Chapter 1

General Introduction

1.1 Introduction

The control and maintenance of inventory is a common problem in all enterprises in any sector of economy. For example, inventories must be maintained in Agriculture, Industry, Military, Retail establishment etc. There are several reasons why organizations maintain inventories. The main reason is that it is physically or economically impossible to have goods arrive in a given system precisely at the time when demands for them occur. In absence of inventories customers would have to wait until their orders were filled from a source. However customer cannot be allowed to wait for long period of time. There are several other reasons for holding inventories as well. For example, the price of some raw materials used by a manufacturer may exhibit considerable seasonal fluctuations. When the price is low, it is profitable to procure a sufficient quantity of it to last through the high priced season and to keep it in inventories to be used when need arises.

The two basic questions that must be answered in controlling the inventory of any physical goods, are when to replenish the inventory and how much to order for replenishments. The problem of inventory management of any organization or firm which stocks different items for sale to the customers is related to the above two questions. As sales occur, the stock is depleted over time. The firm has its own mechanism of placing orders and obtaining fresh replenishments. The manner in which this reorder option is exercised is termed as the firm’s inventory policy. The problem of making optimal decisions with regard to the above questions is called an inventory problem. In a different language, an inventory problem deals with making of decisions that minimizes the total cost of the inventory system or maximizes the profit gained while meeting customers’ demands. Hence, the solution to an inventory problem is a set of specific values of the variables that minimizes the total cost of the system or maximizes the total profit of the system.

In every sector of economy, the control of the material flow from suppliers of raw materials to final customers is an important phenomenon. To materialize this situation of economic sector, different fruitful strategies are given topmost priority by the management.

The total investment in inventories and the control of capital tied up in a raw material, work-in-progress, and finished goods are dealt with newer scientific approach day by day. Advances in technological world have drastically changed the possibilities to apply more efficient inventory control techniques. Modern inventory control is based on advanced and complex decision models, which may recognize considerable computational efforts.

There are great differences between existing inventory systems. They differ in size and complexity; in the types of items they carry, in the costs associated with operating of the system, in the nature of stochastic process associated with the system and in the nature of the information available to decision makers at any given point of time. All these differences can be considered to reflect variations in the structure of the inventory system. These variations can have important bearings on the types of operating doctrines that should be used in controlling the system.

To make the solution of an inventory problem easier, it is necessary to build a mathematical model, which describes the inventory situation. As complete accuracy is not possible in this real world, so some approximations and simplifications must be made for building the mathematical model.

All organization keeps a supply of inventory for the following reasons:

1. *To maintain independence of operations***:** A supply of materials at a work center allows that center flexibility in operations. For example, as there are costs for making each new production setup, this inventory allows management to reduce the number of setups. Independence of workstations is desirable on assembly lines as well. The time that it takes to do identical operations will naturally vary from one unit to the next.

Therefore, it is desirable to have a cushion of several parts within the workstation so that shorter performance times can compensate for longer performance times. This way the average output can be fairly stable.

2. *To meet variation in product demand*: If the demand for the product is known precisely, it may be possible (though not necessarily economical) to produce the product to exactly meet the demand. Usually, however, demand is not completely known, and a safety or buffer stock must be maintained to absorb variation.

3. *To allow flexibility in production scheduling*: A stock of inventory relieves the pressure on the production system to get the goods out. This causes longer lead times, which permit production planning for smoother flow and lower-cost operation through larger lot-size production. High setup costs, for example, favour producing a larger number of units once the setup has been made.

4. *To take advantage of economic purchase order size*: There are costs to place an order, labour, phone calls, typing, postage, and so on. Therefore, the larger each order is, the fewer the orders that need to be written. Also, shipping costs favour larger orders the larger the shipment, the lower the per-unit cost.

There are several other reasons for holding inventories as well. For example, the price of some raw materials used by a manufacturer may exhibit considerable seasonal fluctuations. When the price is low, it is profitable to procure a sufficient quantity of it to last through the high priced season and to keep it in inventories to be used when need arises.

