CHAPTER-6

STOCHASTIC ANALYSIS OF A COLD STANDBY SYSTEM WITH CONDITIONAL ARRIVAL TIME OF SERVER SUBJECTED TO FAILURE

6.1 Introduction

There is no doubt that the method of redundancy in cold standby is helpful in making the repairable system more practicable to use for a considerable period of time. As a result of which several studies have been conducted on stochastic modeling of cold standby systems with different repair policies. The previous chapters of the thesis are also witnessed to the study of these models under the concepts of preventive maintenance of the unit (chapters 2 to 4), server failure, treatment to the server, preference in repair disciplines (chapters 3 & 4) and arrival time of the server/repairman (chapter 5th). It is a fact that arrival time of the server is not a visible provision on the part of the owner when objective is to make the system more available to use. But, due to non availability of the server immediately to the system, there is no option other than to deploye the server with the condition that he has to attend the system immediately in emergency. Malik and Bhardwaj (2007) probed a 2-out-of-3 redundant system with conditional arrival time of server. Malik (2013) has studied a computer system with conditional arrival time of server.

Therefore, the main concentration in this chapter is on the analysis of a stochastic model for a two unit cold standby system of identical units with conditional arrival time of the server. There are two modes for each unit operative and complete failure. There is a single server who allowed to take some time to reach at the system with the condition that he has to attend the system for rectification of faults when system has no standby unit to work. The server is subjected to failure while performing jobs and goes for treatment. And, the failed unit waits for repair till the server becomes in good condition after receiving treatment. The unit works as new after repair. The random variables associated with the failure rate, treatment and arrival rates are statistically independent. The distributions for failure time of the unit and the server are taken as negative exponential while that of repair rate, treatment rate and arrival time of the server are arbitrary with different probability density functions. The SMP and RPT are adopted to derive the expressions for some measures of system effectiveness in steady state such as transition
probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server, expected number of repairs and treatments and profit functions. The numerical and graphical behavior of some important reliability characteristic has been observed for arbitrary values of the parameters and costs. The profit of the present model is compared with that of the model discussed in chapter 5th.

6.2 System Description

A stochastic model of a system consisting two identical units with conditional arrival and server failure during repair has been developed. The possible transition diagram of the system model is shown in Fig.: 6.1

**Fig.: 6.1 State Transition Diagram**

The system model (figure 6.1) has the following transition states:

- Regenerative States: \( S_0, S_1, S_2, S_3 \) and \( S_4 \),
- Non-regenerative: \( S_5, S_6, S_7 \) and \( S_8 \)
The following are the possible transition states of the system:

S₀ = (O, Cs), S₁ = (O, FWr), S₂ = (O, FUr),
S₃ = (FWr, FUr), S₄ = (O, FWr, SFUt), S₅ = (FWr, FWR, SFUt),
S₆ = (FUr, FWR), S₇ = (FWr, FUR), S₈ = (FWr, FWR, SFUt)

6.3 Transition Probabilities (TP)

The differential transition probabilities are given by

\[ dQ_{01}(t) = \lambda e^{-\lambda t} dt \]
\[ dQ_{12}(t) = w(t)e^{-\lambda t} dt \]
\[ dQ_{13}(t) = \lambda e^{-\lambda t}W(t)dt \]
\[ dQ_{20}(t) = e^{-(\mu+\lambda)t} g(t)dt \]
\[ dQ_{24}(t) = \mu e^{-(\mu+\lambda)t} G(t)dt \]
\[ dQ_{26}(t) = \lambda e^{-(\lambda+\mu)t} G(t)dt \]
\[ dQ_{32}(t) = e^{-\mu t} g(t)dt \]
\[ dQ_{37}(t) = \mu e^{-\mu t} G(t)dt \]
\[ dQ_{42}(t) = f(t)e^{-\lambda t} dt \]
\[ dQ_{45}(t) = \lambda e^{-\lambda t} F(t)dt \]
\[ dQ_{58}(t) = f(t)dt \]
\[ dQ_{62}(t) = e^{-\mu t} g(t)dt \]
\[ dQ_{67}(t) = \mu e^{-\mu t} G(t)dt \]
\[ dQ_{78}(t) = f(t)dt \]
\[ dQ_{82}(t) = e^{-\mu t} g(t)dt \]
\[ dQ_{87}(t) = \mu e^{-\mu t} G(t)dt \]

