5.1 GENERAL

It is revealed by several investigators that aluminum/aluminum alloy matrix composites containing SiC whiskers or fibers have anisotropic properties, which is attributed to the different orientation of whiskers. In other words, creep properties obtained by conducting uniaxial test on specimens taken from the same composite material may differ depending on the orientation of specimens. In short fiber or whisker reinforced composites, material flow during processing such as forging and extrusion often leads to preferential alignment of short fibers or whiskers, resulting in anisotropic mechanical properties. As a result, the composite containing aligned fibers or whiskers exhibit different yield stresses in the direction of alignment and transverse direction.

The experimental studies also reveal that mechanical strength of composite depends on the direction considered, because of the orientation of whiskers that contribute to reinforcing the composite to different degrees depending on their alignment in the direction of application of force (Lederich and Sastry, 1982; Crove et al, 1985; McDanels, 1985). The comparison of analytical results reported for creep in isotropic thick-walled cylinder subjected to internal pressure with the experimental results show discrepancies, which has been attributed to the development of anisotropy during creep (Davis, 1960).
With these forethoughts, it is decided to investigate the consequence of anisotropy on the steady state creep behaviour of a long thick-walled circular cylinder made of FGM. For this purpose, the steady state creep is analyzed in a FGM cylinder composed of transversely isotropic 6061Al-SiCw composite, in which the SiC whiskers are assumed to align in the tangential direction. The content of SiCw in the FGM cylinder is assumed to decrease linearly from the inner to outer radius. The creep stresses and creep rates are estimated in the FGM cylinder for two different operating conditions: (i) cylinder subjected to internal pressure alone and (ii) cylinder subjected to both internal and external pressures. The maximum SiCw content (\(V_{\text{max}}\)) in the FGM cylinder is assumed as 20 vol\% at the inner radius while keeping the average SiCw content (\(V_{\text{avg}}\)) as 15 vol\%. The results are estimated for varying degree of anisotropy, characterized by the ratio of radial (or axial) and tangential yield strength of the FGM. As a benchmark, the results are also estimated for a similar cylinder but made of isotropic FGM.

5.2 DISTRIBUTION OF REINFORCEMENT AND ESTIMATION OF CREEP PARAMETERS

The content of SiCw in the FGM cylinder is assumed to decrease linearly from the inner \((a)\) to outer radius \((b)\), similar to those described in chapter 4 for FG Al-SiCp cylinder. The content of SiCw, \(V(r)\), at any radius \(r\) of the FGM cylinder may be estimated from Eqn. 4.1. Similarly the average and minimum SiCw contents in the FGM cylinder may be estimated respectively from Eqs. 4.4 and 4.5.

In order to obtain the values of creep parameters \(M\) and \(\sigma_0\) for FGM cylinder made of 6061Al-SiCw, the regression equations developed in chapter 4,
which are applicable to AlSiCp composites, are modified. In order to account for the effect of changing matrix from Al to 6061Al on the creep parameters, the terms \( dM_1 \) and \( d\sigma_{o_1} \) are added in Eqs. (4.6) and (4.7) respectively. The terms \( dM_1 \) and \( d\sigma_{o_1} \) compensate the effect of incorporating SiC particles in 6061Al matrix, rather than pure Al matrix. To determine the values of \( dM_1 \) and \( d\sigma_{o_1} \), the experimental creep data reported by Nieh et al (1988) for 6061Al-SiCp,w composites are represented on \( \dot{\varepsilon}^{1/5} \) verses \( \sigma \) plots in Fig. 5.1. The slope and intercepts of these graphs yield the values of creep parameters \( M \) and \( \sigma_o \) for 6061Al-SiCp,w, as reported in (Table 5.1).

<table>
<thead>
<tr>
<th>Composite</th>
<th>Temperature (°C)</th>
<th>( M ) (( \dot{\varepsilon}^{1/5}/\text{MPa} ))</th>
<th>( \sigma_o ) (MPa)</th>
<th>Coefficient of correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6061Al-30 vol% SiCp</td>
<td>288</td>
<td>1.796E-03</td>
<td>35.94</td>
<td>0.981</td>
</tr>
<tr>
<td>6061Al-20 vol% SiCw</td>
<td>232</td>
<td>1.457E-03</td>
<td>88.34</td>
<td>0.972</td>
</tr>
<tr>
<td>6061Al-20 vol% SiCw</td>
<td>288</td>
<td>2.710E-03</td>
<td>61.90</td>
<td>0.905</td>
</tr>
<tr>
<td>6061Al-20 vol% SiCw</td>
<td>343</td>
<td>3.937E-03</td>
<td>41.73</td>
<td>0.980</td>
</tr>
</tbody>
</table>