Researcher are engaged in developing the subject with more scientific approach starting from second world war to this age of advanced information technology using the modern techniques. The books by (Donald Waters, 2003, Sven Axsater, 2006, Naddor,1966, Muckstadt and Sapra, 2010) are worth mentioning in this regard.

We all can recognize the necessity of inventories to sustain operations within an economy. One major role of the management is to determine polices that create and distribute inventories most effectively. Though there are many factors or forces that may affect the policies selected by the management. Then environment plays an important role in materializing the selected polices.

Agricultural output is dictated by the growing seasons for crops in a particular location. The harvested crops may not be consumed for several years. Thus inventories may be created because of capacity limitations, seasonal cycles, different new features of the modern economy etc.

Inventory is also used to meet the current demand from stock which was created earlier because of the cyclic nature of the incoming supply of inventory.

An inventory system provides the organizational structure and the operating policies for maintaining and controlling goods to be stocked. The system is responsible for ordering and receipt of goods.

1.2 System Parameters

The properties of inventory system depend on several components or parameters such as, Demands, Replenishments, constraints and Inventory Costs etc.

Demands are units taken from inventory and can be categorized according to their size. Demand size refers to the magnitude of demand and has the dimension of quantity. When demand size is known the system is known as **deterministic**. When the demand size is not known, the system is known as **probabilistic** or **stochastic**. The **Demand rate** is the demand size per unit time and demand patterns refer how units are withdrawn from inventory.

Replenishments are units put into inventory and can be categories according to size, pattern, and lead time. **Replenishment size** means the quantity of the order to be received into inventory. They may be constant, variable, depending on the type of inventory system.

Replenishment pattern refers to, how the units are added to inventory. Replenishment pattern are generally instantaneous, uniform or batch. **Replenishment lead time** is the length of time between the decision to replenish to order an item and its actual arrival in stock. It may be constant or a variable. In case of variable lead time probability distribution are used.

Constraints are limitations which are imposed on inventory system. Warehouse space constraint may limit the amount of inventory held, Capital constraint restrict the amount of money invested in inventories. Similarly facilities, equipment and personal constraint may check the supply capability and operating level of an organization.

1.3 Inventory Costs

As we know cost effective control of inventories can cut costs significantly, and contribute to the effective flow of goods and prompt services in the good economy.

Inventory costs are associated with the operation of an inventory system and result from action or lack of action on the part of management in establishing the system. They are basic economic parameters to any inventory decision model.

The **purchase or unit cost** of an item is the unit purchase price if it is obtained from external source. The **ordering** or **a setup** or **a replenishment** cost originates from the expense of issuing a purchase order to on outside supplier and from internal production setup costs. This cost is directly varied with the number of orders or setups placed. Order cost includes such items as making requisitions, analyzing vendors writing purchase orders, receiving materials inspecting materials, following up of orders and doing the paperwork to complete the transaction.

Holding or a **carrying cost** is the cost associated with investing in inventory and maintaining the physical investment in storage. It is item dependent parameters with dimension of dollars or rupees per unit period. It includes insurances, taxes, obsolescence, warehouse, rental, light, heat etc.

The **shortage** or **stock out cost** is the penalty incurred for being unable to meet a demand when it occurs. The cost parameter depends on items but not on the source of replenishment. It has two wings internal or external shortage internal shortage occur when an order of a group or department within the organization is not filled and external shortage occur when customers order is not filled. External shortage can incur back order costs, present profit loss and future profit loss. Internal shortage can result in lost production and delay in a completion date.

1.4 Area of work

Most inventory control packages are based on the methodology of inventory theory. Inventory theory had its roots in the well known EOQ formula by Ford Harris (Harris, 1915) approximately 100 years ago. Harris was working as an engineer at the Westing house Corporation in Pittsburgh and he was able to show that an optimal production batch-size could be obtained by properly balancing the holding costs and the set-up costs. The EOQ formula first derived by (Harris, 1915) still serves as an effective base model for more and more complex models.

Researchers were engaged to develop the basic EOQ inventory model assuming the demand of the items to be **constant**, **linearly increasing** or **decreasing** or **exponentially increasing** or **decreasing with time**, **stock-dependent** etc. Later, this has been realized that the above demand patterns do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles etc; for which the demand increases with time as they are launched into the market and after some time it become constant. In order to consider the demand of such types, the concept of ramp-type demand is introduced. Ramp-type demand depicts a demand, which increases up to a certain time after which it stabilizes and become constant.