We have the following expressions for transition probabilities:

\[ p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \]

\[ p_{01} = 1 \quad p_{12} = w^*(\lambda) \quad p_{13} = 1 - w^*(\lambda) \]
\[ p_{20} = g^*(\mu + \lambda) \quad p_{24} = \frac{\mu}{\mu+\lambda}\{1 - g^*(\mu + \lambda)\} \quad p_{26} = \frac{\lambda}{\mu+\lambda}\{1 - g^*(\mu + \lambda)\} \]
\[ p_{32} = g^*(\mu) \quad p_{37} = 1 - g^*(\mu) \quad p_{42} = f^*(\lambda) \]
\[ p_{45} = 1 - f^*(\lambda) \quad p_{58} = f^*(0) \quad p_{62} = g^*(\mu) \]
\[ p_{67} = 1 - g^*(\mu) \quad p_{78} = f^*(0) \quad p_{82} = g^*(\mu) \]
\[ p_{87} = 1 - g^*(\mu) \]

For \( w(t) = \theta e^{-\theta t}, g(t) = \beta e^{-\beta t} \) and \( f(t) = \alpha e^{-\alpha t} \)

For a perfect distribution

\[ p_{01} = p_{12} + p_{13} = p_{20} + p_{24} + p_{26} = p_{20} + p_{24} + p_{22.6} + p_{22.6(7,8)^n} = p_{32} + p_{37} = p_{32} + p_{32(7,8)^n} = p_{42} + p_{45} = p_{58} = p_{62} + p_{67} = p_{78} = p_{82} + p_{87} = 1 \]

(6.2)
6.4 Mean Sojourn Times (MST)

\[
\begin{align*}
\mu_0 &= 1 \\
\mu_1 &= \frac{1}{\theta + \lambda} \\
\mu_2 &= \frac{1}{\beta + \lambda + \mu} \\
\mu_3 &= \frac{1}{\beta + \mu} \\
\mu_4 &= \frac{1}{\alpha + \lambda} \\
\mu_5 &= 1 \\
\mu_6 &= \frac{1}{\beta + \mu} \\
\mu_7 &= 1 \\
\mu_8 &= \frac{1}{\beta + \mu}
\end{align*}
\]

Also

\[
\begin{align*}
\mu_0 &= m_{01} \\
\mu_1 &= m_{12} + m_{13} \\
\mu_2 &= m_{20} + m_{24} + m_{26} \\
\mu_3 &= m_{32} + m_{37} \\
\mu_4 &= m_{42} + m_{45} \\
\mu_2' &= m_{20} + m_{24} + m_{22.6} + m_{22.6(7,8)^n} \\
\mu_3' &= m_{32} + m_{32.(7,8)^n} \\
\mu_4' &= m_{42} + m_{42.58} + m_{42.5(8,7)^n}
\end{align*}
\]

(6.3)

6.5 Reliability Measures

The following reliability measures have been evaluated for the system model

6.5.1 Reliability and Mean Time To System Failure (MTSF)

The expressions for \( \pi_i(t) \) are as follows:

\[
\begin{align*}
\pi_0(t) &= Q_{01}(t) & & \pi_1(t) \\
\pi_1(t) &= Q_{12}(t) & & \pi_2(t) + Q_{13}(t) \\
\pi_2(t) &= Q_{20}(t) & & \pi_0(t) + Q_{24}(t) + \pi_4(t) + Q_{26}(t) \\
\pi_4(t) &= Q_{42}(t) & & \pi_2(t) + Q_{45}(t)
\end{align*}
\]

(6.4)

Let us take LST of above relations (3.4) and solving for \( \pi_0^{**}(s) \)

We have

\[
R^*(s) = \frac{1 - \pi_0^{**}(s)}{s}
\]

(6.5)

And

\[
MTSF = \lim_{s \to 0} \frac{1 - \pi_0^{**}(s)}{s} = \frac{N_1}{D_1}
\]

(6.6)

Where,

\[
N_1 = (1 - p_{24}p_{42})(\mu_0 + \mu_1) + p_{12}(\mu_2 + p_{24}\mu_4) \quad \text{and} \quad D_1 = 1 - p_{24}p_{42} - p_{20}p_{12}p_{10}
\]

(6.7)
6.5.2 Long Run (Steady State) Availability

The expressions for $A_i(t)$ are given as under:

$A_0(t) = M_0(t) + q_{01}(t)\circ A_1(t)$

$A_1(t) = M_1(t) + q_{12}(t)\circ A_2(t) + q_{13}(t)\circ A_3(t)$

$A_2(t) = M_2(t) + q_{20}(t)\circ A_0(t) + \left(q_{22.6}(t) + q_{22.6(7,8)n}(t)\right)\circ A_2(t) + q_{24}(t)\circ A_4(t)$

$A_3(t) = \left(q_{32}(t) + q_{32(7,8)n}(t)\right)\circ A_2(t)$

$A_4(t) = M_4(t) + \left(q_{42}(t) + q_{42.58}(t) + q_{42.5(8,7)n}(t)\right)\circ A_2(t)$

(6.8)

Where,

$M_0(t) = e^{-\lambda t}$

$M_1(t) = e^{-\lambda t}W(t)$

$M_2(t) = e^{-(\mu+\lambda)t}G(t)$

$M_4(t) = e^{-\mu t}F(t)$

Let us take LT of relations (6.8) and solving for $A_0^*(s)$.

The steady state availability is given by

$A_0(\infty) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2}$

(6.9)

Where, $N_2 = p_{20}(\mu_0 + \mu_1 + \mu_3p_{13}) + \mu_2 + \mu_4p_{24}$

And $D_2 = p_{20}(\mu_0 + \mu_1 + p_{13}\mu_3') + \mu_2' + p_{24}\mu_4'$

(6.10)

6.5.3 Busy Period of the Server Due to Repair in Long Run

The expressions for $B_i^R(t)$ are given as under:

$B_0^R(t) = q_{01}(t)\circ B_1^R(t)$

$B_1^R(t) = q_{12}(t)\circ B_2^R(t) + q_{13}(t)\circ B_3^R(t)$

$B_2^R(t) = W_2^R(t) + q_{20}(t)\circ B_0^R(t) + \left(q_{22.6}(t) + q_{22.6(7,8)n}(t)\right)\circ B_2^R(t) + q_{24}(t)\circ B_4^R(t)$

$B_3^R(t) = W_3^R(t) + (q_{32}(t) + q_{32(7,8)n}(t))\circ B_2^R(t)$

$B_4^R(t) = \left(q_{42}(t) + q_{42.58}(t) + q_{42.5(8,7)n}(t)\right)\circ B_2^R(t)$

(6.11)

Where, $W_2^R(t) = e^{-(\mu+\lambda)t}G(t) + (\lambda e^{-(\mu+\lambda)t} \circ 1)G(t)$

$W_3^R(t) = e^{-\mu t}G(t) + (g(t)e^{-\mu t} \circ 1)$

Let us take LT of relations (6.11) and solving for $B_0^{R*}(s)$.

We get, $B_0^R(\infty) = \lim_{s \to 0} s B_0^{R*}(s) = \frac{N_3}{D_2}$

(6.12)

$N_3 = p_{01}(W_2^*(0) + W_3^*(0)p_{13}p_{20})$ and $D_2$ is already specified.

(6.13)
6.5.4 Expected Number of Repairs (ENR) of the unit in the Long Run

The expressions for $R_i(t)$ are given as under:

\[
R_0(t) = Q_{01}(t)S R_1(t) \\
R_1(t) = Q_{12}(t)S R_2(t) + Q_{13}(t)S R_3(t) \\
R_2(t) = Q_{20}(t)S(1 + R_0(t)) + \left(Q_{22.6}(t) + Q_{22.6(7,8)}^n(t)\right)S(1 + R_2(t)) + Q_{24}(t)S R_4(t) \\
R_3(t) = Q_{32}(t) + \left(Q_{32(7,8)}^n(t)\right)S(1 + R_2(t)) \\
R_4(t) = Q_{42}(t)S R_2(t) + \left(Q_{42.58}(t) + Q_{42.5(8,7)}^n(t)\right)S(1 + R_2(t))
\]  \hspace{1cm} (6.14)

Let us take LST of relations (6.12) and solving for $R_0^{**}(s)$.

