Chapter 2

Review of literature

2.1 A Review on Inventory Models of Deteriorating Items with Shortages

The application of systematic quantitative methods to the solution of inventory problem begins with the advent of scientific management movement. In the year 1915 Harris, formulated and optimized a simple inventory model which resulted in the well known economic lot size formula. The same formula was develop by R.H. Wilson after few years and so the above formula was renamed as Harris- Wilson's formula or Wilson's formula given by

$$q^* = \left[\frac{2RC_3}{C_1}\right]^{\frac{1}{2}}$$

where q* is the optimal lot size. R is the rate of demand, C_1 is the holding cost per unit per unit time and C_3 is the setup cost per setup.

This model has been modified and extended by many researchers. In 1926, Benjamin Cooper attempted to develop an inventory model assuming the rate of production to be finite. Later in the year 1931 (F.E Raymond,1931) extended the model of Harris and Wilson with a view to generalize it incorporating a large variety of inventory solution.

After World War II, with the gradual development of management science and operation research attempts were made to consider stochastic version of inventory problem. Prior to the most of the investigations in this area were deterministic in nature, except for a few isolated studies such as (Fry, 1928) in 1928 where some attempts were made to include probabilistic consideration .During the war, such type of model was developed known as **Christmas tree model**.

The stochastic version of the classical lot size model was studied by (Whitin, 1957). One of the most developed fields of Operations Research is inventory modeling. Inventory has been defined as idle resources that posses economic value (Monks, 1987). Usually, it is an important component of the investment portfolio of any production system. Keeping an inventory for future sales or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. Usually the demand rate is decided by the amount of the stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons motivate the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as shortages will fetch loss to the producers. The shortage or stock out cost is the penalty incurred for being unable to meet the demand when occurs.

It has two wings internal and external shortage. Internal shortage occurs when an order of a group or department within the organization is not filled. External shortages can incur backorder cost, present profit loss and future profit loss. Internal shortages can result in lost production and delay in a completion date. On the other hand, deterioration is an important natural phenomenon and the consequent loss due to decay of items may be quite significant. Mainly when, physical goods are stocked for future use, in some items such as medicines, foodstuff, dairy items, volatile liquids, the process of deterioration is observed. Hence effect of deterioration is very important in many inventory systems. (Nahmis, 2011) has discussed about the perishable inventory in details.

Research in this direction began with the work of (Whitin**,** 1957) who considered fashion goods deteriorating at the end of a prescribed storage period. (Ghar and Schrader**,** 1963) developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by (Shah and Jaiswal, 1977). In all these models, the demand rate and the deterioration rate were constant and shortages were not allowed.

An interesting subset of inventory theory is the mathematical modeling of deteriorating items with shortage. The literature related to deteriorating items with shortages is scattered and no comprehensive up to date discussion of these models is available. This paper presents a complete survey of the published literature in mathematical modeling of deteriorating items with shortages and suggests future research directions needed in this field.

Deterioration is defined as decay, spoilage, loss of utility of the product as defined by (Shah and Shukla, 2009, Resh et. al, 1976 and Donaldson, 1977) are the first researchers who considered an inventory model with a linear trend in demand. Thereafter, numerous researches have been carried out incorporating time demand patterns into inventory models. The time dependent demand patterns, used in existing models are, mainly,

- linearly time dependent
- exponentially time dependent

The time dependent demand patterns reported above are linear, that is, the demand increases continuously with time or decrease continuously along with the time. (Dave and Patel**,** 1981) considered time proportional demand. (Goyal, 1986) considered linear trend in demand. (Hariga and Benkherouf, 1994) considered exponential time varying demand for deteriorating items.

The works done by (Roy**,** 2008, Sabahno**,** 2009, Mirzazadeh, 2010, Gayen and Pal**,** 2009) are some of the models for deteriorating items based on different realistic situations.

Deterioration is defined as decay, spoilage, loss of utility of the product as defined by (Shah and Shukla, 2009). Product such as vegetables, fish, medicine, blood, radio-active chemicals have finite self life and start to deteriorate once they are produced.

Obsolescence refers to inventories that become obsolete at a certain time, for instance because of rapid changes in technology, or the introduction of a new product by a competitor. If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. (Moon et. al, 2005) considered ameliorating/deteriorating items on an inventory model with time varying demand pattern. Against obsolescing and ameliorating items, many researchers has found in the deteriorating inventory area in recent years.

In the literature survey by (Raafat, 1991, Shah and Shah, 2000, Goyal and Giri, 2001), the details can be found**.** (Dave, 1985) modified the model of (Dave and Pandya,1985) with special sales. Later (Dutta and Pal, 1991) designed the similar problem with shortages where the deterioration rate of the items is constant and deterioration is assumed to start only after a fixed period of time from the instant of their arrival in stock.