Researches on this field continue with (Mandal and Pal, 1998, Wu et. al, 1999, Wu and Ouyang, 2000, Wu, 2001) who studied inventory models with linearly increasing, up to its stabilization point under various assumptions. In the above cited papers, the determination of the optimal replenishment policy requires the determination of the time point, when inventory level falls to zero. So the following two cases should be examined: (i) this time point occurs before the point where the demand is stabilized, and (ii) this time point occurs after the point where the demand is stabilized. Almost all of the researchers examine only the first case. (Deng et. al, 2007) reconsidered the inventory model of (Mandal and Pal, 1998 and Wu and Ouyang, 2000) and studied it exploring these two cases. (Skouri et. al, 2009) extend the work of (Deng et. al, 2007) by introducing a general ramp-type demand rate considering weibull distribution deterioration rate.

In the studies of inventory models, unlimited warehouse capacity is often assumed. However, in busy market places, such as super markets, corporation markets etc. the storage area for items may be limited. When an attractive price discount for bulk purchase is available or the cost of procuring goods is higher than the other inventory related cost or demand for items is very high or when the item under consideration is a seasonable product such as the yield of a harvest or there are some problems in frequent procurement, the procurement of large amount of items at a time is decided. These items cannot be accommodated in the existing store house (the Own warehouse, denoted as OW). In this situation, in order to store the excess items, an additional warehouse (the rented warehouse, denoted as RW), which may be located at a short distance from OW, is hired on a rental basis. It is generally assumed that the holding cost in RW is higher than that in OW due to additional cost of maintenance. To reduce the inventory costs, it will be cost-effective to consume the goods of RW at first.

Maximum physical goods undergo decay or deterioration over time. Decay (or exponential decay) means that a fixed fraction of the inventory is lost in every planning period. Fruits, vegetables etc. suffer from depletion by direct spoilage while stored. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain etc. deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature. (Nahmias,2011) has given an excellent idea considering the large number of perishable items in the economy.

In most of the inventory models, holding cost is known and is considered as constant. But this may not be a real life situation. The traditional parameters of holding cost may be assumed to be time varying. As the changes in the time value of money and in the price, index, holding cost may not remain constant over time. Holding cost may also be dependent on ordering quantity as well.

1.5 Overview of the problems in the thesis

The accumulation and depletion of inventory is a major factor in the cyclical behavior of any business activity. Reports of business conditions reveal that inventory fluctuation is a phenomenon which requires close monitoring. An inventory problem deals with the making decisions that minimizes the total cost of the inventory system or maximizes the profit gained while meeting the customers' demands. The application of systematic quantitative methods to the solution of inventory problems began with the advent of scientific management. In the year 1915, Harris formulated and optimized a simple inventory model which resulted in the well known classical economic lot size formula. Since then, the above mentioned model has been modified and extended by researchers all over the world with a view to make it more realistic by changing the basic assumptions suitably.

Interest in the study of inventory problems has increased since World War ll and numerous publications have been devoted solely to this subject. Excellent reviews of the development of the subject can be found in the books of (Naddor, 1966, Peterson and Silver, 1985, etc.)

The present thesis has been divided into six Chapters with the following subtitles.

Chapter 1

General introduction.

Chapter 2

Review of literature.

Chapter 3

Inventory model for deteriorating items with stock dependent demand and time varying holding cost and shortages.

Chapter 4

Inventory models with two-component demand and nonlinear holding cost with shortages.

Chapter 5

Inventory models with ramp-type demand for deteriorating items with partial backlogging and time-varying holding cost.

Chapter 6

Two warehouse inventory models for deteriorating products with ramp-type demand and shortages.

In chapter 2, review the existing literature of the subject of the inventory models with different types of demand patterns on different parameters under various circumstances.

In chapters 3, 4, 5 and 6 there are ***eight problems*** and ***twenty-two numbers of cases***. In these problems the customers' demand has been considered to be stock-dependent and ramp-type. Recently, it has been observed that the displayed stock level (on hand inventory) has a great influence on the customers demand, i.e. the demand rate may go up or down if the on hand inventory level increases or decreases.

