CHAPTER 3

AN INVENTORY MODEL WITH LOT SIZE DEPENDENT CARRYING / HOLDING COST
3.1 Introduction

In the classical Harris-Wilson (1915) inventory model all the cost associated with the formula was taken to be constant and which also does not depend on any quantity. There are many practical situations where this is not true. This article considers an inventory model where the carrying cost depends on lot-size and increases in steps as the lot size increases. An algorithm is developed to determine the economic order quantity along with numerical examples.

One of the challenging problems for the marketing researchers and practitioners is to study and analyse the inventory systems as the proper inventory systems cannot only reduce the costs, but also reduce stock-outs and improve customer satisfactions. Finding an appropriate system for a given inventory condition requires an expansive overview of inventory literature with thorough understanding of the assumptions of each model. Thus proper inventory systems can improve the profitability and help in the survival of an organization.

When items are received they are to be checked, inspected etc. These costs would generally increases as the size of the lot received increases and it would costs the organization a certain fixed cost such as wages per hour or per shift both for loading and unloading the items. The labour charges would increase as the run-size increases.

The Harris-Wilson model has attracted many researchers to the inventory modeling area. The Wilson Formula for determining the optimum lot-size, the quantity in which an item of inventory should be purchased or produced for stock is

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\[ q = \sqrt{\frac{2 \times D \times C_3}{C_1}} \]  

... (3.1)

Where, \( D \) = annual sales or demand.

\[ C_3 = \text{set-up cost per order}. \]

\[ C_1 = \text{the carrying cost for a unit for one year}. \]

### 3.2 Assumptions and Notations

This chapter considers a situation where lot-size depends on the carrying / holding cost. If the carrying costs are incorrectly stated, then in this case if any inventory model was developed by someone with same assumption the cost of carrying will be very high or very low, which results in the increase of the cost of the goods. by this assumptions, an inventory model has been developed where the carrying cost depends on lot-size. As the lot-size increases stepwise, the carrying cost also increases accordingly. However, in the proposed model all other assumptions of the Harris-Wilson EOQ model remain valid.

In this model, assuming that the carrying cost increases stepwise as the lot-size increases. The following notations are used.

1. \( D \) = Total / Annual Demand.
2. \( C_3 \) = Setup cost per order.
3. \( H_i \) = Carrying cost for the lot-size

\[ Q_i \text{ if } q_{i-1} \leq Q_i \leq q_i \]
Where \( j = 1, 2, 3, \ldots, m \), \( q_0 = 0 \) and \( q_\infty = \infty \)

Also assuming \( H_1 < H_2 < H_3 < \ldots < H_m \).

This formula follows from the propositions,

i) The optimal quantity minimizes the sum of the annual set-up and carrying costs,

\[
TC(q) = \frac{q \times C_1}{2} + \frac{D \times C_3}{q}
\]

... (3.2)

The Wilson formula is popular for its simplicity i.e. the costs considered here are assumed to be constant.

But for the wholesalers and manufacturers, an item for which sales are of unit size and at a constant rate is the exception. When sales vary in size, take place irregularly, and the time and the size of the sales are uncertain equation (3.2) is not a correct statement of annual inventory costs.

### 3.3 Model Development and Analysis

For carrying cost \( H_j \), Harris-Wilson EOQ is given by

\[
Q_j = \sqrt{\frac{2 \times D \times C_3}{H_j}}
\]

... (3.3)
If \( Q_j \) does not lie within the interval \([q_{j-1}, q_j]\), i.e. is not order feasible, then the optimal lot-size will be determined by

\[
q_{j-1} \text{ if } Q_j \leq q_{j-1}
\]

... (3.4.1)

\[
q_j \text{ if } Q_j \geq q_{j-1}
\]

... (3.4.2)

With the known value of \( Q_j \) thus obtained from the equation (3.3) or (3.4.1) or (3.4.2), \( TC(Q_j) \) can be calculated at the carrying cost by

\[
TC(Q_j) = \frac{Q_j \times H_i}{2} + \frac{D \times C_i}{Q_j}
\]

... (3.5)

If \( Q_j \) is order feasible then \( TC(Q_j) \) will be the optimal lot-size, otherwise the value of \( Q_j \) thus obtained by equations (3.4.1) and (3.4.2), are calculated by equation (3.5). Thus among all the calculated value of \( TC(Q_j) \), the smallest value will be the optimal cost i.e. \( TC(Q_{opt}) \) and the respective \( Q_j \) will be the optimal lot-size i.e. \( Q_{opt} \).

### 3.4 Algorithm

1. Input \( j, n \) = no. of lot-size.
2. Set \( j=1 \).
3. Calculate $Q_j = \sqrt{\frac{2D_tC_3}{H_j}}$.

4. If $Q_j$ is order feasible, calculate $TC(Q_j) = \frac{Q_jH_j}{2} + \frac{D_tC_3}{Q_j}$, by equation (3.5).
   Go to step-8.