Some consumer goods for which on stock-dependent demand pattern can be noticed are subject to deterioration. (Datta and Pal, 1990) investigated a model assuming the demand rate to be linear function of the on-hand inventory incorporating deterioration effect and allowing shortages which are completely backlogged for both infinite and finite time-horizon.

(Deb and Choudhuri, 1987) were the first to develop the inventory problem with linearly increasing demand to allow shortages. Mainly there are two types of shortages, viz

- inventory followed by shortages (IFS)
- shortages followed by inventory (SFI)

In IFS policy, it is assumed that each of the (n-1) cycles start with replenishment; the inventory is held for a certain period and then shortages are allowed to occur. Shortages are not permitted in the last replenishment. This IFS policy was then studied by (Bahari - kasani,1989, Goswami and Choudhuri,1991,Chung and Ting, 1993, Hariga,1995, Jalan et.

al,1996, Giri and Chaudhuri,1997, Chakrabarti and Chaudhuri,1998) and others. They have been devoted to incorporating a time varying demand into their models for deteriorating items under a variety of circumstances. SFI policy has emerged in recent years. In the SFI policy, each cycle starts with a shortage and ends with zero stock. The SFI policy was first discussed by (Goyal et. al, 1992) who suggested a new replenishment policy in which shortages are allowed in every cycle. Each cycle starts with a shortage that accumulates until replenishment is made to clear the backlog. They also prove that the system cost in the SFI policy would be less than that in IFS policy. This policy was adopted in the model of (Chakrabarti and Chaudhuri,1997) for an inventory of a perishable commodity with linear trend in demand. Recently (Jalan and Chaudhuri,1999) proposed an EOQ model for deteriorating items with exponentially declining demand under SFI policy. (Wu et. al, 1999) have been developed a deterministic inventory model for deteriorating items with time varying demand and shortage under SFI policy.

(Samanta and Roy, 2004)**,** have developed a continuous production control inventory model for deteriorating items with shortages. It is assumed that the demand rate and production rate are constants and the distribution of the time to deterioration of an item follows the exponential distribution.

The literature available till 1975 in the field of inventory management does not take into account the effect of inflation and time value of money. This has happened because of the belief that inflation would not influence the policy variables to any significant degree. But during the last forty-five years the monetary situation in most of the countries, affluent or otherwise have changed to such an extent due to large scale inflation, consequently sharp decline in purchasing power of money that has not been possible to ignore the effect of inflation. The major attempt in this direction was by (Buzacot,1975) in the year 1975. In his paper he dealt with economic order quantity model with inflation subject to different types of pricing policies. After the above paper several other researchers has extended their approach to various interesting situations taking into consideration like the value of money, different inflation rates for the internal and external costs, finite and infinite replenishment rate with or without shortage. The concept of inflation and time value of money was employed by (Wee and Law,1999) into a model where the demand is price

dependent and shortage is allowed. A production environment with a finite replenishment rate was considered in this model. A production inventory system for deteriorating items with time varying demands and completely backlogged shortages was modeled by (Hariga,1995), describing the impact of inflation and time value of money on an inventory model characterized by time-dependent demand rate and fixed shortage cost.

The production schedules for this system were constructed using traditional scheduling strategy in which each cycle starts with replenishment end with shortages. The stock dependent demand rate models are prepared with some researchers. (Hou and Lin, 2004) developed an inventory model under inflation and time discounting for deteriorating items with stock dependent selling rate. The selling rate is assumed to be a function of the current inventory level and the rate of deterioration is assumed to be constant. A deterministic economic order quantity (EOQ) inventory model taking into account inflation and time value of money developed for deteriorating items with price and stock dependent selling rates by (Hou and Lin, 2006). (Dutta and Pal, 1991) investigated the effect of inflation and time-value of money on an inventory model with linear time dependent rate and shortages are allowed in their model.

(Wee et. al, 2004) developed an inventory model for deteriorating items with shortage occurring at the supplier involving a supply chain between the producer and buyer. In inventory management, deteriorating items with shortages, lot of discussion is made by different researchers. There is a long way to go in this direction. (Dutta Choudhuri and Datta, 2010) developed a model with stock dependent demand rate and dual storage facility. From the above discussion it is clear that in real life situations deterioration may occur and hence shortages. (Paul et. al,1996) developed a model with two component demand rate allowing shortages. Thus the demand may be stock dependent up to certain time after that it is constant due to some good will of the retailer. This model can be considered with deteriorating items.