The particle size of SiC in 6061Al-30 vol% SiCp composite, reported in Table 5.1, has not been mentioned in the study of Nieh et al (1988). However, they reported that 6061Al-20 vol% SiCw composite (Table 5.1) consists of cylindrical whisker having diameter 0.5 \( \mu \text{m} \) and an aspect ratio of 10. Therefore, the size of SiCp, which is assumed to be spherical, is estimated as 1.23 \( \mu \text{m} \), by equating the volume of cylindrical SiCw (in 6061Al-20 vol% SiCw) and spherical SiCp (in 6061Al-30 vol% SiCp).
The creep parameters $M$ and $\sigma_0$ for Al-SiCp composite are calculated from Eqs. (4.6) and (4.7), developed in chapter 4, by taking $P = 1.23 \, \mu m$, $V = 30 \, vol\%$ and $T = 288 \, ^{\circ}C$. The creep parameters, thus obtained, for Al-30 vol\% SiCp composite are compared with those reported for 6061Al-30 vol\% SiCp in Table 5.1. The comparison of these parameters is given in Table 5.2.

Table 5.2: Comparison of creep parameters of Al-30 vol\% SiCp and 6061Al-30 vol\% SiCp ($P = 1.23 \, \mu m; \, T = 288 \, ^{\circ}C; \, V = 30 \, vol\%$)

<table>
<thead>
<tr>
<th>$M_{Al}$ (s$^{-1/2}$/MPa)</th>
<th>$M_{6061Al}$ (s$^{-1/2}$/MPa)</th>
<th>$dM_1 = M_{6061Al} - M_{Al}$</th>
<th>$\sigma_0^{Al}$ (MPa)</th>
<th>$\sigma_0^{6061Al}$ (MPa)</th>
<th>$d\sigma_{o_1} = \sigma_0^{6061Al} - \sigma_0^{Al}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.314E-03</td>
<td>1.795E-03</td>
<td>0.004109</td>
<td>44.65</td>
<td>35.94</td>
<td>8.71</td>
</tr>
</tbody>
</table>

The creep parameters $M$ and $\sigma_0$ for 6061Al-SiCp composites can now be obtained as below by adding the values of terms $dM_1$ and $d\sigma_{o_1}$, reported in Table 5.2, in Eqs. (4.6) and (4.7) respectively.

$$M = 0.02876 - \frac{0.00879}{P} - \frac{14.02666}{T} + \frac{0.03224}{V(r)} + dM_1$$  \hspace{1cm} (5.1)

$$\sigma_0 = -0.084P - 0.023T + 1.185(V(r)) + 22.207 + d\sigma_{o_1}$$  \hspace{1cm} (5.2)

In order to obtain the creep parameters $M$ and $\sigma_0$ for 6061Al-SiCw composites we have further added the terms $dM_2$ and $d\sigma_{o_2}$ respectively to Eqs. (5.1) and (5.2), which are applicable for 6061Al-SiCp composites. The additions of $dM_2$ and $d\sigma_{o_2}$ compensate the effect of incorporating SiCw, rather than SiCp, in a matrix of 6061Al. This procedure of finding the creep parameters of 6061Al-SiCw composite has been followed due to limited experimental creep data available for whisker reinforced composites. The creep parameters $M$ and $\sigma_0$
have been calculated for 6061Al-20 vol% SiCp from Eqs. (5.1) and (5.2) by taking
$P = 1.23 \mu m$, $V = 20$ vol% and $T = 288 \degree C$. The creep parameter, thus obtained, for
6061Al-20 vol% SiCp composite are compared with those estimated for 6061Al-
20 vol% SiCw in Table 5.3.

**Table 5.3: Comparison of creep parameters of 6061Al-20 vol% SiCp and**

<table>
<thead>
<tr>
<th>$M^p$ $(s^{-1/5}/MPa)$</th>
<th>$M^w$ $(s^{-1/5}/MPa)$</th>
<th>$dM_2 = M^w - M^p$</th>
<th>$\sigma^{w}_o$ (MPa)</th>
<th>$\sigma^{w}_o$ (MPa)</th>
<th>$d\sigma_{o2} = \sigma^{w}_o - \sigma^{p}_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.332E-03</td>
<td>2.710E-03</td>
<td>3.78E-04</td>
<td>41.5</td>
<td>61.90</td>
<td>20.40</td>
</tr>
</tbody>
</table>

Finally, the creep parameters $M$ and $\sigma_o$ for 6061Al-SiCw composites can
be obtained from the following equations, obtained by adding the values of terms
$dM_2$ and $d\sigma_{o2}$, reported in Table 5.3, in Eqs. (5.1) and (5.2).

$$M = 0.02876 - \frac{0.00879}{P} - \frac{14.02666}{T} + \frac{0.03224}{V(r)} + dM_1 + dM_2$$  \hspace{1cm} (5.3)

$$\sigma_o = -0.084P - 0.0232T + 1.1853(V(r)) + 22.207 + d\sigma_{o1} + d\sigma_{o2}$$  \hspace{1cm} (5.4)

