CHAPTER 1

GENERAL INTRODUCTION
1.1 Introduction

The first formal activities of Operations research (OR) were initiated in England during the World War II. A team of British Scientist sought out some scientifically based decision regarding the best utilization of the war material in the war. After then the activities of Operations research has become increasingly important in the face of fast moving technology and have been proved to be a very efficient technique for optimization in different real life applications such as Airlines, Finance, Military, Energy Systems, Production Systems, Project Management, Quality control, Reliability, Supply chain Management, Water Management, E-Commerce, civilian sector etc. The introduction of computers and the computing facility flourished the field of OR.

Inventory means a physical stock of material or goods or commodities or other economic resources that are stored or reserved or kept in stock or in hand for smooth and efficient running of future affairs of an organization at the minimum cost in the form of materials or goods, and the function of directing the movement of goods through the entire manufacturing cycle from the requisition of raw materials to the inventory of finished goods in an orderly manner to meet the objectives of maximum customer service with minimum investment and efficient (low cost) plant operation is termed as inventory control. Inventories are classified as

- **Direct Inventories** - which play direct role during manufacturing of the product.
  - Raw material inventories - The inventory of raw materials is the materials used in the manufacture of product.
  - Work-in-process inventories - These inventories are of semi-finished type, which accumulate between operations or facilities.
- **Finished goods inventories** - After finishing the production process and packing, the finished products are stocked in stock room known as finished goods inventory.

- **Spare parts inventories** - Any product sold to the customer, will be subjected to wear and tear due to usage, hence the manufacturers always calculate the life of the various components of the product and try to supply the spare components to the market to help after sales service, known as spare parts inventory.

- **Scrap or waste inventory** - While processing the materials, certain wastages and certain bad components (scrap), which are of no use. These may be used by some other industries as raw material

- **Indirect Inventories** - which are required for manufacturing but not as a part of production of the product.
  - **Fluctuation Inventories** - These inventories are carried out to safeguard the fluctuation in demand, non-delivery of material in time due to extended lead-time.
  - **Anticipation Inventory** - When a large stock of material is stored for future use to meet the demand. Such inventories, which are stocked in anticipation of raising demand or raising rises, are known as anticipation inventories.
  - **Lot size inventory or Cycle inventories** - This situation happens in batch production system. In this system products are produced in economic batch quantities. It sometime happens that the materials are procured in quantities larger than the economic quantities to meet the fluctuation in demand. In such cases the excess materials are stocked, which are known as lot size or cycle inventories.
  - **Transportation Inventories** - When an item is ordered and purchased they are to be received or the furnished goods are to be delivered to the
customers, such inventories, which are in transit, are known as Transportation inventories.

- **Decoupling Inventories** - These inventories are stocked in the manufacturing plant as a precaution, in case the semi-finished from one machine does not come to the next machine, this stock is used to continue production. Such items are known as decoupling inventories.

Inventory modeling deals with determining the level of a commodity that a business must maintain to ensure smooth operations. The control and maintenance of inventory is a common problem in all enterprises in any sector of economy. Proper inventory system cannot only reduce the cost but also reduces stock-out and improve customer satisfactions. As a result, statistical, linear and non-linear optimization in different fields grew in a rapid manner. (Gass, 1997) opined that the future challenges for OR will come from real time decision support and Control problems such as electricity generation and transmission, air traffic control, and high way traffic control etc. Researcher are engaged in developing the subject with more scientific approach starting from World War II, to this age of advanced information technology using the modern techniques. The books by (Donald Waters, 2003, Sven Axsater, 2006, Naddor,1966, Muckstadt and Sapra, 2010) are worth mentioning in this regard.