2.2 Review of inventory models with time varying holding cost

This broad category includes the costs for storage facilities, handling, insurance, breakage, obsolescence, depreciation, taxes, and the opportunity cost of capital. Obviously, high holding costs tend to favour low inventory levels and frequent replenishment.

In most models, holding cost is known and constant. But holding cost may not always be constant. In generalization of EOQ models, various functions describing holding cost were considered by several researchers like (Naddor,1966,Van Der Veen,1967, Muhlemann and Valtis-Spanopoulos,1980, Weiss,1982 and Goh,1994). (Giri and Chaudhuri,1998) treated the holding cost as a non-linear function of the length of the time for which the item is held in stock and as a functional form of the amount of the on-hand inventory. (Roy, 2008) developed an EOQ model for deteriorating items where deterioration rate and holding cost are expressed as linearly increasing functions of time and demand rate is a function of selling price and shortages are allowed and completely backlogged.

2.3 Review of inventory models with ramp-type demand

Inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role for the best inventory policy.

Researchers were engaged to develop the inventory models assuming the demand of the items to be constant, linearly increasing or decreasing demand or exponential increasing or decreasing with time, stock-dependent etc. Later, this has been realized

that the above demand patterns do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles etc; for which the demand increases with time as they are launched into the market and after some time it become constant. In order to consider the demand of such types, the concept of ramp-type demand is introduced. Ramp-type demand depicts a demand, which increases up to a certain time after which it stabilizes and become constant.

Lifetime for some products is assumed to be a random variable with its probability distribution represented by gamma, weibull, exponential or any other distribution pattern. Though studies incorporating finite lifetime of items in inventory started over four decades ago, the number of literatures on EOQ models for deteriorating items is few when compared with the number of literatures on economic order quantity EOQ models for the same class of items (Goyal and Giri 2001). The search for a pattern that will be representative of demand behaviour in real world inventory situations has made researchers to focus on several non-uniform demand patterns. Of these patterns, the time-dependent demand pattern has received considerable attention in recent time, especially in dealing with inventory of deteriorating items. The incorporation of time-dependent demand patterns in inventory models for deteriorating items started with the consideration of linear trend by (Dave and Patel,1981, Sachan,1984, Bahari-kashani, 1989) etc. (Wee ,1995) studied the exponential trend in demand patterns. (Hariga, 1995) developed inventory lot-sizing models for deteriorating items having general continuous log-concave demand functions. In later developments, the time-dependent demand function assumed the form of a general continuous function which can be represented by any continuously increasing or decreasing function of time (e.g. Lee and Hsu, 2009). The above mentioned demand patterns are unidirectional. They are either continuously increasing or continuously decreasing.

For some class of deteriorating items, demand initially increases with time up to a point. It then becomes steady and finally decreases with time and becomes asymptotic. This type of demand behavior can be observed in some seasonal items like fruits, fish, winter cosmetics, fashion apparel, etc. The increasing-steady-

decreasing demand pattern can be represented by a three-phase ramp-type demand pattern that allows three-phase variation in demand. These phases will represent the growth, the steady and the decline phases commonly experienced by the demand of many products during their life cycle in the market. (Panda et. al, 2008) used this type of pattern to generate optimal replenishment policies for perishable seasonal products over a finite time horizon. Another form of this pattern, called trapezoidal demand pattern, was used by (Cheng and Wang,2009) in developing an economic order quantity model for deteriorating items.

(Hill,1995) proposed a time dependent demand pattern by considering it as the combination of linearly time dependent and exponentially time dependent of demand in two successive time periods over the entire time horizon and termed as "Ramp-type" time dependent demand pattern. Then, inventory models with ramp type demand rate also studied by (Mandal and Pal, 1998, Wu et. al,1999, Wu and Ouyang,2000, Wu,2001, Ahmed et.al, 2013), are worth mentioning. In these papers, the determination of the optimal replenishment policy requires the determination of the time point, when the inventory level falls to zero. So the following two cases should be examined:

- this time point occurs before the point, where the demand is stabilized,
- this time point occurs after the point, where the demand is stabilized.

Almost all of the researchers examine only the first case. (Deng et. al,2007) reconsidered the inventory model of (Mandal and Pal,1998) and the models of (Wu and Ouyang,2002) and studied it exploring these two cases. (Skouri et. al,2009) extended the work of (Deng et. al,2007) by introducing a general ramp type demand and considering Weibull distribution deterioration rate. The works done by (Roy,2008, Sabahno,2009, Mirzazadeh,2010, Gayen and Pal,2009) are some of the models for deteriorating items based on different realistic situations.

