frac{(b + c)(\beta - 1) C_2}{2} t_4 = 0
\]

\[ \text{... (5.15.3)} \]

\[ \frac{\partial \tau_{eq}}{\partial t_4} = 0, \text{ gives,} \]

\[
=> \frac{a^{-\gamma}(b + c)(1 - \gamma) \beta \delta}{2} + a(b + c) C_2 - \left( \frac{(b + c)(\beta - 1) C_2}{2} - a \beta C_3 \right) \left( \frac{t_1}{t_4^2} - \frac{t_3}{t_4^2} \right) - \left( a^{-\gamma}(b + c)(1 - \gamma) \beta \delta + \left( \frac{a C_3 \beta}{2} \right) \frac{t_1}{t_4^2} + \left( \frac{b + c C_3 \beta}{2} \right) \frac{t_2}{t_4^2} \right) - \left( \frac{b + c C_3 \beta}{6} \right) \frac{t_1}{t_4^2} + \left( \frac{a C_1 \alpha}{12} \right) \frac{t_1^3}{t_4^2} - \left( \frac{a C_1 \alpha}{40} \right) \frac{t_1^4}{t_4^2} - \left( \frac{a C_1 + a C_2 - (b + c) C_1}{2} \right) \frac{t_1}{t_4^2} + \left( \frac{(b + c) C_1}{2} \right) \frac{t_1 t_2}{t_4^2} - \left( \frac{b + c C_2 \alpha}{12} \right) \frac{t_1^2 t_2}{t_4^2} - \left( \frac{a C_1 \alpha}{12} \right) \frac{t_1^3}{t_4^2} - \left( \frac{(b + c) C_2}{15} \right) \frac{t_2^5}{t_4^2} + \left( a C_2 \right) \frac{t_2^3}{t_4^2} - \frac{(b + c)(\beta - 1) C_2}{2} \frac{t_3^3}{t_4^2} + \frac{(b + c)(\beta - 1) C_2}{3} \frac{t_3}{t_4^2} = 0
\]

\[ \text{... (5.15.4)} \]
5.4 Solution Procedure

The total average cost \(tcq(t_1, t_2, t_3, t_4)\) given above, is a function of the four variables \(t_1, t_2, t_3\) and \(t_4\). The necessary condition for \(tcq\) to be minimum is given by the equations (5.15.1) to (5.15.4) respectively. These equations are highly non-linear in nature, which can be easily solved by Newton Method of higher variables when the values of the parameters are prescribed. The optimal solutions of the equations (5.15.1) to (5.15.4) are \(t_1^*, t_2^*, t_3^*, t_4^*\) provided these values of \(t_i^*\) (\(i = 1, 2, 3, 4\)) satisfy the conditions \(H_i > 0\) (\(i = 1, 2, 3, 4\)). Subsisting these values in equation number (5.13), the optimal average cost \(tcq(t_1^*, t_2^*, t_3^*, t_4^*)\) can be obtained.

5.5 Numerical examples

To illustrate the obtained result following are some examples given in Table - 5.A

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
<th>(\Lambda)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\delta)</th>
<th>(t_1^*)</th>
<th>(t_2^*)</th>
<th>(t_3^*)</th>
<th>(t_4^*)</th>
<th>(tcq^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.01</td>
<td>2</td>
<td>0.01</td>
<td>10</td>
<td>1.70</td>
<td>1.15</td>
<td>1.81</td>
<td>2.51</td>
<td>47.59</td>
</tr>
<tr>
<td>0 &lt; a, b, (c \leq 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>0.2</td>
<td>0.7</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.01</td>
<td>2</td>
<td>0.01</td>
<td>10</td>
<td>1.26</td>
<td>1.24</td>
<td>1.31</td>
<td>1.79</td>
<td>48.55</td>
</tr>
<tr>
<td>0 &lt; a; 0 &lt; b, (c \leq 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>0.9</td>
<td>1</td>
<td>2</td>
<td>4.8</td>
<td>0.01</td>
<td>2</td>
<td>0.01</td>
<td>10</td>
<td>1.09</td>
<td>0.83</td>
<td>1.17</td>
<td>2.16</td>
<td>76.81</td>
</tr>
<tr>
<td>(c = 0)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.83</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0.01</td>
<td>2</td>
<td>0.01</td>
<td>10</td>
<td>0.43</td>
<td>0.13</td>
<td>0.39</td>
<td>1.32</td>
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</tr>
<tr>
<td>1.3</td>
<td>0.9</td>
<td>0</td>
<td>1</td>
<td>2.9</td>
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<td>2</td>
<td>0.01</td>
<td>9</td>
<td>0.88</td>
<td>0.2</td>
<td>0.97</td>
<td>2.31</td>
<td>52.18</td>
</tr>
</tbody>
</table>
5.6 Sensitivity Analysis