In a FGM cylinder, with known SiCw gradient, both the creep parameters
$M$ and $\sigma_o$ will be functions of radial distance ($r$) alone. The values of $M$ and $\sigma_o$
for 6061Al-SiCw, at a given radial distance ($r$), could be estimated from Eqs. (5.3)
and (5.4) by substituting the content, $V(r)$, of SiCw at that location.
5.3 ANALYSIS OF CREEP IN TRANSVERSELY ISOTROPIC FGM CYLINDER

The analysis of steady state creep in thick-walled FGM cylinder made of transversely isotropic composite may be carried out in a similar way as described in chapter 4 for isotropic FGM cylinder. However, in this analysis the reinforcements are assumed to be in the form of whiskers rather than particles and the matrix is taken as 6061Al instead of pure Al. The assumptions made in the present study are similar to those described in chapter 4 for isotropic cylinder with an exception that in this study the FGM chosen is transversely isotropic i.e. its mechanical properties are same in the radial and axial directions but are different in the tangential direction.

The deformation compatibility equation is given by (refer Eqn. 3.5),

\[ r \frac{d\dot{\varepsilon}_r}{dr} = \dot{\varepsilon}_r - \dot{\varepsilon}_\theta \]  

(5.5)

Assuming the FGM cylinder to operate under the following boundary conditions,

At \( r = a \) \( \sigma_r = -p \) \( (5.6) \)

At \( r = b \) \( \sigma_r = -q \) \( (5.7) \)

where the negative sign implies compressive nature of radial stress.

The equilibrium equation, as derived in chapter 3 (Eqn. 3.9), is written as,

\[ r \frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r \]  

(5.8)

The constitutive equations for an anisotropic material are given by (Bhatnagar and Gupta, 1969),

\[ \dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{(G + H)\sigma_e} [G(\sigma_r - \sigma_\theta) + H(\sigma_r - \sigma_\theta)] \]  

(5.9)
\[
\dot{e}_\theta = \frac{\dot{\varepsilon}_e}{(G + H)\sigma_e} \left[ F(\sigma_\theta - \sigma_z) + H(\sigma_\theta - \sigma_r) \right] \quad (5.10)
\]

\[
\dot{e}_z = \frac{\dot{\varepsilon}_e}{(G + H)\sigma_e} \left[ F(\sigma_z - \sigma_\theta) + G(\sigma_z - \sigma_r) \right] \quad (5.11)
\]

where \(F, G\) and \(H\) are the anisotropic constants, and \(\dot{e}_s\) and \(\sigma_e\) are respectively the effective strain rate and the effective stress.

When the principal axes of anisotropy are the axes of reference, the Hill’s yield criterion (Dieter, 1988) for anisotropic material is given by,

\[
\sigma_e = \left[ \frac{1}{G + H} \left\{ F(\sigma_\theta - \sigma_z)^2 + G(\sigma_z - \sigma_r)^2 + H(\sigma_r - \sigma_\theta)^2 \right\} \right]^{1/2} \quad (5.12)
\]

Under the assumption of incompressibility and plane strain condition, Eqn. (5.11) yields,

\[
\sigma_z = \frac{G\sigma_r + F\sigma_\theta}{(F + G)} \quad (5.13)
\]

Substituting \(\sigma_z\) from Eqn. (5.13) into Eqn. (5.12) and simplifying, we get,

\[
\sigma_e = \frac{(\sigma_\theta - \sigma_r)}{\sqrt{(H + G)}} \left[ \frac{(FG + GH + HF)}{(F + G)} \right]^{1/2} \quad (5.14)
\]

Putting \(\dot{\varepsilon}_r = -\frac{C}{r^2}\) (Eqn. 3.24) and \(\sigma_z\) from Eqn. (5.13) into Eqn. (5.9) and simplifying, we obtain,

\[
\sigma_\theta - \sigma_r = \frac{(F + G)(H + G)}{FG + GH + HF} \sigma_e C \frac{1}{r^2} \quad (5.15)
\]

Using Eqn. (5.14) and creep law given by Eqn. (3.2) in Eqn. (5.15), we get,

\[
\sigma_\theta - \sigma_r = \left[ \frac{(F + G)(H + G)}{FG + GH + HF} \right]^{2n+1} C^{1/n} \frac{1}{r^{2n}} M + \sigma_0 \frac{1}{M^{1/n}} \left( \frac{(F + G)(H + G)}{FG + GH + HF} \right) \quad (5.16)
\]

or,
\[ \sigma_\theta - \sigma_r = \frac{I_1}{r^{2/n}} + I_2 \quad (5.16) \]

where,
\[ I_1 = \left[ \frac{(F + G)(H + G)}{(FG + GH + HF)} \right]^{n+1} C^{1/n} \frac{r^{2n}}{M} \quad (5.17) \]

and
\[ I_2 = \sigma_0 \frac{2}{(FG + GH + HF)} \sqrt{I} \quad (5.18) \]