There are several reasons why organizations maintain inventories. The main reason is that it is physically or economically impossible to have goods arrive in a given system precisely at the time when demands for them occur. In absence of inventories, customers would have to wait until their orders were filled from a source. However, customer can not be allowed to wait for long period. Also, the price of some raw materials used by a manufacturer may exhibit considerable seasonal fluctuations. When the price is low, it is profitable to procure a sufficient quantity of it to last through the high priced season and to keep it in inventories to be used when need arises. Thus proper inventory system can improve the profitability and help in the survivable of an organization.
Communicating with the customers in a cost effective manner on time is the dream of each enterprise. (Thompson et al., 2005) has thrown light on the need for product design to reach a new level of satisfaction. As costs plummet, every manufacturer wishes to cram the product with as many features as one can. Market research has shown that buyers assign a higher weightage to product capability over its usability at the time of purchase. This results in high-expected utility. Hence, manufacturer need to segment the market better, increase product variety, and invest to learn consumer behavior on an ongoing basis. Similarly, consumer research can scale new heights in the emerging decades. Customer behavior can be tracked easily, where as customer perception is hard to decipher. Correlating the two and deriving meaningful insights will be the focus of market researchers. With reference to physical goods, the backend of mass customization is intelligent manufacturing that can incorporate a feedback loop from the demand side into the manufacturing process. Capacity management and order management are in focus rather than better forecasting and finished goods inventory control.

Inventory problem consists of determining or answering some basic questions or factors. The two basic questions must be answered in controlling the inventory of any physical goods i.e. when to replenish the inventory and how much to order for replenishment; it is associated with the question when to order and how much to order. Thus, the problem of inventory management of any organization or firm, which stocks different items for sale to the customers, is related to the above questions. As sales occur inventory is depleted over-time. The firm has its own mechanism of placing orders and obtaining fresh replenishment. The manner in which this reorder option is exercised is termed as the firms inventory policy. The problem of making optimal decisions with regard to the above questions is called an inventory problem, which deals with making of decision that minimizes the total cost of the inventory system or maximizes the profit gained while meeting the customers demand.
The solution of an inventory system means solutions of the inventory problem. The solution is nothing but a set of specific values of the variables that minimizes the total cost of the system or maximise the total profit in the system. Some specific values cannot be determined due to some reasons such as storage of water in a dam in which one has no control over the replenishment of inventory i.e. the supply of inventory of water within the dam depends on rainfall and organizations operating the dam has to control it. In such cases, the solution is given by a set of decision rules. These rules are called optimal decision rules. To make a solution of an inventory problem easier, it is necessary to design a mathematical model, which describes the inventory situations. As complete accuracy is not possible in this real world, so some approximations and simplifications must be made for the mathematical models.

Constraints are limitations, which are imposed on inventory system. Warehouse space constraint may limit the amount of inventory held, Capital constraint restrict the amount of money invested in inventories. Similarly facilities, equipment and personal constraint may check the supply capability and operating level of an organization.

Inventories differ in size, complexity the item they carry, the cost associated with operating system, nature of stochastic process etc. All these differences can be considered to reflect variation in the structure of inventory system.

All organization keeps a supply of inventory for the following reasons.

1. **To maintain independence of operations**
   A supply of materials at a work center allows that center flexibility in operations. For example, as there are costs for making each new production setup, this inventory allows management to reduce the number of setups.
2. **To meet variation in product demand**

If the demand for the product is known precisely, it may be possible (though not necessarily economical) to produce the product to exactly meet the demand. Usually, demand may not be completely known, and a safety or buffer stock must be maintained to absorb variation.

3. **To allow flexibility in production scheduling**

A stock of inventory relieves the pressure on the production system to get the goods out. This causes longer lead times, which permit production planning for smoother flow and lower-cost operation through larger lot-size production. High setup costs, for example, favour producing a larger number of units once the setup has been made.

4. **To take advantage of economic purchase order size**

There are costs to place an order, labour, phone calls, typing, postage, and so on. Therefore, the larger each order is, the fewer the orders that need to be written. Also, shipping costs favour larger orders the larger the shipment, the lower the per-unit cost.

We all can recognize the necessity of inventories to sustain operations within an economy. One major role of the inventory management is to determine polices that create and distribute inventories most effectively. Though there are many factors or forces that may affect the policies selected by the management. Then environment plays an important role in materializing the selected polices. Agricultural output is dictated by the growing seasons for crops in a particular location. The harvested crops may not be consumed for several years. Thus, inventories may be created because of capacity limitations, seasonal cycles, different new features of the modern economy etc. Inventory is also used to meet the current demand from stock which was created earlier because of the cyclic nature of the incoming supply of inventory. An inventory system provides the organizational structure and the operating policies for maintaining
and controlling goods to be stocked. The system is responsible for ordering and receipt of goods.

1.2 System Parameters used in Inventory Management

The properties of inventory system depend on several components or parameters such as demand, replenishment, constrains, inventory costs etc.