In this section, we study the sensitivity analysis to examine the effect of changes in the input parameters on the optimal results obtained in the first example for $0 < a, b, c \leq 1$. We first find the optimal values of variables $t_1, t_2, t_3, t_4$ and $tcq$ by changing (increasing or decreasing) one parameters by 25% and 50% and all other parameters remains unchanged. Then we calculate the percentage change of $tcq$ with respect to the other values. The following are the conclusions made from the table – F given below:

From the table, we see that the percentage change in the total average cost is almost same for both positive and negative changes of the value $a$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>% Change in Parameters</th>
<th>$t_1^*$ (1.70)</th>
<th>$t_2^*$ (1.15)</th>
<th>$t_3^*$ (1.80)</th>
<th>$t_4^*$ (2.51)</th>
<th>$tcq^*$ (47.59)</th>
<th>% Change in $tcq^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>+50</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
<td>1.34</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>+25</td>
<td>2.55</td>
<td>2.81</td>
<td>2.74</td>
<td>2.74</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-25</td>
<td>1.77</td>
<td>2.10</td>
<td>1.66</td>
<td>2.93</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-50</td>
<td>2.26</td>
<td>2.45</td>
<td>2.06</td>
<td>4.38</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>b</td>
<td>+50</td>
<td>3.49</td>
<td>3.96</td>
<td>3.48</td>
<td>5.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>+25</td>
<td>2.43</td>
<td>3.91</td>
<td>2.24</td>
<td>3.44</td>
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<td>-</td>
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<td>-</td>
<td>-</td>
<td>1.08</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>c</td>
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<td>-25</td>
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<td>5.20</td>
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<td>-</td>
<td>-</td>
</tr>
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<td>2.27</td>
<td>2.53</td>
<td>2.99</td>
<td>19.57</td>
<td>-58.88</td>
</tr>
<tr>
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<td>1.97</td>
<td>1.75</td>
<td>2.08</td>
<td>2.70</td>
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<td>-10.73</td>
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1. Percentage change in tcq cannot be calculated for the change in the values $a$, $C_2$ and $C_3$.

2. The model is highly sensitive to the changes in some of the remaining values $b$, $c$, $C_1$, $\beta$ and $\delta$.

3. The model is very less sensitive to the parameter $\gamma$. The average cost $tcq^*$ decreases (increases) with increase (decrease) in the value of $\gamma$.

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5.7 Conclusion

In this chapter, some realistic features are considered. These features are likely to be associated with an inventory of consumer goods. The assumption of a periodic time-dependent demand rate and production rate is very realistic in the market. The deterioration rate is also assumed to be time-dependent and unit production cost is inversely proportional with the demand rate. The occurrence of shortages in inventory is a very natural phenomenon. Shortages are allowed and backlogged. From the numerical examples shown in the table, we observe that for $0 < a$, $b$, $c < 1$, the total average cost is minimum. If we put $c = 0$ in equation number (5.13), we get the total average cost in $[0, t_4]$ of the article of (Manna and Choudhury, 2001) with demand rate $D(t) = a + bt$, two numerical examples are done in the table – 5.A with the assumption $c = 0$. The sensitivity analysis of the solution to changes in the values of different parameters has been discussed.