5. If $Q_j$ is not order feasible, then by equation (3.4.1) and (3.4.2), $Q_j$ can be made order feasible. Go to Step-4.

6. Set $j = j + 1$, until $j < n$. Go to Step-3.

7. Among all the calculated $TC(Q_j)$, obtain the minimum value of $TC(Q_j)$, and set $TC(Q_{opt}) = \min \{TC(Q_j)\}$.

8. Thus $TC(Q_{opt})$ obtained is the optimal carrying cost, and corresponding $Q_j$ is the $Q_{opt}$ the optimal lot-size.

9. Thus $Q_{opt}$ & $TC(Q_{opt})$ are the required results.

3.5 Numerical Examples

Example-1

Considering the demand $(D) = 1800$ units, $C_3 = Rs 350.00$

Carrying costs are given in the following Table - 3.A
### Table - 3.A

<table>
<thead>
<tr>
<th>j</th>
<th>( Q_j )</th>
<th>Range</th>
<th>Feasible ( Q_j )</th>
<th>( TC(Q_j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>107.02</td>
<td>1-30</td>
<td>30</td>
<td>110</td>
</tr>
<tr>
<td>2.</td>
<td>98.44</td>
<td>31-60</td>
<td>60</td>
<td>130</td>
</tr>
<tr>
<td>3.</td>
<td>91.65</td>
<td>61-90</td>
<td>90</td>
<td>150</td>
</tr>
<tr>
<td>4.</td>
<td>83.67</td>
<td>91-120</td>
<td>91</td>
<td>180</td>
</tr>
<tr>
<td>5.</td>
<td>79.37</td>
<td>121-150</td>
<td>121</td>
<td>200</td>
</tr>
<tr>
<td>6.</td>
<td>75.68</td>
<td>151 or more</td>
<td>151</td>
<td>220</td>
</tr>
</tbody>
</table>

For the lot-size \( Q_{opt} = 90 \) units (approx) the optimal carrying cost is \( TC(Q_j) = Rs. 13750.00 \)

**Example-2**

Considering the demand \((D) = 1000 \) units, \( C_3 = Rs. 350.00 \)

Carrying costs are given in the following Table - 3.B

### Table - 3.B

<table>
<thead>
<tr>
<th>j</th>
<th>( Q_j )</th>
<th>Range</th>
<th>Feasible ( Q_j )</th>
<th>( TC(Q_j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>79.77</td>
<td>1-30</td>
<td>30</td>
<td>110</td>
</tr>
<tr>
<td>2.</td>
<td>73.38</td>
<td>31-60</td>
<td>60</td>
<td>130</td>
</tr>
<tr>
<td>3.</td>
<td>68.31</td>
<td>61-90</td>
<td>68.31</td>
<td>150</td>
</tr>
<tr>
<td>4.</td>
<td>62.36</td>
<td>91-120</td>
<td>91</td>
<td>180</td>
</tr>
<tr>
<td>5.</td>
<td>59.16</td>
<td>121-150</td>
<td>121</td>
<td>200</td>
</tr>
<tr>
<td>6.</td>
<td>56.40</td>
<td>151 or more</td>
<td>151</td>
<td>220</td>
</tr>
</tbody>
</table>

For the lot-size \( Q_{opt} = 68.31 \) units (approx) the optimal carrying cost is \( TC(Q_j) = Rs. 10246.95 \)
Example-3

Considering the demand \((D) = 1000\) units. \(C_3 = Rs\ 200.00\)

Carrying costs are given in the following Table – 3.C

<table>
<thead>
<tr>
<th>(j)</th>
<th>(q_j = \sqrt{\frac{2 \times D 	imes C_3}{H_j}})</th>
<th>Range</th>
<th>Feasible (Q_j)</th>
<th>(Q_j)</th>
<th>(TC(Q_j))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>60.30</td>
<td>1-30</td>
<td>30</td>
<td>110</td>
<td>8316.67</td>
</tr>
<tr>
<td>2.</td>
<td>55.47</td>
<td>31-60</td>
<td>55.47</td>
<td>130</td>
<td>7211.10</td>
</tr>
<tr>
<td>3.</td>
<td>51.63</td>
<td>61-90</td>
<td>61</td>
<td>150</td>
<td>7853.69</td>
</tr>
<tr>
<td>4.</td>
<td>47.14</td>
<td>91-120</td>
<td>91</td>
<td>180</td>
<td>10378.80</td>
</tr>
<tr>
<td>5.</td>
<td>44.72</td>
<td>121-150</td>
<td>121</td>
<td>200</td>
<td>13752.90</td>
</tr>
<tr>
<td>6.</td>
<td>42.64</td>
<td>151 or more</td>
<td>151</td>
<td>220</td>
<td>17934.50</td>
</tr>
</tbody>
</table>

For the lot-size \(Q_{opt} = 55.47\) units (approx) the optimal carrying cost is \(TC(Q_j) = Rs. 7211.10\)

3.6 Conclusion

In this chapter, the classical Harris-Wilson model has been extended with carrying cost depending on lot-size. It is observed and calculated that, if the value of \(Q_j\) lying within the interval i.e. order feasible, it will give the optimal costs. And if no such value of \(Q_j\) can be obtained which is order feasible, then we can make order feasible by equations (3.4.1) and (3.4.2), and out of all the calculated values of the
minimum value will give the optimal carrying costs. Also, it may be observed that the optimal cost depends on the demand required as well as the set-up cost per order.