Demands are units taken from inventory and can be categorized according to their size. Demand size refers to the magnitude of demand and has the dimension of quantity. When demand size is known the system is known as deterministic. When the demand size is not known the system is known as probabilistic or stochastic.

Demand rate is the demand size per unit time and demand patterns refer how units are withdrawn from inventory.

Replenishments are units put into inventory and can be categorized according to size, pattern and lead time.

Replenishment size means the quantity of the order to be received into inventory. They may be constant, variable depending on the type of inventory system.

Replenishment pattern refers to, how the units are added to inventory. Replenishment pattern are generally instantaneous, uniform or in batch.
Replenishment lead time is the length of time between the decision to replenish or to order an item and its actual arrival in stock. It may be constant or a variable. In case of variable lead time probability distribution is used.

1.3 Inventory Costs

Inventory costs are associated with the operation of an inventory system and result from action or lack of action on the part of management in establishing the system. They are basic economic parameters to any inventory decision models. There are four main types of cost.

(i) Holding Cost.
(ii) Shortage Cost.
(iii) Setup Cost.
(iv) Material Cost.

Holding or carrying cost is the cost associated with investing in inventory and maintaining the physical investment in storage. It is item dependent parameters with dimension of dollars or rupees per unit period. It also includes insurance, taxes, obsolescence, warehouse, rental, light, heat etc.

The shortage or stock-out cost is the penalty incurred for being unable to meet the demand, when it occurs. The cost parameter depends on replenishment. It has two wings internal or external shortage.

⇒ If the order of a group, department or enterprise is not filled up, then this type of shortage is known as Internal Shortage, and
If the order of any customer is not filled up, then this type of shortage is known as External Shortage.

The ordering or setup or replenishment cost originates from the expense of issuing a purchase order to an outside supplier or from internal production setup costs. This cost includes advertisement, consumption of stationary, postage, rent for keeping items, telephone charges, transportation, human labor costs.

Material cost is the cost of the material or the items. It is a parameter that depends upon the material or items as well as the source. If the material or the item is obtained from an external source, the material cost is the purchase cost of the material or the item. If the material or the item is produced internally, it is known as the production cost. It should be taken as the cost of the item as it is placed in the inventory. Material costs / purchase costs / production costs are considered when the quantity discount or price fluctuation situation arises.

1.4 Area of Work

There are more than thousands of works involving with different parameters in different inventory models. The researchers are engaged in developing new models for better result in connection with production. In past two decades, inventory problems with deterministic time-varying demand pattern received considerably more attention of many researchers. Many researchers developed, inventory models dealing with various parameters related to inventory management such as carrying cost or holding costs, shortage costs, setup costs, replenishment costs, demand, deterioration, own warehouse, rented warehouse, delay in payments, discount factors etc.
In most of the inventory models, holding cost is known and is considered constant. However, in real life situation this may not happen always. As the time changes the holding cost may not remain constant. The traditional parameters of holding cost may be assumed to be time varying, stock varying etc.

Demand is a major factor in inventory management. Whenever the demand of any item increases or decreases, it directly affects the stock of the item, cost of the item. The effect of discount, permissible delay in payments or the price of the item affects the demand pattern. In inventory models, different types of demand are considered such as constant-demand, time-dependent-demand, stock-dependent-demand, price-dependent-demand, probabilistic-demand and two-component demand etc. After a brief literature survey, it is seen that the model developed by (Bernardi et al., 2009) consider the periodic demand. In real life scenario, the periodic demand, which changes with time, is the most practical approach in comparison to other time-dependent-demand. In the opinion of many authors, an exponential rate or quadratic rate of change of demand is so high, and the market demand of any product cannot undergo such a change, which is so high, as exponential or quadratic rate is.

In general, deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, etc. that results in decreasing the usefulness of the original one. For items such as steel, hardware, glassware, and toys, the rate of deterioration is so low in which there is little need for considering deterioration in the determination of the economic lot size. However, some items such as blood, fish, fruits, alcohol, gasoline, radioactive chemicals, medicine, and food grains (i.e. paddy, wheat, potato, onion, etc.) deteriorate rapidly over-time. Thus, the loss from deterioration cannot be ignored.