Substituting Eqn. (5.16) in equilibrium Eqn. (5.8) and Integrating, we obtain,
\[ \sigma_r = \int_a^r \frac{I_1}{r^{n+2}} dr + \int_a^r \frac{I_2}{r^2} dr - p \quad (5.19) \]

or,
\[ \sigma_r = X_1 + X_2 - p \quad (5.20) \]

where,
\[ X_1 = \int_a^r \frac{I_1}{r^{n+2}} dr \quad \text{and} \quad X_2 = \int_a^r \frac{I_2}{r^2} dr \quad (5.21) \]

Substituting Eqn. (5.19) into Eqn. (5.8), we get,
\[ \sigma_\theta = \int_a^r \frac{I_1}{r^{n+2}} dr + \int_a^r \frac{I_2}{r^2} dr + \int_a^r \frac{I_1}{r^{2/n}} + I_2 - p \]

or,
\[ \sigma_\theta = X_1 + X_2 + I_1 \frac{r^{2/n}}{I_2} + I_2 - p \quad (5.22) \]

To evaluate the value of constant \( C \), required for estimating \( I_1 \) given by Eqn. (5.17), the boundary conditions given in Eqs. (5.6) and (5.7) are substituted in Eqn. (5.19) to obtain,
\[ \int_a^b \frac{I_1}{r^{n+2}} dr + \int_a^b \frac{I_2}{r^2} dr - p = -q \]
Substituting the values of $I_1$ and $I_2$ respectively from Eqs. (5.17) and (5.18) into above equation and simplifying, we get,

$$C = \left[ \frac{p - q - X_4}{X_3} \right]^n$$  \hspace{1cm} (5.23)

where,

$$X_3 = \frac{b}{X} \int_a^{b} \left( K \right) \frac{r^n}{(n+2)^2} \frac{dr}{r}$$,

$$X_4 = \frac{b}{X} \int_a^{b} \frac{K \sigma_o dr}{r}$$

and

$$K = \frac{(F+G)(H+G)}{(FG+GH+HF)}$$  \hspace{1cm} (5.24)

Using Eqs. (5.13) and (5.14) in Eqs. (5.9) and (5.10), we obtain,

$$\dot{\varepsilon}_\theta = -\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{\sqrt{(F+G)(G+H)}} \frac{1}{\sqrt{FG+GH+HF}}$$  \hspace{1cm} (5.25)

The strain rates in an isotropic FGM cylinder may be obtained from the above equation by setting $F = G = H$, as given below,

$$\dot{\varepsilon}_\theta = -\dot{\varepsilon}_r = 0.87\dot{\varepsilon}_e$$  \hspace{1cm} (5.26)

Therefore, the relationship between radial and tangential strain rates, as described above, similar to that obtain in chapter 3 (Eqn. 3.34).

### 5.4 Estimation of Anisotropic Constants

The Hill’s yield criterion for orthotropic material, as given by Eqn. (5.12), involves constants $F$, $G$ and $H$, the values of which are required for estimating creep response of the cylinder. Hill described a procedure for estimating these anisotropic constants, when principal axes of anisotropy are the axes of reference (Hill, 1948) and the form of Eqn. (5.12) remains valid. If $X$, $Y$ and $Z$ are the tensile stresses in the principal directions of anisotropy, then according to Hill (1950), one may write,
\[
\begin{align*}
\frac{1}{X^2} &= G + H \\
\frac{1}{Y^2} &= H + F \\
\frac{1}{Z^2} &= F + G
\end{align*}
\]  
(5.27)

The above set of equations may be solved to obtain the values of anisotropic constants as given below,

\[
\begin{align*}
2F &= \left( \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \right) \\
2G &= \left( \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2} \right) \\
2H &= \left( \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \right)
\end{align*}
\]  
(5.28)

If the material of cylinder is subjected to uniaxial loading in \( r \) and \( \theta \) directions, the corresponding stress invariant may be expressed in terms of observed tensile strength and Hill’s anisotropic constants as given below,

\[
\sigma_e = \sqrt{\frac{G + H}{2}} \sigma_{r_y}
\]  
(5.29)

\[
\sigma_e = \sqrt{\frac{F + H}{2}} \sigma_{\theta_y}
\]  
(5.30)

where \( \sigma_{r_y} \) and \( \sigma_{\theta_y} \) are respectively the yield strength of composite in \( r \) and \( \theta \) directions and \( \sigma_e \) is the isotropic yield stress. For isotropic case the ratio of anisotropic constants is unity \( i.e. F/G = G/H = H/F = 1 \).

If the material of cylinder is tested under uniaxial loading in \( z \) direction, the stress invariant may similarly be written as,
where $\sigma_{yz}$ is the yield strength of composite in $z$ direction.