Demand rate D (t) is a general time-dependent ramp-type function which has the following form:

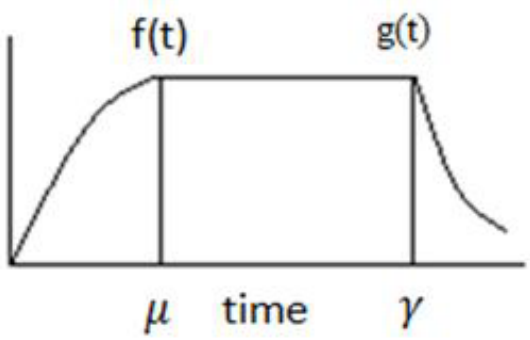

Figure 2.1: Ramp-type Demand pattern

The demand rate $D(t)$ is a ramp-type function of time and is as follows:

$$D(t) = \begin{cases} f(t), & 0 < t < \mu, \\ f(\mu) = g(\gamma), & \mu \leq t \leq \gamma, \\ g(t), & \gamma < t, \end{cases}$$

f(t) is a positive, continuous, and increasing function of t, and g(t) is a positive, continuous decreasing function of t, μ and γ represent the parameter of the ramp-type demand function.

(Manna and Chaudhuri,2003) noted that ramp type demand pattern is generally followed by new brand of consumer goods coming in the market. But for fashionable products as well as for seasonable products, the steady demand will never be continued indefinitely. Rather it would be followed by decrement with respect to time. After a period of time and becomes asymptotic in nature. Thus the demand may be illustrated by three successive time periods that classified time dependent ramp type function, viz,

- first phase the demand increase with time
- after that it becomes steady
- towards the end in the final phase it decreases and becomes asymptotic

In most of the above mentioned papers, the demand during stock out period is totally backlogged. But in real life situations, there are customers who are willing to wait and receive their order at the end of stock-out period due to good will of the retailer or for some reasons while others are not. In the last few years considerable attention has been paid to inventory models with partial backlogging. The first work in which customer's impatience functions are proposed seems to be that by (Abad,1996,2000). (Abad,1996) derived a pricing and ordering policy for a variable rate of deterioration and partially backlogging. The partially backlogging was assumed to be exponential function of waiting time till the next replenishment. (Dye et. al,2007) modified this model taking into consideration the backorder cost and lost sale. (Shah and Shukla,2009) also developed a deterministic inventory model in which items are subject to constant deterioration and shortages are allowed. The unsatisfied demand is backlogged which is a function of time. It is assumed that the backlogged units are proportional to waiting time. Thus in this paper, an optimal replenishment schedule is derived under the assumption of waiting time, backordering when units in an inventory are subject to constant deterioration. Research on models with partial backlogging for deteriorating items continues with (Wang, 2002, Uthya and Parvathi,2006).

2.4 Review of inventory models with dual storage system

In the busy markets like super market, municipality market etc. the storage area of items is limited. When an attractive price discount for bulk purchase is available or the cost of procuring goods is higher than the other inventory related cost or demand of items is very high or there are some problems in frequent procurement, management decide to purchase a large amount of items at a time. These items cannot be accommodated in the existing store house (viz. the Own Warehouse, OW) located

at busy market place. In this situation, for storing the excess items, one additional warehouse (viz. Rented Warehouse, RW) is hired on rental basis, which may be located little away from it.

The capacity of own warehouse (OW) is fixed according to the cost and service level of the company when it is built. When the capacity of our OW cannot store the excess units, they should be stored in a rented warehouse (RW). (Sarma,1997) examined the RW in the form of a central warehousing facility. The RW offers better preserving facility and higher cost than the OW resulting in a lower rate of deterioration for goods. (Sarma,1997) obtained an optimal replenishment scheduled when the demand rate is constant and OW has known capacity.

(Singh et. al, 2009) considered an inventory model for deteriorating item having two warehouses, one is OW of finite dimension, other RW of infinite dimension, under inflation and time value of money. Deterioration rates of items in the two warehouses may be different, which is time dependent, and shortages are allowed. In this model, due to different facilities and storage environment, inventory holding cost is considered different in different warehouses. The demand rate of items is linear with time. (Chiao et. al, 2008) considered a deteriorating inventory model with two storage facilitates, partial backordering and quantity discount.

(Jaggi and Verma, 2010) proposed a model where rented warehouse is used to store the excess units over the capacity of the own warehouse. The stock is being transferred from RW to OW in a continuous release pattern with per unit transportation cost being factored in. The solution obtained in the model helps to decide on the feasibility of renting a warehouse.

This is in short a review of the existing literature of the subject. Researchers are engaged in designing newer inventory models taking into consideration the different aspect of real life situations for the rapid growth of economy.