Maximum physical goods undergo decay or deterioration over time. Decay (or exponential decay) means that a fixed fraction of the inventory is lost in every planning period. Fruits, vegetables etc. suffer from depletion by direct spoilage while
stored. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain etc. deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature. (Nahmias, 2011) has given an excellent idea considering the large number of perishable items in the economy.

Shortages means, when the demand for item exist, and the item is not available in the stores. Then the situation leads to the problem that the organization cannot keep up the delivery promises at proper time. In this case, the organization has to lose the order. Then the following two situations may arises.

- **Backlogged or backorder situation**
- **Lostsale situation**

The concept of permissible delay is not new, even when currency was not in circulation, then also permissible delay was provided by suppliers to buyers. In general practice, suppliers are known to offer their customers a fixed period of time and do not charge any interest for this period. However, a higher interest is charged if the payment is not settled by the end of credit period. So the affect of permissible delay in payment on the optimal inventory system has received the attention from many researchers.

In the studies of inventory models, unlimited warehouse capacity is often assumed. However, in busy market places, such as super markets, corporation markets etc where the storage area for items may be limited. When an attractive price discount for bulk purchase is available or the cost of procuring goods is higher than the other inventory related cost or demand for items is very high or when the item under consideration is a seasonable product such as the yield of a harvest or there are some problems in frequent procurement, the procurement of large amount of items at a time is decided.
These items cannot be accommodated in the existing store house (the Own warehouse, denoted as OW). In this situation, in order to store the excess items, an additional warehouse (the rented warehouse, denoted as RW), which may be located at a short distance from OW, is hired on a rental basis. It is generally assumed that the holding cost in RW is higher than that in OW due to additional cost of maintenance. To reduce the inventory costs, it will be cost-effective to consume the goods of RW at first.

Thus in the present thesis, the emphasis is given on demand specially the Periodic Demand, Quadratic Demand considering Deterioration, Shortages, Two Warehouses with Permissible Delay in Payments in developing the models.

1.5 Overview of the problems in the Thesis

There are many factors, which have influenced many researchers on the inventory. The accumulation and depletion of inventory is a major factor in the cyclical behavior of any business activity. Reports of business conditions reveal that inventory fluctuation is a phenomenon which requires close monitoring. An inventory problem deals with the making decisions that minimizes the total cost of the inventory system or maximizes the profit gained while meeting the customers’ demands. The application of systematic quantitative methods to the solution of inventory problems began with the advent of scientific management. In the year 1915, Harris formulated and optimized a simple inventory model, which resulted in the well known classical economic lot size formula. Since then, the above mentioned model has been modified and extended by researchers all over the world with a view to make it more realistic by changing the basic assumptions suitably.
Interest in the study of inventory problems has increased since World War II and numerous publications have been devoted solely to this subject. Excellent reviews of the development of the subject can be found in the books of (Naddor, 1966, Peterson and Silver, 1985, etc.)

In real world, we are to solve problems for improving our environment to the optimum level of degree with a minimum budget to gain the maximum profit. We as a society are expected to build systems, evolve policies and promote thoughts that accept diversity in totality. This can, however be achieved only in an Economic Model that balances the associated costs with resultant benefits. A more realistic approach would be to consider a periodic-time demand which may represent both accelerated up and retarded down growth in demand.

Thus the present work emphasis mainly on the periodic-time dependent demand, with single warehouse and two warehouses, affect of discount and deterioration. So, in the present thesis, in chapter 1, the general introduction of OR and Inventory Management were discussed. In chapter 2, review the existing literature of the inventory models with different types of demand patterns on different parameters under various circumstances has been discussed in the rest of the chapters in the following models were developed, and in the last Bibliography is given. The thesis is ended with the publications.

Chapter 3

An Inventory Model With Lot Size Dependent Carrying / Holding Cost

In the classical Harris-Wilson (1915) inventory model all the cost associated with the formula was taken to be constant and which also does not depend on any quantity. There are many practical situations where this is not true. This
paper considers an inventory model where the carrying cost depends on lot-size. An algorithm is developed to determine the economic order quantity along with numerical examples.

The emphasis has been given on an inventory model where the carrying cost depends on lot-size, which increases or decreases in steps wise as the lot size increases or decreases. All the cost associated with the problem are assumed to be constant. Shortages are not allowed.