In the present analysis, it is assumed that during processing of FGM cylinder the whiskers get aligned in the tangential ($\theta$) direction, leading to anisotropic behaviour. As a result, in FGM cylinder the direction $\theta$ becomes longitudinal direction and the remaining directions (*i.e.* $r$ and $z$) may be taken as transverse directions. For axisymmetric problems like cylinder, the directions $r$, $\theta$ and $z$ may be taken as the principal directions. Thus, the anisotropic constants given by Eqn. (5.28), may be expressed as,

$$F = \left( \frac{1}{\sigma_{\theta}}^2 + \frac{1}{\sigma_{z}}^2 - \frac{1}{\sigma_{r}}^2 \right) \sigma_e^2$$  \hspace{1cm} (5.32)$$

$$G = \left( \frac{1}{\sigma_{z}}^2 + \frac{1}{\sigma_{r}}^2 - \frac{1}{\sigma_{\theta}}^2 \right) \sigma_e^2$$  \hspace{1cm} (5.33)$$

$$H = \left( \frac{1}{\sigma_{r}}^2 + \frac{1}{\sigma_{\theta}}^2 - \frac{1}{\sigma_{z}}^2 \right) \sigma_e^2$$  \hspace{1cm} (5.34)$$

An orthotropic material having one of its plane as the plane of isotropy is referred as transversely isotropic material. In a transversely isotropic material at every point there is a plane in which the mechanical properties remain same in every direction. Many unidirectional composites with fibers packed in hexagonal array, or close to it, can be considered as transversely isotropic with the direction normal to the whisker as the plane of isotropy (Danial and Ishai, 2005).

For transversely isotropic FGM cylinder, we have $\sigma_r = \sigma_z = \sigma_e$. As a consequence of this, from Eqs. (5.32) and (5.34) we get $F/H = 1$. The yield
strength of the FGM cylinder will be different in circumferential and radial/axial directions. The extent of anisotropy in the FGM cylinder may be characterized by ratio \( \alpha \) as defined below,

\[
\alpha = \frac{\sigma_r}{\sigma_{\theta}}
\] (5.35)

Substituting \( \sigma_r \) and \( \sigma_{\theta} \) respectively from Eqs. (5.29) and Eqn. (5.30), and noting that \( F/H = 1 \), we get,

\[
\frac{G}{H} = 2 \alpha^2 - 1
\] (5.36)

In the present study, the extent of anisotropy \( \alpha \) for FGM is assumed to lie in the range 0.7 to 1.3. The value \( \alpha = 1 \) will correspond to isotropic FGM. The following section investigates the effect of anisotropy \( (i.e. \alpha \text{ deviating from unity}) \) on the distribution of creep stresses and creep rates in the FGM cylinder. For a transversely isotropic FGM cylinder, the ratio \( F/H = 1 \) and the value of \( G/H \) is estimated from Eqn. (5.36) for a given value of \( \alpha \).

5.5 NUMERICAL SCHEME OF COMPUTATION

To start the computation process, the values of \( X_3 \) and \( X_4 \) (Eqs. 5.24) are estimated by putting the value of anisotropic constants \( F, G \) and \( H \) and creep parameters \( M \) and \( \sigma_{o} \) estimated from Eqs. (5.3) and (5.4) respectively. The value of \( X_3 \) and \( X_4 \), thus estimated, are substituted in Eqn. (5.23) to obtain the value of constant \( C \). Thereafter, the values of \( I_1 \) and \( I_2 \) are estimated respectively from Eqs. (5.17) and (5.18). Using \( I_1 \) and \( I_2 \), estimated above, in Eqn. (5.21), we get \( X_1 \) and \( X_2 \). Knowing \( X_1, X_2, X_3 \), and \( X_4 \), the stresses \( \sigma_r \) and \( \sigma_{\theta} \) are estimated respectively from Eqs (5.20) and (5.22). The stresses \( \sigma_r \) and \( \sigma_{\theta} \), thus estimated, are
substituted in Eqn. (5.13) to obtain the distribution of axial stress $\sigma_z$ in the FGM cylinder. Knowing $\sigma_r$, $\sigma_\theta$ and $\sigma_z$, one may obtain $\sigma_e$ and $\dot{\varepsilon}_e$ respectively from Eqs. (5.14) and (3.2). The strain rates $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$ and $\dot{\varepsilon}_z$ in the FGM cylinder are calculated respectively from Eqs. (5.9), (5.10) and (5.11).

5.6 RESULTS AND DISCUSSION

5.6.1 Validation

Before discussing the results obtained by current analytical scheme, it is necessary to validate the analysis as well as software developed. For this purpose, the creep parameters $M$ and $\sigma_o$ are estimated by using present analysis, for an isotropic copper cylinder, the result for which are available in literature (Johnson et al, 1961). The dimensions of the cylinder, operating pressure and temperature, and creep parameters are reported in chapter 3 (refer Table 3.2). These creep parameters have been used in the developed software to estimate the distribution of tangential strain rate in copper cylinder and are compared with those reported by Johnson et al (1961). The results are obtained by substituting $F = G = H$ in the software developed in this study and are compared with the experimental results available for copper cylinder, in Fig. 5.2. An excellent agreement observed between these results, validates the analysis as well as software developed in this study.