We get $R_0 = \lim_{s \to 0} sR_0^{**}(s) = \frac{N_4}{D_2}$  \hspace{1cm} (6.15)

Where, $N_4 = 1 + p_{14}p_{20} - p_{23}p_{32}$ and $D_2$ is already specified.  \hspace{1cm} (6.16)

6.5.5 Expected Number of Treatments (ENT) Given to the Server in the Long Run

The expressions for $T_i(t)$ are given as under:

\[
T_0(t) = Q_{01}(t)S T_1(t) \\
T_1(t) = Q_{12}(t)S T_2(t) + Q_{13}(t)S T_3(t) \\
T_2(t) = Q_{20}(t)S T_0(t) + Q_{22.6}(t)S T_2(t) + Q_{22.6(7,8)}^n(t)S(1 + T_2(t)) + Q_{24}(t)S T_4(t) \\
T_3(t) = Q_{32}(t)S T_2(t) + Q_{32(7,8)}^n(t)S(1 + T_2(t)) \\
T_4(t) = \left(Q_{42}(t) + Q_{42.58}(t) + Q_{42.5(8,7)}^n(t)\right)S(1 + T_2(t))
\]  \hspace{1cm} (6.17)

Let us take LST of above relations (6.17) and solving for $T_0^{**}(s)$.

We get

\[
T_0(\infty) = \lim_{s \to 0} sT_0^{**}(s) = \frac{N_5}{D_2}
\]  \hspace{1cm} (6.18)

Where, $N_5 = p_{01}((1 - p_{20} - p_{22.6}) + p_{13}p_{20}p_{37})$

And $D_2$ is already mentioned.  \hspace{1cm} (6.19)

6.6 Profit Analysis

The profit of the system model can be obtained as:

\[
P_k = K_0A_0 - K_1B_0^R - K_2R_0 - K_3T_0 ; K=6
\]  \hspace{1cm} (6.20)

Where, the variables $P_k$, $K_0$ to $K_3$ are defined under the common notations.

6.7 Particular Cases

Suppose \( w(t) = \theta e^{-\theta t} \), \( g(t) = \beta e^{-\beta t} \) and \( f(t) = \alpha e^{-\alpha t} \)

We can obtain the following results:
\[ \text{MTSF} = \frac{N_1}{D_1}, \quad \text{Availability}(A_0) = \frac{N_2}{D_2} \]
\[ B_0^R = \frac{N_3}{D_2}, \quad R_0 = \frac{N_4}{D_2}, \quad T_0 = \frac{N_5}{D_2} \]

Where,
\[ N_1 = \frac{(\theta + \lambda + 1)((\alpha + \lambda)(\beta + \lambda + \mu) - \mu \alpha) + \theta(\alpha + \lambda + \mu)}{(\theta + \lambda)(\alpha + \lambda)(\beta + \mu + \lambda)} \]
\[ D_1 = \frac{(\theta + \lambda)((\alpha + \lambda)(\beta + \mu + \lambda) - \mu \alpha) - \beta \theta(\alpha + \lambda)}{(\alpha + \lambda)(\beta + \mu + \lambda)(\theta + \lambda)} \]
\[ N_2 = \frac{\beta(\alpha + \lambda)((\beta + \mu)(\theta + \lambda + 1) + \lambda) + (\alpha + \lambda)(\theta + \lambda)(\beta + \mu) + \mu(\theta + \lambda)(\beta + \mu)}{(\alpha + \lambda)(\theta + \lambda)(\beta + \mu + \lambda)(\beta + \mu)} \]
\[ \beta(\alpha + \lambda)\left\{ \beta(\theta + \lambda + 1) + \lambda (1 + \mu) \right\} + (\alpha + \lambda)(\theta + \lambda) \{ \beta + \lambda (1 + \mu) \} + \]
\[ \frac{(\alpha + \lambda)\mu\{\beta + \lambda(\beta + \mu + 1)\}}{\beta(\alpha + \lambda)(\theta + \lambda)(\beta + \mu + \lambda)} \]
\[ D_2 = \frac{(\lambda + \mu)(\beta + \mu)(\theta + \lambda) + \lambda \beta^2}{\beta(\theta + \lambda)(\beta + \mu + \lambda)} \]
\[ N_3 = \frac{(\alpha + \lambda)(\beta + \mu + \lambda) - \mu \alpha + \lambda \beta(\alpha + \lambda)}{(\alpha + \lambda)(\theta + \lambda)(\beta + \mu + \lambda)} \]
\[ N_4 = \frac{(\theta + \lambda)((\beta + \mu)(\beta + \mu + \lambda) - \lambda \beta) - \beta\left\{ (\theta + \lambda)(\beta + \mu) - \mu \lambda \right\}}{(\theta + \lambda)(\beta + \mu)(\beta + \mu + \lambda)} \]