On the other hand, for some class of deteriorating items, demand initially increases with time up to a point. It then becomes steady and finally decreases with time and becomes asymptotic. This type of demand behavior can be observed in some seasonal items like fruits, fish, winter cosmetics, fashion apparel, etc. The increasing-steady-decreasing demand pattern can be represented by a three-phase ramp-type demand pattern that allows three-phase variation in demand. These phases will represent the growth, the steady and the decline phases commonly experienced by the demand of many products during their life cycle in the market.

The first problem of chapter 3, deals with two component demand and time-varying holding cost, for **infinite time-horizon**. Demand is assumed to be stock-dependent up to the level of available stock. After which the demand is considered as constant i.e. during stock-out period. Shortages are allowed and are fully backlogged.

In the second problem of chapter 3, deals with two component demand and time-varying holding cost, for **finite time-horizon**. Demand is assumed to be stock-dependent up to the level of available stock. After which the demand is considered as constant i.e. during stock-out period. Shortages are allowed and are fully backlogged.

The traditional parameters of holding cost may not be constant and it is assumed here to be time varying. As the changes in the time value of money and in the price index, holding cost may not remain constant over time. It is assumed here that the **holding cost is linearly increasing function of time**. In addition, the deterministic demand rate is linear and is assumed to be a function of on hand inventory level with deterioration and shortages for infinite and finite time-horizons and the deterioration has been assumed to begin after a fixed time from the instant of their arrival in stock (like fruits, vegetables etc).

Profits are maximized in both the infinite and finite time horizon cases. Some special cases are also derived from the main models. Lastly the model has been illustrated by numerical examples and sensitivity analysis has been performed to investigate the effect of changes in the parameters on the optimal solutions.

In chapter 4, the demand is assumed to be stock-dependent up to the stock-available time. After which the demand is assumed to be a constant during the stock-out period. Shortages are allowed and are fully backlogged. The first problem in this chapter deals with, traditional parameters of unit item cost and ordering cost are kept constant but holding cost is treated as **nonlinear function of length of time for which the item is held in stock** and the second problem deals with, traditional parameters of unit item cost and ordering cost are kept constant but **holding cost is treated as a functional form of the amount of the on-hand inventory**. Lastly the model has been illustrated by numerical examples and sensitivity analysis has been performed to investigate the effect of changes in the parameters on the optimal solutions.

In chapter 5, of the thesis the emphasis has been given on an inventory model with general ramp-type demand rate, partial backlogging of unsatisfied demand and time-varying holding cost. The first problem in this chapter is studied under the replenishment policy, **starting with no shortages** and the second problem in this chapter is studied under the replenishment policy, **starting with shortages**. The model is fairly general as the demand rate, up to the point of stabilization, is a general function of time. The backlogging rate is any non-increasing function of the waiting time up to the next replenishment. The optimal replenishment policy is derived and an algorithm is provided for both the above mentioned policies. Lastly the model has been illustrated by numerical examples and sensitivity analysis has been performed to investigate the effect of changes in the parameters on the optimal solutions.

In Chapter 6 of the thesis, emphasis has been given on the second storage facility. In retail business, a second storage is often essential to reduce the potential loss (lost sale). Particularly, when the space of the storage is relatively small, the need of a second storage can be realized.

It is our common experience that the demand of certain items such as newly launched fashion goods and cosmetics, garments, automobiles etc; for which the demand increases with time as they are launched into the market and after some time it becomes constant. In order to consider the demand of such items, the concept of ramp type demand is introduced. The ramp-type demand pattern was first introduced by Hill (1995).

The first problem in chapter 6 deals with an inventory system **starting without shortages**. To solve the proposed model, six cases have been derived and an algorithm is provided. The second problem in this chapter deals with an inventory system **starting with shortages**. To solve the proposed model, six cases have been derived and an algorithm is provided. Lastly the model has been illustrated by numerical examples and sensitivity analysis has been performed to investigate the effect of changes in the parameters on the optimal solutions.