5.6.2 Validity of Regression Analysis

The creep parameters $M(r)$ and $\sigma_o(r)$, required for estimating stresses and strain rates in transversely isotropic FGM cylinder, are computed respectively from regression Eqs. (5.3) and (5.4). Therefore, it is imperative to verify the
accuracy of these regression equations for predicting the values of creep parameters. For this purpose, the creep parameters estimated from Eqs. (5.3) and (5.4) are substituted in the creep law given by Eqn. (3.2), to obtain the strain rates corresponding to the experimentally observed stress levels reported by Nieh et al. (1988) for 6061Al-20 vol% SiCw composite subjected to 288 °C. The estimated strain rates are compared with the experimental strain rates reported by Nieh et al. (1988). A good agreement is observed between the estimated and the experimental strain rates, Fig. (5.3).

5.6.3 Distribution of Creep Stresses and Creep Rates

The FGM cylinder chosen in this study is assumed to contain a maximum of 20 vol% SiCw at the inner radius and having an average 15 vol% of SiCw. The content of SiCw in the FGM cylinder decreases linearly from the inner to outer radius, as shown in Fig. 5.4. The numerical results are computed for FGM cylinder subjected to two different boundary conditions: (i) cylinder subjected to only internal pressure, \( p = 85.25 \text{ MPa} \) (ii) cylinder subjected to both internal and external pressures with \( p = 85.25 \text{ MPa} \) and \( q = 21.31 \text{ MPa} \). In both the cases, the dimensions of cylinder used are similar to those reported by Johnson et al. (1961) in their study on copper cylinder (refer Table 3.2).

5.6.3.1 Cylinder Subjected to Internal Pressure

Figures 5.5(a)-(b) show the variation of creep parameters \( M \) and \( \sigma_o \) with radial distance in FGM cylinders. The values of creep parameters in both isotropic and transversely isotropic (referred as anisotropic in this study) cylinders are equal. The value of parameter \( M \) increases with increasing radial distance as shown in Fig. 5.5(a). The increase observed in \( M \) may be attributed to decrease in
particle content $V(r)$, on moving from the inner to outer radius of cylinder. On the other hand, the threshold stress ($\sigma_o$) shown in Fig. 5.5(b) decreases linearly on moving from the inner to outer radius of FGM cylinder. The threshold stress is higher in locations having more amount of SiCw reinforcement compared to locations having lower SiCw content.

The creep stresses and creep rates have been estimated for isotropic FGM cylinder ($\alpha=1$) and anisotropic FGM cylinders having $\alpha = 0.7$ and $\alpha = 1.3$, Figs. 5.6-5.8. The value of $\alpha$ less than or greater than unity implies respectively the strengthening and weakening of FGM cylinder in the tangential direction as compared to radial and axial directions. The radial stress, Fig. 5.6(a), remains compressive throughout the cylinder, with maximum value at the inner radius and zero at the outer radius, under the imposed boundary conditions given in Eqs. (5.6) and (5.7). The magnitude of radial stress changes a little in the presence of anisotropy in the FGM cylinder. Over the entire radius, the radial stress (compressive) increases a little for $\alpha = 0.7$ and decreases a little for $\alpha = 1.3$, when compared with isotropic FGM cylinder having $\alpha = 1$. The tangential stress shown in Fig. 5.6(b) remains tensile throughout and is observed to increase with increasing radius to become maximum at the outer radius of the FGM cylinder. As compared to isotropic FGM cylinder, the presence of anisotropy with $\alpha = 0.7$, the tangential stress decreases near the inner radius but increases towards the outer radius. However, for anisotropic FGM cylinder with $\alpha = 1.3$, the tangential stress increases near the inner radius but decreases towards the outer radius, when compared with isotropic FGM cylinder. The extent of variation in tangential stress observed for anisotropic FGM cylinder with $\alpha = 1.3$, are slightly less than that
observed for $\alpha = 0.7$, when both the cylinders are compared with isotropic FGM cylinder ($\alpha = 1$).

The axial stress, Fig. 5.6(c), changes its nature from compressive to tensile as we move from the inner to outer radius of the isotropic FGM cylinder. For anisotropic FGM cylinder having $\alpha = 0.7$, the magnitude of compressive stress, observed near the inner radius, increases but that of tensile stress, observed near the outer radius, decreases as compared to that observed in isotropic FGM cylinder. However, for anisotropic FGM cylinder having $\alpha = 1.3$, the axial stress becomes tensile over the entire radius and is always higher than that observed for isotropic FGM cylinder. The effect of anisotropy on axial stress is slightly more near the inner radius than that observed towards the outer radius.

The effective stress shown in Fig. 5.6(d) decreases with increasing radial distance. As compared to isotropic FGM cylinder, the presence of anisotropy leads to significant decrease in effective stress over the entire radial distance for $\alpha = 0.7$. Whereas the effective stress increases significantly for $\alpha = 1.3$, when the distribution of effective stress in anisotropic FGM cylinder is compared with that observed in isotropic FGM cylinder.