### 6.8 Numerical and Graphical Representation of Reliability Measures

#### Table 6.1: MTSF Vs Failure Rate

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \mu=0.2, \alpha=2.1, \beta=3.1, \theta=1.1 )</th>
<th>( \mu=0.4 )</th>
<th>( \alpha=3.1 )</th>
<th>( \beta=4.1 )</th>
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Fig. 6.2: MTSF Vs Failure Rate

![MTSF Vs Failure Rate Graph]

Table 6.2: Availability Vs Failure Rate

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<th>$\lambda$</th>
<th>$\mu=0.2, \alpha=2.1, \beta=3.1, \theta=1.1$</th>
<th>$\mu=0.4$</th>
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Fig. 6.3: Availability Vs Failure Rate

![Availability Vs Failure Rate Graph]
Table 6.3: Profit Vs Failure Rate

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<th>$\lambda$</th>
<th>$\mu=0.2, \alpha=2.1, \beta=3.1, \theta=1.1$</th>
<th>$\mu=0.4$</th>
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</table>

Fig.6.4: Profit Vs Failure Rate

Table 6.4: Profit Difference (P5-P6)

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mu=0.2, \alpha=2.1, \beta=3.1, \theta=1.1$</th>
<th>$\mu=0.4$</th>
<th>$\alpha=3.1$</th>
<th>$\beta=4.1$</th>
<th>$\theta=3.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>485.6883</td>
<td>572.529</td>
<td>482.504664</td>
<td>471.086894</td>
<td>661.147222</td>
</tr>
<tr>
<td>0.2</td>
<td>415.106</td>
<td>544.0272</td>
<td>410.46764</td>
<td>404.923559</td>
<td>627.590232</td>
</tr>
<tr>
<td>0.3</td>
<td>300.6142</td>
<td>464.1432</td>
<td>296.869839</td>
<td>306.726192</td>
<td>562.271441</td>
</tr>
<tr>
<td>0.4</td>
<td>159.1852</td>
<td>351.6174</td>
<td>157.771267</td>
<td>187.443279</td>
<td>472.808893</td>
</tr>
<tr>
<td>0.5</td>
<td>1.510359</td>
<td>218.4378</td>
<td>3.28469543</td>
<td>54.1404697</td>
<td>365.19758</td>
</tr>
<tr>
<td>0.6</td>
<td>-165.397</td>
<td>72.58466</td>
<td>-159.95602</td>
<td>-88.3975169</td>
<td>244.177893</td>
</tr>
<tr>
<td>0.7</td>
<td>-336.804</td>
<td>-80.4773</td>
<td>-327.46946</td>
<td>-236.810722</td>
<td>113.511849</td>
</tr>
<tr>
<td>0.8</td>
<td>-509.451</td>
<td>-236.932</td>
<td>-496.16635</td>
<td>-388.674029</td>
<td>-23.8074354</td>
</tr>
<tr>
<td>0.9</td>
<td>-681.068</td>
<td>-394.082</td>
<td>-663.89029</td>
<td>-542.200691</td>
<td>-165.395271</td>
</tr>
</tbody>
</table>
6.6 Conclusion

a) Let us take \( g(t) = \theta e^{-\theta t} \), \( w(t) = \beta e^{-\beta t} \) and \( f(t) = \alpha e^{-\alpha t} \). The values of some important reliability measures including mean time to system failure (MTSF), availability and profit function are obtained for arbitrary values of the parameters. The behavior of these measures with respect to failure rate is shown respectively in figures 6.2, 6.3 and 6.4. It observed that MTSF, availability and profit keep on decreasing with the increase of failure rates of the unit and server. However, they follow an upward trend with the increase of repair rate (\( \theta \)), treatment rate (\( \alpha \)) and arrival rate (\( \beta \)) of the server. Further, it can also be seen that MTSF declines rapidly with a slight positive change in failure rate. Hence, the study reveals that a cold stand by system of two identical units can be made more available and profitable to use either by calling the server immediately to rectify the faults or by increasing the repair rate of the failed unit in case server takes some time to arrive the system.

b) Comparative Study of the Profit of the System Models

The trend of profit difference (P5-P6) of the system models has been shown numerically and graphically respectively in the table 6.4 and figure 6.5. The results show that profit difference goes on decline when we increase of failure rate of the unit, treatment rate of the server, repair rate of the unit while it increases with the increase of arrival rate of the server. It is revealed that the idea of calling the server immediately when system has no unit to work, is useful in making the system more profitable to use, in case failure of the unit is high.