Chapter 4

A Note on Inventory Model

In the classical Harris-Wilson (1915) inventory model all the cost associated with the formula was taken to be constant and which also does not depend on any quantity. This paper considers an inventory model where the carrying cost depends on lot-size and increases in steps as the lot size increases and which is solved by using the similar procedure as in (Gupta, 1994). To illustrate the inventory model result a numerical example is solved.

The emphasis has been given on an inventory model where the carrying cost is linearly depended on lot-size. That is, if the lot size increases, then the carrying cost or holding cost also increases. The demand is assumed to be constant. Shortages are not allowed.

In this chapter and in the chapter 3, the carrying cost is depends on lot-size and increases in steps as the lot-size increases. The numerical result of this chapter is than compared with the numerical result in the chapter 3 “An
Inventory Model with Lotsize Dependent Carrying / Holding Cost”, which is solved by using the calculus method (Maxima & Minima Technique), and is found to be same.

Chapter 5

An EOQ Model for Deteriorating Items with Periodic Time Dependent Demand and Shortages

This paper develops an inventory model for a deteriorating item with a periodic time-dependent demand. It is also assumed that the finite production rate is proportional to the time-dependent demand rate and the deterioration rate is time-proportional. The unit production cost is inversely proportional to the demand rate. To make the model more physical and realistic, storages are assumed. Numerical examples are included and also the results are compared with linear time dependent. Sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

The emphasis has been given on an inventory model where the demand is dependent on periodic time dependent demand. Production rate is proportional to the time-dependent demand rate and the deterioration rate is time-proportional. The unit production cost is inversely proportional to the demand rate. Replenishment rate is finite. Shortages are allowed, and fully backlogged.
Chapter 6

An EOQ model for Two warehouse with Deteriorating Items, Periodic Time Dependent Demand and Shortages

A deterministic inventory model with periodic time dependent demand for deteriorating items with two warehouses is developed when the replenishment rate is finite, demand is at a uniform rate and shortages are allowed. It is assumed that the rates of deterioration of items in the two warehouses are different and time-proportional. To make the model more realistic backlogged are allowed. To illustrate the variation in the optimal inventory level and the optimal cost, a numerical example with sensitive analysis is done.

The emphasis has been given on an inventory model with two warehouses, where the demand is dependent on periodic time dependent demand, when the replenishment rate is finite. Production rate is proportional to the time-dependent demand rate. The unit production cost is inversely proportional to the demand rate. Storages are assumed, and fully backlogged. The deterioration rates of items in the two warehouses are different and time-proportional.

Chapter 7

An Optimal Inventory policy for items having Quadratic Time Dependent Demand and Variable Deterioration rate with Trade Credit

In most of the classical inventory models demand is considered as constant, but in practical situation the demand changes with time, quantity etc. In this model, we developed an order level inventory model with demand rate represented by a
continuous quadratic function of time and constant deterioration rate. The effect of permissible delay is also taken and incorporated in this model. The results are illustrated with numerical examples.

The emphasis has been given on an inventory model with permissible delay in payments, where the demand rate is represented by a continuous quadratic function of time and constant deterioration rate. Shortages are not allowed.

Chapter 8

An EOQ model with Linear Time Dependent Deterioration rate and Periodic Time Dependent Demand under permissible Delay in Payment

An EOQ model is developed with deteriorating item having periodic time dependent demand when delay in payment is permissible. The deterioration rate is assumed to be a linear function of time. Different mathematical models are derived with different situations, i.e. if the credit period is less than, equal to and greater than the cycle time for settling the account. The results are illustrated with numerical examples. Sensitivity analysis of the optimal solution with respect to the different parameters of the system is carried out.

The emphasis has been given on an inventory model with delay in payment where the demand is periodic time dependent demand. The deterioration rate is assumed to be a linear function of time. Different mathematical models are derived with different situations, i.e.

- Credit period is less than the cycle time for settling the account.
- Credit period is equal to the cycle time for settling the account.
- Credit period is greater than the cycle time for settling the account.
In this chapter, we have also formulated the model where the deterioration rate as constant with the other assumptions are same as above. Mathematical models are also derived on the basis of above three conditions. Comparative study of the above two cases i.e. with linear deterioration rate and constant deterioration rate are discussed with the help of the numerical example.

In the above chapters, the inventory models are illustrated by numerical examples and to investigate the effect of changes in the parameters on the optimal solutions, sensitivity analysis has been performed.