It is interesting to observe that there exists a crossover in the distribution of tangential stress, somewhere in the middle of the cylinder, Fig. 5.6(b). In order to investigate the reason for this cross over, the distribution of various terms appearing in the expression of tangential stress (Eqn. 5.22) are plotted for different FGM cylinders and shown in Figs. 5.7(a)-(d). The term $(X_1+X_2)$ is not effected much with varying extent of anisotropy $\alpha$, Fig. 5.7(a). On the other hand, the terms $I_1/r^{2\eta}$ and $I_2$ increases and decreases respectively with increase in $\alpha$ from 0.7.
to 1.3, as evident from Figs. 5.7(b) and 5.7(c). As a result of this, the cumulative effect of $I_1/r^{2n}$ and $I_2$, shown in Fig. 5.7(d), exhibits a cross over with the increase in $\alpha$ from 0.7 to 1.3, which is responsible for crossover in tangential stress shown in Fig. 5.6(b).

It is revealed from the above discussion that the effect of anisotropy on the radial and tangential stresses in the FGM cylinder is not much pronounced. However, both axial and effective stresses are strongly influenced by the presence of anisotropy. The effect of anisotropy with $\alpha < 1$ is just opposite to that observed for $\alpha > 1$.

The strain rates given by Eqs. (5.9) and (5.10) are dependent on the effective strain rate ($\dot{\varepsilon}_e$), which ultimately depend upon the stress difference ($\sigma_e - \sigma_o$), as revealed from creep law given by Eqn. (3.2). Therefore, to investigate the effect of anisotropy on creep rates, the distribution of ($\sigma_e - \sigma_o$) is plotted in Fig. 5.8. The trend of variation observed for ($\sigma_e - \sigma_o$) is similar to those noticed for effective stress in Fig. 5.6(d). The stress difference ($\sigma_e - \sigma_o$) observed for isotropic FGM cylinder is relatively lower over the entire radius than that observed for anisotropic FGM cylinder having $\alpha=1.3$. However, when $\alpha < 1$ (i.e. $\alpha = 0.7$), the stress difference ($\sigma_e - \sigma_o$) decreases significantly over the entire radius when compared with isotropic FGM cylinder. As a result of significantly lower values of ($\sigma_e - \sigma_o$) in anisotropic FGM cylinder having $\alpha = 0.7$, the effective strain rate in this cylinder is lower everywhere compared to any other cylinder, Fig. 5.9(a). On the other hand, the anisotropic FGM cylinder having $\alpha = 1.3$, exhibits significantly higher effective strain rate over the entire radius. The presence of anisotropy affects the radial and tangential strain rates
in a similar way as observed for effective strain rate in Fig. 5.9(a). The radial as well as tangential strain rates decrease by almost two orders of magnitude, Fig. 5.9(b), when $\alpha$ decreases from 1.3 to 0.7. The decrease is less than one order of magnitude on decreasing $\alpha$ from 1.3 to 1.0. Therefore, by increasing the strength of SiCw reinforcement in the tangential direction ($\alpha < 1$), as compared to radial and axial directions, the strain rates in the cylinder are significantly reduced when compared with isotropic FGM cylinder ($\alpha=1$). On the contrary, the FGM cylinder having weaker strength in tangential direction ($\alpha > 1$) shows much higher creep rates than those observed in isotropic FGM cylinder.

The effect of anisotropy on the maximum and minimum values of stresses in the FGM cylinder has also been investigated. The maximum tangential stress, observed at the outer radius of FGM cylinder, decreases with increasing value of anisotropic parameter $\alpha$, Fig. 5.10. Whereas, the minimum tangential stress, observed at the inner radius, increases with increase in extent of anisotropy $\alpha$. The radial stress (compressive) is maximum at the inner radius and minimum at the outer radius, under the imposed boundary conditions given by Eqs. (5.6) and (5.7). As a result of this, the maximum and minimum radial stress in the cylinder will not be affected by the presence of anisotropy. The minimum and maximum values of axial stress, observed respectively at the inner and outer radii, increases with increasing extent of anisotropy $\alpha$ from 0.7 to 1.3, Fig. 5.11. With the increase in $\alpha$, the nature of minimum axial stress changes from compressive to tensile.

The stress inhomogeneity is defined as the difference of maximum and minimum values of stress in the cylinder. The tangential stress inhomogeneity decreases from 45.9 MPa to 32.6 MPa with the increase in $\alpha$ from 0.7 to 1.3, Fig. 5.12. The axial stress inhomogeneity also decreases from 75.6 MPa to 40.7 MPa
with the increase in $\alpha$ from 0.7 to 1.3. However, the inhomogeneity in radial stress remains constant under the imposed boundary conditions, which lead to fixed values of maximum and minimum radial stresses in the FGM cylinder.

The magnitude of maximum and minimum value of strain rates (tangential/radial), observed respectively at the inner and outer radii of the cylinder, increases with the increase in $\alpha$, as shown in Fig. 5.13. The tangential/radial strain rate inhomogeneity increases significantly with the increase in $\alpha$, Fig. 5.14. Though, the inhomogeneity in axial and tangential stresses are minimum corresponding to $\alpha = 1.3$ (Fig. 5.12), but the inhomogeneity in strain rate is the lowest corresponding to $\alpha = 0.7$. Therefore, by increasing the extent of anisotropy i.e. decreasing the strength of FGM cylinder in the tangential direction, the strain rate inhomogeneity in the FGM cylinder increases, which may enhance the chances of deformation in the FGM cylinder.

5.6.3.2 Cylinder Subjected to Internal and External Pressures

This section investigates the effect anisotropy on creep response of the FGM cylinders subjected to both internal and external pressures ($p = 85.25 \text{ MPa}$, $q = 21.31 \text{ MPa}$), the results for which are indicated in Figs. 5.15-5.18. It is observed from Fig. 5.15 that the effect of anisotropy on creep stresses in the FGM cylinder subjected to internal and external pressures is similar to those noticed for FGM cylinder subjected to internal pressure alone, Fig. 5.6. However, the magnitude of stresses in the FGM cylinder are reduced on applying both internal and external pressures, as compared to those observed in FGM cylinder subjected to internal pressure alone.

The influence of anisotropy on strain rates in FGM cylinder subjected to both internal and external pressures, Fig. 5.16, is also similar to those observed in
FGM cylinder under internal pressure alone, Fig. 5.9. Though, the order of strain rates in FGM cylinder subjected to both internal and external pressures is significantly lower than that observed for FGM cylinder operating under internal pressure alone.

By introducing anisotropy with $\alpha = 0.7$, the effective as well as tangential/radial strain rates reduce by about four orders of magnitude as compared to those observed in isotropic FGM cylinder ($\alpha = 1.0$). However, on increasing $\alpha$ from 1.0 to 1.3, the order of strain rates increases by about one order of magnitude. The stress inhomogeneity in FGM cylinder, operating under both internal and external pressures, decreases with the increase in extent of anisotropy, Fig. 5.17. As compared to FGM cylinder operating under internal pressure alone, the axial stress inhomogeneity is significantly lower in FGM cylinder subjected to both internal and external pressures (refer Figs. 5.12 and 5.17). The influence of anisotropy on tangential/radial strain rate inhomogeneity in FGM cylinder, operating under both internal and external pressures, is similar to those noticed for FGM cylinder subjected to internal pressure alone (refer Fig. 5.14 and 5.18). However, the simultaneous presence of internal and external pressures significantly reduces the inhomogeneity in strain rates as compared to that observed for FGM cylinder subjected to internal pressure alone. Therefore, the chances of distortion in an anisotropic FGM cylinder operating under internal and external pressures will reduce as compared to that observed in a similar cylinder subjected to internal pressure alone.
Fig. 5.1: Variation of $\dot{\varepsilon}^{1/5}$ versus $\sigma$ for 6061Al and 6061Al-SiCp,w composites (Nieh et al., 1988).

Fig. 5.2: Comparison of tangential strain rates in copper cylinder estimated from current analysis and reported by Johnson et al. (1961).
Fig. 5.3: Comparison of experimental (Nieh et al, 1988) and theoretically estimated strain rates in 6061 Al-20 vol% SiCw composite

\( (P = 1.23 \mu m; V = 20 \text{ vol\%} \text{ and } T = 288^\circ C) \).

Fig. 5.4: Variation of SiCw content in FGM cylinders.
Fig. 5.5: Variation of creep parameters in FGM cylinders.
Fig. 5.6: Variation of creep stresses in FGM cylinders operating at 288 °C.
Fig. 5.7: Variation of terms \((X_1+X_2), I_2/r^{2/n}, I_2\) and \((I_1/r^{2/n}+I_2)\) in FGM cylinders.
**Fig. 5.8:** Variation of stress difference in FGM cylinders operating at 288°C.

**Fig. 5.9:** Variation of strain rates in FGM cylinders operating at 288°C.
Fig. 5.10: Effect of anisotropy on maximum and minimum tangential stress in FGM cylinders.

Fig. 5.11: Effect of anisotropy on maximum and minimum axial stress in FGM cylinders.
Fig. 5.12: Effect of anisotropy on stress inhomogeneity in FGM cylinders.

Fig. 5.13: Effect of anisotropy on maximum and minimum strain rates in FGM cylinders.
Fig. 5.14: Effect of anisotropy on tangential / radial strain rate inhomogenity in FGM cylinders.
Fig. 5.15: Effect of anisotropy on creep stresses in FGM cylinders subjected to both internal and external pressures and operating at 288°C.
Fig. 5.16: Effect of anisotropy on strain rates in FGM cylinders subjected to both internal and external pressures and operating at 288°C.
Fig. 5.17: Effect of anisotropy on stress inhomogeneity in FGM cylinders subjected to both internal and external pressures.

Fig. 5.18: Effect of anisotropy on tangential/radial strain rate inhomogeneity in FGM cylinders subjected to both internal and external pressures.